Computer algebra independent integration tests

4-Trig-functions/4.5-Secant/4.5.1.3-d-sin-^n-a+b-sec-^m

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [306]. This is test number [119].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.67 (305)	% 0.33 (1)
Mathematica	% 98.37 (301)	% 1.63 (5)
Maple	% 87.25 (267)	% 12.75 (39)
Maxima	% 57.19 (175)	% 42.81 (131)
Fricas	% 62.42 (191)	% 37.58 (115)
Sympy	% 0.65 (2)	% 99.35 (304)
Giac	% 62.42 (191)	% 37.58 (115)

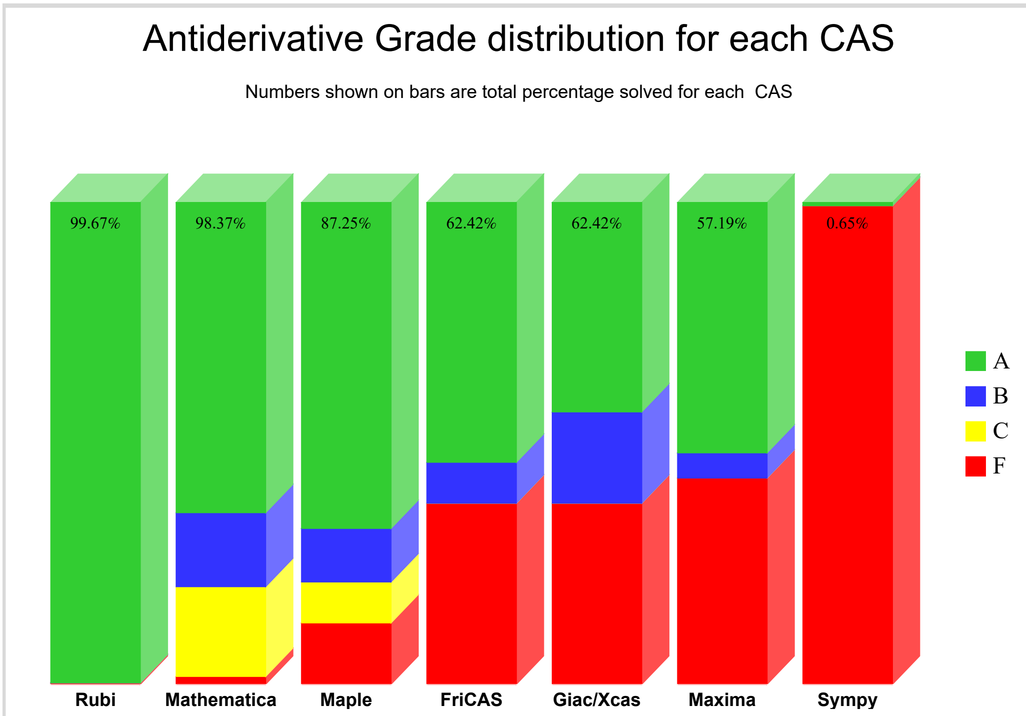
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

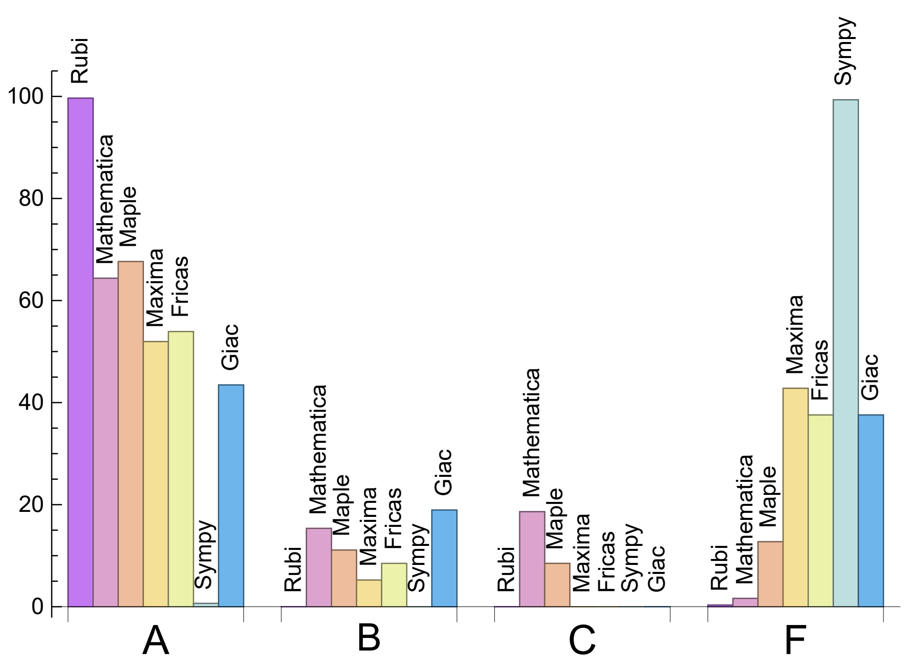
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	99.67	0.	0.	0.33
Mathematica	64.38	15.36	18.63	1.63
Maple	67.65	11.11	8.5	12.75
Maxima	51.96	5.23	0.	42.81
Fricas	53.92	8.5	0.	37.58
Sympy	0.65	0.	0.	99.35
Giac	43.46	18.95	0.	37.58

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.35	169.1	0.96	127.	1.
Mathematica	3.36	323.08	1.77	135.	1.06
Maple	0.61	354.16	1.68	157.	1.21
Maxima	1.03	171.74	1.46	143.	1.32
Fricas	1.83	511.26	3.52	319.	3.01
Sympy	0.	0.	0.	0.	0.
Giac	1.31	330.8	2.28	201.	1.92

1.4 list of integrals that has no closed form antiderivative

{263, 264, 265, 266, 267, 273, 274, 277, 278, 279, 280}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {35, 37, 56, 114, 115, 116, 117, 118, 119, 140, 141, 142, 143, 144, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 254, 255, 260, 261, 262, 272, 275, 276, 287, 288, 289, 290, 291, 292}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-s

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

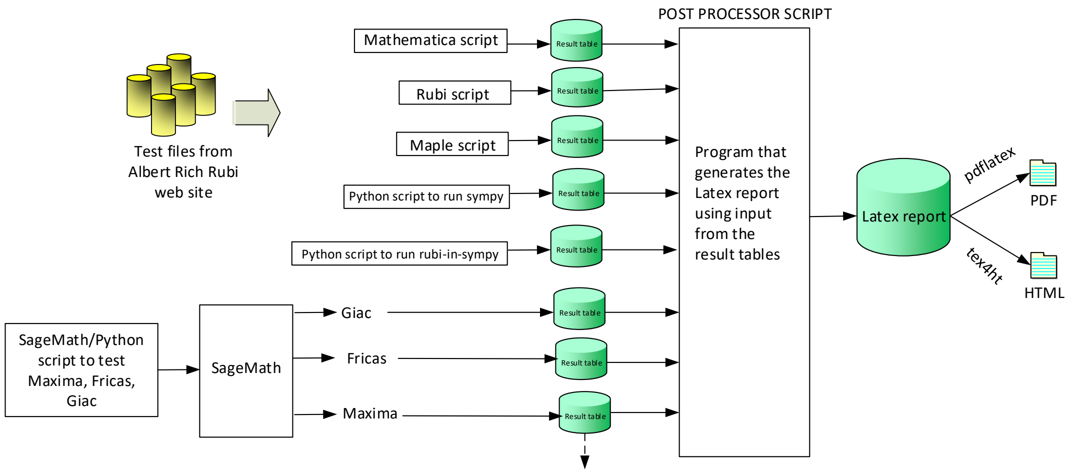
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

B grade: { }

C grade: { }

F grade: { 276}

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 7, 8, 9, 10, 11, 12, 13, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 120, 122, 124, 126, 127, 129, 131, 133, 136, 145, 146, 147, 148, 150, 151, 154, 155, 160, 161, 162, 163, 165, 166, 167, 168, 169, 170, 174, 175, 176, 177, 179, 180, 181, 185, 186, 187, 188, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 228, 230, 231, 232, 249, 250, 252, 253, 254, 256, 257, 258, 259, 263, 264, 265, 266, 267, 269, 270, 271, 273, 274, 277, 278, 279, 280, 281, 283, 285, 293, 295, 297, 299, 300, 302, 304, 306 }

B grade: { 6, 32, 33, 34, 35, 36, 37, 51, 52, 53, 54, 55, 56, 70, 71, 72, 73, 140, 141, 142, 143, 144, 149, 156, 157, 158, 159, 164, 178, 182, 183, 184, 189, 190, 191, 192, 193, 194, 195, 229, 260, 261, 262, 268, 272, 275, 276 }

C grade: { 14, 15, 16, 17, 18, 114, 115, 116, 117, 118, 119, 121, 123, 125, 128, 130, 132, 152, 153, 171, 172, 173, 226, 227, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 251, 255, 282, 284, 286, 287, 288, 289, 290, 291, 292, 294, 296, 298, 301, 303, 305 }

F grade: { 134, 135, 137, 138, 139}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 62, 63, 64, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 219, 220, 221, 222, 223, 224, 225, 226, 227, 231, 232, 239, 245, 246, 247, 248, 249, 253, 263, 264, 265, 266, 267, 273, 274, 277, 278, 279, 280 }

B grade: { 55, 56, 65, 66, 67, 68, 84, 100, 203, 204, 205, 216, 217, 218, 228, 229, 230, 233, 234, 235, 236, 237, 238, 240, 241, 242, 243, 244, 250, 251, 252, 254, 255, 256 }

C grade: { 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

F grade: { 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 257, 258, 259, 260, 261, 262, 268, 269, 270, 271, 272, 275, 276 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 69, 70, 74, 75, 76, 77, 78, 79, 80, 81, 82, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 103, 104, 105, 106, 107, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 209, 210, 211, 212, 213, 214, 221, 222, 223, 224, 225, 226, 263, 264, 265, 266, 267, 273, 274, 277, 278, 279, 280

B grade: { 65, 66, 67, 68, 71, 72, 73, 83, 84, 85, 86, 100, 101, 102, 215, 227 }

C grade: { }

F grade: { 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 203, 204, 205, 206, 207, 208, 216, 217, 218, 219, 220, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 269, 270, 271, 272, 275, 276, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 29, 30, 31, 32, 33, 34, 35, 36, 38, 39, 40, 41, 42, 43, 44, 45, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 100, 101, 102, 103, 104, 105, 106, 107, 160, 161, 162, 163, 164, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 203, 204, 205, 206, 207, 209, 210, 211, 212, 213, 216, 217, 218, 219, 220, 221, 222, 223, 224, 228, 229, 230, 231, 232, 263, 264, 265, 266, 267, 273, 274, 277, 278, 279, 280 }

B grade: { 9, 16, 17, 18, 28, 37, 46, 47, 64, 71, 72, 73, 82, 97, 98, 99, 165, 166, 167, 202, 208, 214, 215, 225, 226, 227 }

C grade: { }

F grade: { 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 269, 270, 271, 272, 275, 276, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

2.1.6 Sympy

A grade: { 265, 279 }

B grade: { }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

2.1.7 Giac

A grade: { 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 22, 23, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 39, 41, 42, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 78, 79, 80, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 162, 163, 168, 175, 176, 177, 180, 186, 187, 191, 193, 194, 195, 198, 199, 200, 201, 206, 211, 212, 216, 217, 218, 219, 220, 223, 224, 229, 231, 232, 263, 264, 265, 266, 267, 273, 274, 277, 278, 279, 280 }

B grade: { 1, 2, 3, 19, 20, 21, 24, 38, 40, 43, 44, 76, 77, 81, 93, 160, 161, 164, 165, 166, 167, 169, 170, 171, 172, 173, 174, 178, 179, 181, 182, 183, 184, 185, 188, 189, 190, 192, 196, 197, 202, 203, 204, 205, 207, 208, 209, 210, 213, 214, 215, 221, 222, 225, 226, 227, 228, 230 }

C grade: { }

F grade: { 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 268, 269, 270, 271, 272, 275, 276, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	106	163	153	339	0	396
normalized size	1	1.	0.7	1.07	1.01	2.23	0.	2.61
time (sec)	N/A	0.107	0.21	0.092	1.103	1.891	0.	1.495

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	86	129	123	261	0	333
normalized size	1	1.	0.72	1.08	1.03	2.19	0.	2.8
time (sec)	N/A	0.097	0.131	0.088	1.068	1.833	0.	1.506

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	83	95	93	193	0	271
normalized size	1	1.	0.95	1.09	1.07	2.22	0.	3.11
time (sec)	N/A	0.087	0.084	0.086	1.109	1.799	0.	1.425

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	57	61	63	126	0	89
normalized size	1	1.	0.98	1.05	1.09	2.17	0.	1.53
time (sec)	N/A	0.077	0.046	0.084	1.125	1.823	0.	1.428

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	37	28	31	59	0	43
normalized size	1	1.	1.42	1.08	1.19	2.27	0.	1.65
time (sec)	N/A	0.031	0.019	0.02	1.139	1.763	0.	1.461

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	63	15	35	81	0	78
normalized size	1	1.	2.1	0.5	1.17	2.7	0.	2.6
time (sec)	N/A	0.058	0.036	0.033	1.032	1.687	0.	1.448

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	114	48	70	246	0	138
normalized size	1	1.	1.56	0.66	0.96	3.37	0.	1.89
time (sec)	N/A	0.095	0.821	0.057	1.093	1.8	0.	1.474

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	164	80	128	512	0	201
normalized size	1	1.	1.39	0.68	1.08	4.34	0.	1.7
time (sec)	N/A	0.12	0.337	0.068	1.076	1.765	0.	1.468

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	165	112	184	802	0	265
normalized size	1	1.	1.01	0.69	1.13	4.92	0.	1.63
time (sec)	N/A	0.15	0.398	0.083	1.048	1.856	0.	1.675

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	106	164	171	378	0	235
normalized size	1	1.	0.64	0.99	1.04	2.29	0.	1.42
time (sec)	N/A	0.146	0.33	0.094	0.99	1.906	0.	1.512

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	86	130	143	290	0	197
normalized size	1	1.	0.68	1.02	1.13	2.28	0.	1.55
time (sec)	N/A	0.128	0.185	0.085	0.999	1.878	0.	1.484

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	86	96	109	217	0	159
normalized size	1	1.	0.97	1.08	1.22	2.44	0.	1.79
time (sec)	N/A	0.111	0.116	0.089	1.012	1.83	0.	1.482

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	54	62	80	143	0	119
normalized size	1	1.	1.06	1.22	1.57	2.8	0.	2.33
time (sec)	N/A	0.082	0.052	0.031	0.99	1.739	0.	1.494

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	41	47	68	170	0	68
normalized size	1	1.	1.11	1.27	1.84	4.59	0.	1.84
time (sec)	N/A	0.093	0.029	0.082	1.016	1.7	0.	1.532

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	81	103	282	0	107
normalized size	1	1.	1.	1.17	1.49	4.09	0.	1.55
time (sec)	N/A	0.103	0.029	0.104	1.007	1.713	0.	1.633

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	91	115	130	504	0	144
normalized size	1	1.	0.9	1.14	1.29	4.99	0.	1.43
time (sec)	N/A	0.11	0.031	0.141	0.982	1.779	0.	1.769

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	113	149	157	745	0	184
normalized size	1	1.	0.86	1.14	1.2	5.69	0.	1.4
time (sec)	N/A	0.117	0.048	0.122	1.026	1.812	0.	2.067

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	135	183	184	987	0	221
normalized size	1	1.	0.82	1.11	1.12	5.98	0.	1.34
time (sec)	N/A	0.127	0.055	0.126	0.998	1.865	0.	1.921

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	127	206	197	487	0	500
normalized size	1	1.	0.69	1.13	1.08	2.66	0.	2.73
time (sec)	N/A	0.188	0.826	0.046	1.003	1.944	0.	1.63

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	107	168	144	342	0	432
normalized size	1	1.	0.82	1.28	1.1	2.61	0.	3.3
time (sec)	N/A	0.168	0.534	0.044	1.027	1.896	0.	1.423

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	112	112	87	130	127	301	0	365
normalized size	1	1.	0.78	1.16	1.13	2.69	0.	3.26
time (sec)	N/A	0.158	0.288	0.043	1.008	1.788	0.	1.489

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	65	92	76	186	0	100
normalized size	1	1.	1.05	1.48	1.23	3.	0.	1.61
time (sec)	N/A	0.124	0.197	0.04	0.994	1.777	0.	1.402

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	31	46	55	116	0	69
normalized size	1	1.	0.72	1.07	1.28	2.7	0.	1.6
time (sec)	N/A	0.077	0.112	0.02	1.006	1.76	0.	1.383

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	36	32	58	155	0	155
normalized size	1	1.	0.75	0.67	1.21	3.23	0.	3.23
time (sec)	N/A	0.115	0.076	0.035	0.976	1.743	0.	1.394

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	75	50	92	270	0	182
normalized size	1	1.	1.09	0.72	1.33	3.91	0.	2.64
time (sec)	N/A	0.144	0.542	0.059	1.003	1.744	0.	1.398

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	103	85	140	531	0	258
normalized size	1	1.	0.9	0.74	1.22	4.62	0.	2.24
time (sec)	N/A	0.171	1.513	0.072	1.023	1.831	0.	1.524

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	136	121	193	722	0	321
normalized size	1	1.	0.85	0.76	1.21	4.51	0.	2.01
time (sec)	N/A	0.199	1.284	0.069	0.989	1.83	0.	1.452

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	164	157	266	1152	0	393
normalized size	1	1.	0.8	0.77	1.3	5.62	0.	1.92
time (sec)	N/A	0.238	3.254	0.083	1.015	1.921	0.	1.482

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	144	210	290	522	0	304
normalized size	1	1.	0.72	1.06	1.46	2.62	0.	1.53
time (sec)	N/A	0.361	0.883	0.049	1.539	1.955	0.	1.419

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	124	172	235	423	0	261
normalized size	1	1.	0.79	1.1	1.5	2.69	0.	1.66
time (sec)	N/A	0.27	0.547	0.043	1.548	1.971	0.	1.424

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	94	134	170	344	0	217
normalized size	1	1.	0.82	1.17	1.48	2.99	0.	1.89
time (sec)	N/A	0.268	0.279	0.041	1.506	1.884	0.	1.378

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	243	86	109	267	0	173
normalized size	1	1.	3.33	1.18	1.49	3.66	0.	2.37
time (sec)	N/A	0.132	1.192	0.034	1.482	1.764	0.	1.444

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	401	77	100	257	0	122
normalized size	1	1.	7.04	1.35	1.75	4.51	0.	2.14
time (sec)	N/A	0.246	6.154	0.045	1.001	1.746	0.	1.436

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	228	140	153	396	0	140
normalized size	1	1.	2.62	1.61	1.76	4.55	0.	1.61
time (sec)	N/A	0.297	1.671	0.059	1.008	1.677	0.	1.494

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	129	129	317	202	194	525	0	184
normalized size	1	1.	2.46	1.57	1.5	4.07	0.	1.43
time (sec)	N/A	0.226	0.945	0.061	1.034	1.735	0.	1.478

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	428	264	236	694	0	227
normalized size	1	1.	2.63	1.62	1.45	4.26	0.	1.39
time (sec)	N/A	0.243	1.231	0.086	1.038	1.875	0.	1.422

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	201	201	1050	326	275	1026	0	270
normalized size	1	1.	5.22	1.62	1.37	5.1	0.	1.34
time (sec)	N/A	0.258	6.831	0.081	1.028	1.948	0.	1.384

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	148	230	213	539	0	535
normalized size	1	1.	0.73	1.13	1.05	2.66	0.	2.64
time (sec)	N/A	0.196	1.697	0.05	0.997	2.21	0.	1.429

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	106	130	144	316	0	323
normalized size	1	1.	0.81	0.99	1.1	2.41	0.	2.47
time (sec)	N/A	0.168	0.921	0.049	1.008	1.865	0.	1.318

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	108	155	143	346	0	401
normalized size	1	1.	0.81	1.16	1.07	2.58	0.	2.99
time (sec)	N/A	0.167	0.628	0.047	1.012	1.83	0.	1.298

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	86	109	108	259	0	138
normalized size	1	1.	0.88	1.11	1.1	2.64	0.	1.41
time (sec)	N/A	0.096	0.195	0.045	1.01	1.853	0.	1.312

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	65	63	74	158	0	86
normalized size	1	1.	1.05	1.02	1.19	2.55	0.	1.39
time (sec)	N/A	0.091	0.229	0.021	0.986	1.847	0.	1.295

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	81	49	76	197	0	192
normalized size	1	1.	1.21	0.73	1.13	2.94	0.	2.87
time (sec)	N/A	0.125	0.131	0.045	0.986	1.796	0.	1.273

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	88	67	113	319	0	255
normalized size	1	1.	1.	0.76	1.28	3.62	0.	2.9
time (sec)	N/A	0.156	0.861	0.074	1.	1.74	0.	1.375

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	100	85	139	441	0	251
normalized size	1	1.	0.9	0.77	1.25	3.97	0.	2.26
time (sec)	N/A	0.169	0.918	0.085	1.003	1.733	0.	1.418

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	129	120	196	740	0	328
normalized size	1	1.	0.82	0.76	1.25	4.71	0.	2.09
time (sec)	N/A	0.195	1.015	0.089	1.004	1.879	0.	1.306

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	159	156	255	1064	0	394
normalized size	1	1.	0.79	0.77	1.26	5.27	0.	1.95
time (sec)	N/A	0.231	1.194	0.088	1.014	1.851	0.	1.374

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	156	235	393	567	0	329
normalized size	1	1.	0.74	1.12	1.87	2.7	0.	1.57
time (sec)	N/A	0.389	2.017	0.051	1.802	2.049	0.	1.266

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	136	197	324	467	0	286
normalized size	1	1.	0.75	1.08	1.78	2.57	0.	1.57
time (sec)	N/A	0.274	0.869	0.049	1.534	1.978	0.	1.284

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	114	159	246	381	0	243
normalized size	1	1.	0.83	1.15	1.78	2.76	0.	1.76
time (sec)	N/A	0.228	0.451	0.044	1.521	1.908	0.	1.317

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	300	111	171	313	0	138
normalized size	1	1.	3.06	1.13	1.74	3.19	0.	1.41
time (sec)	N/A	0.183	2.495	0.038	1.512	1.839	0.	1.288

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	244	102	185	313	0	143
normalized size	1	1.	3.05	1.27	2.31	3.91	0.	1.79
time (sec)	N/A	0.194	1.065	0.049	1.021	1.784	0.	1.303

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	678	188	254	444	0	166
normalized size	1	1.	6.16	1.71	2.31	4.04	0.	1.51
time (sec)	N/A	0.23	6.234	0.074	1.025	1.73	0.	1.36

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	165	165	353	274	308	575	0	190
normalized size	1	1.	2.14	1.66	1.87	3.48	0.	1.15
time (sec)	N/A	0.436	1.155	0.074	1.047	1.767	0.	1.383

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	430	360	362	698	0	228
normalized size	1	1.	2.24	1.88	1.89	3.64	0.	1.19
time (sec)	N/A	0.314	1.182	0.076	1.016	1.808	0.	1.396

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	A	A	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	232	232	1000	446	416	973	0	273
normalized size	1	1.	4.31	1.92	1.79	4.19	0.	1.18
time (sec)	N/A	0.332	6.68	0.086	1.028	1.921	0.	1.418

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	62	89	120	254	0	190
normalized size	1	1.	0.68	0.98	1.32	2.79	0.	2.09
time (sec)	N/A	0.161	4.234	0.098	0.989	1.743	0.	1.306

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	52	70	93	182	0	161
normalized size	1	1.	0.71	0.96	1.27	2.49	0.	2.21
time (sec)	N/A	0.155	1.556	0.085	1.017	1.718	0.	1.236

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	42	49	66	126	0	131
normalized size	1	1.	0.76	0.89	1.2	2.29	0.	2.38
time (sec)	N/A	0.148	0.334	0.072	0.995	1.667	0.	1.204

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	32	30	39	66	0	43
normalized size	1	1.	0.86	0.81	1.05	1.78	0.	1.16
time (sec)	N/A	0.126	0.111	0.056	0.992	1.662	0.	1.224

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	28	49	41	72	0	46
normalized size	1	1.	0.9	1.58	1.32	2.32	0.	1.48
time (sec)	N/A	0.071	0.08	0.023	1.008	1.71	0.	1.259

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	67	54	63	181	0	76
normalized size	1	1.	1.16	0.93	1.09	3.12	0.	1.31
time (sec)	N/A	0.097	0.096	0.051	1.01	1.662	0.	1.326

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	91	72	116	378	0	174
normalized size	1	1.	1.11	0.88	1.41	4.61	0.	2.12
time (sec)	N/A	0.158	0.369	0.06	1.022	1.709	0.	1.299

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	122	108	176	603	0	246
normalized size	1	1.	1.15	1.02	1.66	5.69	0.	2.32
time (sec)	N/A	0.173	0.474	0.067	1.019	1.785	0.	1.384

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	132	290	486	261	0	188
normalized size	1	1.	1.06	2.32	3.89	2.09	0.	1.5
time (sec)	N/A	0.21	1.195	0.099	1.525	1.791	0.	1.31

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	112	222	375	190	0	153
normalized size	1	1.	1.13	2.24	3.79	1.92	0.	1.55
time (sec)	N/A	0.177	0.695	0.08	1.51	1.701	0.	1.309

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	83	154	265	128	0	117
normalized size	1	1.	1.14	2.11	3.63	1.75	0.	1.6
time (sec)	N/A	0.15	0.579	0.072	1.504	1.674	0.	1.309

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	68	85	151	70	0	78
normalized size	1	1.	1.55	1.93	3.43	1.59	0.	1.77
time (sec)	N/A	0.109	0.268	0.065	1.52	1.678	0.	1.336

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	66	36	66	111	0	50
normalized size	1	1.	1.78	0.97	1.78	3.	0.	1.35
time (sec)	N/A	0.126	0.21	0.049	0.996	1.555	0.	1.308

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	116	62	130	225	0	100
normalized size	1	1.	2.11	1.13	2.36	4.09	0.	1.82
time (sec)	N/A	0.143	0.5	0.057	0.978	1.672	0.	1.329

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	158	88	184	347	0	139
normalized size	1	1.	2.16	1.21	2.52	4.75	0.	1.9
time (sec)	N/A	0.147	0.618	0.062	0.997	1.673	0.	1.312

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	200	114	238	466	0	178
normalized size	1	1.	2.2	1.25	2.62	5.12	0.	1.96
time (sec)	N/A	0.151	0.98	0.065	1.003	1.792	0.	1.284

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	242	140	292	602	0	217
normalized size	1	1.	2.22	1.28	2.68	5.52	0.	1.99
time (sec)	N/A	0.155	1.468	0.065	1.03	1.719	0.	1.338

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	72	88	120	254	0	250
normalized size	1	1.	0.53	0.64	0.88	1.85	0.	1.82
time (sec)	N/A	0.186	4.825	0.112	1.016	1.81	0.	1.341

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	62	79	107	221	0	190
normalized size	1	1.	0.54	0.69	0.94	1.94	0.	1.67
time (sec)	N/A	0.18	3.449	0.102	1.025	1.736	0.	1.315

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	53	50	66	126	0	190
normalized size	1	1.	0.73	0.68	0.9	1.73	0.	2.6
time (sec)	N/A	0.159	1.769	0.086	0.99	1.772	0.	1.343

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	42	39	53	100	0	161
normalized size	1	1.	0.76	0.71	0.96	1.82	0.	2.93
time (sec)	N/A	0.154	0.571	0.069	0.979	1.68	0.	1.32

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	51	82	69	132	0	101
normalized size	1	1.	0.77	1.24	1.05	2.	0.	1.53
time (sec)	N/A	0.163	0.205	0.083	1.009	1.76	0.	1.33

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	64	68	62	159	0	70
normalized size	1	1.	1.23	1.31	1.19	3.06	0.	1.35
time (sec)	N/A	0.102	0.185	0.026	1.006	1.726	0.	1.302

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	83	72	100	294	0	117
normalized size	1	1.	1.38	1.2	1.67	4.9	0.	1.95
time (sec)	N/A	0.127	0.175	0.06	1.005	1.703	0.	1.304

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	38	57	80	142	0	111
normalized size	1	1.	0.9	1.36	1.9	3.38	0.	2.64
time (sec)	N/A	0.127	0.089	0.066	0.979	1.649	0.	1.333

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	152	144	225	734	0	279
normalized size	1	1.	1.04	0.99	1.54	5.03	0.	1.91
time (sec)	N/A	0.217	0.746	0.072	1.029	1.738	0.	1.364

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	131	290	510	270	0	188
normalized size	1	1.	0.78	1.74	3.05	1.62	0.	1.13
time (sec)	N/A	0.44	2.755	0.102	1.531	1.749	0.	1.333

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	111	222	394	192	0	153
normalized size	1	1.	1.07	2.13	3.79	1.85	0.	1.47
time (sec)	N/A	0.311	0.869	0.086	1.546	1.721	0.	1.3

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	91	154	278	135	0	117
normalized size	1	1.	1.05	1.77	3.2	1.55	0.	1.34
time (sec)	N/A	0.233	0.544	0.089	1.523	1.726	0.	1.362

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	121	103	189	158	0	101
normalized size	1	1.	1.75	1.49	2.74	2.29	0.	1.46
time (sec)	N/A	0.316	0.321	0.086	1.539	1.703	0.	1.325

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	105	60	122	180	0	100
normalized size	1	1.	1.44	0.82	1.67	2.47	0.	1.37
time (sec)	N/A	0.2	0.426	0.058	1.013	1.609	0.	1.295

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	149	86	181	267	0	142
normalized size	1	1.	1.64	0.95	1.99	2.93	0.	1.56
time (sec)	N/A	0.345	0.649	0.066	1.002	1.667	0.	1.239

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	191	112	235	423	0	181
normalized size	1	1.	1.75	1.03	2.16	3.88	0.	1.66
time (sec)	N/A	0.351	0.973	0.07	0.997	1.738	0.	1.382

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	233	112	235	521	0	181
normalized size	1	1.	1.86	0.9	1.88	4.17	0.	1.45
time (sec)	N/A	0.367	1.41	0.078	1.007	1.851	0.	1.395

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	120	90	120	267	0	279
normalized size	1	1.	0.86	0.65	0.86	1.92	0.	2.01
time (sec)	N/A	0.195	4.27	0.12	0.998	1.809	0.	1.382

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	100	69	93	194	0	250
normalized size	1	1.	0.92	0.63	0.85	1.78	0.	2.29
time (sec)	N/A	0.179	2.864	0.104	0.986	1.756	0.	1.342

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	80	50	66	128	0	220
normalized size	1	1.	1.1	0.68	0.9	1.75	0.	3.01
time (sec)	N/A	0.165	1.666	0.087	0.985	1.756	0.	1.389

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	73	114	99	201	0	232
normalized size	1	1.	0.72	1.12	0.97	1.97	0.	2.27
time (sec)	N/A	0.182	1.017	0.108	1.012	1.726	0.	1.382

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	99	100	97	228	0	127
normalized size	1	1.	1.11	1.12	1.09	2.56	0.	1.43
time (sec)	N/A	0.184	0.412	0.097	0.977	1.771	0.	1.299

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	103	86	96	255	0	85
normalized size	1	1.	1.37	1.15	1.28	3.4	0.	1.13
time (sec)	N/A	0.117	0.323	0.029	1.024	1.734	0.	1.345

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	97	90	132	416	0	153
normalized size	1	1.	1.18	1.1	1.61	5.07	0.	1.87
time (sec)	N/A	0.151	0.326	0.066	1.007	1.755	0.	1.344

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	138	126	197	640	0	246
normalized size	1	1.	1.1	1.	1.56	5.08	0.	1.95
time (sec)	N/A	0.134	0.577	0.079	1.002	1.779	0.	1.345

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	137	126	254	857	0	313
normalized size	1	1.	1.07	0.98	1.98	6.7	0.	2.45
time (sec)	N/A	0.206	5.17	0.081	1.013	1.862	0.	1.402

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	131	290	510	274	0	188
normalized size	1	1.	0.83	1.85	3.25	1.75	0.	1.2
time (sec)	N/A	0.461	4.532	0.105	1.538	1.803	0.	1.282

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	111	222	394	201	0	153
normalized size	1	1.	0.86	1.72	3.05	1.56	0.	1.19
time (sec)	N/A	0.291	1.837	0.113	1.536	1.763	0.	1.346

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	173	171	306	217	0	136
normalized size	1	1.	1.6	1.58	2.83	2.01	0.	1.26
time (sec)	N/A	0.318	0.646	0.107	1.52	1.701	0.	1.359

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	97	97	177	122	221	259	0	130
normalized size	1	1.	1.82	1.26	2.28	2.67	0.	1.34
time (sec)	N/A	0.31	0.417	0.092	1.797	1.817	0.	1.306

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	137	60	122	240	0	99
normalized size	1	1.	1.54	0.67	1.37	2.7	0.	1.11
time (sec)	N/A	0.367	0.564	0.066	1.036	1.836	0.	1.312

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	175	60	124	359	0	99
normalized size	1	1.	1.7	0.58	1.2	3.49	0.	0.96
time (sec)	N/A	0.379	0.714	0.074	1.124	1.892	0.	1.287

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	223	112	235	495	0	181
normalized size	1	1.	1.76	0.88	1.85	3.9	0.	1.43
time (sec)	N/A	0.408	1.216	0.079	1.13	1.989	0.	1.317

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	265	138	289	567	0	220
normalized size	1	1.	1.83	0.95	1.99	3.91	0.	1.52
time (sec)	N/A	0.419	1.761	0.083	1.155	2.059	0.	1.358

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	106	290	0	0	0	0
normalized size	1	1.	0.68	1.85	0.	0.	0.	0.
time (sec)	N/A	0.201	0.302	1.403	0.	0.	0.	0.

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	170	210	0	0	0	0
normalized size	1	1.	1.1	1.36	0.	0.	0.	0.
time (sec)	N/A	0.2	0.574	1.127	0.	0.	0.	0.

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	69	198	0	0	0	0
normalized size	1	1.	0.66	1.9	0.	0.	0.	0.
time (sec)	N/A	0.15	0.109	1.14	0.	0.	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	201	122	0	0	0	0
normalized size	1	1.	1.95	1.18	0.	0.	0.	0.
time (sec)	N/A	0.152	3.206	0.998	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	143	247	0	0	0	0
normalized size	1	1.	0.92	1.59	0.	0.	0.	0.
time (sec)	N/A	0.199	0.363	1.326	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	120	212	0	0	0	0
normalized size	1	1.	0.75	1.32	0.	0.	0.	0.
time (sec)	N/A	0.201	0.348	1.561	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	194	194	205	265	0	0	0	0
normalized size	1	1.	1.06	1.37	0.	0.	0.	0.
time (sec)	N/A	0.382	16.617	2.388	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	192	192	204	201	0	0	0	0
normalized size	1	1.	1.06	1.05	0.	0.	0.	0.
time (sec)	N/A	0.38	14.674	2.076	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	138	138	168	219	0	0	0	0
normalized size	1	1.	1.22	1.59	0.	0.	0.	0.
time (sec)	N/A	0.307	1.937	2.126	0.	0.	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	139	139	164	163	0	0	0	0
normalized size	1	1.	1.18	1.17	0.	0.	0.	0.
time (sec)	N/A	0.307	64.341	2.059	0.	0.	0.	0.

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	224	224	135	238	0	0	0	0
normalized size	1	1.	0.6	1.06	0.	0.	0.	0.
time (sec)	N/A	0.424	10.616	2.436	0.	0.	0.	0.

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	234	234	169	301	0	0	0	0
normalized size	1	1.	0.72	1.29	0.	0.	0.	0.
time (sec)	N/A	0.419	46.669	2.245	0.	0.	0.	0.

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	122	128	0	0	0	0
normalized size	1	1.	0.88	0.92	0.	0.	0.	0.
time (sec)	N/A	0.28	0.678	1.288	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	232	173	0	0	0	0
normalized size	1	1.	2.23	1.66	0.	0.	0.	0.
time (sec)	N/A	0.22	4.718	1.322	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	69	112	0	0	0	0
normalized size	1	1.	0.68	1.1	0.	0.	0.	0.
time (sec)	N/A	0.222	19.829	1.348	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	249	149	0	0	0	0
normalized size	1	1.	2.62	1.57	0.	0.	0.	0.
time (sec)	N/A	0.208	0.588	1.421	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	77	121	0	0	0	0
normalized size	1	1.	0.76	1.2	0.	0.	0.	0.
time (sec)	N/A	0.211	0.528	1.324	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	124	187	0	0	0	0
normalized size	1	1.	0.92	1.39	0.	0.	0.	0.
time (sec)	N/A	0.248	1.064	1.458	0.	0.	0.	0.

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	91	136	0	0	0	0
normalized size	1	1.	0.67	1.01	0.	0.	0.	0.
time (sec)	N/A	0.25	1.253	1.51	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	94	145	0	0	0	0
normalized size	1	1.	0.58	0.9	0.	0.	0.	0.
time (sec)	N/A	0.551	1.557	1.74	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	249	173	0	0	0	0
normalized size	1	1.	1.33	0.93	0.	0.	0.	0.
time (sec)	N/A	0.596	3.088	1.766	0.	0.	0.	0.

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	119	153	0	0	0	0
normalized size	1	1.	0.63	0.81	0.	0.	0.	0.
time (sec)	N/A	0.594	1.834	1.671	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	188	188	222	205	0	0	0	0
normalized size	1	1.	1.18	1.09	0.	0.	0.	0.
time (sec)	N/A	0.59	1.327	1.688	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	82	148	0	0	0	0
normalized size	1	1.	0.43	0.78	0.	0.	0.	0.
time (sec)	N/A	0.591	1.374	1.707	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	163	213	0	0	0	0
normalized size	1	1.	0.73	0.95	0.	0.	0.	0.
time (sec)	N/A	0.664	1.421	1.753	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	113	160	0	0	0	0
normalized size	1	1.	0.5	0.71	0.	0.	0.	0.
time (sec)	N/A	0.67	0.961	1.817	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	247	247	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.353	2.155	0.942	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	195	195	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.285	0.932	0.79	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	97	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.143	0.145	0.651	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	100	100	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.199	29.822	0.619	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	207	207	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.526	0.674	0.364	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	236	236	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.635	1.211	0.409	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	106	106	1243	0	0	0	0	0
normalized size	1	1.	11.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.375	9.71	0.191	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	107	107	433	0	0	0	0	0
normalized size	1	1.	4.05	0.	0.	0.	0.	0.
time (sec)	N/A	0.314	2.819	0.211	0.	0.	0.	0.

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	115	115	277	0	0	0	0	0
normalized size	1	1.	2.41	0.	0.	0.	0.	0.
time (sec)	N/A	0.325	2.037	0.208	0.	0.	0.	0.

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	120	120	484	0	0	0	0	0
normalized size	1	1.	4.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.374	2.863	0.175	0.	0.	0.	0.

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	130	130	276	0	0	0	0	0
normalized size	1	1.	2.12	0.	0.	0.	0.	0.
time (sec)	N/A	0.277	1.846	0.728	0.	0.	0.	0.

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	113	0	0	0	0	0
normalized size	1	1.	0.63	0.	0.	0.	0.	0.
time (sec)	N/A	0.169	1.524	0.701	0.	0.	0.	0.

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	84	0	0	0	0	0
normalized size	1	1.	0.68	0.	0.	0.	0.	0.
time (sec)	N/A	0.108	0.498	0.658	0.	0.	0.	0.

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	67	0	0	0	0	0
normalized size	1	1.	0.81	0.	0.	0.	0.	0.
time (sec)	N/A	0.073	0.138	0.581	0.	0.	0.	0.

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	42	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.037	0.037	0.337	0.	0.	0.	0.

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	40	40	92	0	0	0	0	0
normalized size	1	1.	2.3	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.769	0.254	0.	0.	0.	0.

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	112	112	123	0	0	0	0	0
normalized size	1	1.	1.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	1.684	0.275	0.	0.	0.	0.

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	240	240	316	0	0	0	0	0
normalized size	1	1.	1.32	0.	0.	0.	0.	0.
time (sec)	N/A	0.224	5.721	0.296	0.	0.	0.	0.

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	230	230	7069	0	0	0	0	0
normalized size	1	1.	30.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.668	23.103	0.66	0.	0.	0.	0.

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	95	95	4297	0	0	0	0	0
normalized size	1	1.	45.23	0.	0.	0.	0.	0.
time (sec)	N/A	0.353	17.317	0.569	0.	0.	0.	0.

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	87	0	0	0	0	0
normalized size	1	1.	0.89	0.	0.	0.	0.	0.
time (sec)	N/A	0.132	1.052	0.276	0.	0.	0.	0.

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	349	349	214	0	0	0	0	0
normalized size	1	1.	0.61	0.	0.	0.	0.	0.
time (sec)	N/A	0.541	6.763	0.321	0.	0.	0.	0.

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	382	0	0	0	0	0
normalized size	1	1.	3.64	0.	0.	0.	0.	0.
time (sec)	N/A	0.262	3.188	0.179	0.	0.	0.	0.

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	214	0	0	0	0	0
normalized size	1	1.	2.04	0.	0.	0.	0.	0.
time (sec)	N/A	0.272	1.341	0.161	0.	0.	0.	0.

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	212	0	0	0	0	0
normalized size	1	1.	2.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.25	1.012	0.166	0.	0.	0.	0.

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	105	105	212	0	0	0	0	0
normalized size	1	1.	2.02	0.	0.	0.	0.	0.
time (sec)	N/A	0.267	1.171	0.16	0.	0.	0.	0.

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	115	129	123	261	0	428
normalized size	1	1.	0.97	1.08	1.03	2.19	0.	3.6
time (sec)	N/A	0.11	0.139	0.041	0.968	1.833	0.	1.36

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	83	95	93	193	0	335
normalized size	1	1.	0.95	1.09	1.07	2.22	0.	3.85
time (sec)	N/A	0.098	0.082	0.035	0.967	1.809	0.	1.379

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	58	58	57	61	63	126	0	89
normalized size	1	1.	0.98	1.05	1.09	2.17	0.	1.53
time (sec)	N/A	0.086	0.045	0.033	0.953	1.783	0.	1.318

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	37	28	31	59	0	43
normalized size	1	1.	1.42	1.08	1.19	2.27	0.	1.65
time (sec)	N/A	0.033	0.026	0.018	0.984	1.723	0.	1.294

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	26	26	63	35	61	149	0	82
normalized size	1	1.	2.42	1.35	2.35	5.73	0.	3.15
time (sec)	N/A	0.073	0.035	0.033	0.953	1.726	0.	1.346

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	114	68	96	316	0	228
normalized size	1	1.	1.78	1.06	1.5	4.94	0.	3.56
time (sec)	N/A	0.104	0.523	0.094	0.973	1.826	0.	1.357

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	164	102	149	529	0	359
normalized size	1	1.	1.64	1.02	1.49	5.29	0.	3.59
time (sec)	N/A	0.124	0.592	0.091	0.959	1.841	0.	1.396

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	216	136	193	749	0	482
normalized size	1	1.	1.54	0.97	1.38	5.35	0.	3.44
time (sec)	N/A	0.144	0.601	0.097	0.965	2.184	0.	1.422

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	118	130	143	290	0	308
normalized size	1	1.	0.93	1.02	1.13	2.28	0.	2.43
time (sec)	N/A	0.128	0.209	0.039	0.986	1.989	0.	1.358

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	86	96	109	217	0	232
normalized size	1	1.	0.97	1.08	1.22	2.44	0.	2.61
time (sec)	N/A	0.111	0.149	0.035	0.958	1.812	0.	1.316

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	54	62	80	143	0	154
normalized size	1	1.	1.06	1.22	1.57	2.8	0.	3.02
time (sec)	N/A	0.083	0.059	0.033	0.947	1.804	0.	1.351

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	41	47	68	170	0	104
normalized size	1	1.	1.11	1.27	1.84	4.59	0.	2.81
time (sec)	N/A	0.096	0.027	0.034	0.952	1.739	0.	1.287

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	69	81	103	319	0	180
normalized size	1	1.	1.	1.17	1.49	4.62	0.	2.61
time (sec)	N/A	0.105	0.024	0.041	0.978	1.734	0.	1.358

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	91	115	130	473	0	262
normalized size	1	1.	0.9	1.14	1.29	4.68	0.	2.59
time (sec)	N/A	0.111	0.026	0.042	0.969	1.797	0.	1.339

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	112	184	142	328	0	564
normalized size	1	1.	0.9	1.48	1.15	2.65	0.	4.55
time (sec)	N/A	0.196	0.363	0.041	0.969	1.803	0.	1.397

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	72	125	96	228	0	135
normalized size	1	1.	0.9	1.56	1.2	2.85	0.	1.69
time (sec)	N/A	0.145	0.176	0.039	0.962	1.812	0.	1.314

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	37	45	54	116	0	68
normalized size	1	1.	0.88	1.07	1.29	2.76	0.	1.62
time (sec)	N/A	0.078	0.057	0.022	0.941	1.752	0.	1.287

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	91	77	99	267	0	167
normalized size	1	1.	1.23	1.04	1.34	3.61	0.	2.26
time (sec)	N/A	0.18	0.151	0.035	0.944	1.794	0.	1.367

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	329	139	161	512	0	424
normalized size	1	1.	2.89	1.22	1.41	4.49	0.	3.72
time (sec)	N/A	0.294	0.616	0.042	0.958	1.867	0.	1.384

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	193	246	234	468	0	512
normalized size	1	1.	1.1	1.41	1.34	2.67	0.	2.93
time (sec)	N/A	0.461	1.666	0.044	1.463	1.951	0.	1.366

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	178	178	157	187	169	371	0	385
normalized size	1	1.	0.88	1.05	0.95	2.08	0.	2.16
time (sec)	N/A	0.556	1.012	0.041	1.499	1.883	0.	1.318

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	121	99	108	279	0	215
normalized size	1	1.	1.57	1.29	1.4	3.62	0.	2.79
time (sec)	N/A	0.131	0.576	0.037	1.547	1.83	0.	1.377

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	138	89	99	267	0	225
normalized size	1	1.	2.34	1.51	1.68	4.53	0.	3.81
time (sec)	N/A	0.414	0.465	0.036	1.023	1.773	0.	1.33

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	259	151	151	451	0	305
normalized size	1	1.	2.59	1.51	1.51	4.51	0.	3.05
time (sec)	N/A	0.322	0.603	0.045	0.968	1.784	0.	1.417

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	368	212	193	629	0	440
normalized size	1	1.	2.57	1.48	1.35	4.4	0.	3.08
time (sec)	N/A	0.408	0.721	0.048	1.04	1.864	0.	1.364

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	170	170	154	266	192	437	0	938
normalized size	1	1.	0.91	1.56	1.13	2.57	0.	5.52
time (sec)	N/A	0.255	0.637	0.049	1.	1.928	0.	1.345

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	102	164	132	300	0	173
normalized size	1	1.	0.88	1.41	1.14	2.59	0.	1.49
time (sec)	N/A	0.128	0.337	0.044	1.002	1.766	0.	1.368

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	56	65	77	163	0	89
normalized size	1	1.	0.88	1.02	1.2	2.55	0.	1.39
time (sec)	N/A	0.101	0.107	0.022	0.962	1.823	0.	1.379

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	89	113	151	354	0	338
normalized size	1	1.	0.87	1.11	1.48	3.47	0.	3.31
time (sec)	N/A	0.219	0.292	0.041	0.967	1.93	0.	1.492

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	162	162	669	201	231	683	0	651
normalized size	1	1.	4.13	1.24	1.43	4.22	0.	4.02
time (sec)	N/A	0.349	6.196	0.053	1.026	1.887	0.	1.543

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	818	354	327	605	0	760
normalized size	1	1.	2.74	1.18	1.09	2.02	0.	2.54
time (sec)	N/A	0.336	6.243	0.051	1.532	2.011	0.	1.548

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	696	276	247	466	0	582
normalized size	1	1.	2.95	1.17	1.05	1.97	0.	2.47
time (sec)	N/A	0.748	6.162	0.047	1.485	1.936	0.	1.537

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	327	167	174	359	0	467
normalized size	1	1.	2.37	1.21	1.26	2.6	0.	3.38
time (sec)	N/A	0.504	0.866	0.042	1.493	1.914	0.	1.589

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	406	158	188	378	0	304
normalized size	1	1.	3.05	1.19	1.41	2.84	0.	2.29
time (sec)	N/A	0.273	0.64	0.042	0.964	1.821	0.	1.301

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	205	205	610	246	257	618	0	487
normalized size	1	1.	2.98	1.2	1.25	3.01	0.	2.38
time (sec)	N/A	0.291	0.916	0.05	1.033	1.866	0.	1.351

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	279	279	812	334	311	857	0	672
normalized size	1	1.	2.91	1.2	1.11	3.07	0.	2.41
time (sec)	N/A	0.316	1.412	0.051	0.997	1.862	0.	1.33

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	282	363	302	504	0	2105
normalized size	1	1.	1.26	1.63	1.35	2.26	0.	9.44
time (sec)	N/A	0.251	1.344	0.049	1.055	1.967	0.	1.289

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	172	216	190	327	0	1170
normalized size	1	1.	1.13	1.42	1.25	2.15	0.	7.7
time (sec)	N/A	0.194	0.362	0.043	1.06	1.907	0.	1.32

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	89	106	108	181	0	138
normalized size	1	1.	1.	1.19	1.21	2.03	0.	1.55
time (sec)	N/A	0.156	0.188	0.041	0.979	1.805	0.	1.301

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	30	53	45	74	0	51
normalized size	1	1.	0.88	1.56	1.32	2.18	0.	1.5
time (sec)	N/A	0.077	0.018	0.027	1.024	1.703	0.	1.304

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	63	75	86	173	0	135
normalized size	1	1.	0.85	1.01	1.16	2.34	0.	1.82
time (sec)	N/A	0.105	0.093	0.048	0.982	1.861	0.	1.343

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	116	116	123	121	178	508	0	273
normalized size	1	1.	1.06	1.04	1.53	4.38	0.	2.35
time (sec)	N/A	0.213	0.595	0.069	1.001	2.172	0.	1.33

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	179	179	207	259	362	1033	0	566
normalized size	1	1.	1.16	1.45	2.02	5.77	0.	3.16
time (sec)	N/A	0.301	5.163	0.067	1.018	3.139	0.	1.442

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	230	230	268	1566	0	1296	0	1054
normalized size	1	1.	1.17	6.81	0.	5.63	0.	4.58
time (sec)	N/A	0.608	2.381	0.074	0.	2.403	0.	1.258

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	172	769	0	902	0	549
normalized size	1	1.	1.07	4.78	0.	5.6	0.	3.41
time (sec)	N/A	0.381	0.812	0.065	0.	2.483	0.	1.338

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	96	269	0	599	0	250
normalized size	1	1.	0.96	2.69	0.	5.99	0.	2.5
time (sec)	N/A	0.207	0.302	0.058	0.	2.277	0.	1.336

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	118	96	0	680	0	174
normalized size	1	1.	1.4	1.14	0.	8.1	0.	2.07
time (sec)	N/A	0.149	0.195	0.058	0.	1.861	0.	1.375

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	162	165	0	1214	0	363
normalized size	1	1.	1.16	1.18	0.	8.67	0.	2.59
time (sec)	N/A	0.306	0.891	0.069	0.	1.921	0.	1.404

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	277	282	0	1917	0	730
normalized size	1	1.	1.38	1.4	0.	9.54	0.	3.63
time (sec)	N/A	0.52	1.245	0.074	0.	2.097	0.	1.324

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	417	456	366	803	0	2512
normalized size	1	1.	1.56	1.71	1.37	3.01	0.	9.41
time (sec)	N/A	0.372	3.657	0.067	1.031	2.419	0.	1.396

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	194	194	280	285	248	563	0	1488
normalized size	1	1.	1.44	1.47	1.28	2.9	0.	7.67
time (sec)	N/A	0.299	1.072	0.061	1.044	2.132	0.	1.379

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	167	153	151	346	0	188
normalized size	1	1.	1.4	1.29	1.27	2.91	0.	1.58
time (sec)	N/A	0.228	0.426	0.063	1.071	1.845	0.	1.343

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	76	75	74	178	0	82
normalized size	1	1.	1.33	1.32	1.3	3.12	0.	1.44
time (sec)	N/A	0.112	0.133	0.036	1.001	1.76	0.	1.25

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	165	106	166	486	0	288
normalized size	1	1.	1.51	0.97	1.52	4.46	0.	2.64
time (sec)	N/A	0.226	0.281	0.067	1.082	2.197	0.	1.329

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	224	224	370	1378	0	616
normalized size	1	1.	1.33	1.33	2.2	8.2	0.	3.67
time (sec)	N/A	0.433	1.3	0.078	1.103	2.815	0.	1.44

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	320	368	690	2653	0	959
normalized size	1	1.	1.24	1.42	2.66	10.24	0.	3.7
time (sec)	N/A	0.741	1.383	0.086	1.048	4.197	0.	1.442

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	473	473	402	1735	0	1871	0	1175
normalized size	1	1.	0.85	3.67	0.	3.96	0.	2.48
time (sec)	N/A	1.713	6.909	0.091	0.	2.536	0.	1.383

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	282	883	0	1347	0	651
normalized size	1	1.	1.08	3.38	0.	5.16	0.	2.49
time (sec)	N/A	0.825	3.093	0.081	0.	2.188	0.	1.287

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	178	325	0	1204	0	324
normalized size	1	1.	1.17	2.14	0.	7.92	0.	2.13
time (sec)	N/A	0.571	1.151	0.072	0.	2.034	0.	1.308

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	128	162	0	1164	0	390
normalized size	1	1.	0.63	0.8	0.	5.73	0.	1.92
time (sec)	N/A	0.436	0.823	0.082	0.	1.952	0.	1.378

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	343	343	281	242	0	2268	0	617
normalized size	1	1.	0.82	0.71	0.	6.61	0.	1.8
time (sec)	N/A	0.547	1.081	0.092	0.	2.258	0.	1.376

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	329	329	550	549	440	1062	0	2903
normalized size	1	1.	1.67	1.67	1.34	3.23	0.	8.82
time (sec)	N/A	0.502	4.691	0.072	0.999	3.076	0.	1.575

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	239	239	388	355	316	792	0	1805
normalized size	1	1.	1.62	1.49	1.32	3.31	0.	7.55
time (sec)	N/A	0.365	2.918	0.071	0.974	2.394	0.	1.394

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	208	200	208	524	0	230
normalized size	1	1.	1.32	1.27	1.32	3.32	0.	1.46
time (sec)	N/A	0.272	0.921	0.062	0.968	2.154	0.	1.449

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	111	96	117	304	0	104
normalized size	1	1.	1.34	1.16	1.41	3.66	0.	1.25
time (sec)	N/A	0.134	0.399	0.033	0.961	1.889	0.	1.34

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	203	206	325	1021	0	610
normalized size	1	1.	1.25	1.26	1.99	6.26	0.	3.74
time (sec)	N/A	0.319	0.555	0.071	0.98	2.605	0.	1.507

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	A	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	332	322	587	2344	0	1080
normalized size	1	1.	1.45	1.41	2.56	10.24	0.	4.72
time (sec)	N/A	0.513	6.307	0.087	1.042	4.016	0.	1.457

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	496	427	954	3970	0	2094
normalized size	1	1.	1.58	1.36	3.05	12.68	0.	6.69
time (sec)	N/A	1.007	4.505	0.093	1.048	5.88	0.	1.607

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	539	539	599	2251	0	2515	0	1391
normalized size	1	1.	1.11	4.18	0.	4.67	0.	2.58
time (sec)	N/A	2.436	12.144	0.096	0.	2.95	0.	1.725

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F(-2)	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	1178	1227	0	2331	0	788
normalized size	1	1.	3.54	3.68	0.	7.	0.	2.37
time (sec)	N/A	1.138	9.265	0.086	0.	2.69	0.	1.499

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	A	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	282	729	0	2184	0	815
normalized size	1	1.	1.06	2.73	0.	8.18	0.	3.05
time (sec)	N/A	0.94	3.879	0.088	0.	2.483	0.	1.511

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	376	376	231	234	0	1854	0	521
normalized size	1	1.	0.61	0.62	0.	4.93	0.	1.39
time (sec)	N/A	0.658	0.919	0.091	0.	2.248	0.	1.355

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F(-2)	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	515	515	388	328	0	3421	0	957
normalized size	1	1.	0.75	0.64	0.	6.64	0.	1.86
time (sec)	N/A	0.775	1.072	0.102	0.	2.724	0.	1.457

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	516	516	2049	1776	0	0	0	0
normalized size	1	1.	3.97	3.44	0.	0.	0.	0.
time (sec)	N/A	1.703	17.301	5.92	0.	0.	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	430	430	853	1195	0	0	0	0
normalized size	1	1.	1.98	2.78	0.	0.	0.	0.
time (sec)	N/A	1.109	14.885	4.602	0.	0.	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	444	444	1959	1120	0	0	0	0
normalized size	1	1.	4.41	2.52	0.	0.	0.	0.
time (sec)	N/A	1.044	16.458	4.273	0.	0.	0.	0.

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	356	356	351	919	0	0	0	0
normalized size	1	1.	0.99	2.58	0.	0.	0.	0.
time (sec)	N/A	0.762	20.079	2.525	0.	0.	0.	0.

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	370	370	546	937	0	0	0	0
normalized size	1	1.	1.48	2.53	0.	0.	0.	0.
time (sec)	N/A	0.781	5.943	2.793	0.	0.	0.	0.

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	430	430	834	1083	0	0	0	0
normalized size	1	1.	1.94	2.52	0.	0.	0.	0.
time (sec)	N/A	1.037	14.273	3.08	0.	0.	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	452	452	1233	681	0	0	0	0
normalized size	1	1.	2.73	1.51	0.	0.	0.	0.
time (sec)	N/A	1.05	12.299	4.983	0.	0.	0.	0.

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	511	511	930	1672	0	0	0	0
normalized size	1	1.	1.82	3.27	0.	0.	0.	0.
time (sec)	N/A	1.377	6.795	3.823	0.	0.	0.	0.

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1070	1070	974	3808	0	0	0	0
normalized size	1	1.	0.91	3.56	0.	0.	0.	0.
time (sec)	N/A	2.795	15.353	9.805	0.	0.	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1101	1101	2095	3412	0	0	0	0
normalized size	1	1.	1.9	3.1	0.	0.	0.	0.
time (sec)	N/A	2.93	16.628	9.819	0.	0.	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	850	850	886	2540	0	0	0	0
normalized size	1	1.	1.04	2.99	0.	0.	0.	0.
time (sec)	N/A	2.126	14.912	7.117	0.	0.	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	882	882	2012	2282	0	0	0	0
normalized size	1	1.	2.28	2.59	0.	0.	0.	0.
time (sec)	N/A	2.168	16.048	7.666	0.	0.	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	809	809	854	1563	0	0	0	0
normalized size	1	1.	1.06	1.93	0.	0.	0.	0.
time (sec)	N/A	1.833	15.045	6.344	0.	0.	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	838	838	1246	1475	0	0	0	0
normalized size	1	1.	1.49	1.76	0.	0.	0.	0.
time (sec)	N/A	1.924	12.879	7.002	0.	0.	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1054	1054	922	2263	0	0	0	0
normalized size	1	1.	0.87	2.15	0.	0.	0.	0.
time (sec)	N/A	2.692	6.859	7.915	0.	0.	0.	0.

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F(-1)	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	1089	1089	1320	2159	0	0	0	0
normalized size	1	1.	1.21	1.98	0.	0.	0.	0.
time (sec)	N/A	2.781	15.21	9.019	0.	0.	0.	0.

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	153	215	0	0	0	0
normalized size	1	1.	1.22	1.72	0.	0.	0.	0.
time (sec)	N/A	0.032	0.276	0.258	0.	0.	0.	0.

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	120	264	0	0	0	0
normalized size	1	1.	0.99	2.18	0.	0.	0.	0.
time (sec)	N/A	0.115	1.193	0.278	0.	0.	0.	0.

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	309	309	882	1199	0	0	0	0
normalized size	1	1.	2.85	3.88	0.	0.	0.	0.
time (sec)	N/A	0.23	6.113	0.3	0.	0.	0.	0.

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	276	850	0	0	0	0
normalized size	1	1.	1.21	3.73	0.	0.	0.	0.
time (sec)	N/A	0.24	11.12	0.333	0.	0.	0.	0.

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	140	178	0	0	0	0
normalized size	1	1.	1.32	1.68	0.	0.	0.	0.
time (sec)	N/A	0.022	0.223	0.255	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	255	255	259	852	0	0	0	0
normalized size	1	1.	1.02	3.34	0.	0.	0.	0.
time (sec)	N/A	0.32	7.61	0.32	0.	0.	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	347	347	1249	1209	0	0	0	0
normalized size	1	1.	3.6	3.48	0.	0.	0.	0.
time (sec)	N/A	0.333	6.142	0.281	0.	0.	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	318	318	259	1065	0	0	0	0
normalized size	1	1.	0.81	3.35	0.	0.	0.	0.
time (sec)	N/A	0.525	7.799	0.258	0.	0.	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	249	249	182	0	0	0	0	0
normalized size	1	1.	0.73	0.	0.	0.	0.	0.
time (sec)	N/A	0.387	0.316	1.569	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	134	0	0	0	0	0
normalized size	1	1.	0.71	0.	0.	0.	0.	0.
time (sec)	N/A	0.839	0.253	1.267	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	98	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.158	0.107	0.619	0.	0.	0.	0.

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	232	232	687	0	0	0	0	0
normalized size	1	1.	2.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.259	5.624	0.575	0.	0.	0.	0.

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	405	405	1494	0	0	0	0	0
normalized size	1	1.	3.69	0.	0.	0.	0.	0.
time (sec)	N/A	0.456	14.964	0.314	0.	0.	0.	0.

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	580	580	2904	0	0	0	0	0
normalized size	1	1.	5.01	0.	0.	0.	0.	0.
time (sec)	N/A	0.592	19.103	0.395	0.	0.	0.	0.

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.068	7.607	0.205	0.	0.	0.	0.

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.062	0.513	0.217	0.	0.	0.	0.

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.07	2.564	0.211	0.	0.	0.	0.

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	27	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.071	2.801	0.202	0.	0.	0.	0.

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	3.195	0.72	0.	0.	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	150	150	562	0	0	0	0	0
normalized size	1	1.	3.75	0.	0.	0.	0.	0.
time (sec)	N/A	0.126	8.162	0.753	0.	0.	0.	0.

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	155	0	0	0	0	0
normalized size	1	1.	1.28	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	1.701	0.635	0.	0.	0.	0.

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	48	48	72	0	0	0	0	0
normalized size	1	1.	1.5	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	0.48	0.319	0.	0.	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	132	0	0	0	0	0
normalized size	1	1.	1.15	0.	0.	0.	0.	0.
time (sec)	N/A	0.119	0.946	0.241	0.	0.	0.	0.

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	231	231	710	0	0	0	0	0
normalized size	1	1.	3.07	0.	0.	0.	0.	0.
time (sec)	N/A	0.195	17.198	0.261	0.	0.	0.	0.

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	14.292	0.684	0.	0.	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	23	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	3.779	0.588	0.	0.	0.	0.

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	136	136	3614	0	0	0	0	0
normalized size	1	1.	26.57	0.	0.	0.	0.	0.
time (sec)	N/A	0.165	18.461	0.252	0.	0.	0.	0.

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	F	B	F	F	F	F(-1)	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD
size	424	0	6403	0	0	0	0	0
normalized size	1	0.	15.1	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	23.726	0.296	0.	0.	0.	0.

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.039	1.749	0.222	0.	0.	0.	0.

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	4.854	0.189	0.	0.	0.	0.

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.04	2.584	0.184	0.	0.	0.	0.

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	N/A	A	A	A	A	A	F(-1)	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	25	0	0	0	0	0	0	0
normalized size	1	0.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.041	2.78	0.188	0.	0.	0.	0.

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	190	190	135	694	0	0	0	0
normalized size	1	1.	0.71	3.65	0.	0.	0.	0.
time (sec)	N/A	0.165	1.435	0.335	0.	0.	0.	0.

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	169	169	146	1526	0	0	0	0
normalized size	1	1.	0.86	9.03	0.	0.	0.	0.
time (sec)	N/A	0.162	1.258	0.246	0.	0.	0.	0.

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	111	288	0	0	0	0
normalized size	1	1.	0.92	2.38	0.	0.	0.	0.
time (sec)	N/A	0.137	0.835	0.209	0.	0.	0.	0.

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	130	1503	0	0	0	0
normalized size	1	1.	1.07	12.32	0.	0.	0.	0.
time (sec)	N/A	0.148	0.809	0.239	0.	0.	0.	0.

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	135	710	0	0	0	0
normalized size	1	1.	0.74	3.9	0.	0.	0.	0.
time (sec)	N/A	0.172	10.727	0.233	0.	0.	0.	0.

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	197	197	165	1565	0	0	0	0
normalized size	1	1.	0.84	7.94	0.	0.	0.	0.
time (sec)	N/A	0.172	1.393	0.227	0.	0.	0.	0.

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	270	270	195	730	0	0	0	0
normalized size	1	1.	0.72	2.7	0.	0.	0.	0.
time (sec)	N/A	0.334	3.748	0.272	0.	0.	0.	0.

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	240	240	195	1559	0	0	0	0
normalized size	1	1.	0.81	6.5	0.	0.	0.	0.
time (sec)	N/A	0.331	4.723	0.197	0.	0.	0.	0.

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	154	154	168	729	0	0	0	0
normalized size	1	1.	1.09	4.73	0.	0.	0.	0.
time (sec)	N/A	0.263	2.436	0.242	0.	0.	0.	0.

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	153	153	287	1610	0	0	0	0
normalized size	1	1.	1.88	10.52	0.	0.	0.	0.
time (sec)	N/A	0.272	8.422	0.231	0.	0.	0.	0.

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	222	222	164	763	0	0	0	0
normalized size	1	1.	0.74	3.44	0.	0.	0.	0.
time (sec)	N/A	0.313	7.617	0.218	0.	0.	0.	0.

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	236	236	152	1600	0	0	0	0
normalized size	1	1.	0.64	6.78	0.	0.	0.	0.
time (sec)	N/A	0.32	10.861	0.239	0.	0.	0.	0.

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	155	155	131	465	0	0	0	0
normalized size	1	1.	0.85	3.	0.	0.	0.	0.
time (sec)	N/A	0.224	0.998	0.224	0.	0.	0.	0.

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	145	145	230	781	0	0	0	0
normalized size	1	1.	1.59	5.39	0.	0.	0.	0.
time (sec)	N/A	0.229	1.34	0.21	0.	0.	0.	0.

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	105	105	60	320	0	0	0	0
normalized size	1	1.	0.57	3.05	0.	0.	0.	0.
time (sec)	N/A	0.203	0.355	0.215	0.	0.	0.	0.

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	95	524	0	0	0	0
normalized size	1	1.	0.96	5.29	0.	0.	0.	0.
time (sec)	N/A	0.211	0.602	0.214	0.	0.	0.	0.

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	70	195	0	0	0	0
normalized size	1	1.	0.66	1.84	0.	0.	0.	0.
time (sec)	N/A	0.227	0.375	0.196	0.	0.	0.	0.

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	100	563	0	0	0	0
normalized size	1	1.	0.83	4.69	0.	0.	0.	0.
time (sec)	N/A	0.222	0.875	0.227	0.	0.	0.	0.

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	149	149	91	221	0	0	0	0
normalized size	1	1.	0.61	1.48	0.	0.	0.	0.
time (sec)	N/A	0.254	0.566	0.254	0.	0.	0.	0.

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	268	268	115	609	0	0	0	0
normalized size	1	1.	0.43	2.27	0.	0.	0.	0.
time (sec)	N/A	0.505	1.098	0.247	0.	0.	0.	0.

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	250	250	247	1044	0	0	0	0
normalized size	1	1.	0.99	4.18	0.	0.	0.	0.
time (sec)	N/A	0.493	1.791	0.233	0.	0.	0.	0.

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	82	474	0	0	0	0
normalized size	1	1.	0.41	2.36	0.	0.	0.	0.
time (sec)	N/A	0.448	0.691	0.24	0.	0.	0.	0.

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	252	793	0	0	0	0
normalized size	1	1.	1.27	3.98	0.	0.	0.	0.
time (sec)	N/A	0.469	1.612	0.236	0.	0.	0.	0.

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	213	213	101	327	0	0	0	0
normalized size	1	1.	0.47	1.54	0.	0.	0.	0.
time (sec)	N/A	0.475	0.537	0.221	0.	0.	0.	0.

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	125	551	0	0	0	0
normalized size	1	1.	0.58	2.56	0.	0.	0.	0.
time (sec)	N/A	0.471	2.135	0.239	0.	0.	0.	0.

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	94	221	0	0	0	0
normalized size	1	1.	0.55	1.28	0.	0.	0.	0.
time (sec)	N/A	0.463	2.276	0.208	0.	0.	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [241] had the largest ratio of [0.64]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	5	4	1.	19	0.21
2	A	5	4	1.	19	0.21
3	A	5	4	1.	19	0.21
4	A	5	4	1.	19	0.21
5	A	4	3	1.	17	0.176
6	A	6	6	1.	17	0.353
7	A	5	4	1.	19	0.21
8	A	5	4	1.	19	0.21
9	A	5	4	1.	19	0.21
10	A	11	7	1.	19	0.368
11	A	10	7	1.	19	0.368
12	A	9	7	1.	19	0.368
13	A	7	7	1.	19	0.368
14	A	7	7	1.	19	0.368
15	A	8	6	1.	19	0.316
16	A	8	6	1.	19	0.316
17	A	8	6	1.	19	0.316
18	A	8	6	1.	19	0.316
19	A	5	4	1.	21	0.19
20	A	5	4	1.	21	0.19
21	A	5	4	1.	21	0.19
22	A	5	4	1.	21	0.19
23	A	5	4	1.	19	0.21
24	A	5	4	1.	19	0.21
25	A	5	4	1.	21	0.19
26	A	5	4	1.	21	0.19
27	A	5	4	1.	21	0.19
28	A	5	4	1.	21	0.19
29	A	27	8	1.	21	0.381
30	A	18	8	1.	21	0.381
31	A	14	8	1.	21	0.381
32	A	9	7	1.	21	0.333
33	A	11	9	1.	21	0.429
34	A	8	8	1.	21	0.381
35	A	12	8	1.	21	0.381
36	A	12	8	1.	21	0.381
37	A	12	8	1.	21	0.381
38	A	5	4	1.	21	0.19
39	A	5	4	1.	21	0.19
40	A	5	4	1.	21	0.19
41	A	4	3	1.	21	0.143
42	A	5	4	1.	19	0.21
43	A	5	4	1.	19	0.21

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
44	A	5	4	1.	21	0.19
45	A	5	4	1.	21	0.19
46	A	5	4	1.	21	0.19
47	A	5	4	1.	21	0.19
48	A	29	9	1.	21	0.429
49	A	18	9	1.	21	0.429
50	A	16	9	1.	21	0.429
51	A	11	8	1.	21	0.381
52	A	9	7	1.	21	0.333
53	A	11	8	1.	21	0.381
54	A	10	9	1.	21	0.429
55	A	17	9	1.	21	0.429
56	A	17	9	1.	21	0.429
57	A	7	6	1.	21	0.286
58	A	7	6	1.	21	0.286
59	A	7	6	1.	21	0.286
60	A	6	5	1.	21	0.238
61	A	5	4	1.	19	0.21
62	A	6	6	1.	19	0.316
63	A	7	7	1.	21	0.333
64	A	8	7	1.	21	0.333
65	A	9	7	1.	21	0.333
66	A	8	7	1.	21	0.333
67	A	7	7	1.	21	0.333
68	A	5	5	1.	21	0.238
69	A	6	5	1.	21	0.238
70	A	7	6	1.	21	0.286
71	A	7	6	1.	21	0.286
72	A	7	6	1.	21	0.286
73	A	7	6	1.	21	0.286
74	A	5	4	1.	21	0.19
75	A	5	4	1.	21	0.19
76	A	5	4	1.	21	0.19
77	A	5	4	1.	21	0.19
78	A	5	4	1.	21	0.19
79	A	5	4	1.	19	0.21
80	A	6	5	1.	19	0.263
81	A	4	4	1.	21	0.19
82	A	6	5	1.	21	0.238
83	A	16	8	1.	21	0.381
84	A	7	6	1.	21	0.286
85	A	11	6	1.	21	0.286
86	A	9	8	1.	21	0.381
87	A	11	6	1.	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
88	A	13	7	1.	21	0.333
89	A	13	7	1.	21	0.333
90	A	13	7	1.	21	0.333
91	A	5	4	1.	21	0.19
92	A	5	4	1.	21	0.19
93	A	5	4	1.	21	0.19
94	A	5	4	1.	21	0.19
95	A	5	4	1.	21	0.19
96	A	5	4	1.	19	0.21
97	A	6	5	1.	19	0.263
98	A	5	4	1.	21	0.19
99	A	6	5	1.	21	0.238
100	A	19	9	1.	21	0.429
101	A	15	6	1.	21	0.286
102	A	13	8	1.	21	0.381
103	A	10	8	1.	21	0.381
104	A	15	8	1.	21	0.381
105	A	16	7	1.	21	0.333
106	A	16	7	1.	21	0.333
107	A	16	7	1.	21	0.333
108	A	11	11	1.	23	0.478
109	A	11	11	1.	23	0.478
110	A	9	9	1.	23	0.391
111	A	9	9	1.	23	0.391
112	A	11	11	1.	23	0.478
113	A	11	11	1.	23	0.478
114	A	15	12	1.	25	0.48
115	A	15	12	1.	25	0.48
116	A	13	10	1.	25	0.4
117	A	13	10	1.	25	0.4
118	A	16	13	1.	25	0.52
119	A	16	13	1.	25	0.52
120	A	8	8	1.	25	0.32
121	A	7	7	1.	25	0.28
122	A	7	7	1.	25	0.28
123	A	7	7	1.	25	0.28
124	A	7	7	1.	25	0.28
125	A	8	8	1.	25	0.32
126	A	8	8	1.	25	0.32
127	A	14	8	1.	25	0.32
128	A	14	9	1.	25	0.36
129	A	14	9	1.	25	0.36
130	A	15	9	1.	25	0.36
131	A	15	9	1.	25	0.36

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
132	A	17	9	1.	25	0.36
133	A	17	9	1.	25	0.36
134	A	9	6	1.	23	0.261
135	A	7	6	1.	23	0.261
136	A	5	5	1.	21	0.238
137	A	5	5	1.	23	0.217
138	A	9	6	1.	23	0.261
139	A	12	7	1.	23	0.304
140	A	5	4	1.	25	0.16
141	A	5	4	1.	25	0.16
142	A	5	4	1.	25	0.16
143	A	5	4	1.	25	0.16
144	A	5	4	1.	23	0.174
145	A	4	4	1.	21	0.19
146	A	4	4	1.	21	0.19
147	A	3	3	1.	21	0.143
148	A	2	2	1.	19	0.105
149	A	2	2	1.	19	0.105
150	A	4	4	1.	21	0.19
151	A	5	5	1.	21	0.238
152	A	11	9	1.	21	0.429
153	A	6	5	1.	21	0.238
154	A	4	4	1.	21	0.19
155	A	7	6	1.	21	0.286
156	A	5	4	1.	23	0.174
157	A	5	4	1.	23	0.174
158	A	5	4	1.	23	0.174
159	A	5	4	1.	23	0.174
160	A	5	4	1.	19	0.21
161	A	5	4	1.	19	0.21
162	A	5	4	1.	19	0.21
163	A	4	3	1.	17	0.176
164	A	5	5	1.	17	0.294
165	A	7	6	1.	19	0.316
166	A	9	7	1.	19	0.368
167	A	10	7	1.	19	0.368
168	A	10	7	1.	19	0.368
169	A	9	7	1.	19	0.368
170	A	7	7	1.	19	0.368
171	A	7	7	1.	19	0.368
172	A	8	6	1.	19	0.316
173	A	8	6	1.	19	0.316
174	A	5	4	1.	21	0.19
175	A	5	4	1.	21	0.19

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
176	A	5	4	1.	19	0.21
177	A	5	4	1.	19	0.21
178	A	6	5	1.	21	0.238
179	A	12	10	1.	21	0.476
180	A	7	7	1.	21	0.333
181	A	10	8	1.	21	0.381
182	A	8	6	1.	21	0.286
183	A	9	6	1.	21	0.286
184	A	9	6	1.	21	0.286
185	A	5	4	1.	21	0.19
186	A	4	3	1.	21	0.143
187	A	5	4	1.	19	0.21
188	A	5	4	1.	19	0.21
189	A	6	5	1.	21	0.238
190	A	21	11	1.	21	0.524
191	A	8	7	1.	21	0.333
192	A	8	8	1.	21	0.381
193	A	15	10	1.	21	0.476
194	A	17	9	1.	21	0.429
195	A	17	9	1.	21	0.429
196	A	5	4	1.	21	0.19
197	A	5	4	1.	21	0.19
198	A	5	4	1.	21	0.19
199	A	5	4	1.	19	0.21
200	A	4	3	1.	19	0.158
201	A	6	5	1.	21	0.238
202	A	7	5	1.	21	0.238
203	A	7	5	1.	21	0.238
204	A	6	5	1.	21	0.238
205	A	5	5	1.	21	0.238
206	A	5	5	1.	21	0.238
207	A	6	5	1.	21	0.238
208	A	7	5	1.	21	0.238
209	A	5	4	1.	21	0.19
210	A	5	4	1.	21	0.19
211	A	5	4	1.	21	0.19
212	A	5	4	1.	19	0.21
213	A	5	4	1.	19	0.21
214	A	6	5	1.	21	0.238
215	A	7	5	1.	21	0.238
216	A	10	8	1.	21	0.381
217	A	8	7	1.	21	0.333
218	A	8	8	1.	21	0.381
219	A	11	7	1.	21	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
220	A	15	8	1.	21	0.381
221	A	5	4	1.	21	0.19
222	A	5	4	1.	21	0.19
223	A	5	4	1.	21	0.19
224	A	5	4	1.	19	0.21
225	A	5	4	1.	19	0.21
226	A	5	4	1.	21	0.19
227	A	7	5	1.	21	0.238
228	A	11	8	1.	21	0.381
229	A	9	7	1.	21	0.333
230	A	9	8	1.	21	0.381
231	A	16	8	1.	21	0.381
232	A	20	9	1.	21	0.429
233	A	15	12	1.	25	0.48
234	A	14	12	1.	25	0.48
235	A	14	12	1.	25	0.48
236	A	13	11	1.	25	0.44
237	A	13	11	1.	25	0.44
238	A	14	12	1.	25	0.48
239	A	14	12	1.	25	0.48
240	A	15	12	1.	25	0.48
241	A	35	16	1.	25	0.64
242	A	35	16	1.	25	0.64
243	A	32	15	1.	25	0.6
244	A	32	15	1.	25	0.6
245	A	27	13	1.	25	0.52
246	A	27	13	1.	25	0.52
247	A	33	16	1.	25	0.64
248	A	33	16	1.	25	0.64
249	A	1	1	1.	14	0.071
250	A	2	2	1.	23	0.087
251	A	5	5	1.	14	0.357
252	A	4	4	1.	23	0.174
253	A	1	1	1.	14	0.071
254	A	6	6	1.	23	0.261
255	A	6	6	1.	14	0.429
256	A	6	6	1.	23	0.261
257	A	9	6	1.	23	0.261
258	A	9	8	1.	23	0.348
259	A	5	5	1.	21	0.238
260	A	4	4	1.	23	0.174
261	A	6	4	1.	23	0.174
262	A	7	4	1.	23	0.174
263	A	0	0	0.	0	0.

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
264	A	0	0	0.	0	0.
265	A	0	0	0.	0	0.
266	A	0	0	0.	0	0.
267	A	0	0	0.	0	0.
268	A	6	3	1.	21	0.143
269	A	3	3	1.	21	0.143
270	A	2	2	1.	19	0.105
271	A	6	4	1.	19	0.21
272	A	9	4	1.	21	0.19
273	A	0	0	0.	0	0.
274	A	0	0	0.	0	0.
275	A	4	4	1.	21	0.19
276	F	0	0	N/A	0	N/A
277	A	0	0	0.	0	0.
278	A	0	0	0.	0	0.
279	A	0	0	0.	0	0.
280	A	0	0	0.	0	0.
281	A	11	11	1.	23	0.478
282	A	11	11	1.	23	0.478
283	A	9	9	1.	23	0.391
284	A	9	9	1.	23	0.391
285	A	11	11	1.	23	0.478
286	A	11	11	1.	23	0.478
287	A	15	13	1.	25	0.52
288	A	15	13	1.	25	0.52
289	A	12	10	1.	25	0.4
290	A	12	10	1.	25	0.4
291	A	14	12	1.	25	0.48
292	A	14	12	1.	25	0.48
293	A	8	8	1.	25	0.32
294	A	8	8	1.	25	0.32
295	A	7	7	1.	25	0.28
296	A	7	7	1.	25	0.28
297	A	7	7	1.	25	0.28
298	A	7	7	1.	25	0.28
299	A	8	8	1.	25	0.32
300	A	16	9	1.	25	0.36
301	A	16	9	1.	25	0.36
302	A	14	9	1.	25	0.36
303	A	14	9	1.	25	0.36
304	A	13	9	1.	25	0.36
305	A	13	9	1.	25	0.36
306	A	13	8	1.	25	0.32

Chapter 3

Listing of integrals

3.1 $\int (a + a\sec(c+dx))\sin^9(c+dx)\,dx$

Optimal. Leaf size=152

$$-\frac{a\cos^9(c+dx)}{9d}-\frac{a\cos^8(c+dx)}{8d}+\frac{4a\cos^7(c+dx)}{7d}+\frac{2a\cos^6(c+dx)}{3d}-\frac{6a\cos^5(c+dx)}{5d}-\frac{3a\cos^4(c+dx)}{2d}+\frac{4a\cos^3}{3}$$

```
[Out] -((a*Cos[c + d*x])/d) + (2*a*Cos[c + d*x]^2)/d + (4*a*Cos[c + d*x]^3)/(3*d)
 - (3*a*Cos[c + d*x]^4)/(2*d) - (6*a*Cos[c + d*x]^5)/(5*d) + (2*a*Cos[c + d
*x]^6)/(3*d) + (4*a*Cos[c + d*x]^7)/(7*d) - (a*Cos[c + d*x]^8)/(8*d) - (a*C
os[c + d*x]^9)/(9*d) - (a*Log[Cos[c + d*x]])/d
```

Rubi [A] time = 0.107119, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3872, 2836, 12, 88}

$$-\frac{a\cos^9(c+dx)}{9d}-\frac{a\cos^8(c+dx)}{8d}+\frac{4a\cos^7(c+dx)}{7d}+\frac{2a\cos^6(c+dx)}{3d}-\frac{6a\cos^5(c+dx)}{5d}-\frac{3a\cos^4(c+dx)}{2d}+\frac{4a\cos^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^9,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (2*a*Cos[c + d*x]^2)/d + (4*a*Cos[c + d*x]^3)/(3*d)
 - (3*a*Cos[c + d*x]^4)/(2*d) - (6*a*Cos[c + d*x]^5)/(5*d) + (2*a*Cos[c + d
*x]^6)/(3*d) + (4*a*Cos[c + d*x]^7)/(7*d) - (a*Cos[c + d*x]^8)/(8*d) - (a*C
os[c + d*x]^9)/(9*d) - (a*Log[Cos[c + d*x]])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int (a+a\sec(c+dx))\sin^9(c+dx)\,dx = -\int (-a-a\cos(c+dx))\sin^8(c+dx)\tan(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a(-a-x)^4(-a+x)^5}{x}\,dx, x, -a\cos(c+dx)\right)}{a^9 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^4(-a+x)^5}{x}\,dx, x, -a\cos(c+dx)\right)}{a^8 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^8-\frac{a^9}{x}+4a^7x-4a^6x^2-6a^5x^3+6a^4x^4+4a^3x^5-4a^2x^6-ax^7+x^8\right)\right.}{a^8 d}$$
$$= -\frac{a\cos(c+dx)}{d}+\frac{2a\cos^2(c+dx)}{d}+\frac{4a\cos^3(c+dx)}{3d}-\frac{3a\cos^4(c+dx)}{2d}-\frac{6a\cos^5}{5}$$

Mathematica [A] time = 0.210393, size = 106, normalized size = 0.7

$$-\frac{a\left(10080\cos^8(c+dx)-53760\cos^6(c+dx)+120960\cos^4(c+dx)-161280\cos^2(c+dx)+39690\cos(c+dx)-8820\,\mathrm{c}\right.}{80640d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^9,x]
```

```
[Out] -(a*(39690*Cos[c + d*x] - 161280*Cos[c + d*x]^2 + 120960*Cos[c + d*x]^4 - 5
3760*Cos[c + d*x]^6 + 10080*Cos[c + d*x]^8 - 8820*Cos[3*(c + d*x)] + 2268*C
os[5*(c + d*x)] - 405*Cos[7*(c + d*x)] + 35*Cos[9*(c + d*x)] + 80640*Log[Co
s[c + d*x]]))/(80640*d)
```

Maple [A] time = 0.092, size = 163, normalized size = 1.1

$$-\frac{128\,a\cos(dx+c)}{315\,d}-\frac{\cos(dx+c)(\sin(dx+c))^8 a}{9\,d}-\frac{8\,a\cos(dx+c)(\sin(dx+c))^6}{63\,d}-\frac{16\,a\cos(dx+c)(\sin(dx+c))^4}{105\,d}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^9,x)
```

```
[Out] -128/315*a*cos(d*x+c)/d-1/9/d*cos(d*x+c)*sin(d*x+c)^8*a-8/63/d*a*cos(d*x+c)
*sin(d*x+c)^6-16/105/d*a*cos(d*x+c)*sin(d*x+c)^4-64/315/d*a*cos(d*x+c)*sin(
d*x+c)^2-1/8/d*a*sin(d*x+c)^8-1/6/d*a*sin(d*x+c)^6-1/4/d*a*sin(d*x+c)^4-1/2
/d*a*sin(d*x+c)^2-a*ln(cos(d*x+c))/d
```

Maxima [A] time = 1.10328, size = 153, normalized size = 1.01

$$-\frac{280\,a\cos\left(dx+c\right)^{9}+315\,a\cos\left(dx+c\right)^{8}-1440\,a\cos\left(dx+c\right)^{7}-1680\,a\cos\left(dx+c\right)^{6}+3024\,a\cos\left(dx+c\right)^{5}+3780\,a}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^9,x, algorithm="maxima")
```

```
[Out] -1/2520*(280*a*cos(d*x + c)^9 + 315*a*cos(d*x + c)^8 - 1440*a*cos(d*x + c)^
7 - 1680*a*cos(d*x + c)^6 + 3024*a*cos(d*x + c)^5 + 3780*a*cos(d*x + c)^4 -
 3360*a*cos(d*x + c)^3 - 5040*a*cos(d*x + c)^2 + 2520*a*cos(d*x + c) + 2520
*a*log(cos(d*x + c)))/d
```

Fricas [A] time = 1.89077, size = 339, normalized size = 2.23

$$-\frac{280\,a\cos\left(dx+c\right)^{9}+315\,a\cos\left(dx+c\right)^{8}-1440\,a\cos\left(dx+c\right)^{7}-1680\,a\cos\left(dx+c\right)^{6}+3024\,a\cos\left(dx+c\right)^{5}+3780\,a}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] -1/2520*(280*a*cos(d*x + c)^9 + 315*a*cos(d*x + c)^8 - 1440*a*cos(d*x + c)^
7 - 1680*a*cos(d*x + c)^6 + 3024*a*cos(d*x + c)^5 + 3780*a*cos(d*x + c)^4 -
 3360*a*cos(d*x + c)^3 - 5040*a*cos(d*x + c)^2 + 2520*a*cos(d*x + c) + 2520
*a*log(-cos(d*x + c)))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**9,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.49467, size = 396, normalized size = 2.61

$$\frac{2520\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-2520\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{9177\,a-\frac{87633\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{375732\,a(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{953988\,a(\cos(dx+c)-}{(\cos(dx+c)+1)^{3}}}{}}{2520\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^9,x, algorithm="giac")
```

```
[Out] 1/2520*(2520*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 2520*
a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (9177*a - 87633*a*
```

```
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 375732*a*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 - 953988*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 1594
782*a*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 1336734*a*(cos(d*x + c) -
 1)^5/(cos(d*x + c) + 1)^5 + 781956*a*(cos(d*x + c) - 1)^6/(cos(d*x + c) +
1)^6 - 302004*a*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 69201*a*(cos(d*
x + c) - 1)^8/(cos(d*x + c) + 1)^8 - 7129*a*(cos(d*x + c) - 1)^9/(cos(d*x +
 c) + 1)^9)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^9)/d
```

3.2 $\int (a + a\sec(c + dx))\sin^7(c + dx)\,dx$

Optimal. Leaf size=119

$$\frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^6(c+dx)}{6d} - \frac{3a\cos^5(c+dx)}{5d} - \frac{3a\cos^4(c+dx)}{4d} + \frac{a\cos^3(c+dx)}{d} + \frac{3a\cos^2(c+dx)}{2d} - \frac{a\cos(c+}{d}$$

```
[Out] -((a*Cos[c + d*x])/d) + (3*a*Cos[c + d*x]^2)/(2*d) + (a*Cos[c + d*x]^3)/d -
 (3*a*Cos[c + d*x]^4)/(4*d) - (3*a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c + d*x]^
6)/(6*d) + (a*Cos[c + d*x]^7)/(7*d) - (a*Log[Cos[c + d*x]])/d
```

Rubi [A] time = 0.0974775, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3872, 2836, 12, 88}

$$\frac{a\cos^7(c+dx)}{7d} + \frac{a\cos^6(c+dx)}{6d} - \frac{3a\cos^5(c+dx)}{5d} - \frac{3a\cos^4(c+dx)}{4d} + \frac{a\cos^3(c+dx)}{d} + \frac{3a\cos^2(c+dx)}{2d} - \frac{a\cos(c+}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^7,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (3*a*Cos[c + d*x]^2)/(2*d) + (a*Cos[c + d*x]^3)/d -
 (3*a*Cos[c + d*x]^4)/(4*d) - (3*a*Cos[c + d*x]^5)/(5*d) + (a*Cos[c + d*x]^
6)/(6*d) + (a*Cos[c + d*x]^7)/(7*d) - (a*Log[Cos[c + d*x]])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int (a+a\sec(c+dx))\sin^7(c+dx)\,dx = -\int (-a-a\cos(c+dx))\sin^6(c+dx)\tan(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a(-a-x)^3(-a+x)^4}{x}\,dx, x, -a\cos(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^3(-a+x)^4}{x}\,dx, x, -a\cos(c+dx)\right)}{a^6 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^6 - \frac{a^7}{x} + 3a^5x - 3a^4x^2 - 3a^3x^3 + 3a^2x^4 + ax^5 - x^6\right) dx, x, -a\cos(c+\right.}{a^6 d}$$
$$= -\frac{a\cos(c+dx)}{d} + \frac{3a\cos^2(c+dx)}{2d} + \frac{a\cos^3(c+dx)}{d} - \frac{3a\cos^4(c+dx)}{4d} - \frac{3a\cos^5(c}{5d}$$

Mathematica [A] time = 0.130659, size = 86, normalized size = 0.72

$$\frac{a\left(1120\cos^6(c+dx) - 5040\cos^4(c+dx) + 10080\cos^2(c+dx) - 3675\cos(c+dx) + 735\cos(3(c+dx)) - 147\cos(5(c+\right.}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^7,x]
```

```
[Out] (a*(-3675*Cos[c + d*x] + 10080*Cos[c + d*x]^2 - 5040*Cos[c + d*x]^4 + 1120*
Cos[c + d*x]^6 + 735*Cos[3*(c + d*x)] - 147*Cos[5*(c + d*x)] + 15*Cos[7*(c
+ d*x)] - 6720*Log[Cos[c + d*x]]))/(6720*d)
```

Maple [A] time = 0.088, size = 129, normalized size = 1.1

$$-\frac{16\,a\cos(dx+c)}{35\,d} - \frac{a\cos(dx+c)(\sin(dx+c))^6}{7\,d} - \frac{6\,a\cos(dx+c)(\sin(dx+c))^4}{35\,d} - \frac{8\,a\cos(dx+c)(\sin(dx+c))^2}{35\,d} - \frac{a\,(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^7,x)
```

```
[Out] -16/35*a*cos(d*x+c)/d-1/7/d*a*cos(d*x+c)*sin(d*x+c)^6-6/35/d*a*cos(d*x+c)*s
in(d*x+c)^4-8/35/d*a*cos(d*x+c)*sin(d*x+c)^2-1/6/d*a*sin(d*x+c)^6-1/4/d*a*s
in(d*x+c)^4-1/2/d*a*sin(d*x+c)^2-a*ln(cos(d*x+c))/d
```

Maxima [A] time = 1.06778, size = 123, normalized size = 1.03

$$\frac{60\,a\cos(dx+c)^7 + 70\,a\cos(dx+c)^6 - 252\,a\cos(dx+c)^5 - 315\,a\cos(dx+c)^4 + 420\,a\cos(dx+c)^3 + 630\,a\cos(dx+c}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/420*(60*a*cos(d*x + c)^7 + 70*a*cos(d*x + c)^6 - 252*a*cos(d*x + c)^5 - 3
15*a*cos(d*x + c)^4 + 420*a*cos(d*x + c)^3 + 630*a*cos(d*x + c)^2 - 420*a*c
os(d*x + c) - 420*a*log(cos(d*x + c)))/d
```

Fricas [A] time = 1.83266, size = 261, normalized size = 2.19

$$\frac{60\,a\cos(dx+c)^7 + 70\,a\cos(dx+c)^6 - 252\,a\cos(dx+c)^5 - 315\,a\cos(dx+c)^4 + 420\,a\cos(dx+c)^3 + 630\,a\cos(dx}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/420*(60*a*cos(d*x + c)^7 + 70*a*cos(d*x + c)^6 - 252*a*cos(d*x + c)^5 - 3
15*a*cos(d*x + c)^4 + 420*a*cos(d*x + c)^3 + 630*a*cos(d*x + c)^2 - 420*a*c
os(d*x + c) - 420*a*log(-cos(d*x + c)))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**7,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.50592, size = 333, normalized size = 2.8

$$\frac{420\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 420\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{1473\,a - \frac{11151\,a(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{36813\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{69475\,a(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} + }{\left(\frac{\mathrm{co}}{\mathrm{co}}\right.}}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/420*(420*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*a*l
og(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (1473*a - 11151*a*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 36813*a*(cos(d*x + c) - 1)^2/(cos(d*x
+ c) + 1)^2 - 69475*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56035*a*(
cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 28749*a*(cos(d*x + c) - 1)^5/(co
s(d*x + c) + 1)^5 + 8463*a*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 1089
*a*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/((cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) - 1)^7)/d
```

3.3 $\int (a+a\sec(c+dx))\sin^5(c+dx)\,dx$

Optimal. Leaf size=87

$$-\frac{a\cos^5(c+dx)}{5d}-\frac{a\cos^4(c+dx)}{4d}+\frac{2a\cos^3(c+dx)}{3d}+\frac{a\cos^2(c+dx)}{d}-\frac{a\cos(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[Out] -((a*Cos[c + d*x])/d) + (a*Cos[c + d*x]^2)/d + (2*a*Cos[c + d*x]^3)/(3*d) -
 (a*Cos[c + d*x]^4)/(4*d) - (a*Cos[c + d*x]^5)/(5*d) - (a*Log[Cos[c + d*x]]
)/d
```

Rubi [A] time = 0.0869581, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3872, 2836, 12, 88}

$$-\frac{a\cos^5(c+dx)}{5d}-\frac{a\cos^4(c+dx)}{4d}+\frac{2a\cos^3(c+dx)}{3d}+\frac{a\cos^2(c+dx)}{d}-\frac{a\cos(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^5,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (a*Cos[c + d*x]^2)/d + (2*a*Cos[c + d*x]^3)/(3*d) -
 (a*Cos[c + d*x]^4)/(4*d) - (a*Cos[c + d*x]^5)/(5*d) - (a*Log[Cos[c + d*x]]
)/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int (a + a\sec(c + dx))\sin^5(c + dx)\,dx = -\int (-a - a\cos(c + dx))\sin^4(c + dx)\tan(c + dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a(-a-x)^2(-a+x)^3}{x}\,dx, x, -a\cos(c + dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^2(-a+x)^3}{x}\,dx, x, -a\cos(c + dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^4 - \frac{a^5}{x} + 2a^3x - 2a^2x^2 - ax^3 + x^4\right)dx, x, -a\cos(c + dx)\right)}{a^4 d}$$
$$= -\frac{a\cos(c + dx)}{d} + \frac{a\cos^2(c + dx)}{d} + \frac{2a\cos^3(c + dx)}{3d} - \frac{a\cos^4(c + dx)}{4d} - \frac{a\cos^5(}{5}$$

Mathematica [A] time = 0.0841183, size = 83, normalized size = 0.95

$$-\frac{5a\cos(c + dx)}{8d} + \frac{5a\cos(3(c + dx))}{48d} - \frac{a\cos(5(c + dx))}{80d} - \frac{a\left(\frac{1}{4}\cos^4(c + dx) - \cos^2(c + dx) + \log(\cos(c + dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^5,x]
```

```
[Out] (-5*a*Cos[c + d*x])/(8*d) + (5*a*Cos[3*(c + d*x)])/(48*d) - (a*Cos[5*(c + d
*x)])/(80*d) - (a*(-Cos[c + d*x]^2 + Cos[c + d*x]^4/4 + Log[Cos[c + d*x]]))
/d
```

Maple [A] time = 0.086, size = 95, normalized size = 1.1

$$-\frac{8\,a\cos(dx + c)}{15\,d} - \frac{a\cos(dx + c)(\sin(dx + c))^4}{5\,d} - \frac{4\,a\cos(dx + c)(\sin(dx + c))^2}{15\,d} - \frac{a(\sin(dx + c))^4}{4\,d} - \frac{a(\sin(dx + c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^5,x)
```

```
[Out] -8/15*a*cos(d*x+c)/d-1/5/d*a*cos(d*x+c)*sin(d*x+c)^4-4/15/d*a*cos(d*x+c)*si
n(d*x+c)^2-1/4/d*a*sin(d*x+c)^4-1/2/d*a*sin(d*x+c)^2-a*ln(cos(d*x+c))/d
```

Maxima [A] time = 1.10881, size = 93, normalized size = 1.07

$$-\frac{12\,a\cos(dx + c)^5 + 15\,a\cos(dx + c)^4 - 40\,a\cos(dx + c)^3 - 60\,a\cos(dx + c)^2 + 60\,a\cos(dx + c) + 60\,a\log(\cos(dx}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(12*a*cos(d*x + c)^5 + 15*a*cos(d*x + c)^4 - 40*a*cos(d*x + c)^3 - 60
*a*cos(d*x + c)^2 + 60*a*cos(d*x + c) + 60*a*log(cos(d*x + c)))/d
```

Fricas [A] time = 1.79931, size = 193, normalized size = 2.22

$$-\frac{12\,a\cos\left(dx+c\right)^{5}+15\,a\cos\left(dx+c\right)^{4}-40\,a\cos\left(dx+c\right)^{3}-60\,a\cos\left(dx+c\right)^{2}+60\,a\cos\left(dx+c\right)+60\,a\log\left(-\cos\left(dx+c\right)\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/60*(12*a*cos(d*x + c)^5 + 15*a*cos(d*x + c)^4 - 40*a*cos(d*x + c)^3 - 60
*a*cos(d*x + c)^2 + 60*a*cos(d*x + c) + 60*a*log(-cos(d*x + c)))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.42544, size = 271, normalized size = 3.11

$$\frac{60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-60\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{201\,a-\frac{1125\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{2610\,a(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{1970\,a(\cos(dx+c)-1)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{805\,a(\cos(dx+c)-1)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{137\,a(\cos(dx+c)-1)^{5}}{(\cos(dx+c)+1)^{5}}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^{5}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (201*a - 1125*a*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 2610*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2 - 1970*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 805*a*(cos(d*x +
 c) - 1)^4/(cos(d*x + c) + 1)^4 - 137*a*(cos(d*x + c) - 1)^5/(cos(d*x + c)
+ 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5)/d
```

3.4 $\int (a + a\sec(c + dx))\sin^3(c + dx)\,dx$

Optimal. Leaf size=58

$$\frac{a\cos^3(c + dx)}{3d} + \frac{a\cos^2(c + dx)}{2d} - \frac{a\cos(c + dx)}{d} - \frac{a\log(\cos(c + dx))}{d}$$

```
[Out] -((a*Cos[c + d*x])/d) + (a*Cos[c + d*x]^2)/(2*d) + (a*Cos[c + d*x]^3)/(3*d)
 - (a*Log[Cos[c + d*x]])/d
```

Rubi [A] time = 0.0773241, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3872, 2836, 12, 75}

$$\frac{a\cos^3(c + dx)}{3d} + \frac{a\cos^2(c + dx)}{2d} - \frac{a\cos(c + dx)}{d} - \frac{a\log(\cos(c + dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^3,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (a*Cos[c + d*x]^2)/(2*d) + (a*Cos[c + d*x]^3)/(3*d)
 - (a*Log[Cos[c + d*x]])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$
\begin{aligned}
\int (a+a\sec(c+dx))\sin^3(c+dx)\,dx &= -\int (-a-a\cos(c+dx))\sin^2(c+dx)\tan(c+dx)\,dx \\
&= \frac{\text{Subst}\left(\int \frac{a(-a-x)(-a+x)^2}{x}\,dx, x, -a\cos(c+dx)\right)}{a^3 d} \\
&= \frac{\text{Subst}\left(\int \frac{(-a-x)(-a+x)^2}{x}\,dx, x, -a\cos(c+dx)\right)}{a^2 d} \\
&= \frac{\text{Subst}\left(\int \left(a^2-\frac{a^3}{x}+ax-x^2\right)dx, x, -a\cos(c+dx)\right)}{a^2 d} \\
&= -\frac{a\cos(c+dx)}{d}+\frac{a\cos^2(c+dx)}{2d}+\frac{a\cos^3(c+dx)}{3d}-\frac{a\log(\cos(c+dx))}{d}
\end{aligned}
$$

Mathematica [A] time = 0.0462975, size = 57, normalized size = 0.98

$$
-\frac{3a\cos(c+dx)}{4d}+\frac{a\cos(3(c+dx))}{12d}-\frac{a\left(\log(\cos(c+dx))-\frac{1}{2}\cos^2(c+dx)\right)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^3,x]
```

```
[Out] (-3*a*Cos[c + d*x])/(4*d) + (a*Cos[3*(c + d*x)])/(12*d) - (a*(-Cos[c + d*x]
^2/2 + Log[Cos[c + d*x]]))/d
```

Maple [A] time = 0.084, size = 61, normalized size = 1.1

$$
-\frac{a\cos(dx+c)(\sin(dx+c))^2}{3d}-\frac{2a\cos(dx+c)}{3d}-\frac{a(\sin(dx+c))^2}{2d}-\frac{a\ln(\cos(dx+c))}{d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^3,x)
```

```
[Out] -1/3/d*a*cos(d*x+c)*sin(d*x+c)^2-2/3*a*cos(d*x+c)/d-1/2/d*a*sin(d*x+c)^2-a*
ln(cos(d*x+c))/d
```

Maxima [A] time = 1.12464, size = 63, normalized size = 1.09

$$
\frac{2a\cos(dx+c)^3+3a\cos(dx+c)^2-6a\cos(dx+c)-6a\log(\cos(dx+c))}{6d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a*cos(d*x + c)^3 + 3*a*cos(d*x + c)^2 - 6*a*cos(d*x + c) - 6*a*log(c
os(d*x + c)))/d
```

Fricas [A] time = 1.82297, size = 126, normalized size = 2.17

$$\frac{2\,a\cos\left(dx+c\right)^{3}+3\,a\cos\left(dx+c\right)^{2}-6\,a\cos\left(dx+c\right)-6\,a\log\left(-\cos\left(dx+c\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*cos(d*x + c)^3 + 3*a*cos(d*x + c)^2 - 6*a*cos(d*x + c) - 6*a*log(-
cos(d*x + c)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\sin^{3}\left(c+dx\right)\sec\left(c+dx\right)\,dx+\int\sin^{3}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**3,x)
```

```
[Out] a*(Integral(sin(c + d*x)**3*sec(c + d*x), x) + Integral(sin(c + d*x)**3, x)
)
```

Giac [A] time = 1.42774, size = 89, normalized size = 1.53

$$-\frac{a\log\left(\frac{\left|\cos\left(dx+c\right)\right|}{\left|d\right|}\right)}{d}+\frac{2\,ad^{2}\cos\left(dx+c\right)^{3}+3\,ad^{2}\cos\left(dx+c\right)^{2}-6\,ad^{2}\cos\left(dx+c\right)}{6\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -a*log(abs(cos(d*x + c))/abs(d))/d + 1/6*(2*a*d^2*cos(d*x + c)^3 + 3*a*d^2*
cos(d*x + c)^2 - 6*a*d^2*cos(d*x + c))/d^3
```

3.5 $\int (a + a\sec(c + dx))\sin(c + dx)\,dx$

Optimal. Leaf size=26

$$-\frac{a\cos(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[Out] -((a*Cos[c + d*x])/d) - (a*Log[Cos[c + d*x]])/d
```

Rubi [A] time = 0.0306214, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3872, 2707, 43}

$$-\frac{a\cos(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x],x]
```

```
[Out] -((a*Cos[c + d*x])/d) - (a*Log[Cos[c + d*x]])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (a+a\sec(c+dx))\sin(c+dx)\,dx &= -\int (-a-a\cos(c+dx))\tan(c+dx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{-a+x}{x}\,dx, x, -a\cos(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(1-\frac{a}{x}\right)dx, x, -a\cos(c+dx)\right)}{d}\\
&= -\frac{a\cos(c+dx)}{d}-\frac{a\log(\cos(c+dx))}{d}
\end{aligned}$$

Mathematica [A] time = 0.0185917, size = 37, normalized size = 1.42

$$\frac{a\sin(c)\sin(dx)}{d}-\frac{a\cos(c)\cos(dx)}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x],x]
```

```
[Out] -((a*Cos[c]*Cos[d*x])/d) - (a*Log[Cos[c + d*x]])/d + (a*Sin[c]*Sin[d*x])/d
```

Maple [A] time = 0.02, size = 28, normalized size = 1.1

$$\frac{a\ln(\sec(dx+c))}{d}-\frac{a}{d\sec(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c),x)
```

```
[Out] 1/d*a*ln(sec(d*x+c))-1/d*a/sec(d*x+c)
```

Maxima [A] time = 1.13873, size = 31, normalized size = 1.19

$$-\frac{a\cos(dx+c)+a\log(\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a*cos(d*x + c) + a*log(cos(d*x + c)))/d
```

Fricas [A] time = 1.76289, size = 59, normalized size = 2.27

$$-\frac{a\cos(dx+c)+a\log(-\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -(a*cos(d*x + c) + a*log(-cos(d*x + c)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\sin(c+dx)\sec(c+dx)\,dx+\int\sin(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c),x)
```

```
[Out] a*(Integral(sin(c + d*x)*sec(c + d*x), x) + Integral(sin(c + d*x), x))
```

Giac [A] time = 1.46129, size = 43, normalized size = 1.65

$$-\frac{a\cos\left(dx+c\right)}{d}-\frac{a\log\left(\frac{\left|\cos\left(dx+c\right)\right|}{\left|d\right|}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -a*cos(d*x + c)/d - a*log(abs(cos(d*x + c))/abs(d))/d
```

3.6 $\int \csc(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=30

$$\frac{a\log(1-\cos(c+dx))}{d}-\frac{a\log(\cos(c+dx))}{d}$$

```
[Out] (a*Log[1 - Cos[c + d*x]])/d - (a*Log[Cos[c + d*x]])/d
```

Rubi [A] time = 0.058121, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3872, 2836, 12, 36, 31, 29}

$$\frac{a\log(1-\cos(c+dx))}{d}-\frac{a\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*Log[1 - Cos[c + d*x]])/d - (a*Log[Cos[c + d*x]])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\int \csc(c+dx)(a+a\sec(c+dx))\,dx = -\int (-a-a\cos(c+dx))\csc(c+dx)\sec(c+dx)\,dx$$
$$= \frac{a\,\text{Subst}\left(\int \frac{a}{(-a-x)x}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{1}{(-a-x)x}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{1}{-a-x}\,dx, x, -a\cos(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a\log(1-\cos(c+dx))}{d} - \frac{a\log(\cos(c+dx))}{d}$$

Mathematica [B] time = 0.035584, size = 63, normalized size = 2.1

$$\frac{a\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{a\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{a(\log(\cos(c+dx))-\log(\sin(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sec[c + d*x]),x]
```

```
[Out] -((a*Log[Cos[c/2 + (d*x)/2]])/d) + (a*Log[Sin[c/2 + (d*x)/2]])/d - (a*(Log[
Cos[c + d*x]] - Log[Sin[c + d*x]]))/d
```

Maple [A] time = 0.033, size = 15, normalized size = 0.5

$$\frac{a\ln(-1+\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/d*a*ln(-1+sec(d*x+c))
```

Maxima [A] time = 1.03157, size = 35, normalized size = 1.17

$$\frac{a\log(\cos(dx+c)-1) - a\log(\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] (a*log(cos(d*x + c) - 1) - a*log(cos(d*x + c)))/d
```

Fricas [A] time = 1.68715, size = 81, normalized size = 2.7

$$-\frac{a\log(-\cos(dx+c)) - a\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(a*log(-cos(d*x + c)) - a*log(-1/2*cos(d*x + c) + 1/2))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \csc(c+dx)\sec(c+dx)\,dx + \int \csc(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(csc(c + d*x)*sec(c + d*x), x) + Integral(csc(c + d*x), x))
```

Giac [A] time = 1.44812, size = 78, normalized size = 2.6

$$\frac{a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a*log(abs(-(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) - 1)))/d
```

3.7 $\int \csc^3(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=73

$$-\frac{a^2}{2d(a-a\cos(c+dx))}+\frac{3a\log(1-\cos(c+dx))}{4d}-\frac{a\log(\cos(c+dx))}{d}+\frac{a\log(\cos(c+dx)+1)}{4d}$$

```
[Out] -a^2/(2*d*(a - a*Cos[c + d*x])) + (3*a*Log[1 - Cos[c + d*x]])/(4*d) - (a*Lo
g[Cos[c + d*x]])/d + (a*Log[1 + Cos[c + d*x]])/(4*d)
```

Rubi [A] time = 0.0954302, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3872, 2836, 12, 72}

$$-\frac{a^2}{2d(a-a\cos(c+dx))}+\frac{3a\log(1-\cos(c+dx))}{4d}-\frac{a\log(\cos(c+dx))}{d}+\frac{a\log(\cos(c+dx)+1)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

```
[Out] -a^2/(2*d*(a - a*Cos[c + d*x])) + (3*a*Log[1 - Cos[c + d*x]])/(4*d) - (a*Lo
g[Cos[c + d*x]])/d + (a*Log[1 + Cos[c + d*x]])/(4*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 72

```
Int[((e_.) + (f_.)*(x_))^(p_.)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
 x_Symbol] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \csc^3(c+dx)(a+a\sec(c+dx))\,dx = -\int (-a-a\cos(c+dx))\csc^3(c+dx)\sec(c+dx)\,dx$$
$$= \frac{a^3\,\mathrm{Subst}\left(\int \frac{a}{(-a-x)^2x(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^4\,\mathrm{Subst}\left(\int \frac{1}{(-a-x)^2x(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^4\,\mathrm{Subst}\left(\int \left(-\frac{1}{4a^3(a-x)}-\frac{1}{a^3x}+\frac{1}{2a^2(a+x)^2}+\frac{3}{4a^3(a+x)}\right)dx, x, -a\cos(c+dx)\right)}{d}$$
$$= -\frac{a^2}{2d(a-a\cos(c+dx))}+\frac{3a\log(1-\cos(c+dx))}{4d}-\frac{a\log(\cos(c+dx))}{d}+\frac{a\log}{}$$

Mathematica [A] time = 0.820608, size = 114, normalized size = 1.56

$$-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a\left(\csc^2(c+dx)-2\log\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + a*Sec[c + d*x]),x]
```

```
[Out] -(a*Csc[(c + d*x)/2]^2)/(8*d) - (a*Log[Cos[(c + d*x)/2]])/(2*d) + (a*Log[Si
n[(c + d*x)/2]])/(2*d) - (a*(Csc[c + d*x]^2 + 2*Log[Cos[c + d*x]] - 2*Log[S
in[c + d*x]]))/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

Maple [A] time = 0.057, size = 48, normalized size = 0.7

$$\frac{a\ln(1+\sec(dx+c))}{4d}-\frac{a}{2d(-1+\sec(dx+c))}+\frac{3a\ln(-1+\sec(dx+c))}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/4/d*a*ln(1+sec(d*x+c))-1/2/d*a/(-1+sec(d*x+c))+3/4/d*a*ln(-1+sec(d*x+c))
```

Maxima [A] time = 1.09293, size = 70, normalized size = 0.96

$$\frac{a\log(\cos(dx+c)+1)+3a\log(\cos(dx+c)-1)-4a\log(\cos(dx+c))+\frac{2a}{\cos(dx+c)-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(a*log(cos(d*x + c) + 1) + 3*a*log(cos(d*x + c) - 1) - 4*a*log(cos(d*x
+ c)) + 2*a/(cos(d*x + c) - 1))/d
```

Fricas [A] time = 1.80028, size = 246, normalized size = 3.37

$$-\frac{4\,(a\cos(dx+c)-a)\log(-\cos(dx+c))-(a\cos(dx+c)-a)\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)-3\,(a\cos(dx+c)-a)\log\left(-\frac{1}{2}\right.}{4\,(d\cos(dx+c)-d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(4*(a*cos(d*x + c) - a)*log(-cos(d*x + c)) - (a*cos(d*x + c) - a)*log(
1/2*cos(d*x + c) + 1/2) - 3*(a*cos(d*x + c) - a)*log(-1/2*cos(d*x + c) + 1/
2) - 2*a)/(d*cos(d*x + c) - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\csc^3(c+dx)\sec(c+dx)\,dx+\int\csc^3(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(csc(c + d*x)**3*sec(c + d*x), x) + Integral(csc(c + d*x)**3, x)
)
```

Giac [A] time = 1.47429, size = 138, normalized size = 1.89

$$\frac{3\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-4\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{\left(a-\frac{3\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(3*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 4*a*log(abs(-(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a - 3*a*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1))/d
```

3.8 $\int \csc^5(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=118

$$-\frac{a^3}{8d(a-a\cos(c+dx))^2}-\frac{a^2}{2d(a-a\cos(c+dx))}-\frac{a^2}{8d(a\cos(c+dx)+a)}+\frac{11a\log(1-\cos(c+dx))}{16d}-\frac{a\log(\cos(c+d}{d}$$

```
[Out] -a^3/(8*d*(a - a*Cos[c + d*x])^2) - a^2/(2*d*(a - a*Cos[c + d*x])) - a^2/(8
*d*(a + a*Cos[c + d*x])) + (11*a*Log[1 - Cos[c + d*x]])/(16*d) - (a*Log[Cos
[c + d*x]])/d + (5*a*Log[1 + Cos[c + d*x]])/(16*d)
```

Rubi [A] time = 0.120413, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3872, 2836, 12, 88}

$$-\frac{a^3}{8d(a-a\cos(c+dx))^2}-\frac{a^2}{2d(a-a\cos(c+dx))}-\frac{a^2}{8d(a\cos(c+dx)+a)}+\frac{11a\log(1-\cos(c+dx))}{16d}-\frac{a\log(\cos(c+d}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + a*Sec[c + d*x]),x]
```

```
[Out] -a^3/(8*d*(a - a*Cos[c + d*x])^2) - a^2/(2*d*(a - a*Cos[c + d*x])) - a^2/(8
*d*(a + a*Cos[c + d*x])) + (11*a*Log[1 - Cos[c + d*x]])/(16*d) - (a*Log[Cos
[c + d*x]])/d + (5*a*Log[1 + Cos[c + d*x]])/(16*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \csc^5(c+dx)(a+a\sec(c+dx))\,dx = -\int (-a-a\cos(c+dx))\csc^5(c+dx)\sec(c+dx)\,dx$$
$$= \frac{a^5\,\text{Subst}\left(\int \frac{a}{(-a-x)^3x(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \frac{1}{(-a-x)^3x(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \left(-\frac{1}{8a^4(a-x)^2}-\frac{5}{16a^5(a-x)}-\frac{1}{a^5x}+\frac{1}{4a^3(a+x)^3}+\frac{1}{2a^4(a+x)^2}+\frac{11}{16a^5(a+x)}\right)dx, x, -\right.}{d}$$
$$= -\frac{a^3}{8d(a-a\cos(c+dx))^2}-\frac{a^2}{2d(a-a\cos(c+dx))}-\frac{a^2}{8d(a+a\cos(c+dx))}+\frac{11a\,\text{lo}}{}$$

Mathematica [A] time = 0.336741, size = 164, normalized size = 1.39

$$-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{3a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{3a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}-\frac{3a\,\text{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-3*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (3*a*Log
[Cos[(c + d*x)/2]])/(8*d) + (3*a*Log[Sin[(c + d*x)/2]])/(8*d) - (a*(2*Csc[c
 + d*x]^2 + Csc[c + d*x]^4 + 4*Log[Cos[c + d*x]] - 4*Log[Sin[c + d*x]]))/(4
*d) + (3*a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d)
```

Maple [A] time = 0.068, size = 80, normalized size = 0.7

$$\frac{a}{8d(1+\sec(dx+c))}+\frac{5a\ln(1+\sec(dx+c))}{16d}-\frac{a}{8d(-1+\sec(dx+c))^2}-\frac{3a}{4d(-1+\sec(dx+c))}+\frac{11a\ln(-1+\sec(d}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/8/d*a/(1+sec(d*x+c))+5/16/d*a*ln(1+sec(d*x+c))-1/8/d*a/(-1+sec(d*x+c))^2-
3/4/d*a/(-1+sec(d*x+c))+11/16/d*a*ln(-1+sec(d*x+c))
```

Maxima [A] time = 1.07625, size = 128, normalized size = 1.08

$$\frac{5a\log(\cos(dx+c)+1)+11a\log(\cos(dx+c)-1)-16a\log(\cos(dx+c))+\frac{2\left(3a\cos(dx+c)^2+a\cos(dx+c)-6a\right)}{\cos(dx+c)^3-\cos(dx+c)^2-\cos(dx+c)+1}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(5*a*log(cos(d*x + c) + 1) + 11*a*log(cos(d*x + c) - 1) - 16*a*log(cos
(d*x + c)) + 2*(3*a*cos(d*x + c)^2 + a*cos(d*x + c) - 6*a)/(cos(d*x + c)^3
```

```
- cos(d*x + c)^2 - cos(d*x + c) + 1))/d
```

Fricas [A] time = 1.76525, size = 512, normalized size = 4.34

$$\frac{6\,a\cos\left(dx+c\right)^{2}+2\,a\cos\left(dx+c\right)-16\left(a\cos\left(dx+c\right)^{3}-a\cos\left(dx+c\right)^{2}-a\cos\left(dx+c\right)+a\right)\log\left(-\cos\left(dx+c\right)\right)+5\ldots}{16\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(6*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - 16*(a*cos(d*x + c)^3 - a*cos(
d*x + c)^2 - a*cos(d*x + c) + a)*log(-cos(d*x + c)) + 5*(a*cos(d*x + c)^3 -
 a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*log(1/2*cos(d*x + c) + 1/2) + 11*(a
*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*log(-1/2*cos(d*x +
 c) + 1/2) - 12*a)/(d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - d*cos(d*x + c) +
d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.46771, size = 201, normalized size = 1.7

$$\frac{22\,a\log\left(\frac{\left|-\cos(dx+c)+1\right|}{\left|\cos(dx+c)+1\right|}\right)-32\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\frac{\left(a-\frac{10\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{33\,a(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}\right)(\cos(dx+c)+1)^{2}}{(\cos(dx+c)-1)^{2}}+\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/32*(22*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 32*a*log(abs
(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a - 10*a*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 33*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos
(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2 + 2*a*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1))/d
```

3.9 $\int \csc^7(c + dx)(a + a\sec(c + dx))\,dx$

Optimal. Leaf size=163

$$-\frac{a^4}{24d(a - a\cos(c + dx))^3} - \frac{5a^3}{32d(a - a\cos(c + dx))^2} - \frac{a^3}{32d(a\cos(c + dx) + a)^2} - \frac{a^2}{2d(a - a\cos(c + dx))} - \frac{3a^2}{16d(a\cos(c + d}$$

```
[Out] -a^4/(24*d*(a - a*Cos[c + d*x])^3) - (5*a^3)/(32*d*(a - a*Cos[c + d*x])^2)
 - a^2/(2*d*(a - a*Cos[c + d*x])) - a^3/(32*d*(a + a*Cos[c + d*x])^2) - (3*a
^2)/(16*d*(a + a*Cos[c + d*x])) + (21*a*Log[1 - Cos[c + d*x]])/(32*d) - (a*
Log[Cos[c + d*x]])/d + (11*a*Log[1 + Cos[c + d*x]])/(32*d)
```

Rubi [A] time = 0.149683, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3872, 2836, 12, 88}

$$-\frac{a^4}{24d(a - a\cos(c + dx))^3} - \frac{5a^3}{32d(a - a\cos(c + dx))^2} - \frac{a^3}{32d(a\cos(c + dx) + a)^2} - \frac{a^2}{2d(a - a\cos(c + dx))} - \frac{3a^2}{16d(a\cos(c + d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^7*(a + a*Sec[c + d*x]),x]
```

```
[Out] -a^4/(24*d*(a - a*Cos[c + d*x])^3) - (5*a^3)/(32*d*(a - a*Cos[c + d*x])^2)
 - a^2/(2*d*(a - a*Cos[c + d*x])) - a^3/(32*d*(a + a*Cos[c + d*x])^2) - (3*a
^2)/(16*d*(a + a*Cos[c + d*x])) + (21*a*Log[1 - Cos[c + d*x]])/(32*d) - (a*
Log[Cos[c + d*x]])/d + (11*a*Log[1 + Cos[c + d*x]])/(32*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \csc^7(c+dx)(a+a\sec(c+dx))\,dx = -\int (-a-a\cos(c+dx))\csc^7(c+dx)\sec(c+dx)\,dx$$
$$= \frac{a^7 \operatorname{Subst}\left(\int \frac{a}{(-a-x)^4x(-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^8 \operatorname{Subst}\left(\int \frac{1}{(-a-x)^4x(-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^8 \operatorname{Subst}\left(\int \left(-\frac{1}{16a^5(a-x)^3} - \frac{3}{16a^6(a-x)^2} - \frac{11}{32a^7(a-x)} - \frac{1}{a^7x} + \frac{1}{8a^4(a+x)^4} + \frac{5}{16a^5(a+x)^3} + \right.\right.}{d}$$
$$= -\frac{a^4}{24d(a-a\cos(c+dx))^3} - \frac{5a^3}{32d(a-a\cos(c+dx))^2} - \frac{a^2}{2d(a-a\cos(c+dx))} -$$

Mathematica [A] time = 0.398065, size = 165, normalized size = 1.01

$$-\frac{a\left(\csc^6\left(\frac{1}{2}(c+dx)\right)+6\csc^4\left(\frac{1}{2}(c+dx)\right)+30\csc^2\left(\frac{1}{2}(c+dx)\right)+64\csc^6(c+dx)+96\csc^4(c+dx)+192\csc^2(c+dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^7*(a + a*Sec[c + d*x]),x]
```

```
[Out] -(a*(30*Csc[(c + d*x)/2]^2 + 6*Csc[(c + d*x)/2]^4 + Csc[(c + d*x)/2]^6 + 19
2*Csc[c + d*x]^2 + 96*Csc[c + d*x]^4 + 64*Csc[c + d*x]^6 + 120*Log[Cos[(c +
 d*x)/2]] + 384*Log[Cos[c + d*x]] - 120*Log[Sin[(c + d*x)/2]] - 384*Log[Sin
[c + d*x]] - 30*Sec[(c + d*x)/2]^2 - 6*Sec[(c + d*x)/2]^4 - Sec[(c + d*x)/2
]^6))/(384*d)
```

Maple [A] time = 0.083, size = 112, normalized size = 0.7

$$-\frac{a}{32\,d\,(1+\sec(dx+c))^2}+\frac{a}{4\,d\,(1+\sec(dx+c))}+\frac{11\,a\ln(1+\sec(dx+c))}{32\,d}-\frac{a}{24\,d\,(-1+\sec(dx+c))^3}-\frac{9}{32\,d\,(-1+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^7*(a+a*sec(d*x+c)),x)
```

```
[Out] -1/32/d*a/(1+sec(d*x+c))^2+1/4/d*a/(1+sec(d*x+c))+11/32/d*a*ln(1+sec(d*x+c)
)-1/24/d*a/(-1+sec(d*x+c))^3-9/32/d*a/(-1+sec(d*x+c))^2-15/16/d*a/(-1+sec(d
*x+c))+21/32/d*a*ln(-1+sec(d*x+c))
```

Maxima [A] time = 1.04834, size = 184, normalized size = 1.13

$$\frac{33\,a\log(\cos(dx+c)+1)+63\,a\log(\cos(dx+c)-1)-96\,a\log(\cos(dx+c))+\frac{2\left(15\,a\cos(dx+c)^4+9\,a\cos(dx+c)^3-49\,a\cos(dx+\right.}{\cos(dx+c)^5-\cos(dx+c)^4-2\,\cos(dx+c)^3+2\,\mathrm{co}}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/96*(33*a*log(cos(d*x + c) + 1) + 63*a*log(cos(d*x + c) - 1) - 96*a*log(co
s(d*x + c)) + 2*(15*a*cos(d*x + c)^4 + 9*a*cos(d*x + c)^3 - 49*a*cos(d*x +
c)^2 - 11*a*cos(d*x + c) + 44*a)/(cos(d*x + c)^5 - cos(d*x + c)^4 - 2*cos(d
*x + c)^3 + 2*cos(d*x + c)^2 + cos(d*x + c) - 1))/d
```

Fricas [B] time = 1.85606, size = 802, normalized size = 4.92

$$\frac{30\,a\cos(dx+c)^4+18\,a\cos(dx+c)^3-98\,a\cos(dx+c)^2-22\,a\cos(dx+c)-96\left(a\cos(dx+c)^5-a\cos(dx+c)^4-2\,a\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(30*a*cos(d*x + c)^4 + 18*a*cos(d*x + c)^3 - 98*a*cos(d*x + c)^2 - 22*
a*cos(d*x + c) - 96*(a*cos(d*x + c)^5 - a*cos(d*x + c)^4 - 2*a*cos(d*x + c)
^3 + 2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a)*log(-cos(d*x + c)) + 33*(a*co
s(d*x + c)^5 - a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 + 2*a*cos(d*x + c)^2 +
 a*cos(d*x + c) - a)*log(1/2*cos(d*x + c) + 1/2) + 63*(a*cos(d*x + c)^5 - a
*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 + 2*a*cos(d*x + c)^2 + a*cos(d*x + c)
- a)*log(-1/2*cos(d*x + c) + 1/2) + 88*a)/(d*cos(d*x + c)^5 - d*cos(d*x + c
)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 + d*cos(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**7*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.67476, size = 265, normalized size = 1.63

$$\frac{252\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-384\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{\left(2\,a-\frac{21\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{132\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{462\,a(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}\right)(\cos(dx+c)+1)^3}{(\cos(dx+c)-1)^3}+\cdots}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/384*(252*a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 384*a*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (2*a - 21*a*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 132*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
 - 462*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1)^3/(c
os(d*x + c) - 1)^3 + 42*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/d
```

3.10 $\int (a + a\sec(c + dx))\sin^8(c + dx)\,dx$

Optimal. Leaf size=165

$$-\frac{a\sin^7(c+dx)}{7d}-\frac{a\sin^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\sin^7(c+dx)\cos(c+dx)}{8d}$$

```
[Out] (35*a*x)/128 + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (35*a*Cos
[c + d*x]*Sin[c + d*x])/(128*d) - (a*Sin[c + d*x]^3)/(3*d) - (35*a*Cos[c +
d*x]*Sin[c + d*x]^3)/(192*d) - (a*Sin[c + d*x]^5)/(5*d) - (7*a*Cos[c + d*x]
*Sin[c + d*x]^5)/(48*d) - (a*Sin[c + d*x]^7)/(7*d) - (a*Cos[c + d*x]*Sin[c
+ d*x]^7)/(8*d)
```

Rubi [A] time = 0.145638, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3872, 2838, 2592, 302, 206, 2635, 8}

$$-\frac{a\sin^7(c+dx)}{7d}-\frac{a\sin^5(c+dx)}{5d}-\frac{a\sin^3(c+dx)}{3d}-\frac{a\sin(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\sin^7(c+dx)\cos(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^8,x]
```

```
[Out] (35*a*x)/128 + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (35*a*Cos
[c + d*x]*Sin[c + d*x])/(128*d) - (a*Sin[c + d*x]^3)/(3*d) - (35*a*Cos[c +
d*x]*Sin[c + d*x]^3)/(192*d) - (a*Sin[c + d*x]^5)/(5*d) - (7*a*Cos[c + d*x]
*Sin[c + d*x]^5)/(48*d) - (a*Sin[c + d*x]^7)/(7*d) - (a*Cos[c + d*x]*Sin[c
+ d*x]^7)/(8*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sec(c+dx))\sin^8(c+dx)\,dx = -\int (-a-a\cos(c+dx))\sin^7(c+dx)\tan(c+dx)\,dx$$
$$= a\int \sin^8(c+dx)\,dx + a\int \sin^7(c+dx)\tan(c+dx)\,dx$$
$$= -\frac{a\cos(c+dx)\sin^7(c+dx)}{8d} + \frac{1}{8}(7a)\int \sin^6(c+dx)\,dx + \frac{a\,\text{Subst}\left(\int \frac{x^8}{1-x^2}\,dx, x, \text{si}\right.}{d}$$
$$= -\frac{7a\cos(c+dx)\sin^5(c+dx)}{48d} - \frac{a\cos(c+dx)\sin^7(c+dx)}{8d} + \frac{1}{48}(35a)\int \sin^4(c+$$
$$= -\frac{a\sin(c+dx)}{d} - \frac{a\sin^3(c+dx)}{3d} - \frac{35a\cos(c+dx)\sin^3(c+dx)}{192d} - \frac{a\sin^5(c+dx)}{5d} -$$
$$= \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d} - \frac{35a\cos(c+dx)\sin(c+dx)}{128d} - \frac{a\sin^3(c+}{3d}$$
$$= \frac{35ax}{128} + \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d} - \frac{35a\cos(c+dx)\sin(c+dx)}{128d} - \frac{a}{}$$

Mathematica [A] time = 0.330181, size = 106, normalized size = 0.64

$$\frac{a\left(-15360\sin^7(c+dx) - 21504\sin^5(c+dx) - 35840\sin^3(c+dx) - 107520\sin(c+dx) + 35(-672\sin(2(c+dx)) + 168\,\text{s}\right.}{107520d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^8,x]
```

```
[Out] (a*(107520*ArcTanh[Sin[c + d*x]] - 107520*Sin[c + d*x] - 35840*Sin[c + d*x]
^3 - 21504*Sin[c + d*x]^5 - 15360*Sin[c + d*x]^7 + 35*(840*c + 840*d*x - 67
2*Sin[2*(c + d*x)] + 168*Sin[4*(c + d*x)] - 32*Sin[6*(c + d*x)] + 3*Sin[8*(
c + d*x)])))/(107520*d)
```

Maple [A] time = 0.094, size = 164, normalized size = 1.

$$-\frac{a\cos(dx+c)(\sin(dx+c))^7}{8d} - \frac{7a\cos(dx+c)(\sin(dx+c))^5}{48d} - \frac{35a\cos(dx+c)(\sin(dx+c))^3}{192d} - \frac{35a\cos(dx+c)\sin}{128d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^8,x)
```

```
[Out] -1/8*a*cos(d*x+c)*sin(d*x+c)^7/d-7/48*a*cos(d*x+c)*sin(d*x+c)^5/d-35/192*a*
cos(d*x+c)*sin(d*x+c)^3/d-35/128*a*cos(d*x+c)*sin(d*x+c)/d+35/128*a*x+35/12
8/d*a*c-1/7*a*sin(d*x+c)^7/d-1/5*a*sin(d*x+c)^5/d-1/3*a*sin(d*x+c)^3/d-a*si
n(d*x+c)/d+1/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.990094, size = 171, normalized size = 1.04

$$-\frac{512\left(30\sin\left(dx+c\right)^{7}+42\sin\left(dx+c\right)^{5}+70\sin\left(dx+c\right)^{3}-105\log\left(\sin\left(dx+c\right)+1\right)+105\log\left(\sin\left(dx+c\right)-1\right)+\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/107520*(512*(30*sin(d*x + c)^7 + 42*sin(d*x + c)^5 + 70*sin(d*x + c)^3 -
 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1) + 210*sin(d*x + c))*
a - 35*(128*sin(2*d*x + 2*c)^3 + 840*d*x + 840*c + 3*sin(8*d*x + 8*c) + 168
*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*a)/d
```

Fricas [A] time = 1.9058, size = 378, normalized size = 2.29

$$\frac{3675\,adx+6720\,a\log\left(\sin\left(dx+c\right)+1\right)-6720\,a\log\left(-\sin\left(dx+c\right)+1\right)+\left(1680\,a\cos\left(dx+c\right)^{7}+1920\,a\cos\left(dx+c\right)^{\cdots}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] 1/13440*(3675*a*d*x + 6720*a*log(sin(d*x + c) + 1) - 6720*a*log(-sin(d*x +
c) + 1) + (1680*a*cos(d*x + c)^7 + 1920*a*cos(d*x + c)^6 - 7000*a*cos(d*x +
 c)^5 - 8448*a*cos(d*x + c)^4 + 11410*a*cos(d*x + c)^3 + 15616*a*cos(d*x +
c)^2 - 9765*a*cos(d*x + c) - 22528*a)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.51213, size = 235, normalized size = 1.42

$$\frac{3675\left(dx+c\right)a+13440\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-13440\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(9765\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+83\cdots\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^8,x, algorithm="giac")
```

```
[Out] 1/13440*(3675*(d*x + c)*a + 13440*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 13
440*a*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(9765*a*tan(1/2*d*x + 1/2*c)^1
5 + 83825*a*tan(1/2*d*x + 1/2*c)^13 + 321013*a*tan(1/2*d*x + 1/2*c)^11 + 72
4649*a*tan(1/2*d*x + 1/2*c)^9 + 1078359*a*tan(1/2*d*x + 1/2*c)^7 + 508683*a
*tan(1/2*d*x + 1/2*c)^5 + 140175*a*tan(1/2*d*x + 1/2*c)^3 + 17115*a*tan(1/2
*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^8)/d
```

3.11 $\int (a + a\sec(c + dx)) \sin^6(c + dx)\, dx$

Optimal. Leaf size=127

$$-\frac{a\sin^5(c+dx)}{5d} - \frac{a\sin^3(c+dx)}{3d} - \frac{a\sin(c+dx)}{d} + \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\sin^5(c+dx)\cos(c+dx)}{6d} - \frac{5a\sin^3(c+d}{2}$$

```
[Out] (5*a*x)/16 + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (5*a*Cos[c
+ d*x]*Sin[c + d*x])/(16*d) - (a*Sin[c + d*x]^3)/(3*d) - (5*a*Cos[c + d*x]*
Sin[c + d*x]^3)/(24*d) - (a*Sin[c + d*x]^5)/(5*d) - (a*Cos[c + d*x]*Sin[c +
 d*x]^5)/(6*d)
```

Rubi [A] time = 0.127833, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3872, 2838, 2592, 302, 206, 2635, 8}

$$-\frac{a\sin^5(c+dx)}{5d} - \frac{a\sin^3(c+dx)}{3d} - \frac{a\sin(c+dx)}{d} + \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\sin^5(c+dx)\cos(c+dx)}{6d} - \frac{5a\sin^3(c+d}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^6,x]
```

```
[Out] (5*a*x)/16 + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (5*a*Cos[c
+ d*x]*Sin[c + d*x])/(16*d) - (a*Sin[c + d*x]^3)/(3*d) - (5*a*Cos[c + d*x]*
Sin[c + d*x]^3)/(24*d) - (a*Sin[c + d*x]^5)/(5*d) - (a*Cos[c + d*x]*Sin[c +
 d*x]^5)/(6*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int (a+a\sec(c+dx))\sin^6(c+dx)\,dx = -\int (-a-a\cos(c+dx))\sin^5(c+dx)\tan(c+dx)\,dx$$
$$= a\int \sin^6(c+dx)\,dx + a\int \sin^5(c+dx)\tan(c+dx)\,dx$$
$$= -\frac{a\cos(c+dx)\sin^5(c+dx)}{6d} + \frac{1}{6}(5a)\int \sin^4(c+dx)\,dx + \frac{a\,\mathrm{Subst}\left(\int \frac{x^6}{1-x^2}\,dx, x, \mathrm{si}\right.}{d}$$
$$= -\frac{5a\cos(c+dx)\sin^3(c+dx)}{24d} - \frac{a\cos(c+dx)\sin^5(c+dx)}{6d} + \frac{1}{8}(5a)\int \sin^2(c+dx$$
$$= -\frac{a\sin(c+dx)}{d} - \frac{5a\cos(c+dx)\sin(c+dx)}{16d} - \frac{a\sin^3(c+dx)}{3d} - \frac{5a\cos(c+dx)\sin}{24d}$$
$$= \frac{5ax}{16} + \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d} - \frac{5a\cos(c+dx)\sin(c+dx)}{16d} - \frac{a\sin}{}$$

Mathematica [A] time = 0.184909, size = 86, normalized size = 0.68

$$\frac{a\left(-192\sin^5(c+dx) - 320\sin^3(c+dx) - 960\sin(c+dx) + 5(-45\sin(2(c+dx)) + 9\sin(4(c+dx)) - \sin(6(c+dx)) + 60\right.}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^6,x]
```

```
[Out] (a*(960*ArcTanh[Sin[c + d*x]] - 960*Sin[c + d*x] - 320*Sin[c + d*x]^3 - 192
*Sin[c + d*x]^5 + 5*(60*c + 60*d*x - 45*Sin[2*(c + d*x)] + 9*Sin[4*(c + d*x
)] - Sin[6*(c + d*x)])))/(960*d)
```

Maple [A] time = 0.085, size = 130, normalized size = 1.

$$-\frac{a\cos(dx+c)(\sin(dx+c))^5}{6d} - \frac{5a\cos(dx+c)(\sin(dx+c))^3}{24d} - \frac{5a\cos(dx+c)\sin(dx+c)}{16d} + \frac{5ax}{16} + \frac{5ac}{16d} - \frac{a(\sin(dx}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^6,x)
```

```
[Out] -1/6*a*cos(d*x+c)*sin(d*x+c)^5/d-5/24*a*cos(d*x+c)*sin(d*x+c)^3/d-5/16*a*co
s(d*x+c)*sin(d*x+c)/d+5/16*a*x+5/16/d*a*c-1/5*a*sin(d*x+c)^5/d-1/3*a*sin(d*
x+c)^3/d-a*sin(d*x+c)/d+1/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.998566, size = 143, normalized size = 1.13

$$-\frac{32\left(6\sin\left(dx+c\right)^{5}+10\sin\left(dx+c\right)^{3}-15\log\left(\sin\left(dx+c\right)+1\right)+15\log\left(\sin\left(dx+c\right)-1\right)+30\sin\left(dx+c\right)\right)a-5\left(\cdots\right.}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] -1/960*(32*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin(d*x + c) + 1)
 + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*a - 5*(4*sin(2*d*x + 2*c)^3
+ 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a)/d
```

Fricas [A] time = 1.87778, size = 290, normalized size = 2.28

$$\frac{75\,adx+120\,a\log\left(\sin\left(dx+c\right)+1\right)-120\,a\log\left(-\sin\left(dx+c\right)+1\right)-\left(40\,a\cos\left(dx+c\right)^{5}+48\,a\cos\left(dx+c\right)^{4}-130\,a\,\cdots\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/240*(75*a*d*x + 120*a*log(sin(d*x + c) + 1) - 120*a*log(-sin(d*x + c) + 1
) - (40*a*cos(d*x + c)^5 + 48*a*cos(d*x + c)^4 - 130*a*cos(d*x + c)^3 - 176
*a*cos(d*x + c)^2 + 165*a*cos(d*x + c) + 368*a)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.48382, size = 197, normalized size = 1.55

$$\frac{75\left(dx+c\right)a+240\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-240\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(165\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1095\,a\tan\left(\frac{1}{2}\cdots\right.}{\cdots}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/240*(75*(d*x + c)*a + 240*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 240*a*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(165*a*tan(1/2*d*x + 1/2*c)^11 + 1095*
a*tan(1/2*d*x + 1/2*c)^9 + 3138*a*tan(1/2*d*x + 1/2*c)^7 + 5118*a*tan(1/2*d
*x + 1/2*c)^5 + 1945*a*tan(1/2*d*x + 1/2*c)^3 + 315*a*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.12 $\int (a + a\sec(c + dx))\sin^4(c + dx)\,dx$

Optimal. Leaf size=89

$$-\frac{a\sin^3(c+dx)}{3d} - \frac{a\sin(c+dx)}{d} + \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\sin^3(c+dx)\cos(c+dx)}{4d} - \frac{3a\sin(c+dx)\cos(c+dx)}{8d} +$$

```
[Out] (3*a*x)/8 + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (3*a*Cos[c +
 d*x]*Sin[c + d*x])/(8*d) - (a*Sin[c + d*x]^3)/(3*d) - (a*Cos[c + d*x]*Sin[
c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.111148, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3872, 2838, 2592, 302, 206, 2635, 8}

$$-\frac{a\sin^3(c+dx)}{3d} - \frac{a\sin(c+dx)}{d} + \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\sin^3(c+dx)\cos(c+dx)}{4d} - \frac{3a\sin(c+dx)\cos(c+dx)}{8d} +$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^4,x]
```

```
[Out] (3*a*x)/8 + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (3*a*Cos[c +
 d*x]*Sin[c + d*x])/(8*d) - (a*Sin[c + d*x]^3)/(3*d) - (a*Cos[c + d*x]*Sin[
c + d*x]^3)/(4*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))\sin^4(c+dx)\,dx &= -\int (-a-a\cos(c+dx))\sin^3(c+dx)\tan(c+dx)\,dx\\ &= a\int \sin^4(c+dx)\,dx + a\int \sin^3(c+dx)\tan(c+dx)\,dx\\ &= -\frac{a\cos(c+dx)\sin^3(c+dx)}{4d} + \frac{1}{4}(3a)\int \sin^2(c+dx)\,dx + \frac{a\,\text{Subst}\left(\int \frac{x^4}{1-x^2}\,dx, x, \text{si}\right.}{d}\\ &= -\frac{3a\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\cos(c+dx)\sin^3(c+dx)}{4d} + \frac{1}{8}(3a)\int 1\,dx + \frac{a\,\text{Sub}}{}\\ &= \frac{3ax}{8} - \frac{a\sin(c+dx)}{d} - \frac{3a\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\sin^3(c+dx)}{3d} - \frac{a\cos(c+dx}{4}\\ &= \frac{3ax}{8} + \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\sin(c+dx)}{d} - \frac{3a\cos(c+dx)\sin(c+dx)}{8d} - \frac{a\sin}{}\end{aligned}$$

Mathematica [A] time = 0.116044, size = 86, normalized size = 0.97

$$\frac{3a(c+dx)}{8d} - \frac{a\sin^3(c+dx)}{3d} - \frac{a\sin(c+dx)}{d} - \frac{a\sin(2(c+dx))}{4d} + \frac{a\sin(4(c+dx))}{32d} + \frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^4,x]
```

```
[Out] (3*a*(c + d*x))/(8*d) + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d -
(a*Sin[c + d*x]^3)/(3*d) - (a*Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)]
)/(32*d)
```

Maple [A] time = 0.089, size = 96, normalized size = 1.1

$$-\frac{a\cos(dx+c)(\sin(dx+c))^3}{4d} - \frac{3a\cos(dx+c)\sin(dx+c)}{8d} + \frac{3ax}{8} + \frac{3ac}{8d} - \frac{a(\sin(dx+c))^3}{3d} - \frac{a\sin(dx+c)}{d} + \frac{a\ln(\text{se}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^4,x)
```

```
[Out] -1/4*a*cos(d*x+c)*sin(d*x+c)^3/d-3/8*a*cos(d*x+c)*sin(d*x+c)/d+3/8*a*x+3/8/
d*a*c-1/3*a*sin(d*x+c)^3/d-a*sin(d*x+c)/d+1/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.01175, size = 109, normalized size = 1.22

$$-\frac{16\left(2\,\sin\left(dx+c\right)^{3}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)+6\,\sin\left(dx+c\right)\right)a-3\left(12\,dx+12\,c+\sin\left(4\,d\right.\right.}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/96*(16*(2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
- 1) + 6*sin(d*x + c))*a - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*
x + 2*c))*a)/d
```

Fricas [A] time = 1.83008, size = 217, normalized size = 2.44

$$\frac{9\,adx+12\,a\log\left(\sin\left(dx+c\right)+1\right)-12\,a\log\left(-\sin\left(dx+c\right)+1\right)+\left(6\,a\cos\left(dx+c\right)^{3}+8\,a\cos\left(dx+c\right)^{2}-15\,a\cos\left(dx\right.\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(9*a*d*x + 12*a*log(sin(d*x + c) + 1) - 12*a*log(-sin(d*x + c) + 1) +
(6*a*cos(d*x + c)^3 + 8*a*cos(d*x + c)^2 - 15*a*cos(d*x + c) - 32*a)*sin(d*
x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.48243, size = 159, normalized size = 1.79

$$\frac{9\left(dx+c\right)a+24\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-24\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+71\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}\ldots\right.}{\left(\tan\left(\frac{1}{2}\,dx+\ldots\right.\right.}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/24*(9*(d*x + c)*a + 24*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*a*log(ab
s(tan(1/2*d*x + 1/2*c) - 1)) - 2*(15*a*tan(1/2*d*x + 1/2*c)^7 + 71*a*tan(1/
2*d*x + 1/2*c)^5 + 137*a*tan(1/2*d*x + 1/2*c)^3 + 33*a*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.13 $\int (a + a\sec(c + dx))\sin^2(c + dx)\,dx$

Optimal. Leaf size=51

$$-\frac{a\sin(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

```
[Out] (a*x)/2 + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (a*Cos[c + d*x
]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0818246, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3872, 2838, 2592, 321, 206, 2635, 8}

$$-\frac{a\sin(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\sin(c+dx)\cos(c+dx)}{2d}+\frac{ax}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sin[c + d*x]^2,x]
```

```
[Out] (a*x)/2 + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (a*Cos[c + d*x
]*Sin[c + d*x])/(2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))\sin^2(c+dx)\,dx &= -\int (-a-a\cos(c+dx))\sin(c+dx)\tan(c+dx)\,dx\\ &= a\int \sin^2(c+dx)\,dx + a\int \sin(c+dx)\tan(c+dx)\,dx\\ &= -\frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}a\int 1\,dx+\frac{a\,\mathrm{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d}\\ &= \frac{ax}{2}-\frac{a\sin(c+dx)}{d}-\frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{a\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d}\\ &= \frac{ax}{2}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\sin(c+dx)}{d}-\frac{a\cos(c+dx)\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0520987, size = 54, normalized size = 1.06

$$\frac{a(c+dx)}{2d}-\frac{a\sin(c+dx)}{d}-\frac{a\sin(2(c+dx))}{4d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sin[c + d*x]^2,x]
```

```
[Out] (a*(c + d*x))/(2*d) + (a*ArcTanh[Sin[c + d*x]])/d - (a*Sin[c + d*x])/d - (a
*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.031, size = 62, normalized size = 1.2

$$-\frac{a\cos(dx+c)\sin(dx+c)}{2d}+\frac{ax}{2}+\frac{ac}{2d}+\frac{a\ln(\sec(dx+c)+\tan(dx+c))}{d}-\frac{a\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*sin(d*x+c)^2,x)
```

```
[Out] -1/2*a*cos(d*x+c)*sin(d*x+c)/d+1/2*a*x+1/2/d*a*c+1/d*a*ln(sec(d*x+c)+tan(d*
x+c))-a*sin(d*x+c)/d
```

Maxima [A] time = 0.989886, size = 80, normalized size = 1.57

$$\frac{(2\,dx + 2\,c - \sin(2\,dx + 2\,c))a + 2\,a(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1) - 2\,\sin(dx + c))}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a + 2*a*(log(sin(d*x + c) + 1) - log(
sin(d*x + c) - 1) - 2*sin(d*x + c)))/d
```

Fricas [A] time = 1.73897, size = 143, normalized size = 2.8

$$\frac{adx + a\log(\sin(dx + c) + 1) - a\log(-\sin(dx + c) + 1) - (a\cos(dx + c) + 2\,a)\sin(dx + c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*x + a*log(sin(d*x + c) + 1) - a*log(-sin(d*x + c) + 1) - (a*cos(d*
x + c) + 2*a)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \sin^2(c + dx)\sec(c + dx)\,dx + \int \sin^2(c + dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)**2,x)
```

```
[Out] a*(Integral(sin(c + d*x)**2*sec(c + d*x), x) + Integral(sin(c + d*x)**2, x)
)
```

Giac [A] time = 1.49379, size = 119, normalized size = 2.33

$$\frac{(dx + c)a + 2\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 2\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + 3\,a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*((d*x + c)*a + 2*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*a*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) - 2*(a*tan(1/2*d*x + 1/2*c)^3 + 3*a*tan(1/2*d*x + 1
/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.14 $\int \csc^2(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=37

$$-\frac{a\cot(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (a*Csc[c + d*x])/d
```

Rubi [A] time = 0.0934906, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3872, 2838, 2621, 321, 207, 3767, 8}

$$-\frac{a\cot(c+dx)}{d}-\frac{a\csc(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (a*Csc[c + d*x])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \csc^2(c+dx)(a+a\sec(c+dx))\,dx &= -\int (-a-a\cos(c+dx))\csc^2(c+dx)\sec(c+dx)\,dx \\
&= a\int \csc^2(c+dx)\,dx + a\int \csc^2(c+dx)\sec(c+dx)\,dx \\
&= -\frac{a\,\text{Subst}\left(\int 1\,dx, x, \cot(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} \\
&= -\frac{a\cot(c+dx)}{d} - \frac{a\csc(c+dx)}{d} - \frac{a\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} \\
&= \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\csc(c+dx)}{d}
\end{aligned}$$

Mathematica [C] time = 0.0292145, size = 41, normalized size = 1.11

$$-\frac{a\csc(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\sin^2(c+dx)\right)}{d} - \frac{a\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sec[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) - (a*Csc[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin
[c + d*x]^2])/d
```

Maple [A] time = 0.082, size = 47, normalized size = 1.3

$$-\frac{a\cot(dx+c)}{d} - \frac{a}{d\sin(dx+c)} + \frac{a\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sec(d*x+c)),x)
```

```
[Out] -a*cot(d*x+c)/d-1/d*a/sin(d*x+c)+1/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.01601, size = 68, normalized size = 1.84

$$-\frac{a\left(\frac{2}{\sin(dx+c)} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + \frac{2a}{\tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(a*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) +
2*a/tan(d*x + c))/d
```

Fricas [A] time = 1.69994, size = 170, normalized size = 4.59

$$\frac{a\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-a\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-2\,a\cos\left(dx+c\right)-2\,a}{2\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(a*log(sin(d*x + c) + 1)*sin(d*x + c) - a*log(-sin(d*x + c) + 1)*sin(d*
x + c) - 2*a*cos(d*x + c) - 2*a)/(d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\csc^{2}\left(c+dx\right)\sec\left(c+dx\right)dx+\int\csc^{2}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sec(d*x+c)),x)
```

```
[Out] a*(Integral(csc(c + d*x)**2*sec(c + d*x), x) + Integral(csc(c + d*x)**2, x)
)
```

Giac [A] time = 1.53184, size = 68, normalized size = 1.84

$$\frac{a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) - a/tan(1/2*d*x + 1/2*c))/d
```

3.15 $\int \csc^4(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=69

$$-\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (a*Cot[c + d*x]^3)/(3*d)
 - (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.102516, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3872, 2838, 2621, 302, 207, 3767}

$$-\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (a*Cot[c + d*x]^3)/(3*d)
 - (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \csc^4(c+dx)(a+a\sec(c+dx))\,dx &= -\int (-a-a\cos(c+dx))\csc^4(c+dx)\sec(c+dx)\,dx \\
&= a\int \csc^4(c+dx)\,dx + a\int \csc^4(c+dx)\sec(c+dx)\,dx \\
&= -\frac{a\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,\cot(c+dx\right.}{d} \\
&= -\frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{a\,\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)\,dx,x,\csc(c+\right.}{d} \\
&= -\frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{a\csc(c+dx)}{d} - \frac{a\csc^3(c+dx)}{3d} - \frac{a\,\text{Subst}\left(\right.}{} \\
&= \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{a\csc(c+dx)}{d} - \frac{a\,\text{cs}}{}
\end{aligned}$$

Mathematica [C] time = 0.0285499, size = 69, normalized size = 1.

$$-\frac{a\csc^3(c+dx)\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\sin^2(c+dx)\right)}{3d} - \frac{2a\cot(c+dx)}{3d} - \frac{a\cot(c+dx)\csc^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*a*Cot[c + d*x])/(3*d) - (a*Cot[c + d*x]*Csc[c + d*x]^2)/(3*d) - (a*Csc[
c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[c + d*x]^2])/(3*d)
```

Maple [A] time = 0.104, size = 81, normalized size = 1.2

$$-\frac{2\,a\cot(dx+c)}{3\,d} - \frac{a\cot(dx+c)(\csc(dx+c))^2}{3\,d} - \frac{a}{3\,d(\sin(dx+c))^3} - \frac{a}{d\sin(dx+c)} + \frac{a\ln(\sec(dx+c)+\tan(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+a*sec(d*x+c)),x)
```

```
[Out] -2/3*a*cot(d*x+c)/d-1/3/d*a*cot(d*x+c)*csc(d*x+c)^2-1/3/d*a/sin(d*x+c)^3-1/
d*a/sin(d*x+c)+1/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.00723, size = 103, normalized size = 1.49

$$-\frac{a\left(\frac{2\left(3\sin(dx+c)^2+1\right)}{\sin(dx+c)^3} - 3\log(\sin(dx+c)+1) + 3\log(\sin(dx+c)-1)\right) + \frac{2\left(3\tan(dx+c)^2+1\right)a}{\tan(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(a*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1)
+ 3*log(sin(d*x + c) - 1)) + 2*(3*tan(d*x + c)^2 + 1)*a/tan(d*x + c)^3)/d
```

Fricas [A] time = 1.71284, size = 282, normalized size = 4.09

$$-\frac{4\,a\cos\left(dx+c\right)^{2}-3\,\left(a\cos\left(dx+c\right)-a\right)\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+3\,\left(a\cos\left(dx+c\right)-a\right)\log\left(-\sin\left(dx+c\right)+1\right)}{6\,\left(d\cos\left(dx+c\right)-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(4*a*cos(d*x + c)^2 - 3*(a*cos(d*x + c) - a)*log(sin(d*x + c) + 1)*sin
(d*x + c) + 3*(a*cos(d*x + c) - a)*log(-sin(d*x + c) + 1)*sin(d*x + c) + 2*
a*cos(d*x + c) - 8*a)/((d*cos(d*x + c) - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.63253, size = 107, normalized size = 1.55

$$\frac{12\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-12\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{12\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/12*(12*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 12*a*log(abs(tan(1/2*d*x +
1/2*c) - 1)) - 3*a*tan(1/2*d*x + 1/2*c) - (12*a*tan(1/2*d*x + 1/2*c)^2 + a)
/tan(1/2*d*x + 1/2*c)^3)/d
```

3.16 $\int \csc^6(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=101

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{2a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(}{d}$$

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (2*a*Cot[c + d*x]^3)/(3*
d) - (a*Cot[c + d*x]^5)/(5*d) - (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*
d) - (a*Csc[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.109502, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3872, 2838, 2621, 302, 207, 3767}

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{2a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{3d}-\frac{a\csc(c+dx)}{d}+\frac{a\tanh^{-1}(\sin(}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (2*a*Cot[c + d*x]^3)/(3*
d) - (a*Cot[c + d*x]^5)/(5*d) - (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*
d) - (a*Csc[c + d*x]^5)/(5*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^6(c+dx)(a+a\sec(c+dx))\,dx &= -\int (-a-a\cos(c+dx))\csc^6(c+dx)\sec(c+dx)\,dx\\ &= a\int \csc^6(c+dx)\,dx + a\int \csc^6(c+dx)\sec(c+dx)\,dx\\ &= -\frac{a\,\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \left(1+2x^2+x^4\right)dx,x,\cot(c+\right.}{d}\\ &= -\frac{a\cot(c+dx)}{d} - \frac{2a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} - \frac{a\,\text{Subst}\left(\int \left(1+x^2+x^4+\right.\right.}{d}\\ &= -\frac{a\cot(c+dx)}{d} - \frac{2a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} - \frac{a\csc(c+dx)}{d} - \frac{a\csc^3(c+d}{3d}\\ &= \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{2a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} - \frac{a\csc}{}\end{aligned}$$

Mathematica [C] time = 0.0310175, size = 91, normalized size = 0.9

$$-\frac{a\csc^5(c+dx)\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\sin^2(c+dx)\right)}{5d} - \frac{8a\cot(c+dx)}{15d} - \frac{a\cot(c+dx)\csc^4(c+dx)}{5d} - \frac{4a\cot(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-8*a*Cot[c + d*x])/(15*d) - (4*a*Cot[c + d*x]*Csc[c + d*x]^2)/(15*d) - (a*
Cot[c + d*x]*Csc[c + d*x]^4)/(5*d) - (a*Csc[c + d*x]^5*Hypergeometric2F1[-5
/2, 1, -3/2, Sin[c + d*x]^2])/(5*d)
```

Maple [A] time = 0.141, size = 115, normalized size = 1.1

$$-\frac{8\,a\cot(dx+c)}{15\,d} - \frac{a\cot(dx+c)(\csc(dx+c))^4}{5\,d} - \frac{4\,a\cot(dx+c)(\csc(dx+c))^2}{15\,d} - \frac{a}{5\,d(\sin(dx+c))^5} - \frac{a}{3\,d(\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+a*sec(d*x+c)),x)
```

```
[Out] -8/15*a*cot(d*x+c)/d-1/5/d*a*cot(d*x+c)*csc(d*x+c)^4-4/15/d*a*cot(d*x+c)*cs
c(d*x+c)^2-1/5/d*a/sin(d*x+c)^5-1/3/d*a/sin(d*x+c)^3-1/d*a/sin(d*x+c)+1/d*a
*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.982248, size = 130, normalized size = 1.29

$$-\frac{a\left(\frac{2\left(15\sin(dx+c)^4+5\sin(dx+c)^2+3\right)}{\sin(dx+c)^5}-15\log\left(\sin\left(dx+c\right)+1\right)+15\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{2\left(15\tan(dx+c)^4+10\tan(dx+c)^2+3\right)a}{\tan(dx+c)^5}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/30*(a*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)/sin(d*x + c)^5 - 15*
log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 2*(15*tan(d*x + c)^4 +
10*tan(d*x + c)^2 + 3)*a/tan(d*x + c)^5)/d
```

Fricas [B] time = 1.77905, size = 504, normalized size = 4.99

$$-\frac{16\,a\cos\left(dx+c\right)^4+14\,a\cos\left(dx+c\right)^3-54\,a\cos\left(dx+c\right)^2-15\left(a\cos\left(dx+c\right)^3-a\cos\left(dx+c\right)^2-a\cos\left(dx+c\right)+a\right)}{30\left(d\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/30*(16*a*cos(d*x + c)^4 + 14*a*cos(d*x + c)^3 - 54*a*cos(d*x + c)^2 - 15
*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*log(sin(d*x + c
) + 1)*sin(d*x + c) + 15*(a*cos(d*x + c)^3 - a*cos(d*x + c)^2 - a*cos(d*x +
 c) + a)*log(-sin(d*x + c) + 1)*sin(d*x + c) - 16*a*cos(d*x + c) + 46*a)/((
d*cos(d*x + c)^3 - d*cos(d*x + c)^2 - d*cos(d*x + c) + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.76922, size = 144, normalized size = 1.43

$$-\frac{5\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-240\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+240\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+90\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/240*(5*a*tan(1/2*d*x + 1/2*c)^3 - 240*a*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) + 240*a*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 90*a*tan(1/2*d*x + 1/2*c) +
```

```
 3*(80*a*tan(1/2*d*x + 1/2*c)^4 + 10*a*tan(1/2*d*x + 1/2*c)^2 + a)/tan(1/2*
d*x + 1/2*c)^5)/d
```

3.17 $\int \csc^8(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=131

$$-\frac{a\cot^7(c+dx)}{7d}-\frac{3a\cot^5(c+dx)}{5d}-\frac{a\cot^3(c+dx)}{d}-\frac{a\cot(c+dx)}{d}-\frac{a\csc^7(c+dx)}{7d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{3d}$$

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (a*Cot[c + d*x]^3)/d - (
3*a*Cot[c + d*x]^5)/(5*d) - (a*Cot[c + d*x]^7)/(7*d) - (a*Csc[c + d*x])/d -
 (a*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^7)/(
7*d)
```

Rubi [A] time = 0.117004, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3872, 2838, 2621, 302, 207, 3767}

$$-\frac{a\cot^7(c+dx)}{7d}-\frac{3a\cot^5(c+dx)}{5d}-\frac{a\cot^3(c+dx)}{d}-\frac{a\cot(c+dx)}{d}-\frac{a\csc^7(c+dx)}{7d}-\frac{a\csc^5(c+dx)}{5d}-\frac{a\csc^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^8*(a + a*Sec[c + d*x]),x]
```


```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (a*Cot[c + d*x]^3)/d - (
3*a*Cot[c + d*x]^5)/(5*d) - (a*Cot[c + d*x]^7)/(7*d) - (a*Csc[c + d*x])/d -
 (a*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d*x]^5)/(5*d) - (a*Csc[c + d*x]^7)/(
7*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^8(c+dx)(a+a\sec(c+dx))\,dx &= -\int(-a-a\cos(c+dx))\csc^8(c+dx)\sec(c+dx)\,dx\\ &= a\int\csc^8(c+dx)\,dx+a\int\csc^8(c+dx)\sec(c+dx)\,dx\\ &= -\frac{a\,\text{Subst}\left(\int\frac{x^8}{-1+x^2}\,dx,x,\csc(c+dx)\right)}{d}-\frac{a\,\text{Subst}\left(\int\left(1+3x^2+3x^4+x^6\right)dx,x,\text{c}\right.}{d}\\ &= -\frac{a\cot(c+dx)}{d}-\frac{a\cot^3(c+dx)}{d}-\frac{3a\cot^5(c+dx)}{5d}-\frac{a\cot^7(c+dx)}{7d}-\frac{a\,\text{Subst}\left(\int\right.}{}\\ &= -\frac{a\cot(c+dx)}{d}-\frac{a\cot^3(c+dx)}{d}-\frac{3a\cot^5(c+dx)}{5d}-\frac{a\cot^7(c+dx)}{7d}-\frac{a\csc(c+}{d}\\ &= \frac{a\tanh^{-1}(\sin(c+dx))}{d}-\frac{a\cot(c+dx)}{d}-\frac{a\cot^3(c+dx)}{d}-\frac{3a\cot^5(c+dx)}{5d}-\frac{a\,\text{co}}{}\end{aligned}$$

Mathematica [C] time = 0.0481466, size = 113, normalized size = 0.86

$$-\frac{a\csc^7(c+dx)\text{Hypergeometric2F1}\left(-\frac{7}{2},1,-\frac{5}{2},\sin^2(c+dx)\right)}{7d}-\frac{16a\cot(c+dx)}{35d}-\frac{a\cot(c+dx)\csc^6(c+dx)}{7d}-\frac{6a\cot(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^8*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-16*a*Cot[c + d*x])/(35*d) - (8*a*Cot[c + d*x]*Csc[c + d*x]^2)/(35*d) - (6
*a*Cot[c + d*x]*Csc[c + d*x]^4)/(35*d) - (a*Cot[c + d*x]*Csc[c + d*x]^6)/(7
*d) - (a*Csc[c + d*x]^7*Hypergeometric2F1[-7/2, 1, -5/2, Sin[c + d*x]^2])/(
7*d)
```

Maple [A] time = 0.122, size = 149, normalized size = 1.1

$$-\frac{16\,a\cot(dx+c)}{35\,d}-\frac{a\cot(dx+c)(\csc(dx+c))^6}{7\,d}-\frac{6\,a\cot(dx+c)(\csc(dx+c))^4}{35\,d}-\frac{8\,a\cot(dx+c)(\csc(dx+c))^2}{35\,d}-\frac{}{7\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^8*(a+a*sec(d*x+c)),x)
```

```
[Out] -16/35*a*cot(d*x+c)/d-1/7/d*a*cot(d*x+c)*csc(d*x+c)^6-6/35/d*a*cot(d*x+c)*c
sc(d*x+c)^4-8/35/d*a*cot(d*x+c)*csc(d*x+c)^2-1/7/d*a/sin(d*x+c)^7-1/5/d*a/s
in(d*x+c)^5-1/3/d*a/sin(d*x+c)^3-1/d*a/sin(d*x+c)+1/d*a*ln(sec(d*x+c)+tan(d
```

```
*x+c))
```

Maxima [A] time = 1.02553, size = 157, normalized size = 1.2

$$-\frac{a\left(\frac{2\left(105\sin(dx+c)^6+35\sin(dx+c)^4+21\sin(dx+c)^2+15\right)}{\sin(dx+c)^7}-105\log\left(\sin\left(dx+c\right)+1\right)+105\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{6\left(35\tan(dx+c)\right.}{}}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/210*(a*(2*(105*sin(d*x + c)^6 + 35*sin(d*x + c)^4 + 21*sin(d*x + c)^2 +
15)/sin(d*x + c)^7 - 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1))
 + 6*(35*tan(d*x + c)^6 + 35*tan(d*x + c)^4 + 21*tan(d*x + c)^2 + 5)*a/tan(
d*x + c)^7)/d
```

Fricas [B] time = 1.81187, size = 745, normalized size = 5.69

$$-\frac{96\,a\cos\left(dx+c\right)^6+114\,a\cos\left(dx+c\right)^5-450\,a\cos\left(dx+c\right)^4-250\,a\cos\left(dx+c\right)^3+670\,a\cos\left(dx+c\right)^2-105\left(a\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/210*(96*a*cos(d*x + c)^6 + 114*a*cos(d*x + c)^5 - 450*a*cos(d*x + c)^4 -
 250*a*cos(d*x + c)^3 + 670*a*cos(d*x + c)^2 - 105*(a*cos(d*x + c)^5 - a*co
s(d*x + c)^4 - 2*a*cos(d*x + c)^3 + 2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a
)*log(sin(d*x + c) + 1)*sin(d*x + c) + 105*(a*cos(d*x + c)^5 - a*cos(d*x +
c)^4 - 2*a*cos(d*x + c)^3 + 2*a*cos(d*x + c)^2 + a*cos(d*x + c) - a)*log(-s
in(d*x + c) + 1)*sin(d*x + c) + 142*a*cos(d*x + c) - 352*a)/((d*cos(d*x + c
)^5 - d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^3 + 2*d*cos(d*x + c)^2 + d*cos(d*
x + c) - d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**8*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.06734, size = 184, normalized size = 1.4

$$-\frac{21\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+280\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6720\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+6720\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.\right.\right.}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6720*(21*a*tan(1/2*d*x + 1/2*c)^5 + 280*a*tan(1/2*d*x + 1/2*c)^3 - 6720*
a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 6720*a*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + 3045*a*tan(1/2*d*x + 1/2*c) + (6720*a*tan(1/2*d*x + 1/2*c)^6 + 1015
*a*tan(1/2*d*x + 1/2*c)^4 + 168*a*tan(1/2*d*x + 1/2*c)^2 + 15*a)/tan(1/2*d*
x + 1/2*c)^7)/d
```

3.18 $\int \csc^{10}(c+dx)(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=165

$$-\frac{a\cot^9(c+dx)}{9d}-\frac{4a\cot^7(c+dx)}{7d}-\frac{6a\cot^5(c+dx)}{5d}-\frac{4a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{a\csc^9(c+dx)}{9d}-\frac{a\csc^7(c+}{7d}$$

```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (4*a*Cot[c + d*x]^3)/(3*
d) - (6*a*Cot[c + d*x]^5)/(5*d) - (4*a*Cot[c + d*x]^7)/(7*d) - (a*Cot[c + d
*x]^9)/(9*d) - (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d
*x]^5)/(5*d) - (a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(9*d)
```

Rubi [A] time = 0.126602, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3872, 2838, 2621, 302, 207, 3767}

$$-\frac{a\cot^9(c+dx)}{9d}-\frac{4a\cot^7(c+dx)}{7d}-\frac{6a\cot^5(c+dx)}{5d}-\frac{4a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{a\csc^9(c+dx)}{9d}-\frac{a\csc^7(c+}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^10*(a + a*Sec[c + d*x]),x]
```


```
[Out] (a*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (4*a*Cot[c + d*x]^3)/(3*
d) - (6*a*Cot[c + d*x]^5)/(5*d) - (4*a*Cot[c + d*x]^7)/(7*d) - (a*Cot[c + d
*x]^9)/(9*d) - (a*Csc[c + d*x])/d - (a*Csc[c + d*x]^3)/(3*d) - (a*Csc[c + d
*x]^5)/(5*d) - (a*Csc[c + d*x]^7)/(7*d) - (a*Csc[c + d*x]^9)/(9*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))\,dx = -\int (-a-a\cos(c+dx))\csc^{10}(c+dx)\sec(c+dx)\,dx$$
$$= a\int \csc^{10}(c+dx)\,dx + a\int \csc^{10}(c+dx)\sec(c+dx)\,dx$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{x^{10}}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right)\right.}{d}$$
$$= -\frac{a\cot(c+dx)}{d} - \frac{4a\cot^3(c+dx)}{3d} - \frac{6a\cot^5(c+dx)}{5d} - \frac{4a\cot^7(c+dx)}{7d} - \frac{a\cot^9(}{9}$$
$$= -\frac{a\cot(c+dx)}{d} - \frac{4a\cot^3(c+dx)}{3d} - \frac{6a\cot^5(c+dx)}{5d} - \frac{4a\cot^7(c+dx)}{7d} - \frac{a\cot^9(}{9}$$
$$= \frac{a\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{4a\cot^3(c+dx)}{3d} - \frac{6a\cot^5(c+dx)}{5d} - \frac{4a}{}$$

Mathematica [C] time = 0.0547749, size = 135, normalized size = 0.82

$$-\frac{a\csc^9(c+dx)\text{Hypergeometric2F1}\left(-\frac{9}{2},1,-\frac{7}{2},\sin^2(c+dx)\right)}{9d} - \frac{128a\cot(c+dx)}{315d} - \frac{a\cot(c+dx)\csc^8(c+dx)}{9d} - \frac{8a\cot}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^10*(a + a*Sec[c + d*x]),x]
```

```
[Out] (-128*a*Cot[c + d*x])/(315*d) - (64*a*Cot[c + d*x]*Csc[c + d*x]^2)/(315*d)
- (16*a*Cot[c + d*x]*Csc[c + d*x]^4)/(105*d) - (8*a*Cot[c + d*x]*Csc[c + d*
x]^6)/(63*d) - (a*Cot[c + d*x]*Csc[c + d*x]^8)/(9*d) - (a*Csc[c + d*x]^9*Hy
pergeometric2F1[-9/2, 1, -7/2, Sin[c + d*x]^2])/(9*d)
```

Maple [A] time = 0.126, size = 183, normalized size = 1.1

$$-\frac{128\,a\cot(dx+c)}{315\,d} - \frac{a\cot(dx+c)(\csc(dx+c))^8}{9\,d} - \frac{8\,a\cot(dx+c)(\csc(dx+c))^6}{63\,d} - \frac{16\,a\cot(dx+c)(\csc(dx+c))^4}{105\,d} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^10*(a+a*sec(d*x+c)),x)
```

```
[Out] -128/315*a*cot(d*x+c)/d-1/9/d*a*cot(d*x+c)*csc(d*x+c)^8-8/63/d*a*cot(d*x+c)
*csc(d*x+c)^6-16/105/d*a*cot(d*x+c)*csc(d*x+c)^4-64/315/d*a*cot(d*x+c)*csc(
d*x+c)^2-1/9/d*a/sin(d*x+c)^9-1/7/d*a/sin(d*x+c)^7-1/5/d*a/sin(d*x+c)^5-1/3
```

```
/d*a/sin(d*x+c)^3-1/d*a/sin(d*x+c)+1/d*a*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.99798, size = 184, normalized size = 1.12

$$-\frac{a\left(\frac{2\left(315\sin(dx+c)^{8}+105\sin(dx+c)^{6}+63\sin(dx+c)^{4}+45\sin(dx+c)^{2}+35\right)}{\sin(dx+c)^{9}}-315\log\left(\sin\left(dx+c\right)+1\right)+315\log\left(\sin\left(dx+c\right)-1\right)\right)+}{630\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/630*(a*(2*(315*sin(d*x + c)^8 + 105*sin(d*x + c)^6 + 63*sin(d*x + c)^4 +
 45*sin(d*x + c)^2 + 35)/sin(d*x + c)^9 - 315*log(sin(d*x + c) + 1) + 315*l
og(sin(d*x + c) - 1)) + 2*(315*tan(d*x + c)^8 + 420*tan(d*x + c)^6 + 378*ta
n(d*x + c)^4 + 180*tan(d*x + c)^2 + 35)*a/tan(d*x + c)^9)/d
```

Fricas [B] time = 1.86453, size = 987, normalized size = 5.98

$$-\frac{256\,a\cos\left(dx+c\right)^{8}+374\,a\cos\left(dx+c\right)^{7}-1526\,a\cos\left(dx+c\right)^{6}-1204\,a\cos\left(dx+c\right)^{5}+3220\,a\cos\left(dx+c\right)^{4}+1316\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/630*(256*a*cos(d*x + c)^8 + 374*a*cos(d*x + c)^7 - 1526*a*cos(d*x + c)^6
 - 1204*a*cos(d*x + c)^5 + 3220*a*cos(d*x + c)^4 + 1316*a*cos(d*x + c)^3 -
2996*a*cos(d*x + c)^2 - 315*(a*cos(d*x + c)^7 - a*cos(d*x + c)^6 - 3*a*cos(
d*x + c)^5 + 3*a*cos(d*x + c)^4 + 3*a*cos(d*x + c)^3 - 3*a*cos(d*x + c)^2 -
 a*cos(d*x + c) + a)*log(sin(d*x + c) + 1)*sin(d*x + c) + 315*(a*cos(d*x +
c)^7 - a*cos(d*x + c)^6 - 3*a*cos(d*x + c)^5 + 3*a*cos(d*x + c)^4 + 3*a*cos
(d*x + c)^3 - 3*a*cos(d*x + c)^2 - a*cos(d*x + c) + a)*log(-sin(d*x + c) +
1)*sin(d*x + c) - 496*a*cos(d*x + c) + 1126*a)/((d*cos(d*x + c)^7 - d*cos(d
*x + c)^6 - 3*d*cos(d*x + c)^5 + 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^3 -
3*d*cos(d*x + c)^2 - d*cos(d*x + c) + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**10*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

<u>Giac</u> [A] time = 1.92052, size = 221, normalized size = 1.34

$$-\frac{45\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+630\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+4830\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-80640\,a\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+806}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/80640*(45*a*tan(1/2*d*x + 1/2*c)^7 + 630*a*tan(1/2*d*x + 1/2*c)^5 + 4830
*a*tan(1/2*d*x + 1/2*c)^3 - 80640*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 80
640*a*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 40950*a*tan(1/2*d*x + 1/2*c) + (
80640*a*tan(1/2*d*x + 1/2*c)^8 + 13650*a*tan(1/2*d*x + 1/2*c)^6 + 2898*a*ta
n(1/2*d*x + 1/2*c)^4 + 450*a*tan(1/2*d*x + 1/2*c)^2 + 35*a)/tan(1/2*d*x + 1
/2*c)^9)/d
```

3.19 $\int (a + a\sec(c + dx))^2 \sin^9(c + dx)\,dx$

Optimal. Leaf size=183

$$-\frac{a^2\cos^9(c+dx)}{9d}-\frac{a^2\cos^8(c+dx)}{4d}+\frac{3a^2\cos^7(c+dx)}{7d}+\frac{4a^2\cos^6(c+dx)}{3d}-\frac{2a^2\cos^5(c+dx)}{5d}-\frac{3a^2\cos^4(c+dx)}{d}-2$$

```
[Out] (3*a^2*Cos[c + d*x])/d + (4*a^2*Cos[c + d*x]^2)/d - (2*a^2*Cos[c + d*x]^3)/
(3*d) - (3*a^2*Cos[c + d*x]^4)/d - (2*a^2*Cos[c + d*x]^5)/(5*d) + (4*a^2*Co
s[c + d*x]^6)/(3*d) + (3*a^2*Cos[c + d*x]^7)/(7*d) - (a^2*Cos[c + d*x]^8)/(
4*d) - (a^2*Cos[c + d*x]^9)/(9*d) - (2*a^2*Log[Cos[c + d*x]])/d + (a^2*Sec[
c + d*x])/d
```

Rubi [A] time = 0.187794, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2836, 12, 88}

$$-\frac{a^2\cos^9(c+dx)}{9d}-\frac{a^2\cos^8(c+dx)}{4d}+\frac{3a^2\cos^7(c+dx)}{7d}+\frac{4a^2\cos^6(c+dx)}{3d}-\frac{2a^2\cos^5(c+dx)}{5d}-\frac{3a^2\cos^4(c+dx)}{d}-2$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^9,x]
```


```
[Out] (3*a^2*Cos[c + d*x])/d + (4*a^2*Cos[c + d*x]^2)/d - (2*a^2*Cos[c + d*x]^3)/
(3*d) - (3*a^2*Cos[c + d*x]^4)/d - (2*a^2*Cos[c + d*x]^5)/(5*d) + (4*a^2*Co
s[c + d*x]^6)/(3*d) + (3*a^2*Cos[c + d*x]^7)/(7*d) - (a^2*Cos[c + d*x]^8)/(
4*d) - (a^2*Cos[c + d*x]^9)/(9*d) - (2*a^2*Log[Cos[c + d*x]])/d + (a^2*Sec[
c + d*x])/d
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\sin^9(c+dx)\,dx = \int (-a-a\cos(c+dx))^2\sin^7(c+dx)\tan^2(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^2(-a-x)^4(-a+x)^6}{x^2}\,dx, x, -a\cos(c+dx)\right)}{a^9 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^4(-a+x)^6}{x^2}\,dx, x, -a\cos(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \left(-3a^8+\frac{a^{10}}{x^2}-\frac{2a^9}{x}+8a^7x+2a^6x^2-12a^5x^3+2a^4x^4+8a^3x^5-3a^2x^6-\right.\right.}{a^7 d}$$
$$= \frac{3a^2\cos(c+dx)}{d}+\frac{4a^2\cos^2(c+dx)}{d}-\frac{2a^2\cos^3(c+dx)}{3d}-\frac{3a^2\cos^4(c+dx)}{d}-\frac{2a^2}{}$$

Mathematica [A] time = 0.82583, size = 127, normalized size = 0.69

$$-\frac{a^2\sec(c+dx)(-361620\cos(2(c+dx))-134820\cos(3(c+dx))+29232\cos(4(c+dx))+24780\cos(5(c+dx))-1458\cos}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^9,x]
```

```
[Out] -(a^2*(-714420 - 361620*Cos[2*(c + d*x)] - 134820*Cos[3*(c + d*x)] + 29232*
Cos[4*(c + d*x)] + 24780*Cos[5*(c + d*x)] - 1458*Cos[6*(c + d*x)] - 3885*Co
s[7*(c + d*x)] - 380*Cos[8*(c + d*x)] + 315*Cos[9*(c + d*x)] + 70*Cos[10*(c
 + d*x)] + 210*Cos[c + d*x]*(205 + 3072*Log[Cos[c + d*x]]))*Sec[c + d*x])/(
322560*d)
```

Maple [A] time = 0.046, size = 206, normalized size = 1.1

$$\frac{1024\,a^2\cos(dx+c)}{315\,d}+\frac{8\,a^2(\sin(dx+c))^8\cos(dx+c)}{9\,d}+\frac{64\,a^2\cos(dx+c)(\sin(dx+c))^6}{63\,d}+\frac{128\,a^2\cos(dx+c)(\sin(dx+}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^9,x)
```

```
[Out] 1024/315*a^2*cos(d*x+c)/d+8/9/d*a^2*sin(d*x+c)^8*cos(d*x+c)+64/63/d*a^2*cos
(d*x+c)*sin(d*x+c)^6+128/105/d*a^2*cos(d*x+c)*sin(d*x+c)^4+512/315/d*a^2*co
s(d*x+c)*sin(d*x+c)^2-1/4/d*a^2*sin(d*x+c)^8-1/3/d*a^2*sin(d*x+c)^6-1/2/d*a
^2*sin(d*x+c)^4-1/d*a^2*sin(d*x+c)^2-2*a^2*ln(cos(d*x+c))/d+1/d*a^2*sin(d*x
+c)^10/cos(d*x+c)
```

Maxima [A] time = 1.00253, size = 197, normalized size = 1.08

$$-\frac{140\,a^2\cos(dx+c)^9+315\,a^2\cos(dx+c)^8-540\,a^2\cos(dx+c)^7-1680\,a^2\cos(dx+c)^6+504\,a^2\cos(dx+c)^5+3780\,a}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^9,x, algorithm="maxima")
```

```
[Out] -1/1260*(140*a^2*cos(d*x + c)^9 + 315*a^2*cos(d*x + c)^8 - 540*a^2*cos(d*x
+ c)^7 - 1680*a^2*cos(d*x + c)^6 + 504*a^2*cos(d*x + c)^5 + 3780*a^2*cos(d*
x + c)^4 + 840*a^2*cos(d*x + c)^3 - 5040*a^2*cos(d*x + c)^2 - 3780*a^2*cos(
d*x + c) + 2520*a^2*log(cos(d*x + c)) - 1260*a^2/cos(d*x + c))/d
```

Fricas [A] time = 1.94418, size = 487, normalized size = 2.66

$$-\frac{17920\,a^2\cos(dx+c)^{10} + 40320\,a^2\cos(dx+c)^9 - 69120\,a^2\cos(dx+c)^8 - 215040\,a^2\cos(dx+c)^7 + 64512\,a^2\cos(d}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] -1/161280*(17920*a^2*cos(d*x + c)^10 + 40320*a^2*cos(d*x + c)^9 - 69120*a^2
*cos(d*x + c)^8 - 215040*a^2*cos(d*x + c)^7 + 64512*a^2*cos(d*x + c)^6 + 48
3840*a^2*cos(d*x + c)^5 + 107520*a^2*cos(d*x + c)^4 - 645120*a^2*cos(d*x +
c)^3 - 483840*a^2*cos(d*x + c)^2 + 322560*a^2*cos(d*x + c)*log(-cos(d*x + c
)) + 197295*a^2*cos(d*x + c) - 161280*a^2)/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**9,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.62951, size = 500, normalized size = 2.73

$$\frac{2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 2520\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{2520\left(2\,a^2+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1} + \frac{1457\,a^2-\frac{20673\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+123}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^9,x, algorithm="giac")
```

```
[Out] 1/1260*(2520*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 252
0*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 2520*(2*a^2 +
a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 1) + (1457*a^2 - 20673*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 123012*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 421428*a^2*(cos(d*x
 + c) - 1)^3/(cos(d*x + c) + 1)^3 + 949662*a^2*(cos(d*x + c) - 1)^4/(cos(d*
x + c) + 1)^4 - 1009134*a^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 666
036*a^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 276804*a^2*(cos(d*x + c
) - 1)^7/(cos(d*x + c) + 1)^7 + 66681*a^2*(cos(d*x + c) - 1)^8/(cos(d*x + c
) + 1)^8 - 7129*a^2*(cos(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9)/((cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 1)^9)/d
```

3.20 $\int (a+a\sec(c+dx))^2\sin^7(c+dx)\,dx$

Optimal. Leaf size=131

$$\frac{a^2\cos^7(c+dx)}{7d}+\frac{a^2\cos^6(c+dx)}{3d}-\frac{2a^2\cos^5(c+dx)}{5d}-\frac{3a^2\cos^4(c+dx)}{2d}+\frac{3a^2\cos^2(c+dx)}{d}+\frac{2a^2\cos(c+dx)}{d}+\frac{a^2\sec(}{}$$

```
[Out] (2*a^2*Cos[c + d*x])/d + (3*a^2*Cos[c + d*x]^2)/d - (3*a^2*Cos[c + d*x]^4)/
(2*d) - (2*a^2*Cos[c + d*x]^5)/(5*d) + (a^2*Cos[c + d*x]^6)/(3*d) + (a^2*Co
s[c + d*x]^7)/(7*d) - (2*a^2*Log[Cos[c + d*x]])/d + (a^2*Sec[c + d*x])/d
```

Rubi [A] time = 0.168269, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2836, 12, 88}

$$\frac{a^2\cos^7(c+dx)}{7d}+\frac{a^2\cos^6(c+dx)}{3d}-\frac{2a^2\cos^5(c+dx)}{5d}-\frac{3a^2\cos^4(c+dx)}{2d}+\frac{3a^2\cos^2(c+dx)}{d}+\frac{2a^2\cos(c+dx)}{d}+\frac{a^2\sec(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^7,x]
```

```
[Out] (2*a^2*Cos[c + d*x])/d + (3*a^2*Cos[c + d*x]^2)/d - (3*a^2*Cos[c + d*x]^4)/
(2*d) - (2*a^2*Cos[c + d*x]^5)/(5*d) + (a^2*Cos[c + d*x]^6)/(3*d) + (a^2*Co
s[c + d*x]^7)/(7*d) - (2*a^2*Log[Cos[c + d*x]])/d + (a^2*Sec[c + d*x])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\sin^7(c+dx)\,dx = \int (-a-a\cos(c+dx))^2\sin^5(c+dx)\tan^2(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^2(-a-x)^3(-a+x)^5}{x^2}\,dx, x, -a\cos(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^3(-a+x)^5}{x^2}\,dx, x, -a\cos(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \left(-2a^6+\frac{a^8}{x^2}-\frac{2a^7}{x}+6a^5x-6a^3x^3+2a^2x^4+2ax^5-x^6\right)dx, x, -a\text{ co}\right.}{a^5 d}$$
$$= \frac{2a^2\cos(c+dx)}{d}+\frac{3a^2\cos^2(c+dx)}{d}-\frac{3a^2\cos^4(c+dx)}{2d}-\frac{2a^2\cos^5(c+dx)}{5d}+$$

Mathematica [A] time = 0.534327, size = 107, normalized size = 0.82

$$\frac{a^2\sec(c+dx)(11760\cos(2(c+dx))+5250\cos(3(c+dx))-588\cos(4(c+dx))-770\cos(5(c+dx))-48\cos(6(c+d}{13440d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^7,x]
```

```
[Out] (a^2*(25725 + 11760*Cos[2*(c + d*x)] + 5250*Cos[3*(c + d*x)] - 588*Cos[4*(c
 + d*x)] - 770*Cos[5*(c + d*x)] - 48*Cos[6*(c + d*x)] + 70*Cos[7*(c + d*x)]
 + 15*Cos[8*(c + d*x)] - 70*Cos[c + d*x]*(5 + 384*Log[Cos[c + d*x]]))*Sec[c
 + d*x])/(13440*d)
```

Maple [A] time = 0.044, size = 168, normalized size = 1.3

$$\frac{96\,a^2\cos(dx+c)}{35\,d}+\frac{6\,a^2\cos(dx+c)(\sin(dx+c))^6}{7\,d}+\frac{36\,a^2\cos(dx+c)(\sin(dx+c))^4}{35\,d}+\frac{48\,a^2\cos(dx+c)(\sin(dx+}{35\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^7,x)
```

```
[Out] 96/35*a^2*cos(d*x+c)/d+6/7/d*a^2*cos(d*x+c)*sin(d*x+c)^6+36/35/d*a^2*cos(d*
x+c)*sin(d*x+c)^4+48/35/d*a^2*cos(d*x+c)*sin(d*x+c)^2-1/3/d*a^2*sin(d*x+c)^
6-1/2/d*a^2*sin(d*x+c)^4-1/d*a^2*sin(d*x+c)^2-2*a^2*ln(cos(d*x+c))/d+1/d*a^
2*sin(d*x+c)^8/cos(d*x+c)
```

Maxima [A] time = 1.02748, size = 144, normalized size = 1.1

$$\frac{30\,a^2\cos(dx+c)^7+70\,a^2\cos(dx+c)^6-84\,a^2\cos(dx+c)^5-315\,a^2\cos(dx+c)^4+630\,a^2\cos(dx+c)^2+420\,a^2\text{co}}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/210*(30*a^2*cos(d*x + c)^7 + 70*a^2*cos(d*x + c)^6 - 84*a^2*cos(d*x + c)^
5 - 315*a^2*cos(d*x + c)^4 + 630*a^2*cos(d*x + c)^2 + 420*a^2*cos(d*x + c)
```

```
- 420*a^2*log(cos(d*x + c)) + 210*a^2/cos(d*x + c))/d
```

Fricas [A] time = 1.89551, size = 342, normalized size = 2.61

$$\frac{120\,a^2\cos(dx+c)^8+280\,a^2\cos(dx+c)^7-336\,a^2\cos(dx+c)^6-1260\,a^2\cos(dx+c)^5+2520\,a^2\cos(dx+c)^3+1680\,a}{840\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/840*(120*a^2*cos(d*x + c)^8 + 280*a^2*cos(d*x + c)^7 - 336*a^2*cos(d*x +
c)^6 - 1260*a^2*cos(d*x + c)^5 + 2520*a^2*cos(d*x + c)^3 + 1680*a^2*cos(d*x
 + c)^2 - 1680*a^2*cos(d*x + c)*log(-cos(d*x + c)) - 875*a^2*cos(d*x + c) +
 840*a^2)/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**7,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.42313, size = 432, normalized size = 3.3

$$\frac{420\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-420\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{420\left(2\,a^2+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}+\frac{357\,a^2-\frac{3759\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{16737\,a^2(\cos(dx}{(\cos(dx+c)+}}{}}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/210*(420*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*a
^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 420*(2*a^2 + a^2*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 1) + (357*a^2 - 3759*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 16737
*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 42595*a^2*(cos(d*x + c) -
1)^3/(cos(d*x + c) + 1)^3 + 43855*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) +
1)^4 - 25389*a^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 8043*a^2*(cos(
d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 1089*a^2*(cos(d*x + c) - 1)^7/(cos(d
*x + c) + 1)^7)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^7)/d
```

3.21 $\int (a + a\sec(c + dx))^2 \sin^5(c + dx)\,dx$

Optimal. Leaf size=112

$$-\frac{a^2\cos^5(c+dx)}{5d}-\frac{a^2\cos^4(c+dx)}{2d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{2a^2\cos^2(c+dx)}{d}+\frac{a^2\cos(c+dx)}{d}+\frac{a^2\sec(c+dx)}{d}-\frac{2a^2\log}{}$$

```
[Out] (a^2*Cos[c + d*x])/d + (2*a^2*Cos[c + d*x]^2)/d + (a^2*Cos[c + d*x]^3)/(3*d
) - (a^2*Cos[c + d*x]^4)/(2*d) - (a^2*Cos[c + d*x]^5)/(5*d) - (2*a^2*Log[Co
s[c + d*x]])/d + (a^2*Sec[c + d*x])/d
```

Rubi [A] time = 0.157634, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2836, 12, 88}

$$-\frac{a^2\cos^5(c+dx)}{5d}-\frac{a^2\cos^4(c+dx)}{2d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{2a^2\cos^2(c+dx)}{d}+\frac{a^2\cos(c+dx)}{d}+\frac{a^2\sec(c+dx)}{d}-\frac{2a^2\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^5,x]
```

```
[Out] (a^2*Cos[c + d*x])/d + (2*a^2*Cos[c + d*x]^2)/d + (a^2*Cos[c + d*x]^3)/(3*d
) - (a^2*Cos[c + d*x]^4)/(2*d) - (a^2*Cos[c + d*x]^5)/(5*d) - (2*a^2*Log[Co
s[c + d*x]])/d + (a^2*Sec[c + d*x])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\sin^5(c+dx)\,dx = \int (-a-a\cos(c+dx))^2\sin^3(c+dx)\tan^2(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^2(-a-x)^2(-a+x)^4}{x^2}dx, x, -a\cos(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^2(-a+x)^4}{x^2}dx, x, -a\cos(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^4+\frac{a^6}{x^2}-\frac{2a^5}{x}+4a^3x-a^2x^2-2ax^3+x^4\right)dx, x, -a\cos(c+dx)\right)}{a^3d}$$
$$= \frac{a^2\cos(c+dx)}{d}+\frac{2a^2\cos^2(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}-\frac{a^2\cos^4(c+dx)}{2d}-\frac{a^2\cos^5}{5}$$

Mathematica [A] time = 0.288114, size = 87, normalized size = 0.78

$$-\frac{a^2\sec(c+dx)(-275\cos(2(c+dx))-165\cos(3(c+dx))-2\cos(4(c+dx))+15\cos(5(c+dx))+3\cos(6(c+dx))+30}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^5,x]
```

```
[Out] -(a^2*(-750 - 275*Cos[2*(c + d*x)] - 165*Cos[3*(c + d*x)] - 2*Cos[4*(c + d*
x)] + 15*Cos[5*(c + d*x)] + 3*Cos[6*(c + d*x)] + 30*Cos[c + d*x]*(-3 + 32*L
og[Cos[c + d*x]]))*Sec[c + d*x])/(480*d)
```

Maple [A] time = 0.043, size = 130, normalized size = 1.2

$$\frac{32a^2\cos(dx+c)}{15d}+\frac{4a^2\cos(dx+c)(\sin(dx+c))^4}{5d}+\frac{16a^2\cos(dx+c)(\sin(dx+c))^2}{15d}-\frac{a^2(\sin(dx+c))^4}{2d}-\frac{a^2(\sin(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^5,x)
```

```
[Out] 32/15*a^2*cos(d*x+c)/d+4/5/d*a^2*cos(d*x+c)*sin(d*x+c)^4+16/15/d*a^2*cos(d*
x+c)*sin(d*x+c)^2-1/2/d*a^2*sin(d*x+c)^4-1/d*a^2*sin(d*x+c)^2-2*a^2*ln(cos(
d*x+c))/d+1/d*a^2*sin(d*x+c)^6/cos(d*x+c)
```

Maxima [A] time = 1.00812, size = 127, normalized size = 1.13

$$-\frac{6a^2\cos(dx+c)^5+15a^2\cos(dx+c)^4-10a^2\cos(dx+c)^3-60a^2\cos(dx+c)^2-30a^2\cos(dx+c)+60a^2\log(\cos(d}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/30*(6*a^2*cos(d*x + c)^5 + 15*a^2*cos(d*x + c)^4 - 10*a^2*cos(d*x + c)^3
 - 60*a^2*cos(d*x + c)^2 - 30*a^2*cos(d*x + c) + 60*a^2*log(cos(d*x + c)) -
```

```
30*a^2/cos(d*x + c))/d
```

Fricas [A] time = 1.7875, size = 301, normalized size = 2.69

$$-\frac{48\,a^{2}\cos\left(dx+c\right)^{6}+120\,a^{2}\cos\left(dx+c\right)^{5}-80\,a^{2}\cos\left(dx+c\right)^{4}-480\,a^{2}\cos\left(dx+c\right)^{3}-240\,a^{2}\cos\left(dx+c\right)^{2}+480\,a^{2}}{240\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/240*(48*a^2*cos(d*x + c)^6 + 120*a^2*cos(d*x + c)^5 - 80*a^2*cos(d*x + c
)^4 - 480*a^2*cos(d*x + c)^3 - 240*a^2*cos(d*x + c)^2 + 480*a^2*cos(d*x + c
)*log(-cos(d*x + c)) + 195*a^2*cos(d*x + c) - 240*a^2)/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.48864, size = 365, normalized size = 3.26

$$\frac{60\,a^{2}\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-60\,a^{2}\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{60\left(2\,a^{2}+\frac{a^{2}(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}+\frac{69\,a^{2}-\frac{525\,a^{2}(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{1650\,a^{2}(\cos(dx+c)-}{(\cos(dx+c)+1)^{2}}}{}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/30*(60*a^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a^2*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 60*(2*a^2 + a^2*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 1) + (69*a^2 - 525*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1650*a^2*(c
os(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1610*a^2*(cos(d*x + c) - 1)^3/(co
s(d*x + c) + 1)^3 + 745*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 137
*a^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - 1)^5)/d
```

3.22 $\int (a+a\sec(c+dx))^2\sin^3(c+dx)\,dx$

Optimal. Leaf size=62

$$\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos^2(c+dx)}{d}+\frac{a^2\sec(c+dx)}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

```
[Out] (a^2*Cos[c + d*x]^2)/d + (a^2*Cos[c + d*x]^3)/(3*d) - (2*a^2*Log[Cos[c + d*
x]])/d + (a^2*Sec[c + d*x])/d
```

Rubi [A] time = 0.123591, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2836, 12, 75}

$$\frac{a^2\cos^3(c+dx)}{3d}+\frac{a^2\cos^2(c+dx)}{d}+\frac{a^2\sec(c+dx)}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^3,x]
```

```
[Out] (a^2*Cos[c + d*x]^2)/d + (a^2*Cos[c + d*x]^3)/(3*d) - (2*a^2*Log[Cos[c + d*
x]])/d + (a^2*Sec[c + d*x])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\sin^3(c+dx)\,dx = \int (-a-a\cos(c+dx))^2\sin(c+dx)\tan^2(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^2(-a-x)(-a+x)^3}{x^2}\,dx, x, -a\cos(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)(-a+x)^3}{x^2}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^4}{x^2}-\frac{2a^3}{x}+2ax-x^2\right)dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{a^2\cos^2(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}-\frac{2a^2\log(\cos(c+dx))}{d}+\frac{a^2\sec(c+dx)}{d}$$

Mathematica [A] time = 0.197132, size = 65, normalized size = 1.05

$$\frac{a^2\sec(c+dx)(4\cos(2(c+dx))+6\cos(3(c+dx))+\cos(4(c+dx))-6\cos(c+dx)(8\log(\cos(c+dx))+1)+27)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^3,x]
```

```
[Out] (a^2*(27 + 4*Cos[2*(c + d*x)] + 6*Cos[3*(c + d*x)] + Cos[4*(c + d*x)] - 6*C
os[c + d*x]*(1 + 8*Log[Cos[c + d*x]]))*Sec[c + d*x])/(24*d)
```

Maple [A] time = 0.04, size = 92, normalized size = 1.5

$$\frac{2a^2\cos(dx+c)(\sin(dx+c))^2}{3d}+\frac{4a^2\cos(dx+c)}{3d}-\frac{a^2(\sin(dx+c))^2}{d}-2\frac{a^2\ln(\cos(dx+c))}{d}+\frac{a^2(\sin(dx+c))^4}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^3,x)
```

```
[Out] 2/3/d*a^2*cos(d*x+c)*sin(d*x+c)^2+4/3*a^2*cos(d*x+c)/d-1/d*a^2*sin(d*x+c)^2
-2*a^2*ln(cos(d*x+c))/d+1/d*a^2*sin(d*x+c)^4/cos(d*x+c)
```

Maxima [A] time = 0.993747, size = 76, normalized size = 1.23

$$\frac{a^2\cos(dx+c)^3+3a^2\cos(dx+c)^2-6a^2\log(\cos(dx+c))+\frac{3a^2}{\cos(dx+c)}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(a^2*cos(d*x + c)^3 + 3*a^2*cos(d*x + c)^2 - 6*a^2*log(cos(d*x + c)) +
3*a^2/cos(d*x + c))/d
```

Fricas [A] time = 1.77674, size = 186, normalized size = 3.

$$\frac{2 a^2 \cos(dx+c)^4 + 6 a^2 \cos(dx+c)^3 - 12 a^2 \cos(dx+c) \log(-\cos(dx+c)) - 3 a^2 \cos(dx+c) + 6 a^2}{6 d \cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^2*cos(d*x + c)^4 + 6*a^2*cos(d*x + c)^3 - 12*a^2*cos(d*x + c)*log(
-cos(d*x + c)) - 3*a^2*cos(d*x + c) + 6*a^2)/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4018, size = 100, normalized size = 1.61

$$-\frac{2 a^2 \log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d} + \frac{a^2}{d \cos(dx+c)} + \frac{a^2 d^5 \cos(dx+c)^3 + 3 a^2 d^5 \cos(dx+c)^2}{3 d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -2*a^2*log(abs(cos(d*x + c))/abs(d))/d + a^2/(d*cos(d*x + c)) + 1/3*(a^2*d^
5*cos(d*x + c)^3 + 3*a^2*d^5*cos(d*x + c)^2)/d^6
```

3.23 $\int (a+a\sec(c+dx))^2\sin(c+dx)\,dx$

Optimal. Leaf size=43

$$-\frac{a^2\cos(c+dx)}{d}+\frac{a^2\sec(c+dx)}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

```
[Out] -((a^2*Cos[c + d*x])/d) - (2*a^2*Log[Cos[c + d*x]])/d + (a^2*Sec[c + d*x])/
d
```

Rubi [A] time = 0.0767932, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3872, 2833, 12, 43}

$$-\frac{a^2\cos(c+dx)}{d}+\frac{a^2\sec(c+dx)}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x],x]
```

```
[Out] -((a^2*Cos[c + d*x])/d) - (2*a^2*Log[Cos[c + d*x]])/d + (a^2*Sec[c + d*x])/
d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\sin(c+dx)\,dx = \int (-a-a\cos(c+dx))^2\sec(c+dx)\tan(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^2(-a+x)^2}{x^2}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{(-a+x)^2}{x^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(1+\frac{a^2}{x^2}-\frac{2a}{x}\right)dx, x, -a\cos(c+dx)\right)}{d}$$
$$= -\frac{a^2\cos(c+dx)}{d} - \frac{2a^2\log(\cos(c+dx))}{d} + \frac{a^2\sec(c+dx)}{d}$$

Mathematica [A] time = 0.112216, size = 31, normalized size = 0.72

$$\frac{a^2(\sin(c+dx)\tan(c+dx)-2\log(\cos(c+dx))+1)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x],x]
```

```
[Out] (a^2*(1 - 2*Log[Cos[c + d*x]] + Sin[c + d*x]*Tan[c + d*x]))/d
```

Maple [A] time = 0.02, size = 46, normalized size = 1.1

$$\frac{a^2\sec(dx+c)}{d} + 2\,\frac{a^2\ln(\sec(dx+c))}{d} - \frac{a^2}{d\sec(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c),x)
```

```
[Out] a^2*sec(d*x+c)/d+2/d*a^2*ln(sec(d*x+c))-1/d*a^2/sec(d*x+c)
```

Maxima [A] time = 1.00563, size = 55, normalized size = 1.28

$$-\frac{a^2\cos(dx+c) + 2\,a^2\log(\cos(dx+c)) - \frac{a^2}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a^2*cos(d*x + c) + 2*a^2*log(cos(d*x + c)) - a^2/cos(d*x + c))/d
```

Fricas [A] time = 1.75952, size = 116, normalized size = 2.7

$$-\frac{a^2\cos(dx+c)^2 + 2\,a^2\cos(dx+c)\log(-\cos(dx+c)) - a^2}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c)*log(-cos(d*x + c)) - a^2)/(d*cos(
d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int 2\sin(c+dx)\sec(c+dx)\,dx+\int\sin(c+dx)\sec^2(c+dx)\,dx+\int\sin(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c),x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)*sec(c + d*x), x) + Integral(sin(c + d*x)*sec(
c + d*x)**2, x) + Integral(sin(c + d*x), x))
```

Giac [A] time = 1.38293, size = 69, normalized size = 1.6

$$-\frac{a^2\cos(dx+c)}{d}-\frac{2\,a^2\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{a^2}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -a^2*cos(d*x + c)/d - 2*a^2*log(abs(cos(d*x + c))/abs(d))/d + a^2/(d*cos(d*
x + c))
```

3.24 $\int \csc(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=48

$$\frac{a^2\sec(c+dx)}{d}+\frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

```
[Out] (2*a^2*Log[1 - Cos[c + d*x]])/d - (2*a^2*Log[Cos[c + d*x]])/d + (a^2*Sec[c
+ d*x])/d
```

Rubi [A] time = 0.115148, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3872, 2836, 12, 77}

$$\frac{a^2\sec(c+dx)}{d}+\frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*Log[1 - Cos[c + d*x]])/d - (2*a^2*Log[Cos[c + d*x]])/d + (a^2*Sec[c
+ d*x])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \csc(c+dx)(a+a\sec(c+dx))^2\,dx = \int (-a-a\cos(c+dx))^2\csc(c+dx)\sec^2(c+dx)\,dx$$
$$= \frac{a\,\text{Subst}\left(\int \frac{a^2(-a+x)}{(-a-x)x^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{-a+x}{(-a-x)x^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{x^2}-\frac{2}{ax}+\frac{2}{a(a+x)}\right)dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{2a^2\log(1-\cos(c+dx))}{d} - \frac{2a^2\log(\cos(c+dx))}{d} + \frac{a^2\sec(c+dx)}{d}$$

Mathematica [A] time = 0.0755605, size = 36, normalized size = 0.75

$$\frac{a^2\left(\sec(c+dx)+4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-2\log(\cos(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sec[c + d*x])^2,x]

[Out] (a^2*(-2*Log[Cos[c + d*x]] + 4*Log[Sin[(c + d*x)/2]] + Sec[c + d*x]))/d
```

Maple [A] time = 0.035, size = 32, normalized size = 0.7

$$\frac{a^2\sec(dx+c)}{d}+2\,\frac{a^2\ln(-1+\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sec(d*x+c))^2,x)

[Out] a^2*sec(d*x+c)/d+2/d*a^2*ln(-1+sec(d*x+c))
```

Maxima [A] time = 0.975907, size = 58, normalized size = 1.21

$$\frac{2\,a^2\log(\cos(dx+c)-1)-2\,a^2\log(\cos(dx+c))+\frac{a^2}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] (2*a^2*log(cos(d*x + c) - 1) - 2*a^2*log(cos(d*x + c)) + a^2/cos(d*x + c))/
d
```

Fricas [A] time = 1.7431, size = 155, normalized size = 3.23

$$-\frac{2\,a^2\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)-2\,a^2\cos\left(dx+c\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-a^2}{d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(2*a^2*cos(d*x + c)*log(-cos(d*x + c)) - 2*a^2*cos(d*x + c)*log(-1/2*cos(d
*x + c) + 1/2) - a^2)/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int 2\csc\left(c+dx\right)\sec\left(c+dx\right)dx+\int\csc\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int\csc\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*csc(c + d*x)*sec(c + d*x), x) + Integral(csc(c + d*x)*sec(
c + d*x)**2, x) + Integral(csc(c + d*x), x))
```

Giac [B] time = 1.39435, size = 155, normalized size = 3.23

$$\frac{2\left(a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{2\,a^2+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - a^2*log(abs(-(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (2*a^2 + a^2*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

3.25 $\int \csc^3(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=69

$$-\frac{a^3}{d(a-a\cos(c+dx))}+\frac{a^2\sec(c+dx)}{d}+\frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

```
[Out] -(a^3/(d*(a - a*Cos[c + d*x]))) + (2*a^2*Log[1 - Cos[c + d*x]])/d - (2*a^2*
Log[Cos[c + d*x]])/d + (a^2*Sec[c + d*x])/d
```

Rubi [A] time = 0.144059, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2836, 12, 44}

$$-\frac{a^3}{d(a-a\cos(c+dx))}+\frac{a^2\sec(c+dx)}{d}+\frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{2a^2\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^3/(d*(a - a*Cos[c + d*x]))) + (2*a^2*Log[1 - Cos[c + d*x]])/d - (2*a^2*
Log[Cos[c + d*x]])/d + (a^2*Sec[c + d*x])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^2\,dx = \int (-a-a\cos(c+dx))^2\csc^3(c+dx)\sec^2(c+dx)\,dx$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{a^2}{(-a-x)^2x^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^5\,\text{Subst}\left(\int \frac{1}{(-a-x)^2x^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^5\,\text{Subst}\left(\int \left(\frac{1}{a^2x^2}-\frac{2}{a^3x}+\frac{1}{a^2(a+x)^2}+\frac{2}{a^3(a+x)}\right)dx, x, -a\cos(c+dx)\right)}{d}$$
$$= -\frac{a^3}{d(a-a\cos(c+dx))}+\frac{2a^2\log(1-\cos(c+dx))}{d}-\frac{2a^2\log(\cos(c+dx))}{d}+\frac{a^2\text{se}}{}$$

Mathematica [A] time = 0.542483, size = 75, normalized size = 1.09

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\csc^2\left(\frac{1}{2}(c+dx)\right)-2\sec(c+dx)-8\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\log(\cos(c+dx))\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(Csc[(c + d*x)/2]^2 + 4*Log[C
os[c + d*x]] - 8*Log[Sin[(c + d*x)/2]] - 2*Sec[c + d*x]))/(8*d)
```

Maple [A] time = 0.059, size = 50, normalized size = 0.7

$$\frac{a^2\sec(dx+c)}{d}-\frac{a^2}{d(-1+\sec(dx+c))}+2\frac{a^2\ln(-1+\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+a*sec(d*x+c))^2,x)
```

```
[Out] a^2*sec(d*x+c)/d-1/d*a^2/(-1+sec(d*x+c))+2/d*a^2*ln(-1+sec(d*x+c))
```

Maxima [A] time = 1.00331, size = 92, normalized size = 1.33

$$\frac{2a^2\log(\cos(dx+c)-1)-2a^2\log(\cos(dx+c))+\frac{2a^2\cos(dx+c)-a^2}{\cos(dx+c)^2-\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (2*a^2*log(cos(d*x + c) - 1) - 2*a^2*log(cos(d*x + c)) + (2*a^2*cos(d*x + c
) - a^2)/(cos(d*x + c)^2 - cos(d*x + c)))/d
```

Fricas [A] time = 1.74388, size = 270, normalized size = 3.91

$$\frac{2\,a^2\cos\left(dx+c\right) - a^2 - 2\left(a^2\cos\left(dx+c\right)^2 - a^2\cos\left(dx+c\right)\right)\log\left(-\cos\left(dx+c\right)\right) + 2\left(a^2\cos\left(dx+c\right)^2 - a^2\cos\left(dx+c\right)\right.}{d\cos\left(dx+c\right)^2 - d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (2*a^2*cos(d*x + c) - a^2 - 2*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(-
cos(d*x + c)) + 2*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(-1/2*cos(d*x
+ c) + 1/2))/(d*cos(d*x + c)^2 - d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39769, size = 182, normalized size = 2.64

$$\frac{4\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 4\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{a^2+\frac{5\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 4*a^2*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a^2 + 5*a^2*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + (cos(d*x
 + c) - 1)^2/(cos(d*x + c) + 1)^2))/d
```

3.26 $\int \csc^5(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=115

$$-\frac{a^4}{4d(a-a\cos(c+dx))^2}-\frac{5a^3}{4d(a-a\cos(c+dx))}+\frac{a^2\sec(c+dx)}{d}+\frac{17a^2\log(1-\cos(c+dx))}{8d}-\frac{2a^2\log(\cos(c+dx))}{d}-$$

```
[Out] -a^4/(4*d*(a - a*Cos[c + d*x])^2) - (5*a^3)/(4*d*(a - a*Cos[c + d*x])) + (1
7*a^2*Log[1 - Cos[c + d*x]])/(8*d) - (2*a^2*Log[Cos[c + d*x]])/d - (a^2*Log
[1 + Cos[c + d*x]])/(8*d) + (a^2*Sec[c + d*x])/d
```

Rubi [A] time = 0.171222, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2836, 12, 88}

$$-\frac{a^4}{4d(a-a\cos(c+dx))^2}-\frac{5a^3}{4d(a-a\cos(c+dx))}+\frac{a^2\sec(c+dx)}{d}+\frac{17a^2\log(1-\cos(c+dx))}{8d}-\frac{2a^2\log(\cos(c+dx))}{d}-$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -a^4/(4*d*(a - a*Cos[c + d*x])^2) - (5*a^3)/(4*d*(a - a*Cos[c + d*x])) + (1
7*a^2*Log[1 - Cos[c + d*x]])/(8*d) - (2*a^2*Log[Cos[c + d*x]])/d - (a^2*Log
[1 + Cos[c + d*x]])/(8*d) + (a^2*Sec[c + d*x])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^2\,dx = \int (-a-a\cos(c+dx))^2\csc^5(c+dx)\sec^2(c+dx)\,dx$$
$$= \frac{a^5\operatorname{Subst}\left(\int \frac{a^2}{(-a-x)^3x^2(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^7\operatorname{Subst}\left(\int \frac{1}{(-a-x)^3x^2(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^7\operatorname{Subst}\left(\int \left(\frac{1}{8a^5(a-x)}+\frac{1}{a^4x^2}-\frac{2}{a^5x}+\frac{1}{2a^3(a+x)^3}+\frac{5}{4a^4(a+x)^2}+\frac{17}{8a^5(a+x)}\right)dx, x, -a\right.}{d}$$
$$= -\frac{a^4}{4d(a-a\cos(c+dx))^2}-\frac{5a^3}{4d(a-a\cos(c+dx))}+\frac{17a^2\log(1-\cos(c+dx))}{8d}$$

Mathematica [A] time = 1.51333, size = 103, normalized size = 0.9

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(\csc^4\left(\frac{1}{2}(c+dx)\right)+10\csc^2\left(\frac{1}{2}(c+dx)\right)+4\left(-4\sec(c+dx)-17\log\left(\sin\left(\frac{1}{2}(c\right.\right.\right.\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(10*Csc[(c + d*x)/2]^2 + Csc[
(c + d*x)/2]^4 + 4*(Log[Cos[(c + d*x)/2]] + 8*Log[Cos[c + d*x]] - 17*Log[Si
n[(c + d*x)/2]] - 4*Sec[c + d*x])))/(64*d)
```

Maple [A] time = 0.072, size = 85, normalized size = 0.7

$$\frac{a^2\sec(dx+c)}{d}-\frac{a^2\ln(1+\sec(dx+c))}{8d}-\frac{a^2}{4d(-1+\sec(dx+c))^2}-\frac{7a^2}{4d(-1+\sec(dx+c))}+\frac{17a^2\ln(-1+\sec(dx}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+a*sec(d*x+c))^2,x)
```

```
[Out] a^2*sec(d*x+c)/d-1/8/d*a^2*ln(1+sec(d*x+c))-1/4/d*a^2/(-1+sec(d*x+c))^2-7/4
/d*a^2/(-1+sec(d*x+c))+17/8/d*a^2*ln(-1+sec(d*x+c))
```

Maxima [A] time = 1.0235, size = 140, normalized size = 1.22

$$-\frac{a^2\log(\cos(dx+c)+1)-17a^2\log(\cos(dx+c)-1)+16a^2\log(\cos(dx+c))-\frac{2\left(9a^2\cos(dx+c)^2-14a^2\cos(dx+c)+4a^2\right)}{\cos(dx+c)^3-2\cos(dx+c)^2+\cos(dx+c)}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(a^2*log(cos(d*x + c) + 1) - 17*a^2*log(cos(d*x + c) - 1) + 16*a^2*log
(cos(d*x + c)) - 2*(9*a^2*cos(d*x + c)^2 - 14*a^2*cos(d*x + c) + 4*a^2)/(co
```

```
s(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c)))/d
```

Fricas [A] time = 1.83125, size = 531, normalized size = 4.62

$$\frac{18\,a^2\cos\left(dx+c\right)^2 - 28\,a^2\cos\left(dx+c\right) + 8\,a^2 - 16\left(a^2\cos\left(dx+c\right)^3 - 2\,a^2\cos\left(dx+c\right)^2 + a^2\cos\left(dx+c\right)\right)\log\left(-\cos\left(dx+\cdots\right.\right.}{8\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/8*(18*a^2*cos(d*x + c)^2 - 28*a^2*cos(d*x + c) + 8*a^2 - 16*(a^2*cos(d*x
+ c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*log(-cos(d*x + c)) - (a^2
*cos(d*x + c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*log(1/2*cos(d*x
+ c) + 1/2) + 17*(a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x +
 c))*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^3 - 2*d*cos(d*x + c)^2 +
 d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.52379, size = 258, normalized size = 2.24

$$\frac{34\,a^2\log\left(\frac{\left|-\cos(dx+c)+1\right|}{\left|\cos(dx+c)+1\right|}\right) - 32\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) - \frac{\left(a^2-\frac{12\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{51\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)^2}{(\cos(dx+c)-1)^2} + \frac{32\left(2\,a^2+\frac{a^2(\cos(dx+c\cdots}{\cos(dx+c)\cdots}\right.}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\cdots}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(34*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 32*a^2*log
(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a^2 - 12*a^2*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 51*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
1)^2)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2 + 32*(2*a^2 + a^2*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/
d
```

3.27 $\int \csc^7(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=160

$$-\frac{a^5}{12d(a-a\cos(c+dx))^3}-\frac{3a^4}{8d(a-a\cos(c+dx))^2}-\frac{23a^3}{16d(a-a\cos(c+dx))}+\frac{a^3}{16d(a\cos(c+dx)+a)}+\frac{a^2\sec(c+dx)}{d}$$

```
[Out] -a^5/(12*d*(a - a*Cos[c + d*x])^3) - (3*a^4)/(8*d*(a - a*Cos[c + d*x])^2) -
 (23*a^3)/(16*d*(a - a*Cos[c + d*x])) + a^3/(16*d*(a + a*Cos[c + d*x])) + (
9*a^2*Log[1 - Cos[c + d*x]])/(4*d) - (2*a^2*Log[Cos[c + d*x]])/d - (a^2*Log
[1 + Cos[c + d*x]])/(4*d) + (a^2*Sec[c + d*x])/d
```

Rubi [A] time = 0.199207, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2836, 12, 88}

$$-\frac{a^5}{12d(a-a\cos(c+dx))^3}-\frac{3a^4}{8d(a-a\cos(c+dx))^2}-\frac{23a^3}{16d(a-a\cos(c+dx))}+\frac{a^3}{16d(a\cos(c+dx)+a)}+\frac{a^2\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^7*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -a^5/(12*d*(a - a*Cos[c + d*x])^3) - (3*a^4)/(8*d*(a - a*Cos[c + d*x])^2) -
 (23*a^3)/(16*d*(a - a*Cos[c + d*x])) + a^3/(16*d*(a + a*Cos[c + d*x])) + (
9*a^2*Log[1 - Cos[c + d*x]])/(4*d) - (2*a^2*Log[Cos[c + d*x]])/d - (a^2*Log
[1 + Cos[c + d*x]])/(4*d) + (a^2*Sec[c + d*x])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^2\,dx = \int (-a-a\cos(c+dx))^2\csc^7(c+dx)\sec^2(c+dx)\,dx$$
$$= \frac{a^7\,\text{Subst}\left(\int \frac{a^2}{(-a-x)^4x^2(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^9\,\text{Subst}\left(\int \frac{1}{(-a-x)^4x^2(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^9\,\text{Subst}\left(\int \left(\frac{1}{16a^6(a-x)^2}+\frac{1}{4a^7(a-x)}+\frac{1}{a^6x^2}-\frac{2}{a^7x}+\frac{1}{4a^4(a+x)^4}+\frac{3}{4a^5(a+x)^3}+\frac{23}{16a^6(a+x)^2}\right.\right.}{d}$$
$$= -\frac{a^5}{12d(a-a\cos(c+dx))^3}-\frac{3a^4}{8d(a-a\cos(c+dx))^2}-\frac{23a^3}{16d(a-a\cos(c+dx))}+\frac{}{16}$$

Mathematica [A] time = 1.28363, size = 136, normalized size = 0.85

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(36\csc^4\left(\frac{1}{2}(c+dx)\right)+120\csc^2\left(\frac{1}{2}(c+dx)\right)+\csc^6\left(\frac{1}{2}(c+dx)\right)\left(16-3\sec^2\left(\frac{1}{2}(c+\right.\right.\right.}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^7*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(120*Csc[(c + d*x)/2]^2 + 36*
Csc[(c + d*x)/2]^4 + 48*(Log[Cos[(c + d*x)/2]] + 4*Log[Cos[c + d*x]] - 9*Lo
g[Sin[(c + d*x)/2]]) + Csc[(c + d*x)/2]^6*(16 - 3*Sec[(c + d*x)/2]^2*(3 + 2
*Sec[c + d*x]))))/(384*d)
```

Maple [A] time = 0.069, size = 121, normalized size = 0.8

$$\frac{a^2\sec(dx+c)}{d}-\frac{a^2}{16d(1+\sec(dx+c))}-\frac{a^2\ln(1+\sec(dx+c))}{4d}-\frac{a^2}{12d(-1+\sec(dx+c))^3}-\frac{5a^2}{8d(-1+\sec(dx+c))^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^7*(a+a*sec(d*x+c))^2,x)
```

```
[Out] a^2*sec(d*x+c)/d-1/16/d*a^2/(1+sec(d*x+c))-1/4/d*a^2*ln(1+sec(d*x+c))-1/12/
d*a^2/(-1+sec(d*x+c))^3-5/8/d*a^2/(-1+sec(d*x+c))^2-39/16/d*a^2/(-1+sec(d*x
+c))+9/4/d*a^2*ln(-1+sec(d*x+c))
```

Maxima [A] time = 0.988657, size = 193, normalized size = 1.21

$$-\frac{3a^2\log(\cos(dx+c)+1)-27a^2\log(\cos(dx+c)-1)+24a^2\log(\cos(dx+c))-\frac{2\left(15a^2\cos(dx+c)^4-24a^2\cos(dx+c)^3-7a^2\cos(\right.}{\cos(dx+c)^5-2\cos(dx+c)^4+2\cos(d}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/12*(3*a^2*log(cos(d*x + c) + 1) - 27*a^2*log(cos(d*x + c) - 1) + 24*a^2*
log(cos(d*x + c)) - 2*(15*a^2*cos(d*x + c)^4 - 24*a^2*cos(d*x + c)^3 - 7*a^
2*cos(d*x + c)^2 + 23*a^2*cos(d*x + c) - 6*a^2)/(cos(d*x + c)^5 - 2*cos(d*x
 + c)^4 + 2*cos(d*x + c)^2 - cos(d*x + c)))/d
```

Fricas [A] time = 1.82971, size = 722, normalized size = 4.51

$$\frac{30\,a^2\cos(dx+c)^4 - 48\,a^2\cos(dx+c)^3 - 14\,a^2\cos(dx+c)^2 + 46\,a^2\cos(dx+c) - 12\,a^2 - 24\left(a^2\cos(dx+c)^5 - 2\,a^2 \cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/12*(30*a^2*cos(d*x + c)^4 - 48*a^2*cos(d*x + c)^3 - 14*a^2*cos(d*x + c)^2
 + 46*a^2*cos(d*x + c) - 12*a^2 - 24*(a^2*cos(d*x + c)^5 - 2*a^2*cos(d*x +
c)^4 + 2*a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(-cos(d*x + c)) - 3*(a^2
*cos(d*x + c)^5 - 2*a^2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^2 - a^2*cos(d*x
 + c))*log(1/2*cos(d*x + c) + 1/2) + 27*(a^2*cos(d*x + c)^5 - 2*a^2*cos(d*x
 + c)^4 + 2*a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(-1/2*cos(d*x + c) +
1/2))/(d*cos(d*x + c)^5 - 2*d*cos(d*x + c)^4 + 2*d*cos(d*x + c)^2 - d*cos(d
*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**7*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.45157, size = 321, normalized size = 2.01

$$\frac{216\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 192\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) - \frac{3\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{\left(a^2 - \frac{12\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{90\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{396\,a^2(\cos \cdots}{(\cos(dx \cdots}\right.}{(\cos(dx+c)-1)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/96*(216*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 192*a^2*l
og(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - 3*a^2*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) + (a^2 - 12*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
) + 90*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 396*a^2*(cos(d*x + c
) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1)^3/(cos(d*x + c) - 1)^3 +
192*(2*a^2 + a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)
/(cos(d*x + c) + 1) + 1))/d
```

3.28 $\int \csc^9(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=205

$$-\frac{a^6}{32d(a-a\cos(c+dx))^4}-\frac{7a^5}{48d(a-a\cos(c+dx))^3}-\frac{15a^4}{32d(a-a\cos(c+dx))^2}+\frac{a^4}{64d(a\cos(c+dx)+a)^2}-\frac{51a^3}{32d(a-a\cos}$$

```
[Out] -a^6/(32*d*(a - a*Cos[c + d*x])^4) - (7*a^5)/(48*d*(a - a*Cos[c + d*x])^3)
- (15*a^4)/(32*d*(a - a*Cos[c + d*x])^2) - (51*a^3)/(32*d*(a - a*Cos[c + d*
x])) + a^4/(64*d*(a + a*Cos[c + d*x])^2) + (9*a^3)/(64*d*(a + a*Cos[c + d*x
])) + (303*a^2*Log[1 - Cos[c + d*x]])/(128*d) - (2*a^2*Log[Cos[c + d*x]])/d
 - (47*a^2*Log[1 + Cos[c + d*x]])/(128*d) + (a^2*Sec[c + d*x])/d
```

Rubi [A] time = 0.238155, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2836, 12, 88}

$$-\frac{a^6}{32d(a-a\cos(c+dx))^4}-\frac{7a^5}{48d(a-a\cos(c+dx))^3}-\frac{15a^4}{32d(a-a\cos(c+dx))^2}+\frac{a^4}{64d(a\cos(c+dx)+a)^2}-\frac{51a^3}{32d(a-a\cos}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^9*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -a^6/(32*d*(a - a*Cos[c + d*x])^4) - (7*a^5)/(48*d*(a - a*Cos[c + d*x])^3)
- (15*a^4)/(32*d*(a - a*Cos[c + d*x])^2) - (51*a^3)/(32*d*(a - a*Cos[c + d*
x])) + a^4/(64*d*(a + a*Cos[c + d*x])^2) + (9*a^3)/(64*d*(a + a*Cos[c + d*x
])) + (303*a^2*Log[1 - Cos[c + d*x]])/(128*d) - (2*a^2*Log[Cos[c + d*x]])/d
 - (47*a^2*Log[1 + Cos[c + d*x]])/(128*d) + (a^2*Sec[c + d*x])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^2\,dx = \int (-a-a\cos(c+dx))^2\csc^9(c+dx)\sec^2(c+dx)\,dx$$
$$= \frac{a^9\,\text{Subst}\left(\int \frac{a^2}{(-a-x)^5x^2(-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^{11}\,\text{Subst}\left(\int \frac{1}{(-a-x)^5x^2(-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^{11}\,\text{Subst}\left(\int \left(\frac{1}{32a^7(a-x)^3}+\frac{9}{64a^8(a-x)^2}+\frac{47}{128a^9(a-x)}+\frac{1}{a^8x^2}-\frac{2}{a^9x}+\frac{1}{8a^5(a+x)^5}+\frac{}{16a^6}\right.}{d}$$
$$= -\frac{a^6}{32d(a-a\cos(c+dx))^4}-\frac{7a^5}{48d(a-a\cos(c+dx))^3}-\frac{15a^4}{32d(a-a\cos(c+dx))}$$

Mathematica [A] time = 3.25396, size = 164, normalized size = 0.8

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(3\csc^8\left(\frac{1}{2}(c+dx)\right)+28\csc^6\left(\frac{1}{2}(c+dx)\right)+180\csc^4\left(\frac{1}{2}(c+dx)\right)+1224\csc^2\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^9*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(1224*Csc[(c + d*x)/2]^2 + 18
0*Csc[(c + d*x)/2]^4 + 28*Csc[(c + d*x)/2]^6 + 3*Csc[(c + d*x)/2]^8 - 6*(18
*Sec[(c + d*x)/2]^2 + Sec[(c + d*x)/2]^4 + 4*(-47*Log[Cos[(c + d*x)/2]] - 1
28*Log[Cos[c + d*x]] + 303*Log[Sin[(c + d*x)/2]] + 64*Sec[c + d*x]))))/(614
4*d)
```

Maple [A] time = 0.083, size = 157, normalized size = 0.8

$$\frac{a^2\sec(dx+c)}{d}+\frac{a^2}{64\,d\,(1+\sec(dx+c))^2}-\frac{11\,a^2}{64\,d\,(1+\sec(dx+c))}-\frac{47\,a^2\ln(1+\sec(dx+c))}{128\,d}-\frac{a^2}{32\,d\,(-1+\sec(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^9*(a+a*sec(d*x+c))^2,x)
```

```
[Out] a^2*sec(d*x+c)/d+1/64/d*a^2/(1+sec(d*x+c))^2-11/64/d*a^2/(1+sec(d*x+c))-47/
128/d*a^2*ln(1+sec(d*x+c))-1/32/d*a^2/(-1+sec(d*x+c))^4-13/48/d*a^2/(-1+sec
(d*x+c))^3-35/32/d*a^2/(-1+sec(d*x+c))^2-99/32/d*a^2/(-1+sec(d*x+c))+303/12
8/d*a^2*ln(-1+sec(d*x+c))
```

Maxima [A] time = 1.01514, size = 266, normalized size = 1.3

$$-\frac{141\,a^2\log(\cos(dx+c)+1)-909\,a^2\log(\cos(dx+c)-1)+768\,a^2\log(\cos(dx+c))-\frac{2\left(525\,a^2\cos(dx+c)^6-858\,a^2\cos(dx+\right.}{\cos(dx+c)^7-2\cos(dx}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/384*(141*a^2*log(cos(d*x + c) + 1) - 909*a^2*log(cos(d*x + c) - 1) + 768
*a^2*log(cos(d*x + c)) - 2*(525*a^2*cos(d*x + c)^6 - 858*a^2*cos(d*x + c)^5
 - 734*a^2*cos(d*x + c)^4 + 1654*a^2*cos(d*x + c)^3 - 19*a^2*cos(d*x + c)^2
 - 784*a^2*cos(d*x + c) + 192*a^2)/(cos(d*x + c)^7 - 2*cos(d*x + c)^6 - cos
(d*x + c)^5 + 4*cos(d*x + c)^4 - cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*
x + c)))/d
```

Fricas [B] time = 1.92133, size = 1152, normalized size = 5.62

$$\frac{1050\,a^2\cos(dx+c)^6 - 1716\,a^2\cos(dx+c)^5 - 1468\,a^2\cos(dx+c)^4 + 3308\,a^2\cos(dx+c)^3 - 38\,a^2\cos(dx+c)^2 - 1568\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/384*(1050*a^2*cos(d*x + c)^6 - 1716*a^2*cos(d*x + c)^5 - 1468*a^2*cos(d*x
 + c)^4 + 3308*a^2*cos(d*x + c)^3 - 38*a^2*cos(d*x + c)^2 - 1568*a^2*cos(d*
x + c) + 384*a^2 - 768*(a^2*cos(d*x + c)^7 - 2*a^2*cos(d*x + c)^6 - a^2*cos
(d*x + c)^5 + 4*a^2*cos(d*x + c)^4 - a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c
)^2 + a^2*cos(d*x + c))*log(-cos(d*x + c)) - 141*(a^2*cos(d*x + c)^7 - 2*a^
2*cos(d*x + c)^6 - a^2*cos(d*x + c)^5 + 4*a^2*cos(d*x + c)^4 - a^2*cos(d*x
+ c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/
2) + 909*(a^2*cos(d*x + c)^7 - 2*a^2*cos(d*x + c)^6 - a^2*cos(d*x + c)^5 +
4*a^2*cos(d*x + c)^4 - a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(
d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^7 - 2*d*cos(d*x + c
)^6 - d*cos(d*x + c)^5 + 4*d*cos(d*x + c)^4 - d*cos(d*x + c)^3 - 2*d*cos(d*
x + c)^2 + d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**9*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.48165, size = 393, normalized size = 1.92

$$\frac{3636\,a^2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 3072\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) - \frac{120\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{6\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{\left(3\,a^2 - \frac{40\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1} + 28}{}}{1536\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1536*(3636*a^2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 3072*a
^2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - 120*a^2*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 6*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2 - (3*a^2 - 40*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 282*a^2*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1680*a^2*(cos(d*x + c) - 1)^3/(cos(d
*x + c) + 1)^3 + 7575*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)*(cos(d
*x + c) + 1)^4/(cos(d*x + c) - 1)^4 + 3072*(2*a^2 + a^2*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/d
```

3.29 $\int (a + a\sec(c + dx))^2 \sin^8(c + dx)\, dx$

Optimal. Leaf size=199

$$-\frac{2a^2\sin^7(c+dx)}{7d}-\frac{2a^2\sin^5(c+dx)}{5d}-\frac{2a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (-245*a^2*x)/128 + (2*a^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d
 + (139*a^2*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (11*a^2*Cos[c + d*x]^3*Sin
[c + d*x])/(192*d) - (17*a^2*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) + (a^2*Cos
[c + d*x]^7*Sin[c + d*x])/(8*d) - (2*a^2*Sin[c + d*x]^3)/(3*d) - (2*a^2*Sin
[c + d*x]^5)/(5*d) - (2*a^2*Sin[c + d*x]^7)/(7*d) + (a^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.360779, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2872, 2637, 2635, 8, 2633, 3770, 3767}

$$-\frac{2a^2\sin^7(c+dx)}{7d}-\frac{2a^2\sin^5(c+dx)}{5d}-\frac{2a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^8,x]
```


```
[Out] (-245*a^2*x)/128 + (2*a^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d
 + (139*a^2*Cos[c + d*x]*Sin[c + d*x])/(128*d) + (11*a^2*Cos[c + d*x]^3*Sin
[c + d*x])/(192*d) - (17*a^2*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) + (a^2*Cos
[c + d*x]^7*Sin[c + d*x])/(8*d) - (2*a^2*Sin[c + d*x]^3)/(3*d) - (2*a^2*Sin
[c + d*x]^5)/(5*d) - (2*a^2*Sin[c + d*x]^7)/(7*d) + (a^2*Tan[c + d*x])/d
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\sin^8(c+dx)\,dx = \int (-a-a\cos(c+dx))^2\sin^6(c+dx)\tan^2(c+dx)\,dx$$

$$= \frac{\int \left(-3a^{10}-8a^{10}\cos(c+dx)+2a^{10}\cos^2(c+dx)+12a^{10}\cos^3(c+dx)+2a^{10}\ldots\right.}{\ldots}$$

$$= -3a^2x+a^2\int\cos^8(c+dx)\,dx+a^2\int\sec^2(c+dx)\,dx+\left(2a^2\right)\int\cos^2(c+dx)\ldots$$

$$= -3a^2x+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{8a^2\sin(c+dx)}{d}+\frac{a^2\cos(c+dx)\sin(c+\ldots}{d}$$

$$= -2a^2x+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}+\frac{7a^2\cos(c+dx)\sin(c+\ldots}{4d}$$

$$= -\frac{5a^2x}{4}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}+\frac{13a^2\cos(c+dx)\sin(c\ldots}{16d}$$

$$= -\frac{35a^2x}{16}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}+\frac{139a^2\cos(c+dx)\sin\ldots}{128d}$$

$$= -\frac{245a^2x}{128}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}+\frac{139a^2\cos(c+dx)\sin\ldots}{128d}$$

Mathematica [A] time = 0.883454, size = 144, normalized size = 0.72

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(30720\sin^7(c+dx)+43008\sin^5(c+dx)+71680\sin^3(c+dx)+215040\sin(c\ldots\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^8,x]
```

```
[Out] -(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(168000*c + 168000*d*x + 3780
0*ArcTan[Tan[c + d*x]] - 215040*ArcTanh[Sin[c + d*x]] + 215040*Sin[c + d*x]
 + 71680*Sin[c + d*x]^3 + 43008*Sin[c + d*x]^5 + 30720*Sin[c + d*x]^7 - 554
40*Sin[2*(c + d*x)] + 2520*Sin[4*(c + d*x)] + 560*Sin[6*(c + d*x)] - 105*Si
n[8*(c + d*x)] - 107520*Tan[c + d*x]))/(430080*d)
```

Maple [A] time = 0.049, size = 210, normalized size = 1.1

$$\frac{7\,a^{2} (\sin (dx+c))^{7} \cos (dx+c)}{8\,d}+\frac{49\,a^{2} \cos (dx+c) (\sin (dx+c))^{5}}{48\,d}+\frac{245\,a^{2} \cos (dx+c) (\sin (dx+c))^{3}}{192\,d}+\frac{245\,a^{2} \cos (dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^8,x)
```

```
[Out] 7/8/d*a^2*sin(d*x+c)^7*cos(d*x+c)+49/48/d*a^2*cos(d*x+c)*sin(d*x+c)^5+245/1
92/d*a^2*cos(d*x+c)*sin(d*x+c)^3+245/128*a^2*cos(d*x+c)*sin(d*x+c)/d-245/12
8*a^2*x-245/128/d*a^2*c-2/7*a^2*sin(d*x+c)^7/d-2/5*a^2*sin(d*x+c)^5/d-2/3*a
^2*sin(d*x+c)^3/d-2*a^2*sin(d*x+c)/d+2/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/d*
a^2*sin(d*x+c)^9/cos(d*x+c)
```

Maxima [A] time = 1.53875, size = 290, normalized size = 1.46

$$-\frac{1024\left(30 \sin (dx+c)^{7}+42 \sin (dx+c)^{5}+70 \sin (dx+c)^{3}-105 \log (\sin (dx+c)+1)+105 \log (\sin (dx+c)-1)+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/107520*(1024*(30*sin(d*x + c)^7 + 42*sin(d*x + c)^5 + 70*sin(d*x + c)^3
- 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1) + 210*sin(d*x + c))
*a^2 - 35*(128*sin(2*d*x + 2*c)^3 + 840*d*x + 840*c + 3*sin(8*d*x + 8*c) +
168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*a^2 + 2240*(105*d*x + 105*c -
(87*tan(d*x + c)^5 + 136*tan(d*x + c)^3 + 57*tan(d*x + c))/(tan(d*x + c)^6
+ 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 + 1) - 48*tan(d*x + c))*a^2)/d
```

Fricas [A] time = 1.95531, size = 522, normalized size = 2.62

$$-\frac{25725\,a^{2} dx \cos (dx+c)-13440\,a^{2} \cos (dx+c) \log (\sin (dx+c)+1)+13440\,a^{2} \cos (dx+c) \log (-\sin (dx+c)+1)-\big($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/13440*(25725*a^2*d*x*cos(d*x + c) - 13440*a^2*cos(d*x + c)*log(sin(d*x +
 c) + 1) + 13440*a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) - (1680*a^2*cos(d*
x + c)^8 + 3840*a^2*cos(d*x + c)^7 - 4760*a^2*cos(d*x + c)^6 - 16896*a^2*co
s(d*x + c)^5 + 770*a^2*cos(d*x + c)^4 + 31232*a^2*cos(d*x + c)^3 + 14595*a^
2*cos(d*x + c)^2 - 45056*a^2*cos(d*x + c) + 13440*a^2)*sin(d*x + c))/(d*cos
(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41883, size = 304, normalized size = 1.53

$$-\frac{25725\,(dx+c)a^2 - 26880\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + 26880\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{26880\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/13440*(25725*(d*x + c)*a^2 - 26880*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)
) + 26880*a^2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 26880*a^2*tan(1/2*d*x +
1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(39165*a^2*tan(1/2*d*x + 1/2*c)^15
+ 300265*a^2*tan(1/2*d*x + 1/2*c)^13 + 989261*a^2*tan(1/2*d*x + 1/2*c)^11 +
 1791073*a^2*tan(1/2*d*x + 1/2*c)^9 + 1814943*a^2*tan(1/2*d*x + 1/2*c)^7 +
670131*a^2*tan(1/2*d*x + 1/2*c)^5 + 147735*a^2*tan(1/2*d*x + 1/2*c)^3 + 145
95*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^8)/d
```

3.30 $\int (a+a\sec(c+dx))^2\sin^6(c+dx)\,dx$

Optimal. Leaf size=157

$$-\frac{2a^2\sin^5(c+dx)}{5d}-\frac{2a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^2\sin(c+dx)\cos}{6d}$$

```
[Out] (-25*a^2*x)/16 + (2*a^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d +
 (7*a^2*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (7*a^2*Cos[c + d*x]^3*Sin[c + d
*x])/(24*d) - (a^2*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (2*a^2*Sin[c + d*x]
^3)/(3*d) - (2*a^2*Sin[c + d*x]^5)/(5*d) + (a^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.269959, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2872, 2637, 2633, 2635, 8, 3770, 3767}

$$-\frac{2a^2\sin^5(c+dx)}{5d}-\frac{2a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^2\sin(c+dx)\cos}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^6,x]
```


```
[Out] (-25*a^2*x)/16 + (2*a^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d +
 (7*a^2*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (7*a^2*Cos[c + d*x]^3*Sin[c + d
*x])/(24*d) - (a^2*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (2*a^2*Sin[c + d*x]
^3)/(3*d) - (2*a^2*Sin[c + d*x]^5)/(5*d) + (a^2*Tan[c + d*x])/d
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^2\sin^6(c+dx)\,dx &= \int (-a-a\cos(c+dx))^2\sin^4(c+dx)\tan^2(c+dx)\,dx\\
&= \frac{\int \left(-2a^8-6a^8\cos(c+dx)+6a^8\cos^3(c+dx)+2a^8\cos^4(c+dx)-2a^8\cos^5(c\right.}{a^6}\\
&= -2a^2x-a^2\int\cos^6(c+dx)\,dx+a^2\int\sec^2(c+dx)\,dx+\left(2a^2\right)\int\cos^4(c+dx)\\
&= -2a^2x+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{6a^2\sin(c+dx)}{d}+\frac{a^2\cos^3(c+dx)\sin(c+}{2d}\\
&= -2a^2x+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}+\frac{3a^2\cos(c+dx)\sin(c+}{4d}\\
&= -\frac{5a^2x}{4}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}+\frac{7a^2\cos(c+dx)\sin(c}{16d}\\
&= -\frac{25a^2x}{16}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}+\frac{7a^2\cos(c+dx)\sin(c}{16d}\end{aligned}$$

Mathematica [A] time = 0.547296, size = 124, normalized size = 0.79

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(384\sin^5(c+dx)+640\sin^3(c+dx)+1920\sin(c+dx)-255\sin(2(c+dx))-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^6,x]
```

```
[Out] -(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(1080*c + 1080*d*x + 420*ArcT
an[Tan[c + d*x]] - 1920*ArcTanh[Sin[c + d*x]] + 1920*Sin[c + d*x] + 640*Sin
[c + d*x]^3 + 384*Sin[c + d*x]^5 - 255*Sin[2*(c + d*x)] - 15*Sin[4*(c + d*x
)] + 5*Sin[6*(c + d*x)] - 960*Tan[c + d*x]))/(3840*d)
```

Maple [A] time = 0.043, size = 172, normalized size = 1.1

$$\frac{5a^2\cos(dx+c)(\sin(dx+c))^5}{6d}+\frac{25a^2\cos(dx+c)(\sin(dx+c))^3}{24d}+\frac{25a^2\cos(dx+c)\sin(dx+c)}{16d}-\frac{25a^2x}{16}-\frac{25a^2c}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^6,x)
```

```
[Out] 5/6/d*a^2*cos(d*x+c)*sin(d*x+c)^5+25/24/d*a^2*cos(d*x+c)*sin(d*x+c)^3+25/16
*a^2*cos(d*x+c)*sin(d*x+c)/d-25/16*a^2*x-25/16/d*a^2*c-2/5*a^2*sin(d*x+c)^5
/d-2/3*a^2*sin(d*x+c)^3/d-2*a^2*sin(d*x+c)/d+2/d*a^2*ln(sec(d*x+c)+tan(d*x+
c))+1/d*a^2*sin(d*x+c)^7/cos(d*x+c)
```

Maxima [A] time = 1.54821, size = 235, normalized size = 1.5

$$-\frac{64\left(6\sin(dx+c)^5+10\sin(dx+c)^3-15\log(\sin(dx+c)+1)+15\log(\sin(dx+c)-1)+30\sin(dx+c)\right)a^2-5\left(4\,\mathrm{s}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] -1/960*(64*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin(d*x + c) + 1)
 + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*a^2 - 5*(4*sin(2*d*x + 2*c)^
3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^2 + 120*(15
*d*x + 15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d
*x + c)^2 + 1) - 8*tan(d*x + c))*a^2)/d
```

Fricas [A] time = 1.97077, size = 423, normalized size = 2.69

$$-\frac{375\,a^2dx\cos(dx+c)-240\,a^2\cos(dx+c)\log(\sin(dx+c)+1)+240\,a^2\cos(dx+c)\log(-\sin(dx+c)+1)+\left(40\,a^2\,\mathrm{c}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/240*(375*a^2*d*x*cos(d*x + c) - 240*a^2*cos(d*x + c)*log(sin(d*x + c) +
1) + 240*a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + (40*a^2*cos(d*x + c)^6 +
 96*a^2*cos(d*x + c)^5 - 70*a^2*cos(d*x + c)^4 - 352*a^2*cos(d*x + c)^3 - 1
05*a^2*cos(d*x + c)^2 + 736*a^2*cos(d*x + c) - 240*a^2)*sin(d*x + c))/(d*co
s(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.42387, size = 261, normalized size = 1.66

$$-\frac{375\,(dx+c)a^2 - 480\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + 480\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{480\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} + \frac{2\left(615\right.}{}}{240\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="giac")
```

```
[Out] -1/240*(375*(d*x + c)*a^2 - 480*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 48
0*a^2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 480*a^2*tan(1/2*d*x + 1/2*c)/(ta
n(1/2*d*x + 1/2*c)^2 - 1) + 2*(615*a^2*tan(1/2*d*x + 1/2*c)^11 + 3485*a^2*t
an(1/2*d*x + 1/2*c)^9 + 7926*a^2*tan(1/2*d*x + 1/2*c)^7 + 8586*a^2*tan(1/2*
d*x + 1/2*c)^5 + 2595*a^2*tan(1/2*d*x + 1/2*c)^3 + 345*a^2*tan(1/2*d*x + 1/
2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.31 $\int (a+a\sec(c+dx))^2\sin^4(c+dx)\,dx$

Optimal. Leaf size=115

$$-\frac{2a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}-\frac{a^2\sin(\cdots}{}$$

```
[Out] (-9*a^2*x)/8 + (2*a^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d - (
a^2*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(4
*d) - (2*a^2*Sin[c + d*x]^3)/(3*d) + (a^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.268433, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2872, 2637, 2635, 8, 2633, 3770, 3767}

$$-\frac{2a^2\sin^3(c+dx)}{3d}-\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2\sin(c+dx)\cos^3(c+dx)}{4d}-\frac{a^2\sin(\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^4,x]
```



```
[Out] (-9*a^2*x)/8 + (2*a^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d - (
a^2*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*Cos[c + d*x]^3*Sin[c + d*x])/(4
*d) - (2*a^2*Sin[c + d*x]^3)/(3*d) + (a^2*Tan[c + d*x])/d
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\sin^4(c+dx)\,dx = \int (-a-a\cos(c+dx))^2\sin^2(c+dx)\tan^2(c+dx)\,dx$$
$$= \frac{\int \left(-a^6-4a^6\cos(c+dx)-a^6\cos^2(c+dx)+2a^6\cos^3(c+dx)+a^6\cos^4(c+d\right.}{a^4}$$
$$= -a^2x-a^2\int \cos^2(c+dx)\,dx+a^2\int \cos^4(c+dx)\,dx+a^2\int \sec^2(c+dx)\,dx+$$
$$= -a^2x+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{4a^2\sin(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c+d}{2d}$$
$$= -\frac{3a^2x}{2}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c+}{8d}$$
$$= -\frac{9a^2x}{8}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c+}{8d}$$

Mathematica [A] time = 0.27868, size = 94, normalized size = 0.82

$$-\frac{a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(64\sin^3(c+dx)+192\sin(c+dx)-3\sin(4(c+dx))+60\tan^{-1}(\tan(c+dx))-\right.}{384d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^4,x]
```

```
[Out] -(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(48*c + 48*d*x + 60*ArcTan[Ta
n[c + d*x]] - 192*ArcTanh[Sin[c + d*x]] + 192*Sin[c + d*x] + 64*Sin[c + d*x
]^3 - 3*Sin[4*(c + d*x)] - 96*Tan[c + d*x]))/(384*d)
```

Maple [A] time = 0.041, size = 134, normalized size = 1.2

$$\frac{3\,a^2\cos(dx+c)(\sin(dx+c))^3}{4\,d}+\frac{9\,a^2\cos(dx+c)\sin(dx+c)}{8\,d}-\frac{9\,a^2x}{8}-\frac{9\,a^2c}{8\,d}-\frac{2\,a^2(\sin(dx+c))^3}{3\,d}-2\,\frac{a^2\sin(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^4,x)
```

```
[Out] 3/4/d*a^2*cos(d*x+c)*sin(d*x+c)^3+9/8*a^2*cos(d*x+c)*sin(d*x+c)/d-9/8*a^2*x
-9/8/d*a^2*c-2/3*a^2*sin(d*x+c)^3/d-2*a^2*sin(d*x+c)/d+2/d*a^2*ln(sec(d*x+c
)+tan(d*x+c))+1/d*a^2*sin(d*x+c)^5/cos(d*x+c)
```

Maxima [A] time = 1.506, size = 170, normalized size = 1.48

$$-\frac{32\left(2\,\sin\left(dx+c\right)^{3}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)+6\,\sin\left(dx+c\right)\right)a^{2}-3\left(12\,dx+12\,c+\sin\left(4\,dx+\right.\right.}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/96*(32*(2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
- 1) + 6*sin(d*x + c))*a^2 - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*
d*x + 2*c))*a^2 + 48*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*t
an(d*x + c))*a^2)/d
```

Fricas [A] time = 1.88363, size = 344, normalized size = 2.99

$$-\frac{27\,a^{2}dx\cos\left(dx+c\right)-24\,a^{2}\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)+24\,a^{2}\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)-\left(6\,a^{2}\cos\left(dx\right.\right.}{24\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/24*(27*a^2*d*x*cos(d*x + c) - 24*a^2*cos(d*x + c)*log(sin(d*x + c) + 1)
+ 24*a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) - (6*a^2*cos(d*x + c)^4 + 16*a
^2*cos(d*x + c)^3 - 3*a^2*cos(d*x + c)^2 - 64*a^2*cos(d*x + c) + 24*a^2)*si
n(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37841, size = 217, normalized size = 1.89

$$-\frac{27\left(dx+c\right)a^{2}-48\,a^{2}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+48\,a^{2}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{48\,a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}+\frac{2\left(51\,a^{2}\tan\left(\frac{1}{2}\,dx\right.\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/24*(27*(d*x + c)*a^2 - 48*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 48*a^
2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 48*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2
*d*x + 1/2*c)^2 - 1) + 2*(51*a^2*tan(1/2*d*x + 1/2*c)^7 + 187*a^2*tan(1/2*d
*x + 1/2*c)^5 + 229*a^2*tan(1/2*d*x + 1/2*c)^3 + 45*a^2*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.32 $\int (a+a\sec(c+dx))^2\sin^2(c+dx)\,dx$

Optimal. Leaf size=73

$$-\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{a^2x}{2}$$

```
[Out] -(a^2*x)/2 + (2*a^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d - (a^
2*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.132185, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2709, 2637, 2635, 8, 3770, 3767}

$$-\frac{2a^2\sin(c+dx)}{d}+\frac{a^2\tan(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}-\frac{a^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^2,x]
```

```
[Out] -(a^2*x)/2 + (2*a^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*Sin[c + d*x])/d - (a^
2*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^2*Tan[c + d*x])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\sin^2(c+dx)\,dx = \int (-a-a\cos(c+dx))^2\tan^2(c+dx)\,dx$$
$$= \frac{\int \left(-2a^4\cos(c+dx)-a^4\cos^2(c+dx)+2a^4\sec(c+dx)+a^4\sec^2(c+dx)\right) d}{a^2}$$
$$= -\left(a^2\int \cos^2(c+dx)\,dx\right)+a^2\int \sec^2(c+dx)\,dx-\left(2a^2\right)\int \cos(c+dx)\,dx+$$
$$= \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}-\frac{1}{2}a$$
$$= -\frac{a^2x}{2}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\sin(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c+a}{2d}$$

Mathematica [B] time = 1.19226, size = 243, normalized size = 3.33

$$\frac{1}{16}a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-\frac{8\sin(c)\cos(dx)}{d}-\frac{\sin(2c)\cos(2dx)}{d}-\frac{8\cos(c)\sin(dx)}{d}-\frac{\cos(2c)\sin(2dx)}{d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sin[c + d*x]^2,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(-2*x - (8*Log[Cos[(c + d*x)/2
] - Sin[(c + d*x)/2]])/d + (8*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d -
 (8*Cos[d*x]*Sin[c])/d - (Cos[2*d*x]*Sin[2*c])/d - (8*Cos[c]*Sin[d*x])/d -
(Cos[2*c]*Sin[2*d*x])/d + (4*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2])) + (4*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])
*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/16
```

Maple [A] time = 0.034, size = 86, normalized size = 1.2

$$-\frac{a^2\cos(dx+c)\sin(dx+c)}{2d}-\frac{a^2x}{2}-\frac{a^2c}{2d}+2\frac{a^2\ln(\sec(dx+c)+\tan(dx+c))}{d}-2\frac{a^2\sin(dx+c)}{d}+\frac{a^2\tan(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*sin(d*x+c)^2,x)
```

```
[Out] -1/2*a^2*cos(d*x+c)*sin(d*x+c)/d-1/2*a^2*x-1/2/d*a^2*c+2/d*a^2*ln(sec(d*x+c
)+tan(d*x+c))-2*a^2*sin(d*x+c)/d+a^2*tan(d*x+c)/d
```

Maxima [A] time = 1.48239, size = 109, normalized size = 1.49

$$\frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))a^2-4\,(dx+c-\tan(dx+c))a^2+4\,a^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)-2\,\sin(}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a^2 - 4*(d*x + c - tan(d*x + c))*a^2
+ 4*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)))/d
```

Fricas [A] time = 1.76393, size = 267, normalized size = 3.66

$$-\frac{a^2dx\cos(dx+c)-2\,a^2\cos(dx+c)\log(\sin(dx+c)+1)+2\,a^2\cos(dx+c)\log(-\sin(dx+c)+1)+\big(a^2\cos(dx+c)^2}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/2*(a^2*d*x*cos(d*x + c) - 2*a^2*cos(d*x + c)*log(sin(d*x + c) + 1) + 2*a
^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + (a^2*cos(d*x + c)^2 + 4*a^2*cos(d*
x + c) - 2*a^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int 2\sin^2(c+dx)\sec(c+dx)\,dx+\int\sin^2(c+dx)\sec^2(c+dx)\,dx+\int\sin^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*sin(d*x+c)**2,x)
```

```
[Out] a**2*(Integral(2*sin(c + d*x)**2*sec(c + d*x), x) + Integral(sin(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(sin(c + d*x)**2, x))
```

Giac [A] time = 1.44367, size = 173, normalized size = 2.37

$$-\frac{(dx+c)a^2-4\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+4\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{4\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\frac{2\big(3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5}{\big(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*((d*x + c)*a^2 - 4*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 4*a^2*log(
abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x +
1/2*c)^2 - 1) + 2*(3*a^2*tan(1/2*d*x + 1/2*c)^3 + 5*a^2*tan(1/2*d*x + 1/2*c
))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.33 $\int \csc^2(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=57

$$\frac{a^2\tan(c+dx)}{d}-\frac{2a^2\cot(c+dx)}{d}-\frac{2a^2\csc(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*Cot[c + d*x])/d - (2*a^2*Csc[c + d
*x])/d + (a^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.246487, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3872, 2873, 3767, 8, 2621, 321, 207, 2620, 14}

$$\frac{a^2\tan(c+dx)}{d}-\frac{2a^2\cot(c+dx)}{d}-\frac{2a^2\csc(c+dx)}{d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```



```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/d - (2*a^2*Cot[c + d*x])/d - (2*a^2*Csc[c + d
*x])/d + (a^2*Tan[c + d*x])/d
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^2\,dx = \int (-a-a\cos(c+dx))^2\csc^2(c+dx)\sec^2(c+dx)\,dx$$
$$= \int \left(a^2\csc^2(c+dx)+2a^2\csc^2(c+dx)\sec(c+dx)+a^2\csc^2(c+dx)\sec^2(c+dx)\right.$$
$$= a^2\int \csc^2(c+dx)\,dx + a^2\int \csc^2(c+dx)\sec^2(c+dx)\,dx + \left(2a^2\right)\int \csc^2(c+dx$$
$$= -\frac{a^2\,\mathrm{Subst}(\int 1\,dx,x,\cot(c+dx))}{d} + \frac{a^2\,\mathrm{Subst}\left(\int \frac{1+x^2}{x^2}\,dx,x,\tan(c+dx)\right)}{d} - \frac{\left(2a^2\right.}{}$$
$$= -\frac{a^2\cot(c+dx)}{d} - \frac{2a^2\csc(c+dx)}{d} + \frac{a^2\,\mathrm{Subst}\left(\int\left(1+\frac{1}{x^2}\right)dx,x,\tan(c+dx)\right)}{d} -$$
$$= \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d} - \frac{2a^2\cot(c+dx)}{d} - \frac{2a^2\csc(c+dx)}{d} + \frac{a^2\tan(c+dx)}{d}$$

Mathematica [B] time = 6.15428, size = 401, normalized size = 7.04

$$\frac{\sin\left(\frac{dx}{2}\right)\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^2}{4d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)} + \frac{\sin\left(\frac{dx}{2}\right)\cos^2(c+dx)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^2}{4d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)} - \frac{\cos^2}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(Cos[c + d*x]^2*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*
x)/2]^4*(a + a*Sec[c + d*x])^2)/(2*d) + (Cos[c + d*x]^2*Log[Cos[c/2 + (d*x)
/2] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(2*d
) + (Cos[c + d*x]^2*Csc[c/2]*Csc[c/2 + (d*x)/2]*Sec[c/2 + (d*x)/2]^4*(a + a
*Sec[c + d*x])^2*Sin[(d*x)/2])/(2*d) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4
*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(4*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 +
```

```
(d*x)/2] - Sin[c/2 + (d*x)/2])) + (Cos[c + d*x]^2*Sec[c/2 + (d*x)/2]^4*(a
+ a*Sec[c + d*x])^2*Sin[(d*x)/2])/(4*d*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*
x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A] time = 0.045, size = 77, normalized size = 1.4

$$-3\,\frac{a^{2}\cot\left(dx+c\right)}{d}-2\,\frac{a^{2}}{d\sin\left(dx+c\right)}+2\,\frac{a^{2}\ln\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)}{d}+\frac{a^{2}}{d\sin\left(dx+c\right)\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -3*a^2*cot(d*x+c)/d-2/d*a^2/sin(d*x+c)+2/d*a^2*ln(sec(d*x+c)+tan(d*x+c))+1/
d*a^2/sin(d*x+c)/cos(d*x+c)
```

Maxima [A] time = 1.0009, size = 100, normalized size = 1.75

$$-\frac{a^{2}\left(\frac{2}{\sin(dx+c)}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+a^{2}\left(\frac{1}{\tan(dx+c)}-\tan\left(dx+c\right)\right)+\frac{a^{2}}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(a^2*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + a^
2*(1/tan(d*x + c) - tan(d*x + c)) + a^2/tan(d*x + c))/d
```

Fricas [A] time = 1.74643, size = 257, normalized size = 4.51

$$\frac{a^{2}\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-a^{2}\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-3\,a^{2}\cos\left(dx+c\right)\ldots}{d\cos\left(dx+c\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (a^2*cos(d*x + c)*log(sin(d*x + c) + 1)*sin(d*x + c) - a^2*cos(d*x + c)*log
(-sin(d*x + c) + 1)*sin(d*x + c) - 3*a^2*cos(d*x + c)^2 - 2*a^2*cos(d*x + c
) + a^2)/(d*cos(d*x + c)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{2}\left(\int 2\csc^{2}\left(c+dx\right)\sec\left(c+dx\right)\,dx+\int\csc^{2}\left(c+dx\right)\sec^{2}\left(c+dx\right)\,dx+\int\csc^{2}\left(c+dx\right)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sec(d*x+c))**2,x)
```

```
[Out] a**2*(Integral(2*csc(c + d*x)**2*sec(c + d*x), x) + Integral(csc(c + d*x)**
2*sec(c + d*x)**2, x) + Integral(csc(c + d*x)**2, x))
```

Giac [A] time = 1.43561, size = 122, normalized size = 2.14

$$\frac{2\left(a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - a^2*log(abs(tan(1/2*d*x + 1/2*c
) - 1)) - (2*a^2*tan(1/2*d*x + 1/2*c)^2 - a^2)/(tan(1/2*d*x + 1/2*c)^3 - ta
n(1/2*d*x + 1/2*c)))/d
```

3.34 $\int \csc^4(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=87

$$\frac{10a^2\tan(c+dx)}{3d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^4\tan(c+dx)}{3d(a-a\cos(c+dx))^2}-\frac{2a^2\tan(c+dx)}{d(1-\cos(c+dx))}$$

```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/d + (10*a^2*Tan[c + d*x])/(3*d) - (2*a^2*Tan[
c + d*x])/(d*(1 - Cos[c + d*x])) - (a^4*Tan[c + d*x])/(3*d*(a - a*Cos[c + d
*x])^2)
```

Rubi [A] time = 0.297441, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2869, 2766, 2978, 2748, 3767, 8, 3770}

$$\frac{10a^2\tan(c+dx)}{3d}+\frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^4\tan(c+dx)}{3d(a-a\cos(c+dx))^2}-\frac{2a^2\tan(c+dx)}{d(1-\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/d + (10*a^2*Tan[c + d*x])/(3*d) - (2*a^2*Tan[
c + d*x])/(d*(1 - Cos[c + d*x])) - (a^4*Tan[c + d*x])/(3*d*(a - a*Cos[c + d
*x])^2)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
```

```
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \csc^4(c+dx)(a+a\sec(c+dx))^2\,dx &= \int (-a-a\cos(c+dx))^2\csc^4(c+dx)\sec^2(c+dx)\,dx \\
&= a^4\int \frac{\sec^2(c+dx)}{(-a+a\cos(c+dx))^2}\,dx \\
&= -\frac{a^4\tan(c+dx)}{3d(a-a\cos(c+dx))^2}+\frac{1}{3}a^2\int \frac{(-4a-2a\cos(c+dx))\sec^2(c+dx)}{-a+a\cos(c+dx)}\,dx \\
&= -\frac{2a^2\tan(c+dx)}{d(1-\cos(c+dx))}-\frac{a^4\tan(c+dx)}{3d(a-a\cos(c+dx))^2}+\frac{1}{3}\int \left(10a^2+6a^2\cos(c+dx)\right)\text{se} \\
&= -\frac{2a^2\tan(c+dx)}{d(1-\cos(c+dx))}-\frac{a^4\tan(c+dx)}{3d(a-a\cos(c+dx))^2}+\left(2a^2\right)\int \sec(c+dx)\,dx+\frac{1}{3}\left(10a\right. \\
&= \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^2\tan(c+dx)}{d(1-\cos(c+dx))}-\frac{a^4\tan(c+dx)}{3d(a-a\cos(c+dx))^2}-\frac{\left(10a^2\right)}{} \\
&= \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}+\frac{10a^2\tan(c+dx)}{3d}-\frac{2a^2\tan(c+dx)}{d(1-\cos(c+dx))}-\frac{a^4\tan(}{3d(a-a\cos}
\end{aligned}$$

Mathematica [B] time = 1.67115, size = 228, normalized size = 2.62

$$a^2(\cos(c+dx)+1)^2\sec^4\left(\frac{1}{2}(c+dx)\right)\left(-\cot\left(\frac{c}{2}\right)\csc^2\left(\frac{1}{2}(c+dx)\right)+6\left(\frac{\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*(-(Cot[c/2]*Csc[(c + d*x)/2]^2
) - (-8 + 7*Cos[c + d*x])*Csc[c/2]*Csc[(c + d*x)/2]^3*Sin[(d*x)/2] + 6*(-2*
```

```
Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*Log[Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2]] + Sin[d*x]/((Cos[c/2] - Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(24
*d)
```

Maple [A] time = 0.059, size = 140, normalized size = 1.6

$$-\frac{10\,a^2\cot(dx+c)}{3\,d}-\frac{a^2\cot(dx+c)(\csc(dx+c))^2}{3\,d}-\frac{2\,a^2}{3\,d(\sin(dx+c))^3}-2\,\frac{a^2}{d\sin(dx+c)}+2\,\frac{a^2\ln(\sec(dx+c)+\mathrm{ta}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -10/3*a^2*cot(d*x+c)/d-1/3/d*a^2*cot(d*x+c)*csc(d*x+c)^2-2/3/d*a^2/sin(d*x+
c)^3-2/d*a^2/sin(d*x+c)+2/d*a^2*ln(sec(d*x+c)+tan(d*x+c))-1/3/d*a^2/sin(d*x
+c)^3/cos(d*x+c)+4/3/d*a^2/sin(d*x+c)/cos(d*x+c)
```

Maxima [A] time = 1.00846, size = 153, normalized size = 1.76

$$-\frac{a^2\left(\frac{2\left(3\,\sin(dx+c)^2+1\right)}{\sin(dx+c)^3}-3\,\log(\sin(dx+c)+1)+3\,\log(\sin(dx+c)-1)\right)+a^2\left(\frac{6\,\tan(dx+c)^2+1}{\tan(dx+c)^3}-3\,\tan(dx+c)\right)+\frac{(3\,\tan}{\mathrm{ta}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(a^2*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1
) + 3*log(sin(d*x + c) - 1)) + a^2*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3 -
 3*tan(d*x + c)) + (3*tan(d*x + c)^2 + 1)*a^2/tan(d*x + c)^3)/d
```

Fricas [A] time = 1.67692, size = 396, normalized size = 4.55

$$-\frac{10\,a^2\cos(dx+c)^3-4\,a^2\cos(dx+c)^2-11\,a^2\cos(dx+c)-3\left(a^2\cos(dx+c)^2-a^2\cos(dx+c)\right)\log(\sin(dx+c)+}{3\left(d\cos(dx+c)^2-d\cos(dx+c)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(10*a^2*cos(d*x + c)^3 - 4*a^2*cos(d*x + c)^2 - 11*a^2*cos(d*x + c) -
3*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(sin(d*x + c) + 1)*sin(d*x + c
) + 3*(a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*log(-sin(d*x + c) + 1)*sin(d*
x + c) + 3*a^2)/((d*cos(d*x + c)^2 - d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.4944, size = 140, normalized size = 1.61

$$\frac{12\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-12\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{12\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-\frac{15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a^2}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(12*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 12*a^2*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) - 12*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1)
- (15*a^2*tan(1/2*d*x + 1/2*c)^2 + a^2)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.35 $\int \csc^6(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=129

$$\frac{a^2\tan(c+dx)}{d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{5a^2\cot^3(c+dx)}{3d}-\frac{4a^2\cot(c+dx)}{d}-\frac{2a^2\csc^5(c+dx)}{5d}-\frac{2a^2\csc^3(c+dx)}{3d}-\frac{2a^2}{}$$

```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/d - (4*a^2*Cot[c + d*x])/d - (5*a^2*Cot[c + d
*x]^3)/(3*d) - (2*a^2*Cot[c + d*x]^5)/(5*d) - (2*a^2*Csc[c + d*x])/d - (2*a
^2*Csc[c + d*x]^3)/(3*d) - (2*a^2*Csc[c + d*x]^5)/(5*d) + (a^2*Tan[c + d*x]
)/d
```

Rubi [A] time = 0.226338, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2873, 3767, 2621, 302, 207, 2620, 270}

$$\frac{a^2\tan(c+dx)}{d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{5a^2\cot^3(c+dx)}{3d}-\frac{4a^2\cot(c+dx)}{d}-\frac{2a^2\csc^5(c+dx)}{5d}-\frac{2a^2\csc^3(c+dx)}{3d}-\frac{2a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/d - (4*a^2*Cot[c + d*x])/d - (5*a^2*Cot[c + d
*x]^3)/(3*d) - (2*a^2*Cot[c + d*x]^5)/(5*d) - (2*a^2*Csc[c + d*x])/d - (2*a
^2*Csc[c + d*x]^3)/(3*d) - (2*a^2*Csc[c + d*x]^5)/(5*d) + (a^2*Tan[c + d*x]
)/d
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \csc^6(c+dx)(a+a\sec(c+dx))^2\,dx &= \int (-a-a\cos(c+dx))^2\csc^6(c+dx)\sec^2(c+dx)\,dx \\
&= \int \left(a^2\csc^6(c+dx)+2a^2\csc^6(c+dx)\sec(c+dx)+a^2\csc^6(c+dx)\sec^2(c+dx)\right. \\
&= a^2\int \csc^6(c+dx)\,dx+a^2\int \csc^6(c+dx)\sec^2(c+dx)\,dx+\left(2a^2\right)\int \csc^6(c+dx \\
&= \frac{a^2\,\mathrm{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^6}\,dx,x,\tan(c+dx)\right)}{d}-\frac{a^2\,\mathrm{Subst}\left(\int \left(1+2x^2+x^4\right)dx,x,\cot(\right.}{d} \\
&= -\frac{a^2\cot(c+dx)}{d}-\frac{2a^2\cot^3(c+dx)}{3d}-\frac{a^2\cot^5(c+dx)}{5d}+\frac{a^2\,\mathrm{Subst}\left(\int \left(1+\frac{1}{x^6}+\right.\right.}{} \\
&= -\frac{4a^2\cot(c+dx)}{d}-\frac{5a^2\cot^3(c+dx)}{3d}-\frac{2a^2\cot^5(c+dx)}{5d}-\frac{2a^2\csc(c+dx)}{d}-2a \\
&= \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d}-\frac{4a^2\cot(c+dx)}{d}-\frac{5a^2\cot^3(c+dx)}{3d}-\frac{2a^2\cot^5(c+d}{5d}
\end{aligned}
$$

Mathematica [B] time = 0.945245, size = 317, normalized size = 2.46

$$
\frac{a^2\cos(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^2\left(\csc(2c)(216\sin(c-dx)-416\sin(c+dx)+624\sin(2(c+dx))-416\sin\right.}{}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^6*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Cos[c + d*x]*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(-3840*Cos[c + d*
x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 3840*Cos[c + d*x]*Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]] + Csc[2*c]*Csc[(c + d*x)/2]^4*Csc[c + d*x]*(3
20*Sin[2*c] - 596*Sin[d*x] + 864*Sin[2*d*x] + 216*Sin[c - d*x] - 416*Sin[c
+ d*x] + 624*Sin[2*(c + d*x)] - 416*Sin[3*(c + d*x)] + 104*Sin[4*(c + d*x)]
 - 596*Sin[2*c + d*x] - 680*Sin[3*c + d*x] + 894*Sin[c + 2*d*x] + 224*Sin[2
```

```
*(c + 2*d*x)] + 894*Sin[3*c + 2*d*x] + 480*Sin[4*c + 2*d*x] - 776*Sin[c + 3
*d*x] - 596*Sin[2*c + 3*d*x] - 596*Sin[4*c + 3*d*x] - 120*Sin[5*c + 3*d*x]
+ 149*Sin[3*c + 4*d*x] + 149*Sin[5*c + 4*d*x])))/(7680*d)
```

Maple [A] time = 0.061, size = 202, normalized size = 1.6

$$-\frac{56\,a^2\cot(dx+c)}{15\,d}-\frac{a^2\cot(dx+c)(\csc(dx+c))^4}{5\,d}-\frac{4\,a^2\cot(dx+c)(\csc(dx+c))^2}{15\,d}-\frac{2\,a^2}{5\,d(\sin(dx+c))^5}-\frac{2}{3\,d(\sin}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -56/15*a^2*cot(d*x+c)/d-1/5/d*a^2*cot(d*x+c)*csc(d*x+c)^4-4/15/d*a^2*cot(d*
x+c)*csc(d*x+c)^2-2/5/d*a^2/sin(d*x+c)^5-2/3/d*a^2/sin(d*x+c)^3-2/d*a^2/sin
(d*x+c)+2/d*a^2*ln(sec(d*x+c)+tan(d*x+c))-1/5/d*a^2/sin(d*x+c)^5/cos(d*x+c)
-2/5/d*a^2/sin(d*x+c)^3/cos(d*x+c)+8/5/d*a^2/sin(d*x+c)/cos(d*x+c)
```

Maxima [A] time = 1.03421, size = 194, normalized size = 1.5

$$-\frac{a^2\left(\frac{2\left(15\,\sin(dx+c)^4+5\,\sin(dx+c)^2+3\right)}{\sin(dx+c)^5}-15\,\log(\sin(dx+c)+1)+15\,\log(\sin(dx+c)-1)\right)+3\,a^2\left(\frac{15\,\tan(dx+c)^4+5\,\tan(dx+c)^2\cdots}{\tan(dx+c)^5}\right.}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/15*(a^2*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)/sin(d*x + c)^5 - 1
5*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 3*a^2*((15*tan(d*x +
c)^4 + 5*tan(d*x + c)^2 + 1)/tan(d*x + c)^5 - 5*tan(d*x + c)) + (15*tan(d*x
 + c)^4 + 10*tan(d*x + c)^2 + 3)*a^2/tan(d*x + c)^5)/d
```

Fricas [A] time = 1.73451, size = 525, normalized size = 4.07

$$-\frac{56\,a^2\cos(dx+c)^4-82\,a^2\cos(dx+c)^3-32\,a^2\cos(dx+c)^2+76\,a^2\cos(dx+c)-15\left(a^2\cos(dx+c)^3-2\,a^2\cos(d\cdots\right.}{15\left(d\cos(\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(56*a^2*cos(d*x + c)^4 - 82*a^2*cos(d*x + c)^3 - 32*a^2*cos(d*x + c)^
2 + 76*a^2*cos(d*x + c) - 15*(a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c)^2 + a
^2*cos(d*x + c))*log(sin(d*x + c) + 1)*sin(d*x + c) + 15*(a^2*cos(d*x + c)^
3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*log(-sin(d*x + c) + 1)*sin(d*x
 + c) - 15*a^2)/((d*cos(d*x + c)^3 - 2*d*cos(d*x + c)^2 + d*cos(d*x + c))*s
in(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.47779, size = 184, normalized size = 1.43

$$\frac{240\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-240\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+15\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{240\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-\frac{345}{}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/120*(240*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 240*a^2*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) + 15*a^2*tan(1/2*d*x + 1/2*c) - 240*a^2*tan(1/2*d*x + 1
/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (345*a^2*tan(1/2*d*x + 1/2*c)^4 + 35*a
^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.36 $\int \csc^8(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=163

$$\frac{a^2\tan(c+dx)}{d}-\frac{2a^2\cot^7(c+dx)}{7d}-\frac{7a^2\cot^5(c+dx)}{5d}-\frac{3a^2\cot^3(c+dx)}{d}-\frac{5a^2\cot(c+dx)}{d}-\frac{2a^2\csc^7(c+dx)}{7d}-\frac{2a^2}{}$$

```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/d - (5*a^2*Cot[c + d*x])/d - (3*a^2*Cot[c + d
*x]^3)/d - (7*a^2*Cot[c + d*x]^5)/(5*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) - (2
*a^2*Csc[c + d*x])/d - (2*a^2*Csc[c + d*x]^3)/(3*d) - (2*a^2*Csc[c + d*x]^5
)/(5*d) - (2*a^2*Csc[c + d*x]^7)/(7*d) + (a^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.242792, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2873, 3767, 2621, 302, 207, 2620, 270}

$$\frac{a^2\tan(c+dx)}{d}-\frac{2a^2\cot^7(c+dx)}{7d}-\frac{7a^2\cot^5(c+dx)}{5d}-\frac{3a^2\cot^3(c+dx)}{d}-\frac{5a^2\cot(c+dx)}{d}-\frac{2a^2\csc^7(c+dx)}{7d}-\frac{2a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^8*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/d - (5*a^2*Cot[c + d*x])/d - (3*a^2*Cot[c + d
*x]^3)/d - (7*a^2*Cot[c + d*x]^5)/(5*d) - (2*a^2*Cot[c + d*x]^7)/(7*d) - (2
*a^2*Csc[c + d*x])/d - (2*a^2*Csc[c + d*x]^3)/(3*d) - (2*a^2*Csc[c + d*x]^5
)/(5*d) - (2*a^2*Csc[c + d*x]^7)/(7*d) + (a^2*Tan[c + d*x])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^2\,dx = \int (-a-a\cos(c+dx))^2\csc^8(c+dx)\sec^2(c+dx)\,dx$$
$$= \int \left(a^2\csc^8(c+dx)+2a^2\csc^8(c+dx)\sec(c+dx)+a^2\csc^8(c+dx)\sec^2(c+dx)\right)$$
$$= a^2\int \csc^8(c+dx)\,dx + a^2\int \csc^8(c+dx)\sec^2(c+dx)\,dx + \left(2a^2\right)\int \csc^8(c+dx$$
$$= \frac{a^2\,\mathrm{Subst}\left(\int \frac{\left(1+x^2\right)^4}{x^8}\,dx, x, \tan(c+dx)\right)}{d} - \frac{a^2\,\mathrm{Subst}\left(\int \left(1+3x^2+3x^4+x^6\right)dx, x\right.}{d}$$
$$= -\frac{a^2\cot(c+dx)}{d} - \frac{a^2\cot^3(c+dx)}{d} - \frac{3a^2\cot^5(c+dx)}{5d} - \frac{a^2\cot^7(c+dx)}{7d} + \frac{a^2\,\mathrm{Su}}{}$$
$$= -\frac{5a^2\cot(c+dx)}{d} - \frac{3a^2\cot^3(c+dx)}{d} - \frac{7a^2\cot^5(c+dx)}{5d} - \frac{2a^2\cot^7(c+dx)}{7d} - 2$$
$$= \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d} - \frac{5a^2\cot(c+dx)}{d} - \frac{3a^2\cot^3(c+dx)}{d} - \frac{7a^2\cot^5(c+d}{5d}$$

Mathematica [B] time = 1.23118, size = 428, normalized size = 2.63

$$\frac{a^2\cos(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^2\left(-32\csc(2c)(-7264\sin(c-dx)+14208\sin(c+dx)-19536\sin(2(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^8*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (a^2*Cos[c + d*x]*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(-6881280*Cos[c +
 d*x]*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6881280*Cos[c + d*x]*Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]] - 32*Csc[2*c]*Csc[(c + d*x)/2]^4*Csc[c
+ d*x]^3*(-9856*Sin[2*c] + 17288*Sin[d*x] - 29056*Sin[2*d*x] - 7264*Sin[c -
 d*x] + 14208*Sin[c + d*x] - 19536*Sin[2*(c + d*x)] + 7104*Sin[3*(c + d*x)]
 + 7104*Sin[4*(c + d*x)] - 7104*Sin[5*(c + d*x)] + 1776*Sin[6*(c + d*x)] +
```

```
17288*Sin[2*c + d*x] + 20384*Sin[3*c + d*x] - 23771*Sin[c + 2*d*x] + 7104*S
in[2*(c + 2*d*x)] - 23771*Sin[3*c + 2*d*x] - 8960*Sin[4*c + 2*d*x] + 19984*
Sin[c + 3*d*x] + 8644*Sin[2*c + 3*d*x] + 8644*Sin[4*c + 3*d*x] - 6160*Sin[5
*c + 3*d*x] + 8644*Sin[3*c + 4*d*x] + 8644*Sin[5*c + 4*d*x] + 6720*Sin[6*c
+ 4*d*x] - 12144*Sin[3*c + 5*d*x] - 8644*Sin[4*c + 5*d*x] - 8644*Sin[6*c +
5*d*x] - 1680*Sin[7*c + 5*d*x] + 3456*Sin[4*c + 6*d*x] + 2161*Sin[5*c + 6*d
*x] + 2161*Sin[7*c + 6*d*x])))/(13762560*d)
```

Maple [A] time = 0.086, size = 264, normalized size = 1.6

$$-\frac{144\,a^2\cot(dx+c)}{35\,d}-\frac{a^2\cot(dx+c)\,(\csc(dx+c))^6}{7\,d}-\frac{6\,a^2\cot(dx+c)\,(\csc(dx+c))^4}{35\,d}-\frac{8\,a^2\cot(dx+c)\,(\csc(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^8*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -144/35*a^2*cot(d*x+c)/d-1/7/d*a^2*cot(d*x+c)*csc(d*x+c)^6-6/35/d*a^2*cot(d
*x+c)*csc(d*x+c)^4-8/35/d*a^2*cot(d*x+c)*csc(d*x+c)^2-2/7/d*a^2/sin(d*x+c)^
7-2/5/d*a^2/sin(d*x+c)^5-2/3/d*a^2/sin(d*x+c)^3-2/d*a^2/sin(d*x+c)+2/d*a^2*
ln(sec(d*x+c)+tan(d*x+c))-1/7/d*a^2/sin(d*x+c)^7/cos(d*x+c)-8/35/d*a^2/sin(
d*x+c)^5/cos(d*x+c)-16/35/d*a^2/sin(d*x+c)^3/cos(d*x+c)+64/35/d*a^2/sin(d*x
+c)/cos(d*x+c)
```

Maxima [A] time = 1.03769, size = 236, normalized size = 1.45

$$-\frac{a^2\left(\frac{2\left(105\,\sin(dx+c)^6+35\,\sin(dx+c)^4+21\,\sin(dx+c)^2+15\right)}{\sin(dx+c)^7}-105\,\log(\sin(dx+c)+1)+105\,\log(\sin(dx+c)-1)\right)+3\,a^2\left(\frac{140\,\tan}{}\right.}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/105*(a^2*(2*(105*sin(d*x + c)^6 + 35*sin(d*x + c)^4 + 21*sin(d*x + c)^2
+ 15)/sin(d*x + c)^7 - 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1
)) + 3*a^2*((140*tan(d*x + c)^6 + 70*tan(d*x + c)^4 + 28*tan(d*x + c)^2 + 5
)/tan(d*x + c)^7 - 35*tan(d*x + c)) + 3*(35*tan(d*x + c)^6 + 35*tan(d*x + c
)^4 + 21*tan(d*x + c)^2 + 5)*a^2/tan(d*x + c)^7)/d
```

Fricas [A] time = 1.87472, size = 694, normalized size = 4.26

$$-\frac{432\,a^2\cos(dx+c)^6-654\,a^2\cos(dx+c)^5-636\,a^2\cos(dx+c)^4+1226\,a^2\cos(dx+c)^3+74\,a^2\cos(dx+c)^2-562}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/105*(432*a^2*cos(d*x + c)^6 - 654*a^2*cos(d*x + c)^5 - 636*a^2*cos(d*x +
 c)^4 + 1226*a^2*cos(d*x + c)^3 + 74*a^2*cos(d*x + c)^2 - 562*a^2*cos(d*x +
```

```
 c) - 105*(a^2*cos(d*x + c)^5 - 2*a^2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^2
 - a^2*cos(d*x + c))*log(sin(d*x + c) + 1)*sin(d*x + c) + 105*(a^2*cos(d*x
+ c)^5 - 2*a^2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^2 - a^2*cos(d*x + c))*lo
g(-sin(d*x + c) + 1)*sin(d*x + c) + 105*a^2)/((d*cos(d*x + c)^5 - 2*d*cos(d
*x + c)^4 + 2*d*cos(d*x + c)^2 - d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**8*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.42212, size = 227, normalized size = 1.39

$$\frac{35\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6720\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6720\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+945\,a^2\tan\left(\frac{1}{2}\,dx+\right.}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/3360*(35*a^2*tan(1/2*d*x + 1/2*c)^3 + 6720*a^2*log(abs(tan(1/2*d*x + 1/2*
c) + 1)) - 6720*a^2*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 945*a^2*tan(1/2*d*
x + 1/2*c) - 6720*a^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (
10710*a^2*tan(1/2*d*x + 1/2*c)^6 + 1330*a^2*tan(1/2*d*x + 1/2*c)^4 + 189*a^
2*tan(1/2*d*x + 1/2*c)^2 + 15*a^2)/tan(1/2*d*x + 1/2*c)^7)/d
```

3.37 $\int \csc^{10}(c+dx)(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=201

$$\frac{a^2\tan(c+dx)}{d}-\frac{2a^2\cot^9(c+dx)}{9d}-\frac{9a^2\cot^7(c+dx)}{7d}-\frac{16a^2\cot^5(c+dx)}{5d}-\frac{14a^2\cot^3(c+dx)}{3d}-\frac{6a^2\cot(c+dx)}{d}-2$$

```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/d - (6*a^2*Cot[c + d*x])/d - (14*a^2*Cot[c +
d*x]^3)/(3*d) - (16*a^2*Cot[c + d*x]^5)/(5*d) - (9*a^2*Cot[c + d*x]^7)/(7*d
) - (2*a^2*Cot[c + d*x]^9)/(9*d) - (2*a^2*Csc[c + d*x])/d - (2*a^2*Csc[c +
d*x]^3)/(3*d) - (2*a^2*Csc[c + d*x]^5)/(5*d) - (2*a^2*Csc[c + d*x]^7)/(7*d)
 - (2*a^2*Csc[c + d*x]^9)/(9*d) + (a^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.258413, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2873, 3767, 2621, 302, 207, 2620, 270}

$$\frac{a^2\tan(c+dx)}{d}-\frac{2a^2\cot^9(c+dx)}{9d}-\frac{9a^2\cot^7(c+dx)}{7d}-\frac{16a^2\cot^5(c+dx)}{5d}-\frac{14a^2\cot^3(c+dx)}{3d}-\frac{6a^2\cot(c+dx)}{d}-2$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^10*(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (2*a^2*ArcTanh[Sin[c + d*x]])/d - (6*a^2*Cot[c + d*x])/d - (14*a^2*Cot[c +
d*x]^3)/(3*d) - (16*a^2*Cot[c + d*x]^5)/(5*d) - (9*a^2*Cot[c + d*x]^7)/(7*d
) - (2*a^2*Cot[c + d*x]^9)/(9*d) - (2*a^2*Csc[c + d*x])/d - (2*a^2*Csc[c +
d*x]^3)/(3*d) - (2*a^2*Csc[c + d*x]^5)/(5*d) - (2*a^2*Csc[c + d*x]^7)/(7*d)
 - (2*a^2*Csc[c + d*x]^9)/(9*d) + (a^2*Tan[c + d*x])/d
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^2\,dx = \int (-a-a\cos(c+dx))^2\csc^{10}(c+dx)\sec^2(c+dx)\,dx$$
$$= \int \left(a^2\csc^{10}(c+dx)+2a^2\csc^{10}(c+dx)\sec(c+dx)+a^2\csc^{10}(c+dx)\sec^2(c+\right.$$
$$= a^2\int \csc^{10}(c+dx)\,dx + a^2\int \csc^{10}(c+dx)\sec^2(c+dx)\,dx + \left(2a^2\right)\int \csc^{10}(c+$$
$$= \frac{a^2\,\mathrm{Subst}\left(\int \frac{\left(1+x^2\right)^5}{x^{10}}\,dx, x, \tan(c+dx)\right)}{d} - \frac{a^2\,\mathrm{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x\right.\right.}{d}$$
$$= -\frac{a^2\cot(c+dx)}{d} - \frac{4a^2\cot^3(c+dx)}{3d} - \frac{6a^2\cot^5(c+dx)}{5d} - \frac{4a^2\cot^7(c+dx)}{7d} - \frac{a^2}{}$$
$$= -\frac{6a^2\cot(c+dx)}{d} - \frac{14a^2\cot^3(c+dx)}{3d} - \frac{16a^2\cot^5(c+dx)}{5d} - \frac{9a^2\cot^7(c+dx)}{7d} -$$
$$= \frac{2a^2\tanh^{-1}(\sin(c+dx))}{d} - \frac{6a^2\cot(c+dx)}{d} - \frac{14a^2\cot^3(c+dx)}{3d} - \frac{16a^2\cot^5(c+}{5d}$$

Mathematica [B] time = 6.83056, size = 1050, normalized size = 5.22

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^10*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-6899*Cos[c + d*x]^2*Cot[c/2]*Csc[c/2 + (d*x)/2]^2*Sec[c/2 + (d*x)/2]^4*(a
 + a*Sec[c + d*x])^2)/(80640*d) - (193*Cos[c + d*x]^2*Cot[c/2]*Csc[c/2 + (d
*x)/2]^4*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(13440*d) - (71*Cos[c
 + d*x]^2*Cot[c/2]*Csc[c/2 + (d*x)/2]^6*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c +
 d*x])^2)/(32256*d) - (Cos[c + d*x]^2*Cot[c/2]*Csc[c/2 + (d*x)/2]^8*Sec[c/2
 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(4608*d) - (Cos[c + d*x]^2*Log[Cos[c/
2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x]
```

```
)^2)/(2*d) + (Cos[c + d*x]^2*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*S
ec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2)/(2*d) + (123041*Cos[c + d*x]^2*
Csc[c/2]*Csc[c/2 + (d*x)/2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin
[(d*x)/2])/(161280*d) + (6899*Cos[c + d*x]^2*Csc[c/2]*Csc[c/2 + (d*x)/2]^3*
Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(80640*d) + (193*
Cos[c + d*x]^2*Csc[c/2]*Csc[c/2 + (d*x)/2]^5*Sec[c/2 + (d*x)/2]^4*(a + a*Se
c[c + d*x])^2*Sin[(d*x)/2])/(13440*d) + (71*Cos[c + d*x]^2*Csc[c/2]*Csc[c/2
 + (d*x)/2]^7*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(32
256*d) + (Cos[c + d*x]^2*Csc[c/2]*Csc[c/2 + (d*x)/2]^9*Sec[c/2 + (d*x)/2]^4
*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(4608*d) + (803*Cos[c + d*x]^2*Sec[c/
2]*Sec[c/2 + (d*x)/2]^5*(a + a*Sec[c + d*x])^2*Sin[(d*x)/2])/(7680*d) + (49
*Cos[c + d*x]^2*Sec[c/2]*Sec[c/2 + (d*x)/2]^7*(a + a*Sec[c + d*x])^2*Sin[(d
*x)/2])/(7680*d) + (Cos[c + d*x]^2*Sec[c/2]*Sec[c/2 + (d*x)/2]^9*(a + a*Sec
[c + d*x])^2*Sin[(d*x)/2])/(2560*d) + (Cos[c + d*x]*Sec[c]*Sec[c/2 + (d*x)/
2]^4*(a + a*Sec[c + d*x])^2*Sin[d*x])/(4*d) + (49*Cos[c + d*x]^2*Sec[c/2 +
(d*x)/2]^6*(a + a*Sec[c + d*x])^2*Tan[c/2])/(7680*d) + (Cos[c + d*x]^2*Sec[
c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^2*Tan[c/2])/(2560*d)
```

Maple [A] time = 0.081, size = 326, normalized size = 1.6

$$-\frac{1408\,a^2\cot(dx+c)}{315\,d}-\frac{a^2\cot(dx+c)\,(\csc(dx+c))^8}{9\,d}-\frac{8\,a^2\cot(dx+c)\,(\csc(dx+c))^6}{63\,d}-\frac{16\,a^2\cot(dx+c)\,(\csc(dx\cdots}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^10*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1408/315*a^2*cot(d*x+c)/d-1/9/d*a^2*cot(d*x+c)*csc(d*x+c)^8-8/63/d*a^2*cot
(d*x+c)*csc(d*x+c)^6-16/105/d*a^2*cot(d*x+c)*csc(d*x+c)^4-64/315/d*a^2*cot(
d*x+c)*csc(d*x+c)^2-2/9/d*a^2/sin(d*x+c)^9-2/7/d*a^2/sin(d*x+c)^7-2/5/d*a^2
/sin(d*x+c)^5-2/3/d*a^2/sin(d*x+c)^3-2/d*a^2/sin(d*x+c)+2/d*a^2*ln(sec(d*x+
c)+tan(d*x+c))-1/9/d*a^2/sin(d*x+c)^9/cos(d*x+c)-10/63/d*a^2/sin(d*x+c)^7/c
os(d*x+c)-16/63/d*a^2/sin(d*x+c)^5/cos(d*x+c)-32/63/d*a^2/sin(d*x+c)^3/cos(
d*x+c)+128/63/d*a^2/sin(d*x+c)/cos(d*x+c)
```

Maxima [A] time = 1.02784, size = 275, normalized size = 1.37

$$-\frac{a^2\left(\frac{2\left(315\sin(dx+c)^8+105\sin(dx+c)^6+63\sin(dx+c)^4+45\sin(dx+c)^2+35\right)}{\sin(dx+c)^9}-315\log(\sin(dx+c)+1)+315\log(\sin(dx+c)-1)\right)\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/315*(a^2*(2*(315*sin(d*x + c)^8 + 105*sin(d*x + c)^6 + 63*sin(d*x + c)^4
 + 45*sin(d*x + c)^2 + 35)/sin(d*x + c)^9 - 315*log(sin(d*x + c) + 1) + 315
*log(sin(d*x + c) - 1)) + 5*a^2*((315*tan(d*x + c)^8 + 210*tan(d*x + c)^6 +
 126*tan(d*x + c)^4 + 45*tan(d*x + c)^2 + 7)/tan(d*x + c)^9 - 63*tan(d*x +
c)) + (315*tan(d*x + c)^8 + 420*tan(d*x + c)^6 + 378*tan(d*x + c)^4 + 180*t
an(d*x + c)^2 + 35)*a^2/tan(d*x + c)^9)/d
```

Fricas [B] time = 1.94829, size = 1026, normalized size = 5.1

$$-\frac{1408\,a^2\cos(dx+c)^8 - 2186\,a^2\cos(dx+c)^7 - 3372\,a^2\cos(dx+c)^6 + 6200\,a^2\cos(dx+c)^5 + 2060\,a^2\cos(dx+c)^4 - 5\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/315*(1408*a^2*cos(d*x + c)^8 - 2186*a^2*cos(d*x + c)^7 - 3372*a^2*cos(d*
x + c)^6 + 6200*a^2*cos(d*x + c)^5 + 2060*a^2*cos(d*x + c)^4 - 5784*a^2*cos
(d*x + c)^3 + 268*a^2*cos(d*x + c)^2 + 1756*a^2*cos(d*x + c) - 315*(a^2*cos
(d*x + c)^7 - 2*a^2*cos(d*x + c)^6 - a^2*cos(d*x + c)^5 + 4*a^2*cos(d*x + c
)^4 - a^2*cos(d*x + c)^3 - 2*a^2*cos(d*x + c)^2 + a^2*cos(d*x + c))*log(sin
(d*x + c) + 1)*sin(d*x + c) + 315*(a^2*cos(d*x + c)^7 - 2*a^2*cos(d*x + c)^
6 - a^2*cos(d*x + c)^5 + 4*a^2*cos(d*x + c)^4 - a^2*cos(d*x + c)^3 - 2*a^2*
cos(d*x + c)^2 + a^2*cos(d*x + c))*log(-sin(d*x + c) + 1)*sin(d*x + c) - 31
5*a^2)/((d*cos(d*x + c)^7 - 2*d*cos(d*x + c)^6 - d*cos(d*x + c)^5 + 4*d*cos
(d*x + c)^4 - d*cos(d*x + c)^3 - 2*d*cos(d*x + c)^2 + d*cos(d*x + c))*sin(d
*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**10*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38362, size = 270, normalized size = 1.34

$$\frac{63\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 1155\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 80640\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 80640\,a^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\cdots\right.\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/40320*(63*a^2*tan(1/2*d*x + 1/2*c)^5 + 1155*a^2*tan(1/2*d*x + 1/2*c)^3 +
80640*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 80640*a^2*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) + 17955*a^2*tan(1/2*d*x + 1/2*c) - 80640*a^2*tan(1/2*d*x +
 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (139545*a^2*tan(1/2*d*x + 1/2*c)^8 +
 19635*a^2*tan(1/2*d*x + 1/2*c)^6 + 3591*a^2*tan(1/2*d*x + 1/2*c)^4 + 495*a
^2*tan(1/2*d*x + 1/2*c)^2 + 35*a^2)/tan(1/2*d*x + 1/2*c)^9)/d
```

3.38 $\int (a + a\sec(c + dx))^3 \sin^9(c + dx)\, dx$

Optimal. Leaf size=203

$$-\frac{a^3\cos^9(c+dx)}{9d}-\frac{3a^3\cos^8(c+dx)}{8d}+\frac{a^3\cos^7(c+dx)}{7d}+\frac{11a^3\cos^6(c+dx)}{6d}+\frac{6a^3\cos^5(c+dx)}{5d}-\frac{7a^3\cos^4(c+dx)}{2d}-$$

```
[Out] (11*a^3*Cos[c + d*x])/d + (3*a^3*Cos[c + d*x]^2)/d - (14*a^3*Cos[c + d*x]^3
)/(3*d) - (7*a^3*Cos[c + d*x]^4)/(2*d) + (6*a^3*Cos[c + d*x]^5)/(5*d) + (11
*a^3*Cos[c + d*x]^6)/(6*d) + (a^3*Cos[c + d*x]^7)/(7*d) - (3*a^3*Cos[c + d*
x]^8)/(8*d) - (a^3*Cos[c + d*x]^9)/(9*d) + (a^3*Log[Cos[c + d*x]])/d + (3*a
^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.195633, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2836, 12, 88}

$$-\frac{a^3\cos^9(c+dx)}{9d}-\frac{3a^3\cos^8(c+dx)}{8d}+\frac{a^3\cos^7(c+dx)}{7d}+\frac{11a^3\cos^6(c+dx)}{6d}+\frac{6a^3\cos^5(c+dx)}{5d}-\frac{7a^3\cos^4(c+dx)}{2d}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^9,x]
```


```
[Out] (11*a^3*Cos[c + d*x])/d + (3*a^3*Cos[c + d*x]^2)/d - (14*a^3*Cos[c + d*x]^3
)/(3*d) - (7*a^3*Cos[c + d*x]^4)/(2*d) + (6*a^3*Cos[c + d*x]^5)/(5*d) + (11
*a^3*Cos[c + d*x]^6)/(6*d) + (a^3*Cos[c + d*x]^7)/(7*d) - (3*a^3*Cos[c + d*
x]^8)/(8*d) - (a^3*Cos[c + d*x]^9)/(9*d) + (a^3*Log[Cos[c + d*x]])/d + (3*a
^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\sin^9(c+dx)\,dx = -\int (-a-a\cos(c+dx))^3\sin^6(c+dx)\tan^3(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^3(-a-x)^4(-a+x)^7}{x^3}\,dx, x, -a\cos(c+dx)\right)}{a^9 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^4(-a+x)^7}{x^3}\,dx, x, -a\cos(c+dx)\right)}{a^6 d}$$
$$= \frac{\text{Subst}\left(\int \left(-11a^8 - \frac{a^{11}}{x^3} + \frac{3a^{10}}{x^2} + \frac{a^9}{x} + 6a^7x + 14a^6x^2 - 14a^5x^3 - 6a^4x^4 + 11a^3x^5 -\right.\right.}{a^6 d}$$
$$= \frac{11a^3\cos(c+dx)}{d} + \frac{3a^3\cos^2(c+dx)}{d} - \frac{14a^3\cos^3(c+dx)}{3d} - \frac{7a^3\cos^4(c+dx)}{2d} + \frac{6}{}$$

Mathematica [A] time = 1.69703, size = 148, normalized size = 0.73

$$\frac{a^3\sec^2(c+dx)(11624760\cos(c+dx)+2188872\cos(3(c+dx))+41160\cos(4(c+dx))-204156\cos(5(c+dx))-35805}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^9,x]
```

```
[Out] (a^3*(471450 + 11624760*Cos[c + d*x] + 2188872*Cos[3*(c + d*x)] + 41160*Cos
[4*(c + d*x)] - 204156*Cos[5*(c + d*x)] - 35805*Cos[6*(c + d*x)] + 22972*Co
s[7*(c + d*x)] + 9030*Cos[8*(c + d*x)] - 820*Cos[9*(c + d*x)] - 945*Cos[10*
(c + d*x)] - 140*Cos[11*(c + d*x)] + 645120*Log[Cos[c + d*x]] + 210*Cos[2*(
c + d*x)]*(-413 + 3072*Log[Cos[c + d*x]]))*Sec[c + d*x]^2)/(1290240*d)
```

Maple [A] time = 0.05, size = 230, normalized size = 1.1

$$\frac{3328\,a^3\cos(dx+c)}{315\,d} + \frac{26\,a^3(\sin(dx+c))^8\cos(dx+c)}{9\,d} + \frac{208\,a^3\cos(dx+c)(\sin(dx+c))^6}{63\,d} + \frac{416\,a^3\cos(dx+c)(\sin(d}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^9,x)
```

```
[Out] 3328/315*a^3*cos(d*x+c)/d+26/9/d*a^3*sin(d*x+c)^8*cos(d*x+c)+208/63/d*a^3*c
os(d*x+c)*sin(d*x+c)^6+416/105/d*a^3*cos(d*x+c)*sin(d*x+c)^4+1664/315/d*a^3
*cos(d*x+c)*sin(d*x+c)^2+1/8/d*a^3*sin(d*x+c)^8+1/6/d*a^3*sin(d*x+c)^6+1/4/
d*a^3*sin(d*x+c)^4+1/2/d*a^3*sin(d*x+c)^2+a^3*ln(cos(d*x+c))/d+3/d*a^3*sin(
d*x+c)^10/cos(d*x+c)+1/2/d*a^3*sin(d*x+c)^10/cos(d*x+c)^2
```

Maxima [A] time = 0.997478, size = 213, normalized size = 1.05

$$-\frac{280\,a^3\cos(dx+c)^9 + 945\,a^3\cos(dx+c)^8 - 360\,a^3\cos(dx+c)^7 - 4620\,a^3\cos(dx+c)^6 - 3024\,a^3\cos(dx+c)^5 + 8820}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^9,x, algorithm="maxima")
```

```
[Out] -1/2520*(280*a^3*cos(d*x + c)^9 + 945*a^3*cos(d*x + c)^8 - 360*a^3*cos(d*x
+ c)^7 - 4620*a^3*cos(d*x + c)^6 - 3024*a^3*cos(d*x + c)^5 + 8820*a^3*cos(d
*x + c)^4 + 11760*a^3*cos(d*x + c)^3 - 7560*a^3*cos(d*x + c)^2 - 27720*a^3*
cos(d*x + c) - 2520*a^3*log(cos(d*x + c)) - 1260*(6*a^3*cos(d*x + c) + a^3)
/cos(d*x + c)^2)/d
```

Fricas [A] time = 2.21011, size = 539, normalized size = 2.66

$$-\frac{35840\,a^3\cos(dx+c)^{11}+120960\,a^3\cos(dx+c)^{10}-46080\,a^3\cos(dx+c)^9-591360\,a^3\cos(dx+c)^8-387072\,a^3\cos\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^9,x, algorithm="fricas")
```

```
[Out] -1/322560*(35840*a^3*cos(d*x + c)^11 + 120960*a^3*cos(d*x + c)^10 - 46080*a
^3*cos(d*x + c)^9 - 591360*a^3*cos(d*x + c)^8 - 387072*a^3*cos(d*x + c)^7 +
 1128960*a^3*cos(d*x + c)^6 + 1505280*a^3*cos(d*x + c)^5 - 967680*a^3*cos(d
*x + c)^4 - 3548160*a^3*cos(d*x + c)^3 - 322560*a^3*cos(d*x + c)^2*log(-cos
(d*x + c)) + 212205*a^3*cos(d*x + c)^2 - 967680*a^3*cos(d*x + c) - 161280*a
^3)/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**9,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.42928, size = 535, normalized size = 2.64

$$-\frac{2520\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-2520\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\frac{1260\left(9\,a^3+\frac{2\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{3\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}+\frac{45257\,a^3-39\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^9,x, algorithm="giac")
```

```
[Out] -1/2520*(2520*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 25
20*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - 1260*(9*a^3 +
 2*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a^3*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2 + (45257
*a^3 - 392193*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1467972*a^3*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3001908*a^3*(cos(d*x + c) - 1)^3/(co
s(d*x + c) + 1)^3 + 3232782*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 -
 2359854*a^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 1190196*a^3*(cos(d
*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 397764*a^3*(cos(d*x + c) - 1)^7/(cos(
```

```
d*x + c) + 1)^7 + 79281*a^3*(cos(d*x + c) - 1)^8/(cos(d*x + c) + 1)^8 - 712
9*a^3*(cos(d*x + c) - 1)^9/(cos(d*x + c) + 1)^9)/((cos(d*x + c) - 1)/(cos(d
*x + c) + 1) - 1)^9)/d
```

3.39 $\int (a + a\sec(c + dx))^3 \sin^7(c + dx)\,dx$

Optimal. Leaf size=131

$$\frac{a^3\cos^7(c+dx)}{7d}+\frac{a^3\cos^6(c+dx)}{2d}-\frac{2a^3\cos^4(c+dx)}{d}-\frac{2a^3\cos^3(c+dx)}{d}+\frac{3a^3\cos^2(c+dx)}{d}+\frac{8a^3\cos(c+dx)}{d}+a^3 s$$

```
[Out] (8*a^3*Cos[c + d*x])/d + (3*a^3*Cos[c + d*x]^2)/d - (2*a^3*Cos[c + d*x]^3)/
d - (2*a^3*Cos[c + d*x]^4)/d + (a^3*Cos[c + d*x]^6)/(2*d) + (a^3*Cos[c + d*
x]^7)/(7*d) + (3*a^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.16816, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2836, 12, 88}

$$\frac{a^3\cos^7(c+dx)}{7d}+\frac{a^3\cos^6(c+dx)}{2d}-\frac{2a^3\cos^4(c+dx)}{d}-\frac{2a^3\cos^3(c+dx)}{d}+\frac{3a^3\cos^2(c+dx)}{d}+\frac{8a^3\cos(c+dx)}{d}+a^3 s$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^7,x]
```


```
[Out] (8*a^3*Cos[c + d*x])/d + (3*a^3*Cos[c + d*x]^2)/d - (2*a^3*Cos[c + d*x]^3)/
d - (2*a^3*Cos[c + d*x]^4)/d + (a^3*Cos[c + d*x]^6)/(2*d) + (a^3*Cos[c + d*
x]^7)/(7*d) + (3*a^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\sin^7(c+dx)\,dx = -\int (-a-a\cos(c+dx))^3\sin^4(c+dx)\tan^3(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^3(-a-x)^3(-a+x)^6}{x^3}\,dx, x, -a\cos(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^3(-a+x)^6}{x^3}\,dx, x, -a\cos(c+dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(-8a^6 - \frac{a^9}{x^3} + \frac{3a^8}{x^2} + 6a^5x + 6a^4x^2 - 8a^3x^3 + 3ax^5 - x^6\right) dx, x, -a\cos(c\right.}{a^4 d}$$
$$= \frac{8a^3\cos(c+dx)}{d} + \frac{3a^3\cos^2(c+dx)}{d} - \frac{2a^3\cos^3(c+dx)}{d} - \frac{2a^3\cos^4(c+dx)}{d} + \frac{a^3}{}$$

Mathematica [A] time = 0.920869, size = 106, normalized size = 0.81

$$\frac{a^3(14014\cos(c+dx) - 210\cos(2(c+dx)) + 2548\cos(3(c+dx)) + 196\cos(4(c+dx)) - 188\cos(5(c+dx)) - 56\cos(6(c}{1792d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^7,x]
```

```
[Out] (a^3*(427 + 14014*Cos[c + d*x] - 210*Cos[2*(c + d*x)] + 2548*Cos[3*(c + d*x
)] + 196*Cos[4*(c + d*x)] - 188*Cos[5*(c + d*x)] - 56*Cos[6*(c + d*x)] + 9*
Cos[7*(c + d*x)] + 7*Cos[8*(c + d*x)] + Cos[9*(c + d*x)])*Sec[c + d*x]^2)/(
1792*d)
```

Maple [A] time = 0.049, size = 130, normalized size = 1.

$$\frac{64\,a^3\cos(dx+c)}{7\,d} + \frac{20\,a^3\cos(dx+c)(\sin(dx+c))^6}{7\,d} + \frac{24\,a^3\cos(dx+c)(\sin(dx+c))^4}{7\,d} + \frac{32\,a^3\cos(dx+c)(\sin(dx+c}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^7,x)
```

```
[Out] 64/7*a^3*cos(d*x+c)/d+20/7/d*a^3*cos(d*x+c)*sin(d*x+c)^6+24/7/d*a^3*cos(d*x
+c)*sin(d*x+c)^4+32/7/d*a^3*cos(d*x+c)*sin(d*x+c)^2+3/d*a^3*sin(d*x+c)^8/co
s(d*x+c)+1/2/d*a^3*sin(d*x+c)^8/cos(d*x+c)^2
```

Maxima [A] time = 1.00762, size = 144, normalized size = 1.1

$$\frac{2\,a^3\cos(dx+c)^7 + 7\,a^3\cos(dx+c)^6 - 28\,a^3\cos(dx+c)^4 - 28\,a^3\cos(dx+c)^3 + 42\,a^3\cos(dx+c)^2 + 112\,a^3\cos(dx+c}{14\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/14*(2*a^3*cos(d*x + c)^7 + 7*a^3*cos(d*x + c)^6 - 28*a^3*cos(d*x + c)^4 -
 28*a^3*cos(d*x + c)^3 + 42*a^3*cos(d*x + c)^2 + 112*a^3*cos(d*x + c) + 7*(
```

```
6*a^3*cos(d*x + c) + a^3)/cos(d*x + c)^2)/d
```

Fricas [A] time = 1.86464, size = 316, normalized size = 2.41

$$\frac{32\,a^3\cos\left(dx+c\right)^9+112\,a^3\cos\left(dx+c\right)^8-448\,a^3\cos\left(dx+c\right)^6-448\,a^3\cos\left(dx+c\right)^5+672\,a^3\cos\left(dx+c\right)^4+1792\,a^3}{224\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/224*(32*a^3*cos(d*x + c)^9 + 112*a^3*cos(d*x + c)^8 - 448*a^3*cos(d*x + c
)^6 - 448*a^3*cos(d*x + c)^5 + 672*a^3*cos(d*x + c)^4 + 1792*a^3*cos(d*x +
c)^3 - 203*a^3*cos(d*x + c)^2 + 672*a^3*cos(d*x + c) + 112*a^3)/(d*cos(d*x
+ c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**7,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31763, size = 323, normalized size = 2.47

$$\frac{2\left(\frac{7\left(3\,a^3+\frac{2\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}-\frac{43\,a^3-\frac{273\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{672\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{630\,a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{343\,a^3(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{105\,a^3(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}+\frac{14\,a^3(\cos(dx+c)}{(\cos(dx+c)}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^7}\right.}{7\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 2/7*(7*(3*a^3 + 2*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + 1)^2 - (43*a^3 - 273*a^3*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) + 672*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 630*a^
3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 343*a^3*(cos(d*x + c) - 1)^4/
(cos(d*x + c) + 1)^4 - 105*a^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 +
14*a^3*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6)/((cos(d*x + c) - 1)/(cos(
d*x + c) + 1) - 1)^7)/d
```

3.40 $\int (a+a\sec(c+dx))^3 \sin^5(c+dx)\,dx$

Optimal. Leaf size=134

$$-\frac{a^3\cos^5(c+dx)}{5d}-\frac{3a^3\cos^4(c+dx)}{4d}-\frac{a^3\cos^3(c+dx)}{3d}+\frac{5a^3\cos^2(c+dx)}{2d}+\frac{5a^3\cos(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\mathrm{sec}}{}$$

```
[Out] (5*a^3*Cos[c + d*x])/d + (5*a^3*Cos[c + d*x]^2)/(2*d) - (a^3*Cos[c + d*x]^3
)/(3*d) - (3*a^3*Cos[c + d*x]^4)/(4*d) - (a^3*Cos[c + d*x]^5)/(5*d) - (a^3*
Log[Cos[c + d*x]])/d + (3*a^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.166756, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2836, 12, 88}

$$-\frac{a^3\cos^5(c+dx)}{5d}-\frac{3a^3\cos^4(c+dx)}{4d}-\frac{a^3\cos^3(c+dx)}{3d}+\frac{5a^3\cos^2(c+dx)}{2d}+\frac{5a^3\cos(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\mathrm{sec}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^5,x]
```

```
[Out] (5*a^3*Cos[c + d*x])/d + (5*a^3*Cos[c + d*x]^2)/(2*d) - (a^3*Cos[c + d*x]^3
)/(3*d) - (3*a^3*Cos[c + d*x]^4)/(4*d) - (a^3*Cos[c + d*x]^5)/(5*d) - (a^3*
Log[Cos[c + d*x]])/d + (3*a^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\sin^5(c+dx)\,dx = -\int (-a-a\cos(c+dx))^3\sin^2(c+dx)\tan^3(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^3(-a-x)^2(-a+x)^5}{x^3}\,dx, x, -a\cos(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^2(-a+x)^5}{x^3}\,dx, x, -a\cos(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(-5a^4-\frac{a^7}{x^3}+\frac{3a^6}{x^2}-\frac{a^5}{x}+5a^3x+a^2x^2-3ax^3+x^4\right)dx, x, -a\cos(c+\right.}{a^2 d}$$
$$= \frac{5a^3\cos(c+dx)}{d}+\frac{5a^3\cos^2(c+dx)}{2d}-\frac{a^3\cos^3(c+dx)}{3d}-\frac{3a^3\cos^4(c+dx)}{4d}-\frac{a}{}$$

Mathematica [A] time = 0.628204, size = 108, normalized size = 0.81

$$-\frac{a^3\sec^2(c+dx)(-12350\cos(c+dx)-2074\cos(3(c+dx))-330\cos(4(c+dx))+82\cos(5(c+dx))+45\cos(6(c+d}{1920d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^5,x]
```

```
[Out] -(a^3*(-120 - 12350*Cos[c + d*x] - 2074*Cos[3*(c + d*x)] - 330*Cos[4*(c + d
*x)] + 82*Cos[5*(c + d*x)] + 45*Cos[6*(c + d*x)] + 6*Cos[7*(c + d*x)] + 960
*Log[Cos[c + d*x]] + 15*Cos[2*(c + d*x)]*(31 + 64*Log[Cos[c + d*x]]))*Sec[c
 + d*x]^2)/(1920*d)
```

Maple [A] time = 0.047, size = 155, normalized size = 1.2

$$\frac{112\,a^3\cos(dx+c)}{15\,d}+\frac{14\,a^3\cos(dx+c)(\sin(dx+c))^4}{5\,d}+\frac{56\,a^3\cos(dx+c)(\sin(dx+c))^2}{15\,d}-\frac{a^3(\sin(dx+c))^4}{4\,d}-\frac{a^3(\text{si}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^5,x)
```

```
[Out] 112/15*a^3*cos(d*x+c)/d+14/5/d*a^3*cos(d*x+c)*sin(d*x+c)^4+56/15/d*a^3*cos(
d*x+c)*sin(d*x+c)^2-1/4/d*a^3*sin(d*x+c)^4-1/2/d*a^3*sin(d*x+c)^2-a^3*ln(co
s(d*x+c))/d+3/d*a^3*sin(d*x+c)^6/cos(d*x+c)+1/2/d*a^3*sin(d*x+c)^6/cos(d*x+
c)^2
```

Maxima [A] time = 1.01187, size = 143, normalized size = 1.07

$$-\frac{12\,a^3\cos(dx+c)^5+45\,a^3\cos(dx+c)^4+20\,a^3\cos(dx+c)^3-150\,a^3\cos(dx+c)^2-300\,a^3\cos(dx+c)+60\,a^3\log}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(12*a^3*cos(d*x + c)^5 + 45*a^3*cos(d*x + c)^4 + 20*a^3*cos(d*x + c)^
3 - 150*a^3*cos(d*x + c)^2 - 300*a^3*cos(d*x + c) + 60*a^3*log(cos(d*x + c)
) - 30*(6*a^3*cos(d*x + c) + a^3)/cos(d*x + c)^2)/d
```

Fricas [A] time = 1.8304, size = 346, normalized size = 2.58

$$-\frac{96\,a^3\cos\left(dx+c\right)^7+360\,a^3\cos\left(dx+c\right)^6+160\,a^3\cos\left(dx+c\right)^5-1200\,a^3\cos\left(dx+c\right)^4-2400\,a^3\cos\left(dx+c\right)^3+480\,a^3}{480\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/480*(96*a^3*cos(d*x + c)^7 + 360*a^3*cos(d*x + c)^6 + 160*a^3*cos(d*x +
c)^5 - 1200*a^3*cos(d*x + c)^4 - 2400*a^3*cos(d*x + c)^3 + 480*a^3*cos(d*x
+ c)^2*log(-cos(d*x + c)) + 465*a^3*cos(d*x + c)^2 - 1440*a^3*cos(d*x + c)
- 240*a^3)/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29833, size = 401, normalized size = 2.99

$$\frac{60\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-60\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{30\left(15\,a^3+\frac{14\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}-\frac{399\,a^3-\frac{1395\,a^3(\cos(dx+c)-}{\cos(dx+c)+1}}{}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*a^3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a^3*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 30*(15*a^3 + 14*a^3*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a^3*(cos(d*x + c) - 1)^2/(cos(d*x
 + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2 - (399*a^3 - 13
95*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 390*a^3*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 + 650*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 -
 565*a^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 137*a^3*(cos(d*x + c)
- 1)^5/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5)
/d
```

3.41 $\int (a+a\sec(c+dx))^3\sin^3(c+dx)\,dx$

Optimal. Leaf size=98

$$\frac{a^3\cos^3(c+dx)}{3d}+\frac{3a^3\cos^2(c+dx)}{2d}+\frac{2a^3\cos(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}-\frac{2a^3\log(\cos(c+dx))}{d}$$

```
[Out] (2*a^3*Cos[c + d*x])/d + (3*a^3*Cos[c + d*x]^2)/(2*d) + (a^3*Cos[c + d*x]^3
)/(3*d) - (2*a^3*Log[Cos[c + d*x]])/d + (3*a^3*Sec[c + d*x])/d + (a^3*Sec[c
 + d*x]^2)/(2*d)
```

Rubi [A] time = 0.0959318, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3872, 2707, 75}

$$\frac{a^3\cos^3(c+dx)}{3d}+\frac{3a^3\cos^2(c+dx)}{2d}+\frac{2a^3\cos(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}-\frac{2a^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^3,x]
```

```
[Out] (2*a^3*Cos[c + d*x])/d + (3*a^3*Cos[c + d*x]^2)/(2*d) + (a^3*Cos[c + d*x]^3
)/(3*d) - (2*a^3*Log[Cos[c + d*x]])/d + (3*a^3*Sec[c + d*x])/d + (a^3*Sec[c
 + d*x]^2)/(2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^3\sin^3(c+dx)\,dx &= -\int (-a-a\cos(c+dx))^3\tan^3(c+dx)\,dx \\ &= \frac{\text{Subst}\left(\int \frac{(-a-x)(-a+x)^4}{x^3}\,dx, x, -a\cos(c+dx)\right)}{d} \\ &= \frac{\text{Subst}\left(\int \left(-2a^2-\frac{a^5}{x^3}+\frac{3a^4}{x^2}-\frac{2a^3}{x}+3ax-x^2\right)dx, x, -a\cos(c+dx)\right)}{d} \\ &= \frac{2a^3\cos(c+dx)}{d}+\frac{3a^3\cos^2(c+dx)}{2d}+\frac{a^3\cos^3(c+dx)}{3d}-\frac{2a^3\log(\cos(c+dx))}{d}+\end{aligned}$$

Mathematica [A] time = 0.194758, size = 86, normalized size = 0.88

$$\frac{a^3\sec^2(c+dx)(226\cos(c+dx)+29\cos(3(c+dx))+9\cos(4(c+dx))+\cos(5(c+dx))-48\log(\cos(c+dx))-8\cos(2(c}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^3,x]
```

```
[Out] (a^3*(-41 + 226*Cos[c + d*x] + 29*Cos[3*(c + d*x)] + 9*Cos[4*(c + d*x)] + C
os[5*(c + d*x)] - 48*Log[Cos[c + d*x]] - 8*Cos[2*(c + d*x)]*(7 + 6*Log[Cos[
c + d*x]]))*Sec[c + d*x]^2)/(48*d)
```

Maple [A] time = 0.045, size = 109, normalized size = 1.1

$$\frac{8a^3\cos(dx+c)(\sin(dx+c))^2}{3d}+\frac{16a^3\cos(dx+c)}{3d}-\frac{3a^3(\sin(dx+c))^2}{2d}-2\frac{a^3\ln(\cos(dx+c))}{d}+3\frac{a^3(\sin(dx+c))^4}{d\cos(dx+c)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^3,x)
```

```
[Out] 8/3/d*a^3*cos(d*x+c)*sin(d*x+c)^2+16/3*a^3*cos(d*x+c)/d-3/2/d*a^3*sin(d*x+c
)^2-2*a^3*ln(cos(d*x+c))/d+3/d*a^3*sin(d*x+c)^4/cos(d*x+c)+1/2/d*a^3*tan(d*
x+c)^2
```

Maxima [A] time = 1.01041, size = 108, normalized size = 1.1

$$\frac{2a^3\cos(dx+c)^3+9a^3\cos(dx+c)^2+12a^3\cos(dx+c)-12a^3\log(\cos(dx+c))+\frac{3(6a^3\cos(dx+c)+a^3)}{\cos(dx+c)^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a^3*cos(d*x + c)^3 + 9*a^3*cos(d*x + c)^2 + 12*a^3*cos(d*x + c) - 12
*a^3*log(cos(d*x + c)) + 3*(6*a^3*cos(d*x + c) + a^3)/cos(d*x + c)^2)/d
```

Fricas [A] time = 1.85308, size = 259, normalized size = 2.64

$$\frac{4\,a^3\cos\left(dx+c\right)^5+18\,a^3\cos\left(dx+c\right)^4+24\,a^3\cos\left(dx+c\right)^3-24\,a^3\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right)-9\,a^3\cos\left(dx+c\right.}{12\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*a^3*cos(d*x + c)^5 + 18*a^3*cos(d*x + c)^4 + 24*a^3*cos(d*x + c)^3
- 24*a^3*cos(d*x + c)^2*log(-cos(d*x + c)) - 9*a^3*cos(d*x + c)^2 + 36*a^3*
cos(d*x + c) + 6*a^3)/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31223, size = 138, normalized size = 1.41

$$-\frac{2\,a^3\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{6\,a^3\cos\left(dx+c\right)+a^3}{2\,d\cos\left(dx+c\right)^2}+\frac{2\,a^3d^8\cos\left(dx+c\right)^3+9\,a^3d^8\cos\left(dx+c\right)^2+12\,a^3d^8\cos\left(dx+c\right)}{6\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -2*a^3*log(abs(cos(d*x + c))/abs(d))/d + 1/2*(6*a^3*cos(d*x + c) + a^3)/(d*
cos(d*x + c)^2) + 1/6*(2*a^3*d^8*cos(d*x + c)^3 + 9*a^3*d^8*cos(d*x + c)^2
+ 12*a^3*d^8*cos(d*x + c))/d^9
```

3.42 $\int (a + a\sec(c + dx))^3 \sin(c + dx)\, dx$

Optimal. Leaf size=62

$$-\frac{a^3\cos(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}-\frac{3a^3\log(\cos(c+dx))}{d}$$

```
[Out] -((a^3*Cos[c + d*x])/d) - (3*a^3*Log[Cos[c + d*x]])/d + (3*a^3*Sec[c + d*x]
)/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.0914808, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3872, 2833, 12, 43}

$$-\frac{a^3\cos(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}-\frac{3a^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x],x]
```

```
[Out] -((a^3*Cos[c + d*x])/d) - (3*a^3*Log[Cos[c + d*x]])/d + (3*a^3*Sec[c + d*x]
)/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\sin(c+dx)\,dx = -\int (-a-a\cos(c+dx))^3\sec^2(c+dx)\tan(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^3(-a+x)^3}{x^3}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{(-a+x)^3}{x^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(1-\frac{a^3}{x^3}+\frac{3a^2}{x^2}-\frac{3a}{x}\right)dx, x, -a\cos(c+dx)\right)}{d}$$
$$= -\frac{a^3\cos(c+dx)}{d}-\frac{3a^3\log(\cos(c+dx))}{d}+\frac{3a^3\sec(c+dx)}{d}+\frac{a^3\sec^2(c+dx)}{2d}$$

Mathematica [A] time = 0.228813, size = 65, normalized size = 1.05

$$-\frac{a^3\sec^2(c+dx)(-9\cos(c+dx)+\cos(3(c+dx))+6\log(\cos(c+dx))+\cos(2(c+dx))(6\log(\cos(c+dx))-2)-4)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x],x]
```

```
[Out] -(a^3*(-4 - 9*Cos[c + d*x] + Cos[3*(c + d*x)] + 6*Log[Cos[c + d*x]] + Cos[2
*(c + d*x)]*(-2 + 6*Log[Cos[c + d*x]]))*Sec[c + d*x]^2)/(4*d)
```

Maple [A] time = 0.021, size = 63, normalized size = 1.

$$\frac{a^3(\sec(dx+c))^2}{2d}+3\frac{a^3\sec(dx+c)}{d}+3\frac{a^3\ln(\sec(dx+c))}{d}-\frac{a^3}{d\sec(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c),x)
```

```
[Out] 1/2*a^3*sec(d*x+c)^2/d+3*a^3*sec(d*x+c)/d+3/d*a^3*ln(sec(d*x+c))-1/d*a^3/se
c(d*x+c)
```

Maxima [A] time = 0.986018, size = 74, normalized size = 1.19

$$-\frac{2\,a^3\cos(dx+c)+6\,a^3\log(\cos(dx+c))-\frac{6\,a^3}{\cos(dx+c)}-\frac{a^3}{\cos(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a^3*cos(d*x + c) + 6*a^3*log(cos(d*x + c)) - 6*a^3/cos(d*x + c) - a
^3/cos(d*x + c)^2)/d
```

Fricas [A] time = 1.84729, size = 158, normalized size = 2.55

$$-\frac{2\,a^{3}\cos\left(dx+c\right)^{3}+6\,a^{3}\cos\left(dx+c\right)^{2}\log\left(-\cos\left(dx+c\right)\right)-6\,a^{3}\cos\left(dx+c\right)-a^{3}}{2\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*cos(d*x + c)^3 + 6*a^3*cos(d*x + c)^2*log(-cos(d*x + c)) - 6*a^
3*cos(d*x + c) - a^3)/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{3}\left(\int 3\sin\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3\sin\left(c+dx\right)\sec^{2}\left(c+dx\right)dx+\int\sin\left(c+dx\right)\sec^{3}\left(c+dx\right)dx+\int\sin\left(c+dx\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c),x)
```

```
[Out] a**3*(Integral(3*sin(c + d*x)*sec(c + d*x), x) + Integral(3*sin(c + d*x)*se
c(c + d*x)**2, x) + Integral(sin(c + d*x)*sec(c + d*x)**3, x) + Integral(si
n(c + d*x), x))
```

Giac [A] time = 1.29489, size = 86, normalized size = 1.39

$$-\frac{a^{3}\cos\left(dx+c\right)}{d}-\frac{3\,a^{3}\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{6\,a^{3}\cos\left(dx+c\right)+a^{3}}{2\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -a^3*cos(d*x + c)/d - 3*a^3*log(abs(cos(d*x + c))/abs(d))/d + 1/2*(6*a^3*co
s(d*x + c) + a^3)/(d*cos(d*x + c)^2)
```

3.43 $\int \csc(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=67

$$\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}+\frac{4a^3\log(1-\cos(c+dx))}{d}-\frac{4a^3\log(\cos(c+dx))}{d}$$

```
[Out] (4*a^3*Log[1 - Cos[c + d*x]])/d - (4*a^3*Log[Cos[c + d*x]])/d + (3*a^3*Sec[
c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.125251, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3872, 2836, 12, 88}

$$\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}+\frac{4a^3\log(1-\cos(c+dx))}{d}-\frac{4a^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (4*a^3*Log[1 - Cos[c + d*x]])/d - (4*a^3*Log[Cos[c + d*x]])/d + (3*a^3*Sec[
c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \csc(c+dx)(a+a\sec(c+dx))^3\,dx = -\int (-a-a\cos(c+dx))^3\csc(c+dx)\sec^3(c+dx)\,dx$$
$$= \frac{a\,\text{Subst}\left(\int \frac{a^3(-a+x)^2}{(-a-x)x^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^4\,\text{Subst}\left(\int \frac{(-a+x)^2}{(-a-x)x^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^4\,\text{Subst}\left(\int \left(-\frac{a}{x^3}+\frac{3}{x^2}-\frac{4}{ax}+\frac{4}{a(a+x)}\right)dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{4a^3\log(1-\cos(c+dx))}{d} - \frac{4a^3\log(\cos(c+dx))}{d} + \frac{3a^3\sec(c+dx)}{d} + \frac{a^3\sec^2(c+}{2d}$$

Mathematica [A] time = 0.131181, size = 81, normalized size = 1.21

$$\frac{a^3\sec^2(c+dx)\left(6\cos(c+dx)+8\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-4\log(\cos(c+dx))-4\cos(2(c+dx))\left(\log(\cos(c+dx))-2\log\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + 6*Cos[c + d*x] - 4*Log[Cos[c + d*x]] - 4*Cos[2*(c + d*x)]*(Log[Co
s[c + d*x]] - 2*Log[Sin[(c + d*x)/2]]) + 8*Log[Sin[(c + d*x)/2]])*Sec[c + d
*x]^2)/(2*d)
```

Maple [A] time = 0.045, size = 49, normalized size = 0.7

$$\frac{a^3(\sec(dx+c))^2}{2d}+3\frac{a^3\sec(dx+c)}{d}+4\frac{a^3\ln(-1+\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/2*a^3*sec(d*x+c)^2/d+3*a^3*sec(d*x+c)/d+4/d*a^3*ln(-1+sec(d*x+c))
```

Maxima [A] time = 0.98626, size = 76, normalized size = 1.13

$$\frac{8a^3\log(\cos(dx+c)-1)-8a^3\log(\cos(dx+c))+\frac{6a^3\cos(dx+c)+a^3}{\cos(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(8*a^3*log(cos(d*x + c) - 1) - 8*a^3*log(cos(d*x + c)) + (6*a^3*cos(d*x
 + c) + a^3)/cos(d*x + c)^2)/d
```

Fricas [A] time = 1.79563, size = 197, normalized size = 2.94

$$-\frac{8\,a^{3}\cos\left(dx+c\right)^{2}\log\left(-\cos\left(dx+c\right)\right)-8\,a^{3}\cos\left(dx+c\right)^{2}\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-6\,a^{3}\cos\left(dx+c\right)-a^{3}}{2\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(8*a^3*cos(d*x + c)^2*log(-cos(d*x + c)) - 8*a^3*cos(d*x + c)^2*log(-1
/2*cos(d*x + c) + 1/2) - 6*a^3*cos(d*x + c) - a^3)/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^{3}\left(\int 3\csc\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3\csc\left(c+dx\right)\sec^{2}\left(c+dx\right)dx+\int\csc\left(c+dx\right)\sec^{3}\left(c+dx\right)dx+\int\csc\left(c+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))**3,x)
```

```
[Out] a**3*(Integral(3*csc(c + d*x)*sec(c + d*x), x) + Integral(3*csc(c + d*x)*se
c(c + d*x)**2, x) + Integral(csc(c + d*x)*sec(c + d*x)**3, x) + Integral(cs
c(c + d*x), x))
```

Giac [B] time = 1.27331, size = 192, normalized size = 2.87

$$\frac{2\left(2\,a^{3}\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-2\,a^{3}\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{6\,a^{3}+\frac{8\,a^{3}(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a^{3}(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^{2}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 2*(2*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 2*a^3*log(abs(
-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (6*a^3 + 8*a^3*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + 3*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)
/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

3.44 $\int \csc^3(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=88

$$-\frac{2a^4}{d(a-a\cos(c+dx))}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}+\frac{5a^3\log(1-\cos(c+dx))}{d}-\frac{5a^3\log(\cos(c+dx))}{d}$$

```
[Out] (-2*a^4)/(d*(a - a*Cos[c + d*x])) + (5*a^3*Log[1 - Cos[c + d*x]])/d - (5*a^
3*Log[Cos[c + d*x]])/d + (3*a^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d
)
```

Rubi [A] time = 0.156347, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2836, 12, 77}

$$-\frac{2a^4}{d(a-a\cos(c+dx))}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}+\frac{5a^3\log(1-\cos(c+dx))}{d}-\frac{5a^3\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-2*a^4)/(d*(a - a*Cos[c + d*x])) + (5*a^3*Log[1 - Cos[c + d*x]])/d - (5*a^
3*Log[Cos[c + d*x]])/d + (3*a^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d
)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^3\,dx = -\int (-a-a\cos(c+dx))^3\csc^3(c+dx)\sec^3(c+dx)\,dx$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{a^3(-a+x)}{(-a-x)^2x^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \frac{-a+x}{(-a-x)^2x^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^6\,\text{Subst}\left(\int \left(-\frac{1}{ax^3}+\frac{3}{a^2x^2}-\frac{5}{a^3x}+\frac{2}{a^2(a+x)^2}+\frac{5}{a^3(a+x)}\right)dx, x, -a\cos(c+dx)\right)}{d}$$
$$= -\frac{2a^4}{d(a-a\cos(c+dx))} + \frac{5a^3\log(1-\cos(c+dx))}{d} - \frac{5a^3\log(\cos(c+dx))}{d} + \frac{3}{}$$

Mathematica [A] time = 0.861069, size = 88, normalized size = 1.

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(2\csc^2\left(\frac{1}{2}(c+dx)\right)-\sec^2(c+dx)-6\sec(c+dx)+10\left(\log(\cos(c+dx))-2\,\text{lo}\right.\right.}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(2*Csc[(c + d*x)/2]^2 + 10*(L
og[Cos[c + d*x]] - 2*Log[Sin[(c + d*x)/2]]) - 6*Sec[c + d*x] - Sec[c + d*x]
^2))/(16*d)
```

Maple [A] time = 0.074, size = 67, normalized size = 0.8

$$\frac{a^3(\sec(dx+c))^2}{2d} + 3\frac{a^3\sec(dx+c)}{d} - 2\frac{a^3}{d(-1+\sec(dx+c))} + 5\frac{a^3\ln(-1+\sec(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/2*a^3*sec(d*x+c)^2/d+3*a^3*sec(d*x+c)/d-2/d*a^3/(-1+sec(d*x+c))+5/d*a^3*l
n(-1+sec(d*x+c))
```

Maxima [A] time = 1.00001, size = 113, normalized size = 1.28

$$\frac{10a^3\log(\cos(dx+c)-1) - 10a^3\log(\cos(dx+c)) + \frac{10a^3\cos(dx+c)^2-5a^3\cos(dx+c)-a^3}{\cos(dx+c)^3-\cos(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(10*a^3*log(cos(d*x + c) - 1) - 10*a^3*log(cos(d*x + c)) + (10*a^3*cos(
d*x + c)^2 - 5*a^3*cos(d*x + c) - a^3)/(cos(d*x + c)^3 - cos(d*x + c)^2))/d
```

Fricas [A] time = 1.74009, size = 319, normalized size = 3.62

$$\frac{10\,a^3\cos(dx+c)^2-5\,a^3\cos(dx+c)-a^3-10\left(a^3\cos(dx+c)^3-a^3\cos(dx+c)^2\right)\log(-\cos(dx+c))+10\left(a^3\cos(dx+\right.}{2\left(d\cos(dx+c)^3-d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(10*a^3*cos(d*x + c)^2 - 5*a^3*cos(d*x + c) - a^3 - 10*(a^3*cos(d*x + c
)^3 - a^3*cos(d*x + c)^2)*log(-cos(d*x + c)) + 10*(a^3*cos(d*x + c)^3 - a^3
*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^3 - d*cos(d*
x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.37479, size = 255, normalized size = 2.9

$$\frac{10\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-10\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{2\left(a^3-\frac{5\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}+\frac{27\,a^3+\frac{38\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{15\,a^3(\cos(dx+c)-}{(\cos(dx+c)+1)^2}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(10*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 10*a^3*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 2*(a^3 - 5*a^3*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (27*a
^3 + 38*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 15*a^3*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

3.45 $\int \csc^5(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=111

$$-\frac{a^5}{2d(a-a\cos(c+dx))^2}-\frac{3a^4}{d(a-a\cos(c+dx))}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}+\frac{6a^3\log(1-\cos(c+dx))}{d}-\frac{6a^3}{}$$

```
[Out] -a^5/(2*d*(a - a*Cos[c + d*x])^2) - (3*a^4)/(d*(a - a*Cos[c + d*x])) + (6*a
^3*Log[1 - Cos[c + d*x]])/d - (6*a^3*Log[Cos[c + d*x]])/d + (3*a^3*Sec[c +
d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.168826, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2836, 12, 44}

$$-\frac{a^5}{2d(a-a\cos(c+dx))^2}-\frac{3a^4}{d(a-a\cos(c+dx))}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}+\frac{6a^3\log(1-\cos(c+dx))}{d}-\frac{6a^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
```


```
[Out] -a^5/(2*d*(a - a*Cos[c + d*x])^2) - (3*a^4)/(d*(a - a*Cos[c + d*x])) + (6*a
^3*Log[1 - Cos[c + d*x]])/d - (6*a^3*Log[Cos[c + d*x]])/d + (3*a^3*Sec[c +
d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 44

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[
ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &
& NeQ[b*c - a*d, 0] && ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m
 + n + 2, 0])
```

Rubi steps

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^3\,dx = -\int (-a-a\cos(c+dx))^3\csc^5(c+dx)\sec^3(c+dx)\,dx$$
$$= \frac{a^5\,\text{Subst}\left(\int \frac{a^3}{(-a-x)^3x^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^8\,\text{Subst}\left(\int \frac{1}{(-a-x)^3x^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^8\,\text{Subst}\left(\int \left(-\frac{1}{a^3x^3}+\frac{3}{a^4x^2}-\frac{6}{a^5x}+\frac{1}{a^3(a+x)^3}+\frac{3}{a^4(a+x)^2}+\frac{6}{a^5(a+x)}\right)dx, x, -a\cos(c+\right.}{d}$$
$$= -\frac{a^5}{2d(a-a\cos(c+dx))^2} - \frac{3a^4}{d(a-a\cos(c+dx))} + \frac{6a^3\log(1-\cos(c+dx))}{d} - \frac{6a^3}{}$$

Mathematica [A] time = 0.918052, size = 100, normalized size = 0.9

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\csc^4\left(\frac{1}{2}(c+dx)\right)+12\csc^2\left(\frac{1}{2}(c+dx)\right)-4\sec^2(c+dx)-24\sec(c+dx)+48\left(\log\right.\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(12*Csc[(c + d*x)/2]^2 + Csc[
(c + d*x)/2]^4 + 48*(Log[Cos[c + d*x]] - 2*Log[Sin[(c + d*x)/2]]) - 24*Sec[
c + d*x] - 4*Sec[c + d*x]^2))/(64*d)
```

Maple [A] time = 0.085, size = 85, normalized size = 0.8

$$\frac{a^3(\sec(dx+c))^2}{2d} + 3\frac{a^3\sec(dx+c)}{d} - 4\frac{a^3}{d(-1+\sec(dx+c))} + 6\frac{a^3\ln(-1+\sec(dx+c))}{d} - \frac{a^3}{2d(-1+\sec(dx+c))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/2*a^3*sec(d*x+c)^2/d+3*a^3*sec(d*x+c)/d-4/d*a^3/(-1+sec(d*x+c))+6/d*a^3*l
n(-1+sec(d*x+c))-1/2/d*a^3/(-1+sec(d*x+c))^2
```

Maxima [A] time = 1.00343, size = 139, normalized size = 1.25

$$\frac{12a^3\log(\cos(dx+c)-1) - 12a^3\log(\cos(dx+c)) + \frac{12a^3\cos(dx+c)^3-18a^3\cos(dx+c)^2+4a^3\cos(dx+c)+a^3}{\cos(dx+c)^4-2\cos(dx+c)^3+\cos(dx+c)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(12*a^3*log(cos(d*x + c) - 1) - 12*a^3*log(cos(d*x + c)) + (12*a^3*cos(
d*x + c)^3 - 18*a^3*cos(d*x + c)^2 + 4*a^3*cos(d*x + c) + a^3)/(cos(d*x + c
```

```
)^4 - 2*cos(d*x + c)^3 + cos(d*x + c)^2))/d
```

Fricas [A] time = 1.73261, size = 441, normalized size = 3.97

$$\frac{12\,a^3\cos\left(dx+c\right)^3-18\,a^3\cos\left(dx+c\right)^2+4\,a^3\cos\left(dx+c\right)+a^3-12\left(a^3\cos\left(dx+c\right)^4-2\,a^3\cos\left(dx+c\right)^3+a^3\cos\left(dx\right.\right.}{2\left(d\cos\left(dx+c\right)^4-2\,d\cos\left(d\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(12*a^3*cos(d*x + c)^3 - 18*a^3*cos(d*x + c)^2 + 4*a^3*cos(d*x + c) + a
^3 - 12*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2)*lo
g(-cos(d*x + c)) + 12*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^3 + a^3*cos(
d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^4 - 2*d*cos(d*x +
 c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41761, size = 251, normalized size = 2.26

$$\frac{48\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-48\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\frac{a^3-\frac{12\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{75\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{46\,a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(48*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 48*a^3*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a^3 - 12*a^3*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) - 75*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2 - 46*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/((cos(d*x + c) - 1)
/(cos(d*x + c) + 1) + (cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)^2)/d
```

3.46 $\int \csc^7(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=157

$$-\frac{a^6}{6d(a-a\cos(c+dx))^3}-\frac{7a^5}{8d(a-a\cos(c+dx))^2}-\frac{31a^4}{8d(a-a\cos(c+dx))}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}+\frac{111a^3\,\mathrm{lo}}{}$$

```
[Out] -a^6/(6*d*(a - a*Cos[c + d*x])^3) - (7*a^5)/(8*d*(a - a*Cos[c + d*x])^2) -
(31*a^4)/(8*d*(a - a*Cos[c + d*x])) + (111*a^3*Log[1 - Cos[c + d*x]])/(16*d
) - (7*a^3*Log[Cos[c + d*x]])/d + (a^3*Log[1 + Cos[c + d*x]])/(16*d) + (3*a
^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.195268, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2836, 12, 88}

$$-\frac{a^6}{6d(a-a\cos(c+dx))^3}-\frac{7a^5}{8d(a-a\cos(c+dx))^2}-\frac{31a^4}{8d(a-a\cos(c+dx))}+\frac{a^3\sec^2(c+dx)}{2d}+\frac{3a^3\sec(c+dx)}{d}+\frac{111a^3\,\mathrm{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^7*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -a^6/(6*d*(a - a*Cos[c + d*x])^3) - (7*a^5)/(8*d*(a - a*Cos[c + d*x])^2) -
(31*a^4)/(8*d*(a - a*Cos[c + d*x])) + (111*a^3*Log[1 - Cos[c + d*x]])/(16*d
) - (7*a^3*Log[Cos[c + d*x]])/d + (a^3*Log[1 + Cos[c + d*x]])/(16*d) + (3*a
^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \csc^7(c+dx)(a+a\sec(c+dx))^3\,dx = -\int(-a-a\cos(c+dx))^3\csc^7(c+dx)\sec^3(c+dx)\,dx$$
$$= \frac{a^7\,\text{Subst}\left(\int \frac{a^3}{(-a-x)^4x^3(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^{10}\,\text{Subst}\left(\int \frac{1}{(-a-x)^4x^3(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^{10}\,\text{Subst}\left(\int\left(-\frac{1}{16a^7(a-x)} - \frac{1}{a^5x^3} + \frac{3}{a^6x^2} - \frac{7}{a^7x} + \frac{1}{2a^4(a+x)^4} + \frac{7}{4a^5(a+x)^3} + \frac{31}{8a^6(a+x)^2}\right.\right.}{d}$$
$$= -\frac{a^6}{6d(a-a\cos(c+dx))^3} - \frac{7a^5}{8d(a-a\cos(c+dx))^2} - \frac{31a^4}{8d(a-a\cos(c+dx))} + \cdots$$

Mathematica [A] time = 1.0151, size = 129, normalized size = 0.82

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(2\csc^6\left(\frac{1}{2}(c+dx)\right)+21\csc^4\left(\frac{1}{2}(c+dx)\right)+186\csc^2\left(\frac{1}{2}(c+dx)\right)-12\left(4\sec^2(c\right.\right.}{768d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^7*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(186*Csc[(c + d*x)/2]^2 + 21*
Csc[(c + d*x)/2]^4 + 2*Csc[(c + d*x)/2]^6 - 12*(Log[Cos[(c + d*x)/2]] - 56*
Log[Cos[c + d*x]] + 111*Log[Sin[(c + d*x)/2]] + 24*Sec[c + d*x] + 4*Sec[c +
 d*x]^2)))/(768*d)
```

Maple [A] time = 0.089, size = 120, normalized size = 0.8

$$\frac{a^3(\sec(dx+c))^2}{2d} + 3\frac{a^3\sec(dx+c)}{d} + \frac{a^3\ln(1+\sec(dx+c))}{16d} - \frac{a^3}{6d(-1+\sec(dx+c))^3} - \frac{11a^3}{8d(-1+\sec(dx+c))^2} - \cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^7*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/2*a^3*sec(d*x+c)^2/d+3*a^3*sec(d*x+c)/d+1/16/d*a^3*ln(1+sec(d*x+c))-1/6/d
*a^3/(-1+sec(d*x+c))^3-11/8/d*a^3/(-1+sec(d*x+c))^2-49/8/d*a^3/(-1+sec(d*x+
c))+111/16/d*a^3*ln(-1+sec(d*x+c))
```

Maxima [A] time = 1.00432, size = 196, normalized size = 1.25

$$\frac{3a^3\log(\cos(dx+c)+1) + 333a^3\log(\cos(dx+c)-1) - 336a^3\log(\cos(dx+c)) + \frac{2\left(165a^3\cos(dx+c)^4 - 411a^3\cos(dx+c)^3 + \cdots\right.}{\cos(dx+c)^5 - 3\cos(dx+c)^4 \cdots}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/48*(3*a^3*log(cos(d*x + c) + 1) + 333*a^3*log(cos(d*x + c) - 1) - 336*a^3
*log(cos(d*x + c)) + 2*(165*a^3*cos(d*x + c)^4 - 411*a^3*cos(d*x + c)^3 + 2
98*a^3*cos(d*x + c)^2 - 36*a^3*cos(d*x + c) - 12*a^3)/(cos(d*x + c)^5 - 3*c
os(d*x + c)^4 + 3*cos(d*x + c)^3 - cos(d*x + c)^2))/d
```

Fricas [B] time = 1.87885, size = 740, normalized size = 4.71

$$330\,a^3\cos(dx+c)^4 - 822\,a^3\cos(dx+c)^3 + 596\,a^3\cos(dx+c)^2 - 72\,a^3\cos(dx+c) - 24\,a^3 - 336\left(a^3\cos(dx+c)^5 - 3\,a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/48*(330*a^3*cos(d*x + c)^4 - 822*a^3*cos(d*x + c)^3 + 596*a^3*cos(d*x + c
)^2 - 72*a^3*cos(d*x + c) - 24*a^3 - 336*(a^3*cos(d*x + c)^5 - 3*a^3*cos(d*
x + c)^4 + 3*a^3*cos(d*x + c)^3 - a^3*cos(d*x + c)^2)*log(-cos(d*x + c)) +
3*(a^3*cos(d*x + c)^5 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^3 - a^3*c
os(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + 333*(a^3*cos(d*x + c)^5 - 3*a^
3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^3 - a^3*cos(d*x + c)^2)*log(-1/2*cos(
d*x + c) + 1/2))/(d*cos(d*x + c)^5 - 3*d*cos(d*x + c)^4 + 3*d*cos(d*x + c)^
3 - d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**7*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30583, size = 328, normalized size = 2.09

$$\frac{666\,a^3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 672\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right) + \frac{\left(2\,a^3 - \frac{27\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{234\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{1221\,a^3(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}\right)(\cos(dx+c)+}{(\cos(dx+c)-1)^3}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/96*(666*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 672*a^3*l
og(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (2*a^3 - 27*a^3*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + 234*a^3*(cos(d*x + c) - 1)^2/(cos(d*x +
c) + 1)^2 - 1221*a^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x +
c) + 1)^3/(cos(d*x + c) - 1)^3 + 48*(33*a^3 + 50*a^3*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) + 21*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

3.47 $\int \csc^9(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=202

$$-\frac{a^7}{16d(a-a\cos(c+dx))^4}-\frac{a^6}{3d(a-a\cos(c+dx))^3}-\frac{39a^5}{32d(a-a\cos(c+dx))^2}-\frac{75a^4}{16d(a-a\cos(c+dx))}-\frac{a^4}{32d(a\cos(c+$$

```
[Out] -a^7/(16*d*(a - a*Cos[c + d*x])^4) - a^6/(3*d*(a - a*Cos[c + d*x])^3) - (39
*a^5)/(32*d*(a - a*Cos[c + d*x])^2) - (75*a^4)/(16*d*(a - a*Cos[c + d*x]))
- a^4/(32*d*(a + a*Cos[c + d*x])) + (501*a^3*Log[1 - Cos[c + d*x]])/(64*d)
- (8*a^3*Log[Cos[c + d*x]])/d + (11*a^3*Log[1 + Cos[c + d*x]])/(64*d) + (3*
a^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.230595, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2836, 12, 88}

$$-\frac{a^7}{16d(a-a\cos(c+dx))^4}-\frac{a^6}{3d(a-a\cos(c+dx))^3}-\frac{39a^5}{32d(a-a\cos(c+dx))^2}-\frac{75a^4}{16d(a-a\cos(c+dx))}-\frac{a^4}{32d(a\cos(c+$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^9*(a + a*Sec[c + d*x])^3,x]
```


```
[Out] -a^7/(16*d*(a - a*Cos[c + d*x])^4) - a^6/(3*d*(a - a*Cos[c + d*x])^3) - (39
*a^5)/(32*d*(a - a*Cos[c + d*x])^2) - (75*a^4)/(16*d*(a - a*Cos[c + d*x]))
- a^4/(32*d*(a + a*Cos[c + d*x])) + (501*a^3*Log[1 - Cos[c + d*x]])/(64*d)
- (8*a^3*Log[Cos[c + d*x]])/d + (11*a^3*Log[1 + Cos[c + d*x]])/(64*d) + (3*
a^3*Sec[c + d*x])/d + (a^3*Sec[c + d*x]^2)/(2*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \csc^9(c+dx)(a+a\sec(c+dx))^3\,dx = -\int (-a-a\cos(c+dx))^3\csc^9(c+dx)\sec^3(c+dx)\,dx$$
$$= \frac{a^9\,\text{Subst}\left(\int \frac{a^3}{(-a-x)^5x^3(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^{12}\,\text{Subst}\left(\int \frac{1}{(-a-x)^5x^3(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^{12}\,\text{Subst}\left(\int \left(-\frac{1}{32a^8(a-x)^2} - \frac{11}{64a^9(a-x)} - \frac{1}{a^7x^3} + \frac{3}{a^8x^2} - \frac{8}{a^9x} + \frac{1}{4a^5(a+x)^5} + \frac{1}{a^6(a+x)^4} + \right.\right.}{d}$$
$$= -\frac{a^7}{16d(a-a\cos(c+dx))^4} - \frac{a^6}{3d(a-a\cos(c+dx))^3} - \frac{39a^5}{32d(a-a\cos(c+dx))^2} -$$

Mathematica [A] time = 1.1944, size = 159, normalized size = 0.79

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(3\csc^8\left(\frac{1}{2}(c+dx)\right)+32\csc^6\left(\frac{1}{2}(c+dx)\right)+234\csc^4\left(\frac{1}{2}(c+dx)\right)+1800\csc^2\left(\frac{1}{2}(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^9*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(1800*Csc[(c + d*x)/2]^2 + 23
4*Csc[(c + d*x)/2]^4 + 32*Csc[(c + d*x)/2]^6 + 3*Csc[(c + d*x)/2]^8 - 12*(2
2*Log[Cos[(c + d*x)/2]] - 512*Log[Cos[c + d*x]] + 1002*Log[Sin[(c + d*x)/2]
] - Sec[(c + d*x)/2]^2 + 192*Sec[c + d*x] + 32*Sec[c + d*x]^2)))/(6144*d)
```

Maple [A] time = 0.088, size = 156, normalized size = 0.8

$$\frac{a^3(\sec(dx+c))^2}{2d} + 3\frac{a^3\sec(dx+c)}{d} + \frac{a^3}{32d(1+\sec(dx+c))} + \frac{11a^3\ln(1+\sec(dx+c))}{64d} - \frac{a^3}{16d(-1+\sec(dx+c))^4} -$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^9*(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/2*a^3*sec(d*x+c)^2/d+3*a^3*sec(d*x+c)/d+1/32/d*a^3/(1+sec(d*x+c))+11/64/d
*a^3*ln(1+sec(d*x+c))-1/16/d*a^3/(-1+sec(d*x+c))^4-7/12/d*a^3/(-1+sec(d*x+c
))^3-83/32/d*a^3/(-1+sec(d*x+c))^2-67/8/d*a^3/(-1+sec(d*x+c))+501/64/d*a^3*
ln(-1+sec(d*x+c))
```

Maxima [A] time = 1.01392, size = 255, normalized size = 1.26

$$\frac{33a^3\log(\cos(dx+c)+1) + 1503a^3\log(\cos(dx+c)-1) - 1536a^3\log(\cos(dx+c)) + \frac{2\left(735a^3\cos(dx+c)^6 - 1821a^3\cos(dx+c)^5 \cdots\right.}{\cos(dx+c)^7 - 3\cos(\cdots}}{192d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/192*(33*a^3*log(cos(d*x + c) + 1) + 1503*a^3*log(cos(d*x + c) - 1) - 1536
*a^3*log(cos(d*x + c)) + 2*(735*a^3*cos(d*x + c)^6 - 1821*a^3*cos(d*x + c)^
5 + 563*a^3*cos(d*x + c)^4 + 1695*a^3*cos(d*x + c)^3 - 1376*a^3*cos(d*x + c
)^2 + 144*a^3*cos(d*x + c) + 48*a^3)/(cos(d*x + c)^7 - 3*cos(d*x + c)^6 + 2
*cos(d*x + c)^5 + 2*cos(d*x + c)^4 - 3*cos(d*x + c)^3 + cos(d*x + c)^2))/d
```

Fricas [B] time = 1.85144, size = 1064, normalized size = 5.27

$$\frac{1470\,a^3\cos\left(dx+c\right)^6 - 3642\,a^3\cos\left(dx+c\right)^5 + 1126\,a^3\cos\left(dx+c\right)^4 + 3390\,a^3\cos\left(dx+c\right)^3 - 2752\,a^3\cos\left(dx+c\right)^2 + \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/192*(1470*a^3*cos(d*x + c)^6 - 3642*a^3*cos(d*x + c)^5 + 1126*a^3*cos(d*x
 + c)^4 + 3390*a^3*cos(d*x + c)^3 - 2752*a^3*cos(d*x + c)^2 + 288*a^3*cos(d
*x + c) + 96*a^3 - 1536*(a^3*cos(d*x + c)^7 - 3*a^3*cos(d*x + c)^6 + 2*a^3*
cos(d*x + c)^5 + 2*a^3*cos(d*x + c)^4 - 3*a^3*cos(d*x + c)^3 + a^3*cos(d*x
+ c)^2)*log(-cos(d*x + c)) + 33*(a^3*cos(d*x + c)^7 - 3*a^3*cos(d*x + c)^6
+ 2*a^3*cos(d*x + c)^5 + 2*a^3*cos(d*x + c)^4 - 3*a^3*cos(d*x + c)^3 + a^3*
cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + 1503*(a^3*cos(d*x + c)^7 - 3*
a^3*cos(d*x + c)^6 + 2*a^3*cos(d*x + c)^5 + 2*a^3*cos(d*x + c)^4 - 3*a^3*co
s(d*x + c)^3 + a^3*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x
 + c)^7 - 3*d*cos(d*x + c)^6 + 2*d*cos(d*x + c)^5 + 2*d*cos(d*x + c)^4 - 3*
d*cos(d*x + c)^3 + d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**9*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.37384, size = 394, normalized size = 1.95

$$\frac{6012\,a^3\log\left(\frac{\left|-\cos(dx+c)+1\right|}{\left|\cos(dx+c)+1\right|}\right) - 6144\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{12\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{\left(3\,a^3 - \frac{44\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{348\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - \frac{237\cdots}{\cdots}\cdots\right.}{(\cos\cdots}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^9*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/768*(6012*a^3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 6144*a^
3*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 12*a^3*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - (3*a^3 - 44*a^3*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) + 348*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 2376*a^3*(cos
(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 12525*a^3*(cos(d*x + c) - 1)^4/(cos
(d*x + c) + 1)^4)*(cos(d*x + c) + 1)^4/(cos(d*x + c) - 1)^4 + 1536*(9*a^3 +
 14*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 6*a^3*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

3.48 $\int (a + a\sec(c + dx))^3 \sin^8(c + dx)\, dx$

Optimal. Leaf size=210

$$-\frac{3a^3\sin^7(c+dx)}{7d} - \frac{2a^3\sin^5(c+dx)}{5d} - \frac{a^3\sin^3(c+dx)}{3d} + \frac{3a^3\tan(c+dx)}{d} - \frac{a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^3\sin(c+dx)}{8d}$$

```
[Out] (-805*a^3*x)/128 - (a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (603*a^3*Cos[c + d*x
]*Sin[c + d*x])/(128*d) - (293*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) - (
a^3*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) + (a^3*Cos[c + d*x]^7*Sin[c + d*x])
/(8*d) - (a^3*Sin[c + d*x]^3)/(3*d) - (2*a^3*Sin[c + d*x]^5)/(5*d) - (3*a^3
*Sin[c + d*x]^7)/(7*d) + (3*a^3*Tan[c + d*x])/d + (a^3*Sec[c + d*x]*Tan[c +
 d*x])/(2*d)
```

Rubi [A] time = 0.389447, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 29, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3872, 2872, 2637, 2635, 8, 2633, 3770, 3767, 3768}

$$-\frac{3a^3\sin^7(c+dx)}{7d} - \frac{2a^3\sin^5(c+dx)}{5d} - \frac{a^3\sin^3(c+dx)}{3d} + \frac{3a^3\tan(c+dx)}{d} - \frac{a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a^3\sin(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^8,x]
```



```
[Out] (-805*a^3*x)/128 - (a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (603*a^3*Cos[c + d*x
]*Sin[c + d*x])/(128*d) - (293*a^3*Cos[c + d*x]^3*Sin[c + d*x])/(192*d) - (
a^3*Cos[c + d*x]^5*Sin[c + d*x])/(48*d) + (a^3*Cos[c + d*x]^7*Sin[c + d*x])
/(8*d) - (a^3*Sin[c + d*x]^3)/(3*d) - (2*a^3*Sin[c + d*x]^5)/(5*d) - (3*a^3
*Sin[c + d*x]^7)/(7*d) + (3*a^3*Tan[c + d*x])/d + (a^3*Sec[c + d*x]*Tan[c +
 d*x])/(2*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
```

```
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int (a + a\sec(c+dx))^3 \sin^8(c+dx)\,dx = -\int (-a - a\cos(c+dx))^3 \sin^5(c+dx)\tan^3(c+dx)\,dx$$
$$= -\frac{\int \left(11a^{11} + 6a^{11}\cos(c+dx) - 14a^{11}\cos^2(c+dx) - 14a^{11}\cos^3(c+dx) + 6a^{11}\mathrm{c}\right.}{}$$
$$= -11a^3x - a^3\int \cos^6(c+dx)\,dx + a^3\int \cos^8(c+dx)\,dx - a^3\int \sec(c+dx)\,dx + a$$
$$= -11a^3x - \frac{a^3\tanh^{-1}(\sin(c+dx))}{d} - \frac{6a^3\sin(c+dx)}{d} + \frac{7a^3\cos(c+dx)\sin(c+dx}{d}$$
$$= -4a^3x - \frac{a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{19a^3\cos(c+dx)\sin(c+dx)}{4d} - \frac{41a^3\cos^3(c+}{2}$$
$$= -\frac{25a^3x}{4} - \frac{a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{71a^3\cos(c+dx)\sin(c+dx)}{16d} - \frac{293a^3\cos^3(c}{}$$
$$= -\frac{105a^3x}{16} - \frac{a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{603a^3\cos(c+dx)\sin(c+dx)}{128d} - \frac{293a^3\cos}{}$$
$$= -\frac{805a^3x}{128} - \frac{a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{603a^3\cos(c+dx)\sin(c+dx)}{128d} - \frac{293a^3\cos}{}$$

Mathematica [A] time = 2.01652, size = 156, normalized size = 0.74

$$\frac{a^3\sec^2(c+dx)\left(173600\sin(c+dx) + 1052520\sin(2(c+dx)) - 11648\sin(3(c+dx)) + 175280\sin(4(c+dx)) + 22784\sin\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^8,x]
```

```
[Out] (a^3*Sec[c + d*x]^2*(-1352400*c - 1352400*d*x - 215040*ArcTanh[Sin[c + d*x]
]*Cos[c + d*x]^2 - 1352400*(c + d*x)*Cos[2*(c + d*x)] + 173600*Sin[c + d*x]
 + 1052520*Sin[2*(c + d*x)] - 11648*Sin[3*(c + d*x)] + 175280*Sin[4*(c + d*
x)] + 22784*Sin[5*(c + d*x)] - 18095*Sin[6*(c + d*x)] - 6288*Sin[7*(c + d*x
)] + 770*Sin[8*(c + d*x)] + 720*Sin[9*(c + d*x)] + 105*Sin[10*(c + d*x)]))/
(430080*d)
```

Maple [A] time = 0.051, size = 235, normalized size = 1.1

$$\frac{23\,a^3\left(\sin\left(dx+c\right)\right)^7\cos\left(dx+c\right)}{8\,d}+\frac{161\,a^3\cos\left(dx+c\right)\left(\sin\left(dx+c\right)\right)^5}{48\,d}+\frac{805\,a^3\cos\left(dx+c\right)\left(\sin\left(dx+c\right)\right)^3}{192\,d}+\frac{805\,a^3\,\mathrm{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^8,x)
```

```
[Out] 23/8/d*a^3*sin(d*x+c)^7*cos(d*x+c)+161/48*a^3*cos(d*x+c)*sin(d*x+c)^5/d+805
/192*a^3*cos(d*x+c)*sin(d*x+c)^3/d+805/128*a^3*cos(d*x+c)*sin(d*x+c)/d-805/
128*a^3*x-805/128/d*a^3*c+1/14*a^3*sin(d*x+c)^7/d+1/10*a^3*sin(d*x+c)^5/d+1
/6*a^3*sin(d*x+c)^3/d+1/2*a^3*sin(d*x+c)/d-1/2/d*a^3*ln(sec(d*x+c)+tan(d*x+
c))+3/d*a^3*sin(d*x+c)^9/cos(d*x+c)+1/2/d*a^3*sin(d*x+c)^9/cos(d*x+c)^2
```

Maxima [A] time = 1.80166, size = 393, normalized size = 1.87

$$-\frac{1536\left(30\,\sin\left(dx+c\right)^7+42\,\sin\left(dx+c\right)^5+70\,\sin\left(dx+c\right)^3-105\,\log\left(\sin\left(dx+c\right)+1\right)+105\,\log\left(\sin\left(dx+c\right)-1\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^8,x, algorithm="maxima")
```

```
[Out] -1/107520*(1536*(30*sin(d*x + c)^7 + 42*sin(d*x + c)^5 + 70*sin(d*x + c)^3
- 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1) + 210*sin(d*x + c))
*a^3 - 1792*(12*sin(d*x + c)^5 + 40*sin(d*x + c)^3 - 30*sin(d*x + c)/(sin(d
*x + c)^2 - 1) - 105*log(sin(d*x + c) + 1) + 105*log(sin(d*x + c) - 1) + 18
0*sin(d*x + c))*a^3 - 35*(128*sin(2*d*x + 2*c)^3 + 840*d*x + 840*c + 3*sin(
8*d*x + 8*c) + 168*sin(4*d*x + 4*c) - 768*sin(2*d*x + 2*c))*a^3 + 6720*(105
*d*x + 105*c - (87*tan(d*x + c)^5 + 136*tan(d*x + c)^3 + 57*tan(d*x + c))/(
tan(d*x + c)^6 + 3*tan(d*x + c)^4 + 3*tan(d*x + c)^2 + 1) - 48*tan(d*x + c)
)*a^3)/d
```

Fricas [A] time = 2.04888, size = 567, normalized size = 2.7

$$-\frac{84525\,a^3dx\cos\left(dx+c\right)^2+3360\,a^3\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-3360\,a^3\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^8,x, algorithm="fricas")
```

```
[Out] -1/13440*(84525*a^3*d*x*cos(d*x + c)^2 + 3360*a^3*cos(d*x + c)^2*log(sin(d*
x + c) + 1) - 3360*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - (1680*a^3*co
s(d*x + c)^9 + 5760*a^3*cos(d*x + c)^8 - 280*a^3*cos(d*x + c)^7 - 22656*a^3
*cos(d*x + c)^6 - 20510*a^3*cos(d*x + c)^5 + 32512*a^3*cos(d*x + c)^4 + 633
15*a^3*cos(d*x + c)^3 - 15616*a^3*cos(d*x + c)^2 + 40320*a^3*cos(d*x + c) +
 6720*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**8,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2664, size = 329, normalized size = 1.57

$$-\frac{84525\,(dx+c)a^3 + 6720\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 6720\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{13440\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 7\cdots\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\cdots\right.}\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^8,x, algorithm="giac")
```

```
[Out] -1/13440*(84525*(d*x + c)*a^3 + 6720*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1))
 - 6720*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 13440*(5*a^3*tan(1/2*d*x +
 1/2*c)^3 - 7*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 + 2*
(44205*a^3*tan(1/2*d*x + 1/2*c)^15 + 303065*a^3*tan(1/2*d*x + 1/2*c)^13 + 8
41981*a^3*tan(1/2*d*x + 1/2*c)^11 + 1123793*a^3*tan(1/2*d*x + 1/2*c)^9 + 48
7983*a^3*tan(1/2*d*x + 1/2*c)^7 - 490749*a^3*tan(1/2*d*x + 1/2*c)^5 - 26722
5*a^3*tan(1/2*d*x + 1/2*c)^3 - 44205*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
 + 1/2*c)^2 + 1)^8)/d
```

3.49 $\int (a+a\sec(c+dx))^3\sin^6(c+dx)\,dx$

Optimal. Leaf size=182

$$-\frac{3a^3\sin^5(c+dx)}{5d}-\frac{2a^3\sin^3(c+dx)}{3d}-\frac{a^3\sin(c+dx)}{d}+\frac{3a^3\tan(c+dx)}{d}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\sin(c+dx)\mathrm{c}}{6d}$$

```
[Out] (-85*a^3*x)/16 + (a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (a^3*Sin[c + d*x])/d +
 (43*a^3*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (5*a^3*Cos[c + d*x]^3*Sin[c +
d*x])/(24*d) - (a^3*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (2*a^3*Sin[c + d*x
]^3)/(3*d) - (3*a^3*Sin[c + d*x]^5)/(5*d) + (3*a^3*Tan[c + d*x])/d + (a^3*S
ec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.273681, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 18, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3872, 2872, 2637, 2635, 8, 2633, 3767, 3768, 3770}

$$-\frac{3a^3\sin^5(c+dx)}{5d}-\frac{2a^3\sin^3(c+dx)}{3d}-\frac{a^3\sin(c+dx)}{d}+\frac{3a^3\tan(c+dx)}{d}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\sin(c+dx)\mathrm{c}}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^6,x]
```


```
[Out] (-85*a^3*x)/16 + (a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (a^3*Sin[c + d*x])/d +
 (43*a^3*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (5*a^3*Cos[c + d*x]^3*Sin[c +
d*x])/(24*d) - (a^3*Cos[c + d*x]^5*Sin[c + d*x])/(6*d) - (2*a^3*Sin[c + d*x
]^3)/(3*d) - (3*a^3*Sin[c + d*x]^5)/(5*d) + (3*a^3*Tan[c + d*x])/d + (a^3*S
ec[c + d*x]*Tan[c + d*x])/(2*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\sin^6(c+dx)\,dx = -\int (-a-a\cos(c+dx))^3\sin^3(c+dx)\tan^3(c+dx)\,dx$$
$$= -\frac{\int \left(8a^9+6a^9\cos(c+dx)-6a^9\cos^2(c+dx)-8a^9\cos^3(c+dx)+3a^9\cos^5(c+$$
$$a^6$$
$$= -8a^3x-a^3\int\cos^6(c+dx)\,dx+a^3\int\sec^3(c+dx)\,dx-\left(3a^3\right)\int\cos^5(c+dx)\,dx$$
$$= -8a^3x-\frac{6a^3\sin(c+dx)}{d}+\frac{3a^3\cos(c+dx)\sin(c+dx)}{d}-\frac{a^3\cos^5(c+dx)\sin(c+}{6d}$$
$$= -5a^3x+\frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\sin(c+dx)}{d}+\frac{3a^3\cos(c+dx)\sin(c+dx)}{d}$$
$$= -5a^3x+\frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\sin(c+dx)}{d}+\frac{43a^3\cos(c+dx)\sin(c+dx)}{16d}$$
$$= -\frac{85a^3x}{16}+\frac{a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\sin(c+dx)}{d}+\frac{43a^3\cos(c+dx)\sin(c+d}{16d}$$

Mathematica [A] time = 0.86938, size = 136, normalized size = 0.75

$$-\frac{a^3\sec^2(c+dx)\left(-460\sin(c+dx)-8145\sin(2(c+dx))+1156\sin(3(c+dx))-1120\sin(4(c+dx))-268\sin(5(c+dx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^6,x]
```

```
[Out] -(a^3*Sec[c + d*x]^2*(10200*c + 10200*d*x - 1920*ArcTanh[Sin[c + d*x]]*Cos[
c + d*x]^2 + 10200*(c + d*x)*Cos[2*(c + d*x)] - 460*Sin[c + d*x] - 8145*Sin
```

```
[2*(c + d*x)] + 1156*Sin[3*(c + d*x)] - 1120*Sin[4*(c + d*x)] - 268*Sin[5*(
c + d*x)] + 55*Sin[6*(c + d*x)] + 36*Sin[7*(c + d*x)] + 5*Sin[8*(c + d*x)])
)/(3840*d)
```

Maple [A] time = 0.049, size = 197, normalized size = 1.1

$$\frac{17\,a^3\cos(dx+c)(\sin(dx+c))^5}{6d}+\frac{85\,a^3\cos(dx+c)(\sin(dx+c))^3}{24\,d}+\frac{85\,a^3\cos(dx+c)\sin(dx+c)}{16\,d}-\frac{85\,a^3x}{16}-\frac{85\,a^3}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^6,x)
```

```
[Out] 17/6*a^3*cos(d*x+c)*sin(d*x+c)^5/d+85/24*a^3*cos(d*x+c)*sin(d*x+c)^3/d+85/1
6*a^3*cos(d*x+c)*sin(d*x+c)/d-85/16*a^3*x-85/16/d*a^3*c-1/10*a^3*sin(d*x+c)
^5/d-1/6*a^3*sin(d*x+c)^3/d-1/2*a^3*sin(d*x+c)/d+1/2/d*a^3*ln(sec(d*x+c)+ta
n(d*x+c))+3/d*a^3*sin(d*x+c)^7/cos(d*x+c)+1/2/d*a^3*sin(d*x+c)^7/cos(d*x+c)
^2
```

Maxima [A] time = 1.53424, size = 324, normalized size = 1.78

$$-\frac{96\left(6\sin(dx+c)^5+10\sin(dx+c)^3-15\log(\sin(dx+c)+1)+15\log(\sin(dx+c)-1)+30\sin(dx+c)\right)a^3-5}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] -1/960*(96*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin(d*x + c) + 1)
 + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*a^3 - 5*(4*sin(2*d*x + 2*c)^
3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*sin(2*d*x + 2*c))*a^3 - 80*(4*s
in(d*x + c)^3 - 6*sin(d*x + c)/(sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) +
 1) + 15*log(sin(d*x + c) - 1) + 24*sin(d*x + c))*a^3 + 360*(15*d*x + 15*c
- (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 +
1) - 8*tan(d*x + c))*a^3)/d
```

Fricas [A] time = 1.97831, size = 467, normalized size = 2.57

$$-\frac{1275\,a^3dx\cos(dx+c)^2-60\,a^3\cos(dx+c)^2\log(\sin(dx+c)+1)+60\,a^3\cos(dx+c)^2\log(-\sin(dx+c)+1)+(40}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] -1/240*(1275*a^3*d*x*cos(d*x + c)^2 - 60*a^3*cos(d*x + c)^2*log(sin(d*x + c
) + 1) + 60*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + (40*a^3*cos(d*x + c
)^7 + 144*a^3*cos(d*x + c)^6 + 50*a^3*cos(d*x + c)^5 - 448*a^3*cos(d*x + c)
^4 - 645*a^3*cos(d*x + c)^3 + 544*a^3*cos(d*x + c)^2 - 720*a^3*cos(d*x + c)
 - 120*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28381, size = 286, normalized size = 1.57

$$-\frac{1275\,(dx+c)a^3 - 120\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) + 120\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + \frac{240\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 7\,a^3\tan\left(\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="giac")
```

```
[Out] -1/240*(1275*(d*x + c)*a^3 - 120*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 1
20*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 240*(5*a^3*tan(1/2*d*x + 1/2*c)
^3 - 7*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 + 2*(795*a^
3*tan(1/2*d*x + 1/2*c)^11 + 4025*a^3*tan(1/2*d*x + 1/2*c)^9 + 7614*a^3*tan(
1/2*d*x + 1/2*c)^7 + 5634*a^3*tan(1/2*d*x + 1/2*c)^5 - 345*a^3*tan(1/2*d*x
+ 1/2*c)^3 - 315*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/
d
```

3.50 $\int (a+a\sec(c+dx))^3\sin^4(c+dx)\,dx$

Optimal. Leaf size=138

$$-\frac{a^3\sin^3(c+dx)}{d}-\frac{2a^3\sin(c+dx)}{d}+\frac{3a^3\tan(c+dx)}{d}+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{7a^3}{}$$

```
[Out] (-33*a^3*x)/8 + (3*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (2*a^3*Sin[c + d*x])/
d + (7*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^3*Cos[c + d*x]^3*Sin[c + d
*x])/(4*d) - (a^3*Sin[c + d*x]^3)/d + (3*a^3*Tan[c + d*x])/d + (a^3*Sec[c +
 d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.227665, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3872, 2872, 2637, 2635, 8, 2633, 3770, 3767, 3768}

$$-\frac{a^3\sin^3(c+dx)}{d}-\frac{2a^3\sin(c+dx)}{d}+\frac{3a^3\tan(c+dx)}{d}+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^3\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{7a^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^4,x]
```


```
[Out] (-33*a^3*x)/8 + (3*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (2*a^3*Sin[c + d*x])/
d + (7*a^3*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^3*Cos[c + d*x]^3*Sin[c + d
*x])/(4*d) - (a^3*Sin[c + d*x]^3)/d + (3*a^3*Tan[c + d*x])/d + (a^3*Sec[c +
 d*x]*Tan[c + d*x])/(2*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^3\sin^4(c+dx)\,dx = -\int (-a-a\cos(c+dx))^3\sin(c+dx)\tan^3(c+dx)\,dx$$
$$= -\frac{\int \left(5a^7+5a^7\cos(c+dx)-a^7\cos^2(c+dx)-3a^7\cos^3(c+dx)-a^7\cos^4(c+dx)\right.}{a^4}$$
$$= -5a^3x+a^3\int \cos^2(c+dx)\,dx+a^3\int \cos^4(c+dx)\,dx+a^3\int \sec(c+dx)\,dx+a$$
$$= -5a^3x+\frac{a^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{5a^3\sin(c+dx)}{d}+\frac{a^3\cos(c+dx)\sin(c+dx)}{2d}$$
$$= -\frac{9a^3x}{2}+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{2a^3\sin(c+dx)}{d}+\frac{7a^3\cos(c+dx)\sin(c+d}{8d}$$
$$= -\frac{33a^3x}{8}+\frac{3a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{2a^3\sin(c+dx)}{d}+\frac{7a^3\cos(c+dx)\sin(c+}{8d}$$

Mathematica [A] time = 0.450713, size = 114, normalized size = 0.83

$$\frac{a^3\sec^2(c+dx)\left(-16\sin(c+dx)+225\sin(2(c+dx))-72\sin(3(c+dx))+18\sin(4(c+dx))+8\sin(5(c+dx))+\sin(6(c\right.}{128d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^4,x]
```

```
[Out] (a^3*Sec[c + d*x]^2*(-264*c - 264*d*x + 192*ArcTanh[Sin[c + d*x]]*Cos[c + d
*x]^2 - 264*(c + d*x)*Cos[2*(c + d*x)] - 16*Sin[c + d*x] + 225*Sin[2*(c + d
*x)] - 72*Sin[3*(c + d*x)] + 18*Sin[4*(c + d*x)] + 8*Sin[5*(c + d*x)] + Sin
[6*(c + d*x)]))/(128*d)
```

Maple [A] time = 0.044, size = 159, normalized size = 1.2

$$\frac{11\,a^3\cos(dx+c)(\sin(dx+c))^3}{4d}+\frac{33\,a^3\cos(dx+c)\sin(dx+c)}{8d}-\frac{33\,a^3x}{8}-\frac{33\,a^3c}{8d}-\frac{a^3(\sin(dx+c))^3}{2d}+\frac{3\,a^3\ln(\text{sec}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^4,x)
```

```
[Out] 11/4*a^3*cos(d*x+c)*sin(d*x+c)^3/d+33/8*a^3*cos(d*x+c)*sin(d*x+c)/d-33/8*a^
3*x-33/8/d*a^3*c-1/2*a^3*sin(d*x+c)^3/d+3/2/d*a^3*ln(sec(d*x+c)+tan(d*x+c))
-3/2*a^3*sin(d*x+c)/d+3/d*a^3*sin(d*x+c)^5/cos(d*x+c)+1/2/d*a^3*sin(d*x+c)^
5/cos(d*x+c)^2
```

Maxima [A] time = 1.5212, size = 246, normalized size = 1.78

$$-\frac{16\left(2\sin(dx+c)^3-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)+6\sin(dx+c)\right)a^3-(12\,dx+12\,c+\sin(4\,dx}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] -1/32*(16*(2*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
- 1) + 6*sin(d*x + c))*a^3 - (12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*
x + 2*c))*a^3 + 48*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*tan
(d*x + c))*a^3 + 8*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(d*x
 + c) + 1) - 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)))/d
```

Fricas [A] time = 1.90757, size = 381, normalized size = 2.76

$$-\frac{33\,a^3dx\cos(dx+c)^2-6\,a^3\cos(dx+c)^2\log(\sin(dx+c)+1)+6\,a^3\cos(dx+c)^2\log(-\sin(dx+c)+1)-\left(2\,a^3\cos\right.}{8\,d\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] -1/8*(33*a^3*d*x*cos(d*x + c)^2 - 6*a^3*cos(d*x + c)^2*log(sin(d*x + c) + 1
) + 6*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - (2*a^3*cos(d*x + c)^5 + 8
*a^3*cos(d*x + c)^4 + 7*a^3*cos(d*x + c)^3 - 24*a^3*cos(d*x + c)^2 + 24*a^3
*cos(d*x + c) + 4*a^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3166, size = 243, normalized size = 1.76

$$-\frac{33\,(dx+c)a^3-12\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+12\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{8\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\cdots\right.\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}\cdots}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="giac")
```

```
[Out] -1/8*(33*(d*x + c)*a^3 - 12*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 12*a^3
*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 8*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a
^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 + 2*(25*a^3*tan(1/2
*d*x + 1/2*c)^7 + 81*a^3*tan(1/2*d*x + 1/2*c)^5 + 79*a^3*tan(1/2*d*x + 1/2*
c)^3 + 7*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.51 $\int (a+a\sec(c+dx))^3\sin^2(c+dx)\,dx$

Optimal. Leaf size=98

$$-\frac{3a^3\sin(c+dx)}{d}+\frac{3a^3\tan(c+dx)}{d}+\frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{a^3\tan(c+dx)\sec(c+$$

```
[Out] (-5*a^3*x)/2 + (5*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (3*a^3*Sin[c + d*x])/d
 - (a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (3*a^3*Tan[c + d*x])/d + (a^3*Se
c[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.183177, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2872, 2637, 2635, 8, 3770, 3767, 3768}

$$-\frac{3a^3\sin(c+dx)}{d}+\frac{3a^3\tan(c+dx)}{d}+\frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{a^3\tan(c+dx)\sec(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^2,x]
```


```
[Out] (-5*a^3*x)/2 + (5*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (3*a^3*Sin[c + d*x])/d
 - (a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (3*a^3*Tan[c + d*x])/d + (a^3*Se
c[c + d*x]*Tan[c + d*x])/(2*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^3\sin^2(c+dx)\,dx &= -\int (-a-a\cos(c+dx))^3\sec(c+dx)\tan^2(c+dx)\,dx\\ &= -\frac{\int \left(2a^5+3a^5\cos(c+dx)+a^5\cos^2(c+dx)-2a^5\sec(c+dx)-3a^5\sec^2(c+dx\right.}{a^2}\\ &= -2a^3x-a^3\int \cos^2(c+dx)\,dx+a^3\int \sec^3(c+dx)\,dx+\left(2a^3\right)\int \sec(c+dx)\,dx\\ &= -2a^3x+\frac{2a^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{3a^3\sin(c+dx)}{d}-\frac{a^3\cos(c+dx)\sin(c+dx)}{2d}\\ &= -\frac{5a^3x}{2}+\frac{5a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{3a^3\sin(c+dx)}{d}-\frac{a^3\cos(c+dx)\sin(c+dx}{2d}\end{aligned}$$

Mathematica [B] time = 2.49477, size = 300, normalized size = 3.06

$$\frac{1}{32}a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(-\frac{12\sin(c)\cos(dx)}{d}-\frac{\sin(2c)\cos(2dx)}{d}-\frac{12\cos(c)\sin(dx)}{d}-\frac{\cos(2c)\sin(2dx)}{d}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*Sin[c + d*x]^2,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(-10*x - (10*Log[Cos[(c + d*x)
/2] - Sin[(c + d*x)/2]])/d + (10*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/
d - (12*Cos[d*x]*Sin[c])/d - (Cos[2*d*x]*Sin[2*c])/d - (12*Cos[c]*Sin[d*x])
/d - (Cos[2*c]*Sin[2*d*x])/d + 1/(d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2
) + (12*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c +
 d*x)/2])) - 1/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (12*Sin[(d*x)/
2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/32
```

Maple [A] time = 0.038, size = 111, normalized size = 1.1

$$-\frac{a^3\cos(dx+c)\sin(dx+c)}{2d}-\frac{5a^3x}{2}-\frac{5a^3c}{2d}+\frac{5a^3\ln(\sec(dx+c)+\tan(dx+c))}{2d}-\frac{5a^3\sin(dx+c)}{2d}+3\frac{a^3\tan(dx+c}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*sin(d*x+c)^2,x)
```

```
[Out] -1/2*a^3*cos(d*x+c)*sin(d*x+c)/d-5/2*a^3*x-5/2/d*a^3*c+5/2/d*a^3*ln(sec(d*x
+c)+tan(d*x+c))-5/2*a^3*sin(d*x+c)/d+3*a^3*tan(d*x+c)/d+1/2/d*a^3*sin(d*x+c
)^3/cos(d*x+c)^2
```

Maxima [A] time = 1.5123, size = 171, normalized size = 1.74

$$\frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))a^3-12\,(dx+c-\tan(dx+c))a^3-a^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}+\log(\sin(dx+c)+1)-\log(\sin(dx+\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a^3 - 12*(d*x + c - tan(d*x + c))*a^3
 - a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log(s
in(d*x + c) - 1)) + 6*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) -
2*sin(d*x + c)))/d
```

Fricas [A] time = 1.83919, size = 313, normalized size = 3.19

$$-\frac{10\,a^3dx\cos(dx+c)^2-5\,a^3\cos(dx+c)^2\log(\sin(dx+c)+1)+5\,a^3\cos(dx+c)^2\log(-\sin(dx+c)+1)+2\left(a^3\cos\right.}{4\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(10*a^3*d*x*cos(d*x + c)^2 - 5*a^3*cos(d*x + c)^2*log(sin(d*x + c) + 1
) + 5*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(a^3*cos(d*x + c)^3 + 6
*a^3*cos(d*x + c)^2 - 6*a^3*cos(d*x + c) - a^3)*sin(d*x + c))/(d*cos(d*x +
c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*sin(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28844, size = 138, normalized size = 1.41

$$-\frac{5 (dx + c)a^3 - 5 a^3 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + 1\right|\right) + 5 a^3 \log\left(\left|\tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) - 1\right|\right) + \frac{4\left(5 a^3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^7 - 9 a^3 \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^3\right)}{\left(\tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^4 - 1\right)^2}}{2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*(d*x + c)*a^3 - 5*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 5*a^3*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 4*(5*a^3*tan(1/2*d*x + 1/2*c)^7 - 9*a^3*
tan(1/2*d*x + 1/2*c)^3)/(tan(1/2*d*x + 1/2*c)^4 - 1)^2)/d
```

3.52 $\int \csc^2(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=80

$$\frac{3a^3\tan(c+dx)}{d}+\frac{9a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{4a^3\sin(c+dx)}{d(1-\cos(c+dx))}+\frac{a^3\tan(c+dx)\sec(c+dx)}{2d}$$

```
[Out] (9*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (4*a^3*Sin[c + d*x])/(d*(1 - Cos[c +
d*x])) + (3*a^3*Tan[c + d*x])/d + (a^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.194048, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2872, 2648, 3770, 3767, 8, 3768}

$$\frac{3a^3\tan(c+dx)}{d}+\frac{9a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{4a^3\sin(c+dx)}{d(1-\cos(c+dx))}+\frac{a^3\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (9*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (4*a^3*Sin[c + d*x])/(d*(1 - Cos[c +
d*x])) + (3*a^3*Tan[c + d*x])/d + (a^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^3\,dx = -\int(-a-a\cos(c+dx))^3\csc^2(c+dx)\sec^3(c+dx)\,dx$$
$$= a^2\int\left(\frac{4a}{1-\cos(c+dx)}+4a\sec(c+dx)+3a\sec^2(c+dx)+a\sec^3(c+dx)\right)dx$$
$$= a^3\int\sec^3(c+dx)\,dx+\left(3a^3\right)\int\sec^2(c+dx)\,dx+\left(4a^3\right)\int\frac{1}{1-\cos(c+dx)}\,dx+$$
$$= \frac{4a^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{4a^3\sin(c+dx)}{d(1-\cos(c+dx))}+\frac{a^3\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}$$
$$= \frac{9a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{4a^3\sin(c+dx)}{d(1-\cos(c+dx))}+\frac{3a^3\tan(c+dx)}{d}+\frac{a^3\sec(c+d}{2}$$

Mathematica [B] time = 1.06539, size = 244, normalized size = 3.05

$$a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\frac{12\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{}{\left(\cos\left(\frac{1}{2}\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*(-18*Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]] + 18*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 16*Csc[c/2]
*Csc[(c + d*x)/2]*Sin[(d*x)/2] + (Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^(-2)
 - (Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^(-2) + (12*Sin[d*x])/((Cos[c/2] -
Sin[c/2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]))))/(32*d)
```

Maple [A] time = 0.049, size = 102, normalized size = 1.3

$$-7\,\frac{a^3\cot(dx+c)}{d}-\frac{9\,a^3}{2\,d\sin(dx+c)}+\frac{9\,a^3\ln(\sec(dx+c)+\tan(dx+c))}{2\,d}+3\,\frac{a^3}{d\sin(dx+c)\cos(dx+c)}+\frac{}{2\,d\sin(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -7*a^3*cot(d*x+c)/d-9/2/d*a^3/sin(d*x+c)+9/2/d*a^3*ln(sec(d*x+c)+tan(d*x+c)
)+3/d*a^3/sin(d*x+c)/cos(d*x+c)+1/2/d*a^3/sin(d*x+c)/cos(d*x+c)^2
```

Maxima [A] time = 1.02052, size = 185, normalized size = 2.31

$$-\frac{a^3\left(\frac{2\left(3\sin(dx+c)^2-2\right)}{\sin(dx+c)^3-\sin(dx+c)} - 3\log\left(\sin\left(dx+c\right)+1\right) + 3\log\left(\sin\left(dx+c\right)-1\right)\right) + 6a^3\left(\frac{2}{\sin(dx+c)} - \log\left(\sin\left(dx+c\right)+1\right) + \mathrm{l}\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(a^3*(2*(3*sin(d*x + c)^2 - 2)/(sin(d*x + c)^3 - sin(d*x + c)) - 3*log
(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) + 6*a^3*(2/sin(d*x + c) - log
(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a^3*(1/tan(d*x + c) - tan(
d*x + c)) + 4*a^3/tan(d*x + c))/d
```

Fricas [A] time = 1.78353, size = 313, normalized size = 3.91

$$\frac{9a^3\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right) - 9a^3\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right) - 28a^3\cos}{4d\cos\left(dx+c\right)^2\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(9*a^3*cos(d*x + c)^2*log(sin(d*x + c) + 1)*sin(d*x + c) - 9*a^3*cos(d*
x + c)^2*log(-sin(d*x + c) + 1)*sin(d*x + c) - 28*a^3*cos(d*x + c)^3 - 18*a
^3*cos(d*x + c)^2 + 12*a^3*cos(d*x + c) + 2*a^3)/(d*cos(d*x + c)^2*sin(d*x
+ c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30293, size = 143, normalized size = 1.79

$$\frac{9a^3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - 9a^3\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) - \frac{8a^3}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)} - \frac{2\left(5a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 7a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(9*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 9*a^3*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) - 8*a^3/tan(1/2*d*x + 1/2*c) - 2*(5*a^3*tan(1/2*d*x + 1/2*c)^
3 - 7*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.53 $\int \csc^4(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=110

$$\frac{3a^3\tan(c+dx)}{d}+\frac{11a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{17a^3\sin(c+dx)}{3d(1-\cos(c+dx))}-\frac{2a^3\sin(c+dx)}{3d(1-\cos(c+dx))^2}+\frac{a^3\tan(c+dx)\sec(c+d}{2d}$$

```
[Out] (11*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (2*a^3*Sin[c + d*x])/(3*d*(1 - Cos[c
 + d*x])^2) - (17*a^3*Sin[c + d*x])/(3*d*(1 - Cos[c + d*x])) + (3*a^3*Tan[c
 + d*x])/d + (a^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.229634, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2872, 2650, 2648, 3770, 3767, 8, 3768}

$$\frac{3a^3\tan(c+dx)}{d}+\frac{11a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{17a^3\sin(c+dx)}{3d(1-\cos(c+dx))}-\frac{2a^3\sin(c+dx)}{3d(1-\cos(c+dx))^2}+\frac{a^3\tan(c+dx)\sec(c+d}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (11*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (2*a^3*Sin[c + d*x])/(3*d*(1 - Cos[c
 + d*x])^2) - (17*a^3*Sin[c + d*x])/(3*d*(1 - Cos[c + d*x])) + (3*a^3*Tan[c
 + d*x])/d + (a^3*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^3\,dx = -\int (-a-a\cos(c+dx))^3\csc^4(c+dx)\sec^3(c+dx)\,dx$$
$$= a^4\int \left(\frac{2}{a(1-\cos(c+dx))^2}+\frac{5}{a(1-\cos(c+dx))}+\frac{5\sec(c+dx)}{a}+\frac{3\sec^2(c+dx}{a}\right.$$
$$= a^3\int \sec^3(c+dx)\,dx+\left(2a^3\right)\int \frac{1}{(1-\cos(c+dx))^2}\,dx+\left(3a^3\right)\int \sec^2(c+dx)\,dx$$
$$= \frac{5a^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{2a^3\sin(c+dx)}{3d(1-\cos(c+dx))^2}-\frac{5a^3\sin(c+dx)}{d(1-\cos(c+dx))}+\frac{a^3\sec(c}{}$$
$$= \frac{11a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{2a^3\sin(c+dx)}{3d(1-\cos(c+dx))^2}-\frac{17a^3\sin(c+dx)}{3d(1-\cos(c+dx))}+\frac{3a^3\,\mathrm{ta}}{}$$

Mathematica [B] time = 6.23361, size = 678, normalized size = 6.16

$$\frac{3\sin\left(\frac{dx}{2}\right)\cos^3(c+dx)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^3}{8d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\frac{3\sin\left(\frac{dx}{2}\right)\cos^3(c+dx)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^3}{8d\left(\sin\left(\frac{c}{2}\right)+\cos\left(\frac{c}{2}\right)\right)\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}+\cdots$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(Cos[c + d*x]^3*Cot[c/2]*Csc[c/2 + (d*x)/2]^2*Sec[c/2 + (d*x)/2]^6*(a + a*
Sec[c + d*x])^3)/(24*d) - (11*Cos[c + d*x]^3*Log[Cos[c/2 + (d*x)/2] - Sin[c
/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(16*d) + (11*Co
s[c + d*x]^3*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2
]^6*(a + a*Sec[c + d*x])^3)/(16*d) + (17*Cos[c + d*x]^3*Csc[c/2]*Csc[c/2 +
(d*x)/2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])/(24*d) +
 (Cos[c + d*x]^3*Csc[c/2]*Csc[c/2 + (d*x)/2]^3*Sec[c/2 + (d*x)/2]^6*(a + a*
Sec[c + d*x])^3*Sin[(d*x)/2])/(24*d) + (Cos[c + d*x]^3*Sec[c/2 + (d*x)/2]^6
*(a + a*Sec[c + d*x])^3)/(32*d*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2)
 + (3*Cos[c + d*x]^3*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/
2])/(8*d*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) -
 (Cos[c + d*x]^3*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(32*d*(Cos[c/
```

```
2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^2) + (3*Cos[c + d*x]^3*Sec[c/2 + (d*x)/2
]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])/(8*d*(Cos[c/2] + Sin[c/2])*(Cos[c/
2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A] time = 0.074, size = 188, normalized size = 1.7

$$-\frac{26\,a^3\cot(dx+c)}{3\,d}-\frac{a^3\cot(dx+c)\,(\csc(dx+c))^2}{3\,d}-\frac{a^3}{d\,(\sin(dx+c))^3}-\frac{11\,a^3}{2\,d\sin(dx+c)}+\frac{11\,a^3\ln(\sec(dx+c)+\mathrm{ta}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -26/3*a^3*cot(d*x+c)/d-1/3/d*a^3*cot(d*x+c)*csc(d*x+c)^2-1/d*a^3/sin(d*x+c)
^3-11/2/d*a^3/sin(d*x+c)+11/2/d*a^3*ln(sec(d*x+c)+tan(d*x+c))-1/d*a^3/sin(d
*x+c)^3/cos(d*x+c)+4/d*a^3/sin(d*x+c)/cos(d*x+c)-1/3/d*a^3/sin(d*x+c)^3/cos
(d*x+c)^2+5/6/d*a^3/sin(d*x+c)/cos(d*x+c)^2
```

Maxima [A] time = 1.02512, size = 254, normalized size = 2.31

$$-\frac{a^3\left(\frac{2\left(15\sin(dx+c)^4-10\sin(dx+c)^2-2\right)}{\sin(dx+c)^5-\sin(dx+c)^3}-15\log(\sin(dx+c)+1)+15\log(\sin(dx+c)-1)\right)+6\,a^3\left(\frac{2\left(3\sin(dx+c)^2+1\right)}{\sin(dx+c)^3}-3\log\right.}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/12*(a^3*(2*(15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 - 2)/(sin(d*x + c)^5 -
 sin(d*x + c)^3) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 6
*a^3*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3
*log(sin(d*x + c) - 1)) + 12*a^3*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3 - 3
*tan(d*x + c)) + 4*(3*tan(d*x + c)^2 + 1)*a^3/tan(d*x + c)^3)/d
```

Fricas [A] time = 1.73043, size = 444, normalized size = 4.04

$$-\frac{104\,a^3\cos(dx+c)^4-38\,a^3\cos(dx+c)^3-118\,a^3\cos(dx+c)^2+30\,a^3\cos(dx+c)+6\,a^3-33\left(a^3\cos(dx+c)^3-a^3\right.}{12\left(d\cos(dx+c)^3-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/12*(104*a^3*cos(d*x + c)^4 - 38*a^3*cos(d*x + c)^3 - 118*a^3*cos(d*x + c
)^2 + 30*a^3*cos(d*x + c) + 6*a^3 - 33*(a^3*cos(d*x + c)^3 - a^3*cos(d*x +
c)^2)*log(sin(d*x + c) + 1)*sin(d*x + c) + 33*(a^3*cos(d*x + c)^3 - a^3*cos
(d*x + c)^2)*log(-sin(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^3 - d*co
s(d*x + c)^2)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3604, size = 166, normalized size = 1.51

$$\frac{33\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-33\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{6\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}-\frac{2\left(18\,a^3\tan\left(\frac{1}{2}\,d\right.\right.}{\tan\left(\frac{1}{2}\,dx\right.}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/6*(33*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 33*a^3*log(abs(tan(1/2*d*x
 + 1/2*c) - 1)) - 6*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^3*tan(1/2*d*x + 1/2
*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 - 2*(18*a^3*tan(1/2*d*x + 1/2*c)^2 + a^
3)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.54 $\int \csc^6(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=165

$$\frac{152a^3\tan(c+dx)}{15d}+\frac{13a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{13a^3\tan(c+dx)\sec(c+dx)}{2d}-\frac{76a^6\tan(c+dx)\sec(c+dx)}{15d\left(a^3-a^3\cos(c+dx)\right)}-\frac{a^6\,\mathrm{ta}}{5d}$$

```
[Out] (13*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (152*a^3*Tan[c + d*x])/(15*d) + (13*
a^3*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (a^6*Sec[c + d*x]*Tan[c + d*x])/(5*d
*(a - a*Cos[c + d*x])^3) - (11*a^5*Sec[c + d*x]*Tan[c + d*x])/(15*d*(a - a*
Cos[c + d*x])^2) - (76*a^6*Sec[c + d*x]*Tan[c + d*x])/(15*d*(a^3 - a^3*Cos[
c + d*x]))
```

Rubi [A] time = 0.435617, antiderivative size = 165, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3872, 2869, 2766, 2978, 2748, 3768, 3770, 3767, 8}

$$\frac{152a^3\tan(c+dx)}{15d}+\frac{13a^3\tanh^{-1}(\sin(c+dx))}{2d}+\frac{13a^3\tan(c+dx)\sec(c+dx)}{2d}-\frac{76a^6\tan(c+dx)\sec(c+dx)}{15d\left(a^3-a^3\cos(c+dx)\right)}-\frac{a^6\,\mathrm{ta}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (13*a^3*ArcTanh[Sin[c + d*x]])/(2*d) + (152*a^3*Tan[c + d*x])/(15*d) + (13*
a^3*Sec[c + d*x]*Tan[c + d*x])/(2*d) - (a^6*Sec[c + d*x]*Tan[c + d*x])/(5*d
*(a - a*Cos[c + d*x])^3) - (11*a^5*Sec[c + d*x]*Tan[c + d*x])/(15*d*(a - a*
Cos[c + d*x])^2) - (76*a^6*Sec[c + d*x]*Tan[c + d*x])/(15*d*(a^3 - a^3*Cos[
c + d*x]))
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2766

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])^(n_), x_Symbol] :> Simp[(b^2*Cos[e + f*x]*(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^(n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(
a*(2*m + 1)*(b*c - a*d)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f
*x])^n*Simp[b*c*(m + 1) - a*d*(2*m + n + 2) + b*d*(m + n + 2)*Sin[e + f*x],
 x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && EqQ[
a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] &&  !GtQ[n, 0] && (Integer
sQ[2*m, 2*n] || (IntegerQ[m] && EqQ[c, 0]))
```

Rule 2978

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(
n + 1))/(a*f*(2*m + 1)*(b*c - a*d)), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 2748

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \csc^6(c+dx)(a+a\sec(c+dx))^3\,dx = -\int (-a-a\cos(c+dx))^3\csc^6(c+dx)\sec^3(c+dx)\,dx$$
$$= -\left(a^6\int \frac{\sec^3(c+dx)}{(-a+a\cos(c+dx))^3}\,dx\right)$$
$$= -\frac{a^6\sec(c+dx)\tan(c+dx)}{5d(a-a\cos(c+dx))^3} - \frac{1}{5}a^4\int \frac{(-7a-4a\cos(c+dx))\sec^3(c+dx)}{(-a+a\cos(c+dx))^2}\,dx$$
$$= -\frac{a^6\sec(c+dx)\tan(c+dx)}{5d(a-a\cos(c+dx))^3} - \frac{11a^5\sec(c+dx)\tan(c+dx)}{15d(a-a\cos(c+dx))^2} - \frac{1}{15}a^2\int \frac{(43a^2}{}$$
$$= -\frac{a^6\sec(c+dx)\tan(c+dx)}{5d(a-a\cos(c+dx))^3} - \frac{11a^5\sec(c+dx)\tan(c+dx)}{15d(a-a\cos(c+dx))^2} - \frac{76a^4\sec(c+d}{15d(a-a\,c}$$
$$= -\frac{a^6\sec(c+dx)\tan(c+dx)}{5d(a-a\cos(c+dx))^3} - \frac{11a^5\sec(c+dx)\tan(c+dx)}{15d(a-a\cos(c+dx))^2} - \frac{76a^4\sec(c+d}{15d(a-a\,c}$$
$$= \frac{13a^3\sec(c+dx)\tan(c+dx)}{2d} - \frac{a^6\sec(c+dx)\tan(c+dx)}{5d(a-a\cos(c+dx))^3} - \frac{11a^5\sec(c+dx)}{15d(a-a\cos}$$
$$= \frac{13a^3\tanh^{-1}(\sin(c+dx))}{2d} + \frac{152a^3\tan(c+dx)}{15d} + \frac{13a^3\sec(c+dx)\tan(c+dx}{2d}$$

Mathematica [B] time = 1.15463, size = 353, normalized size = 2.14

$$-\frac{a^3(\cos(c+dx)+1)^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(24960\cos^2(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^2*(24960*Cos[c +
 d*x]^2*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] +
Sin[(c + d*x)/2]]) + Csc[c/2]*Csc[(c + d*x)/2]^5*Sec[c]*(-1235*Sin[(d*x)/2]
 + 3805*Sin[(3*d*x)/2] + 4329*Sin[c - (d*x)/2] - 1989*Sin[c + (d*x)/2] - 35
75*Sin[2*c + (d*x)/2] + 475*Sin[c + (3*d*x)/2] + 2005*Sin[2*c + (3*d*x)/2]
+ 2275*Sin[3*c + (3*d*x)/2] - 2673*Sin[c + (5*d*x)/2] + 105*Sin[2*c + (5*d*
x)/2] - 1593*Sin[3*c + (5*d*x)/2] - 975*Sin[4*c + (5*d*x)/2] + 1325*Sin[2*c
 + (7*d*x)/2] - 255*Sin[3*c + (7*d*x)/2] + 875*Sin[4*c + (7*d*x)/2] + 195*S
in[5*c + (7*d*x)/2] - 304*Sin[3*c + (9*d*x)/2] + 90*Sin[4*c + (9*d*x)/2] -
214*Sin[5*c + (9*d*x)/2])))/(30720*d)
```

Maple [A] time = 0.074, size = 274, normalized size = 1.7

$$-\frac{152\,a^3\cot(dx+c)}{15\,d} - \frac{a^3\cot(dx+c)(\csc(dx+c))^4}{5\,d} - \frac{4\,a^3\cot(dx+c)(\csc(dx+c))^2}{15\,d} - \frac{3\,a^3}{5\,d(\sin(dx+c))^5} - \frac{}{d(\sin(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -152/15*a^3*cot(d*x+c)/d-1/5/d*a^3*cot(d*x+c)*csc(d*x+c)^4-4/15/d*a^3*cot(d
*x+c)*csc(d*x+c)^2-3/5/d*a^3/sin(d*x+c)^5-1/d*a^3/sin(d*x+c)^3-13/2/d*a^3/s
in(d*x+c)+13/2/d*a^3*ln(sec(d*x+c)+tan(d*x+c))-3/5/d*a^3/sin(d*x+c)^5/cos(d
*x+c)-6/5/d*a^3/sin(d*x+c)^3/cos(d*x+c)+24/5/d*a^3/sin(d*x+c)/cos(d*x+c)-1/
5/d*a^3/sin(d*x+c)^5/cos(d*x+c)^2-7/15/d*a^3/sin(d*x+c)^3/cos(d*x+c)^2+7/6/
```

d*a^3/sin(d*x+c)/cos(d*x+c)^2

Maxima [A] time = 1.0469, size = 308, normalized size = 1.87

$$-\frac{a^{3}\left(\frac{2\left(105\sin(dx+c)^{6}-70\sin(dx+c)^{4}-14\sin(dx+c)^{2}-6\right)}{\sin(dx+c)^{7}-\sin(dx+c)^{5}}-105\log\left(\sin\left(dx+c\right)+1\right)+105\log\left(\sin\left(dx+c\right)-1\right)\right)+6a^{3}\left(\frac{2\left(15\sin(dx+\right.}{\mathrm{si}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(a^3*(2*(105*sin(d*x + c)^6 - 70*sin(d*x + c)^4 - 14*sin(d*x + c)^2 -
 6)/(sin(d*x + c)^7 - sin(d*x + c)^5) - 105*log(sin(d*x + c) + 1) + 105*log
(sin(d*x + c) - 1)) + 6*a^3*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)/s
in(d*x + c)^5 - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 36*a
^3*((15*tan(d*x + c)^4 + 5*tan(d*x + c)^2 + 1)/tan(d*x + c)^5 - 5*tan(d*x +
 c)) + 4*(15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 + 3)*a^3/tan(d*x + c)^5)/d
```

Fricas [A] time = 1.76661, size = 575, normalized size = 3.48

$$-\frac{608\,a^{3}\cos\left(dx+c\right)^{5}-826\,a^{3}\cos\left(dx+c\right)^{4}-476\,a^{3}\cos\left(dx+c\right)^{3}+868\,a^{3}\cos\left(dx+c\right)^{2}-120\,a^{3}\cos\left(dx+c\right)-30\,a^{3}-1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(608*a^3*cos(d*x + c)^5 - 826*a^3*cos(d*x + c)^4 - 476*a^3*cos(d*x +
c)^3 + 868*a^3*cos(d*x + c)^2 - 120*a^3*cos(d*x + c) - 30*a^3 - 195*(a^3*co
s(d*x + c)^4 - 2*a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2)*log(sin(d*x + c)
+ 1)*sin(d*x + c) + 195*(a^3*cos(d*x + c)^4 - 2*a^3*cos(d*x + c)^3 + a^3*co
s(d*x + c)^2)*log(-sin(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^4 - 2*d
*cos(d*x + c)^3 + d*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38286, size = 190, normalized size = 1.15

$$\frac{390\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-390\,a^{3}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{60\left(5\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-7\,a^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{2}}-\frac{465\,a^{3}\tan(}{}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(390*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 390*a^3*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) - 60*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^3*tan(1/2*d*x +
 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 - (465*a^3*tan(1/2*d*x + 1/2*c)^4 +
 40*a^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.55 $\int \csc^8(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=192

$$\frac{3a^3\tan(c+dx)}{d}-\frac{4a^3\cot^7(c+dx)}{7d}-\frac{3a^3\cot^5(c+dx)}{d}-\frac{7a^3\cot^3(c+dx)}{d}-\frac{13a^3\cot(c+dx)}{d}-\frac{15a^3\csc^7(c+dx)}{14d}-\frac{3a^3}{}$$

```
[Out] (15*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (13*a^3*Cot[c + d*x])/d - (7*a^3*Cot
[c + d*x]^3)/d - (3*a^3*Cot[c + d*x]^5)/d - (4*a^3*Cot[c + d*x]^7)/(7*d) -
(15*a^3*Csc[c + d*x])/(2*d) - (5*a^3*Csc[c + d*x]^3)/(2*d) - (3*a^3*Csc[c +
 d*x]^5)/(2*d) - (15*a^3*Csc[c + d*x]^7)/(14*d) + (a^3*Csc[c + d*x]^7*Sec[c
 + d*x]^2)/(2*d) + (3*a^3*Tan[c + d*x])/d
```

Rubi [A] time = 0.314372, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3872, 2873, 3767, 2621, 302, 207, 2620, 270, 288}

$$\frac{3a^3\tan(c+dx)}{d}-\frac{4a^3\cot^7(c+dx)}{7d}-\frac{3a^3\cot^5(c+dx)}{d}-\frac{7a^3\cot^3(c+dx)}{d}-\frac{13a^3\cot(c+dx)}{d}-\frac{15a^3\csc^7(c+dx)}{14d}-\frac{3a^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^8*(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (15*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (13*a^3*Cot[c + d*x])/d - (7*a^3*Cot
[c + d*x]^3)/d - (3*a^3*Cot[c + d*x]^5)/d - (4*a^3*Cot[c + d*x]^7)/(7*d) -
(15*a^3*Csc[c + d*x])/(2*d) - (5*a^3*Csc[c + d*x]^3)/(2*d) - (3*a^3*Csc[c +
 d*x]^5)/(2*d) - (15*a^3*Csc[c + d*x]^7)/(14*d) + (a^3*Csc[c + d*x]^7*Sec[c
 + d*x]^2)/(2*d) + (3*a^3*Tan[c + d*x])/d
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \csc^8(c+dx)(a+a\sec(c+dx))^3\,dx = -\int (-a-a\cos(c+dx))^3\csc^8(c+dx)\sec^3(c+dx)\,dx$$
$$= \int \left(a^3\csc^8(c+dx)+3a^3\csc^8(c+dx)\sec(c+dx)+3a^3\csc^8(c+dx)\sec^2(c+$$
$$= a^3\int \csc^8(c+dx)\,dx + a^3\int \csc^8(c+dx)\sec^3(c+dx)\,dx + \left(3a^3\right)\int \csc^8(c+$$
$$= -\frac{a^3\,\mathrm{Subst}\left(\int \frac{x^{10}}{\left(-1+x^2\right)^2}\,dx, x, \csc(c+dx)\right)}{d} - \frac{a^3\,\mathrm{Subst}\left(\int \left(1+3x^2+3x^4+x^6\right)\right.}{d}$$
$$= -\frac{a^3\cot(c+dx)}{d} - \frac{a^3\cot^3(c+dx)}{d} - \frac{3a^3\cot^5(c+dx)}{5d} - \frac{a^3\cot^7(c+dx)}{7d} + \frac{a^3}{}$$
$$= -\frac{13a^3\cot(c+dx)}{d} - \frac{7a^3\cot^3(c+dx)}{d} - \frac{3a^3\cot^5(c+dx)}{d} - \frac{4a^3\cot^7(c+dx)}{7d}$$
$$= \frac{3a^3\tanh^{-1}(\sin(c+dx))}{d} - \frac{13a^3\cot(c+dx)}{d} - \frac{7a^3\cot^3(c+dx)}{d} - \frac{3a^3\cot^5(c}{d}$$
$$= \frac{15a^3\tanh^{-1}(\sin(c+dx))}{2d} - \frac{13a^3\cot(c+dx)}{d} - \frac{7a^3\cot^3(c+dx)}{d} - \frac{3a^3\cot^5(}{d}$$

Mathematica [B] time = 1.18217, size = 430, normalized size = 2.24

$$\frac{a^3\cos(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^3\left(-8\csc(2c)(2776\sin(c-dx)-6080\sin(c+dx)+8816\sin(2(c+dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^8*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (a^3*Cos[c + d*x]*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(-860160*Cos[c +
d*x]^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 860160*Cos[c + d*x]^2*Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 8*Csc[2*c]*Csc[(c + d*x)/2]^6*Csc[c
 + d*x]*(5264*Sin[2*c] - 9580*Sin[d*x] + 8480*Sin[2*d*x] + 2776*Sin[c - d*x
] - 6080*Sin[c + d*x] + 8816*Sin[2*(c + d*x)] - 7904*Sin[3*(c + d*x)] + 486
4*Sin[4*(c + d*x)] - 1824*Sin[5*(c + d*x)] + 304*Sin[6*(c + d*x)] - 9580*Si
n[2*c + d*x] - 10024*Sin[3*c + d*x] + 13891*Sin[c + 2*d*x] + 7720*Sin[2*(c
+ 2*d*x)] + 13891*Sin[3*c + 2*d*x] + 10080*Sin[4*c + 2*d*x] - 10060*Sin[c +
 3*d*x] - 12454*Sin[2*c + 3*d*x] - 12454*Sin[4*c + 3*d*x] - 6580*Sin[5*c +
3*d*x] + 7664*Sin[3*c + 4*d*x] + 7664*Sin[5*c + 4*d*x] + 2520*Sin[6*c + 4*d
*x] - 3420*Sin[3*c + 5*d*x] - 2874*Sin[4*c + 5*d*x] - 2874*Sin[6*c + 5*d*x]
 - 420*Sin[7*c + 5*d*x] + 640*Sin[4*c + 6*d*x] + 479*Sin[5*c + 6*d*x] + 479
*Sin[7*c + 6*d*x])))/(917504*d)
```

Maple [B] time = 0.076, size = 360, normalized size = 1.9

$$-\frac{80\,a^3\cot(dx+c)}{7d}-\frac{a^3\cot(dx+c)(\csc(dx+c))^6}{7d}-\frac{6\,a^3\cot(dx+c)(\csc(dx+c))^4}{35d}-\frac{8\,a^3\cot(dx+c)(\csc(dx+c))^2}{35d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^8*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -80/7*a^3*cot(d*x+c)/d-1/7/d*a^3*cot(d*x+c)*csc(d*x+c)^6-6/35/d*a^3*cot(d*x
+c)*csc(d*x+c)^4-8/35/d*a^3*cot(d*x+c)*csc(d*x+c)^2-3/7/d*a^3/sin(d*x+c)^7-
3/5/d*a^3/sin(d*x+c)^5-1/d*a^3/sin(d*x+c)^3-15/2/d*a^3/sin(d*x+c)+15/2/d*a^
3*ln(sec(d*x+c)+tan(d*x+c))-3/7/d*a^3/sin(d*x+c)^7/cos(d*x+c)-24/35/d*a^3/s
in(d*x+c)^5/cos(d*x+c)-48/35/d*a^3/sin(d*x+c)^3/cos(d*x+c)+192/35/d*a^3/sin
(d*x+c)/cos(d*x+c)-1/7/d*a^3/sin(d*x+c)^7/cos(d*x+c)^2-9/35/d*a^3/sin(d*x+c
)^5/cos(d*x+c)^2-3/5/d*a^3/sin(d*x+c)^3/cos(d*x+c)^2+3/2/d*a^3/sin(d*x+c)/c
os(d*x+c)^2
```

Maxima [A] time = 1.01563, size = 362, normalized size = 1.89

$$-\frac{a^3\left(\frac{2\left(315\sin(dx+c)^8-210\sin(dx+c)^6-42\sin(dx+c)^4-18\sin(dx+c)^2-10\right)}{\sin(dx+c)^9-\sin(dx+c)^7}-315\log(\sin(dx+c)+1)+315\log(\sin(dx+c)-1)\right)+2\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/140*(a^3*(2*(315*sin(d*x + c)^8 - 210*sin(d*x + c)^6 - 42*sin(d*x + c)^4
 - 18*sin(d*x + c)^2 - 10)/(sin(d*x + c)^9 - sin(d*x + c)^7) - 315*log(sin(
d*x + c) + 1) + 315*log(sin(d*x + c) - 1)) + 2*a^3*(2*(105*sin(d*x + c)^6 +
 35*sin(d*x + c)^4 + 21*sin(d*x + c)^2 + 15)/sin(d*x + c)^7 - 105*log(sin(d
*x + c) + 1) + 105*log(sin(d*x + c) - 1)) + 12*a^3*((140*tan(d*x + c)^6 + 7
0*tan(d*x + c)^4 + 28*tan(d*x + c)^2 + 5)/tan(d*x + c)^7 - 35*tan(d*x + c))
 + 4*(35*tan(d*x + c)^6 + 35*tan(d*x + c)^4 + 21*tan(d*x + c)^2 + 5)*a^3/ta
```

```
n(d*x + c)^7)/d
```

Fricas [A] time = 1.80772, size = 698, normalized size = 3.64

$$-\frac{320\,a^3\cos\left(dx+c\right)^6-750\,a^3\cos\left(dx+c\right)^5+170\,a^3\cos\left(dx+c\right)^4+720\,a^3\cos\left(dx+c\right)^3-520\,a^3\cos\left(dx+c\right)^2+42\,a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/28*(320*a^3*cos(d*x + c)^6 - 750*a^3*cos(d*x + c)^5 + 170*a^3*cos(d*x +
c)^4 + 720*a^3*cos(d*x + c)^3 - 520*a^3*cos(d*x + c)^2 + 42*a^3*cos(d*x + c
) + 14*a^3 - 105*(a^3*cos(d*x + c)^5 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x
 + c)^3 - a^3*cos(d*x + c)^2)*log(sin(d*x + c) + 1)*sin(d*x + c) + 105*(a^3
*cos(d*x + c)^5 - 3*a^3*cos(d*x + c)^4 + 3*a^3*cos(d*x + c)^3 - a^3*cos(d*x
 + c)^2)*log(-sin(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^5 - 3*d*cos(
d*x + c)^4 + 3*d*cos(d*x + c)^3 - d*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**8*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39616, size = 228, normalized size = 1.19

$$\frac{840\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-840\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-7\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{112\left(5\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}}{112\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/112*(840*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 840*a^3*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) - 7*a^3*tan(1/2*d*x + 1/2*c) - 112*(5*a^3*tan(1/2*d*x +
 1/2*c)^3 - 7*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 - (1
050*a^3*tan(1/2*d*x + 1/2*c)^6 + 112*a^3*tan(1/2*d*x + 1/2*c)^4 + 14*a^3*ta
n(1/2*d*x + 1/2*c)^2 + a^3)/tan(1/2*d*x + 1/2*c)^7)/d
```

3.56 $\int \csc^{10}(c+dx)(a+a\sec(c+dx))^3\,dx$

Optimal. Leaf size=232

$$\frac{3a^3\tan(c+dx)}{d}-\frac{4a^3\cot^9(c+dx)}{9d}-\frac{19a^3\cot^7(c+dx)}{7d}-\frac{36a^3\cot^5(c+dx)}{5d}-\frac{34a^3\cot^3(c+dx)}{3d}-\frac{16a^3\cot(c+dx)}{d}-1$$

```
[Out] (17*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (16*a^3*Cot[c + d*x])/d - (34*a^3*Co
t[c + d*x]^3)/(3*d) - (36*a^3*Cot[c + d*x]^5)/(5*d) - (19*a^3*Cot[c + d*x]^
7)/(7*d) - (4*a^3*Cot[c + d*x]^9)/(9*d) - (17*a^3*Csc[c + d*x])/(2*d) - (17
*a^3*Csc[c + d*x]^3)/(6*d) - (17*a^3*Csc[c + d*x]^5)/(10*d) - (17*a^3*Csc[c
 + d*x]^7)/(14*d) - (17*a^3*Csc[c + d*x]^9)/(18*d) + (a^3*Csc[c + d*x]^9*Se
c[c + d*x]^2)/(2*d) + (3*a^3*Tan[c + d*x])/d
```

Rubi [A] time = 0.331666, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3872, 2873, 3767, 2621, 302, 207, 2620, 270, 288}

$$\frac{3a^3\tan(c+dx)}{d}-\frac{4a^3\cot^9(c+dx)}{9d}-\frac{19a^3\cot^7(c+dx)}{7d}-\frac{36a^3\cot^5(c+dx)}{5d}-\frac{34a^3\cot^3(c+dx)}{3d}-\frac{16a^3\cot(c+dx)}{d}-1$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^10*(a + a*Sec[c + d*x])^3,x]
```



```
[Out] (17*a^3*ArcTanh[Sin[c + d*x]])/(2*d) - (16*a^3*Cot[c + d*x])/d - (34*a^3*Co
t[c + d*x]^3)/(3*d) - (36*a^3*Cot[c + d*x]^5)/(5*d) - (19*a^3*Cot[c + d*x]^
7)/(7*d) - (4*a^3*Cot[c + d*x]^9)/(9*d) - (17*a^3*Csc[c + d*x])/(2*d) - (17
*a^3*Csc[c + d*x]^3)/(6*d) - (17*a^3*Csc[c + d*x]^5)/(10*d) - (17*a^3*Csc[c
 + d*x]^7)/(14*d) - (17*a^3*Csc[c + d*x]^9)/(18*d) + (a^3*Csc[c + d*x]^9*Se
c[c + d*x]^2)/(2*d) + (3*a^3*Tan[c + d*x])/d
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
```

```
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \csc^{10}(c+dx)(a+a\sec(c+dx))^3\,dx = -\int(-a-a\cos(c+dx))^3\csc^{10}(c+dx)\sec^3(c+dx)\,dx$$

$$= \int\left(a^3\csc^{10}(c+dx)+3a^3\csc^{10}(c+dx)\sec(c+dx)+3a^3\csc^{10}(c+dx)\sec^2$$

$$= a^3\int\csc^{10}(c+dx)\,dx+a^3\int\csc^{10}(c+dx)\sec^3(c+dx)\,dx+\left(3a^3\right)\int\csc^{10}($$

$$= -\frac{a^3\,\mathrm{Subst}\left(\int\frac{x^{12}}{\left(-1+x^2\right)^2}\,dx,x,\csc(c+dx)\right)}{d}-\frac{a^3\,\mathrm{Subst}\left(\int\left(1+4x^2+6x^4+4x\right.\right.}{d}$$

$$= -\frac{a^3\cot(c+dx)}{d}-\frac{4a^3\cot^3(c+dx)}{3d}-\frac{6a^3\cot^5(c+dx)}{5d}-\frac{4a^3\cot^7(c+dx)}{7d}-$$

$$= -\frac{16a^3\cot(c+dx)}{d}-\frac{34a^3\cot^3(c+dx)}{3d}-\frac{36a^3\cot^5(c+dx)}{5d}-\frac{19a^3\cot^7(c+}{7d}$$

$$= \frac{3a^3\tanh^{-1}(\sin(c+dx))}{d}-\frac{16a^3\cot(c+dx)}{d}-\frac{34a^3\cot^3(c+dx)}{3d}-\frac{36a^3\cot}{5}$$

$$= \frac{17a^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{16a^3\cot(c+dx)}{d}-\frac{34a^3\cot^3(c+dx)}{3d}-\frac{36a^3\cot}{}$$

Mathematica [B] time = 6.68008, size = 1000, normalized size = 4.31

$$\frac{\cos^3(c+dx)\csc\left(\frac{c}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(\sec(c+dx)a+a)^3\sin\left(\frac{dx}{2}\right)\csc^9\left(\frac{c}{2}+\frac{dx}{2}\right)}{4608d}-\frac{\cos^3(c+dx)\cot\left(\frac{c}{2}\right)\sec^6\left(\frac{c}{2}+\frac{dx}{2}\right)(\sec(c+}{4608d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^10*(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-9833*Cos[c + d*x]^3*Cot[c/2]*Csc[c/2 + (d*x)/2]^2*Sec[c/2 + (d*x)/2]^6*(a
 + a*Sec[c + d*x])^3)/(80640*d) - (979*Cos[c + d*x]^3*Cot[c/2]*Csc[c/2 + (d
*x)/2]^4*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(53760*d) - (5*Cos[c
+ d*x]^3*Cot[c/2]*Csc[c/2 + (d*x)/2]^6*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c +
d*x])^3)/(2016*d) - (Cos[c + d*x]^3*Cot[c/2]*Csc[c/2 + (d*x)/2]^8*Sec[c/2 +
 (d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(4608*d) - (17*Cos[c + d*x]^3*Log[Cos[c
/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x
])^3)/(16*d) + (17*Cos[c + d*x]^3*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/
2]]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3)/(16*d) + (197147*Cos[c + d
*x]^3*Csc[c/2]*Csc[c/2 + (d*x)/2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])
^3*Sin[(d*x)/2])/(161280*d) + (9833*Cos[c + d*x]^3*Csc[c/2]*Csc[c/2 + (d*x)
/2]^3*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])/(80640*d) +
 (979*Cos[c + d*x]^3*Csc[c/2]*Csc[c/2 + (d*x)/2]^5*Sec[c/2 + (d*x)/2]^6*(a
+ a*Sec[c + d*x])^3*Sin[(d*x)/2])/(53760*d) + (5*Cos[c + d*x]^3*Csc[c/2]*Cs
c[c/2 + (d*x)/2]^7*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2]
)/(2016*d) + (Cos[c + d*x]^3*Csc[c/2]*Csc[c/2 + (d*x)/2]^9*Sec[c/2 + (d*x)/
2]^6*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])/(4608*d) - (35*Cos[c + d*x]^3*Sec
[c/2]*Sec[c/2 + (d*x)/2]^7*(a + a*Sec[c + d*x])^3*Sin[(d*x)/2])/(1536*d) -
(Cos[c + d*x]^3*Sec[c/2]*Sec[c/2 + (d*x)/2]^9*(a + a*Sec[c + d*x])^3*Sin[(d
*x)/2])/(1536*d) + (Cos[c + d*x]*Sec[c]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c +
 d*x])^3*Sin[d*x])/(16*d) + (Cos[c + d*x]^2*Sec[c]*Sec[c/2 + (d*x)/2]^6*(a
+ a*Sec[c + d*x])^3*(Sin[c] + 6*Sin[d*x]))/(16*d) - (Cos[c + d*x]^3*Sec[c/2
 + (d*x)/2]^8*(a + a*Sec[c + d*x])^3*Tan[c/2])/(1536*d)
```

Maple [B] time = 0.086, size = 446, normalized size = 1.9

$$-\frac{3968\,a^3\cot(dx+c)}{315\,d}-\frac{a^3\cot(dx+c)(\csc(dx+c))^8}{9\,d}-\frac{8\,a^3\cot(dx+c)(\csc(dx+c))^6}{63\,d}-\frac{16\,a^3\cot(dx+c)(\csc(dx+c}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^10*(a+a*sec(d*x+c))^3,x)
```

```
[Out] -3968/315*a^3*cot(d*x+c)/d-1/9/d*a^3*cot(d*x+c)*csc(d*x+c)^8-8/63/d*a^3*cot
(d*x+c)*csc(d*x+c)^6-16/105/d*a^3*cot(d*x+c)*csc(d*x+c)^4-64/315/d*a^3*cot(
d*x+c)*csc(d*x+c)^2-1/3/d*a^3/sin(d*x+c)^9-3/7/d*a^3/sin(d*x+c)^7-3/5/d*a^3
/sin(d*x+c)^5-1/d*a^3/sin(d*x+c)^3-17/2/d*a^3/sin(d*x+c)+17/2/d*a^3*ln(sec(
d*x+c)+tan(d*x+c))-1/3/d*a^3/sin(d*x+c)^9/cos(d*x+c)-10/21/d*a^3/sin(d*x+c)
^7/cos(d*x+c)-16/21/d*a^3/sin(d*x+c)^5/cos(d*x+c)-32/21/d*a^3/sin(d*x+c)^3/
cos(d*x+c)+128/21/d*a^3/sin(d*x+c)/cos(d*x+c)-1/9/d*a^3/sin(d*x+c)^9/cos(d*
x+c)^2-11/63/d*a^3/sin(d*x+c)^7/cos(d*x+c)^2-11/35/d*a^3/sin(d*x+c)^5/cos(d
*x+c)^2-11/15/d*a^3/sin(d*x+c)^3/cos(d*x+c)^2+11/6/d*a^3/sin(d*x+c)/cos(d*x
+c)^2
```

Maxima [A] time = 1.02781, size = 416, normalized size = 1.79

$$-\frac{a^3\left(\frac{2\left(3465 \sin(dx+c)^{10}-2310 \sin(dx+c)^8-462 \sin(dx+c)^6-198 \sin(dx+c)^4-110 \sin(dx+c)^2-70\right)}{\sin(dx+c)^{11}-\sin(dx+c)^9} - 3465 \log\left(\sin\left(dx+c\right)+1\right) + 3465 \log\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/1260*(a^3*(2*(3465*sin(d*x + c)^10 - 2310*sin(d*x + c)^8 - 462*sin(d*x +
 c)^6 - 198*sin(d*x + c)^4 - 110*sin(d*x + c)^2 - 70)/(sin(d*x + c)^11 - si
n(d*x + c)^9) - 3465*log(sin(d*x + c) + 1) + 3465*log(sin(d*x + c) - 1)) +
6*a^3*(2*(315*sin(d*x + c)^8 + 105*sin(d*x + c)^6 + 63*sin(d*x + c)^4 + 45*
sin(d*x + c)^2 + 35)/sin(d*x + c)^9 - 315*log(sin(d*x + c) + 1) + 315*log(s
in(d*x + c) - 1)) + 60*a^3*((315*tan(d*x + c)^8 + 210*tan(d*x + c)^6 + 126*
tan(d*x + c)^4 + 45*tan(d*x + c)^2 + 7)/tan(d*x + c)^9 - 63*tan(d*x + c)) +
 4*(315*tan(d*x + c)^8 + 420*tan(d*x + c)^6 + 378*tan(d*x + c)^4 + 180*tan(
d*x + c)^2 + 35)*a^3/tan(d*x + c)^9)/d
```

Fricas [A] time = 1.92144, size = 973, normalized size = 4.19

$$-\frac{15872\,a^3\cos\left(dx+c\right)^8 - 36906\,a^3\cos\left(dx+c\right)^7 - 8322\,a^3\cos\left(dx+c\right)^6 + 73402\,a^3\cos\left(dx+c\right)^5 - 33342\,a^3\cos\left(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/1260*(15872*a^3*cos(d*x + c)^8 - 36906*a^3*cos(d*x + c)^7 - 8322*a^3*cos
(d*x + c)^6 + 73402*a^3*cos(d*x + c)^5 - 33342*a^3*cos(d*x + c)^4 - 34746*a
^3*cos(d*x + c)^3 + 26702*a^3*cos(d*x + c)^2 - 1890*a^3*cos(d*x + c) - 630*
a^3 - 5355*(a^3*cos(d*x + c)^7 - 3*a^3*cos(d*x + c)^6 + 2*a^3*cos(d*x + c)^
5 + 2*a^3*cos(d*x + c)^4 - 3*a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2)*log(s
in(d*x + c) + 1)*sin(d*x + c) + 5355*(a^3*cos(d*x + c)^7 - 3*a^3*cos(d*x +
c)^6 + 2*a^3*cos(d*x + c)^5 + 2*a^3*cos(d*x + c)^4 - 3*a^3*cos(d*x + c)^3 +
 a^3*cos(d*x + c)^2)*log(-sin(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^
7 - 3*d*cos(d*x + c)^6 + 2*d*cos(d*x + c)^5 + 2*d*cos(d*x + c)^4 - 3*d*cos(
d*x + c)^3 + d*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**10*(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41781, size = 273, normalized size = 1.18

$$-\frac{105\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-171360\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+171360\,a^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3780\,a^3\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10*(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/20160*(105*a^3*tan(1/2*d*x + 1/2*c)^3 - 171360*a^3*log(abs(tan(1/2*d*x +
 1/2*c) + 1)) + 171360*a^3*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 3780*a^3*ta
n(1/2*d*x + 1/2*c) + 20160*(5*a^3*tan(1/2*d*x + 1/2*c)^3 - 7*a^3*tan(1/2*d*
x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 + (220185*a^3*tan(1/2*d*x + 1/2*
c)^8 + 26880*a^3*tan(1/2*d*x + 1/2*c)^6 + 4347*a^3*tan(1/2*d*x + 1/2*c)^4 +
 540*a^3*tan(1/2*d*x + 1/2*c)^2 + 35*a^3)/tan(1/2*d*x + 1/2*c)^9)/d
```

3.57 $\int \frac{\sin^9(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{\sin^8(c+dx)}{8ad}-\frac{\cos^9(c+dx)}{9ad}+\frac{3\cos^7(c+dx)}{7ad}-\frac{3\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}$$

```
[Out] Cos[c + d*x]^3/(3*a*d) - (3*Cos[c + d*x]^5)/(5*a*d) + (3*Cos[c + d*x]^7)/(7
*a*d) - Cos[c + d*x]^9/(9*a*d) + Sin[c + d*x]^8/(8*a*d)
```

Rubi [A] time = 0.161202, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3872, 2835, 2564, 30, 2565, 270}

$$\frac{\sin^8(c+dx)}{8ad}-\frac{\cos^9(c+dx)}{9ad}+\frac{3\cos^7(c+dx)}{7ad}-\frac{3\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^9/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cos[c + d*x]^3/(3*a*d) - (3*Cos[c + d*x]^5)/(5*a*d) + (3*Cos[c + d*x]^7)/(7
*a*d) - Cos[c + d*x]^9/(9*a*d) + Sin[c + d*x]^8/(8*a*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^9(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin^9(c+dx)}{-a-a\cos(c+dx)}\,dx \\
&= \frac{\int \cos(c+dx)\sin^7(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^7(c+dx)\,dx}{a} \\
&= \frac{\text{Subst}\left(\int x^7\,dx, x, \sin(c+dx)\right)}{ad} + \frac{\text{Subst}\left(\int x^2\left(1-x^2\right)^3\,dx, x, \cos(c+dx)\right)}{ad} \\
&= \frac{\sin^8(c+dx)}{8ad} + \frac{\text{Subst}\left(\int \left(x^2-3x^4+3x^6-x^8\right)dx, x, \cos(c+dx)\right)}{ad} \\
&= \frac{\cos^3(c+dx)}{3ad} - \frac{3\cos^5(c+dx)}{5ad} + \frac{3\cos^7(c+dx)}{7ad} - \frac{\cos^9(c+dx)}{9ad} + \frac{\sin^8(c+dx)}{8ad}
\end{aligned}$$

Mathematica [A] time = 4.23403, size = 62, normalized size = 0.68

$$\frac{\sin^{10}\left(\frac{1}{2}(c+dx)\right)(6995\cos(c+dx)+3650\cos(2(c+dx))+1085\cos(3(c+dx))+140\cos(4(c+dx))+4258)}{315ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^9/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((4258 + 6995*Cos[c + d*x] + 3650*Cos[2*(c + d*x)] + 1085*Cos[3*(c + d*x)]
+ 140*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]^10)/(315*a*d)
```

Maple [A] time = 0.098, size = 89, normalized size = 1.

$$\frac{1}{da}\left(\frac{1}{3(\sec(dx+c))^3} - \frac{1}{2(\sec(dx+c))^6} - \frac{1}{2(\sec(dx+c))^2} + \frac{1}{8(\sec(dx+c))^8} + \frac{3}{4(\sec(dx+c))^4} - \frac{3}{5(\sec(dx+c))^5}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^9/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/d/a*(1/3/sec(d*x+c)^3-1/2/sec(d*x+c)^6-1/2/sec(d*x+c)^2+1/8/sec(d*x+c)^8+
3/4/sec(d*x+c)^4-3/5/sec(d*x+c)^5+3/7/sec(d*x+c)^7-1/9/sec(d*x+c)^9)
```

Maxima [A] time = 0.988516, size = 120, normalized size = 1.32

$$-\frac{280\cos(dx+c)^9 - 315\cos(dx+c)^8 - 1080\cos(dx+c)^7 + 1260\cos(dx+c)^6 + 1512\cos(dx+c)^5 - 1890\cos(dx+}{2520\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2520*(280*cos(d*x + c)^9 - 315*cos(d*x + c)^8 - 1080*cos(d*x + c)^7 + 12
60*cos(d*x + c)^6 + 1512*cos(d*x + c)^5 - 1890*cos(d*x + c)^4 - 840*cos(d*x
 + c)^3 + 1260*cos(d*x + c)^2)/(a*d)
```

Fricas [A] time = 1.74285, size = 254, normalized size = 2.79

$$-\frac{280\cos(dx+c)^9 - 315\cos(dx+c)^8 - 1080\cos(dx+c)^7 + 1260\cos(dx+c)^6 + 1512\cos(dx+c)^5 - 1890\cos(dx+c)^4 - 840\cos(dx+c)^3 + 1260\cos(dx+c)^2}{2520\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2520*(280*cos(d*x + c)^9 - 315*cos(d*x + c)^8 - 1080*cos(d*x + c)^7 + 12
60*cos(d*x + c)^6 + 1512*cos(d*x + c)^5 - 1890*cos(d*x + c)^4 - 840*cos(d*x
 + c)^3 + 1260*cos(d*x + c)^2)/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**9/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30621, size = 190, normalized size = 2.09

$$\frac{32\left(\frac{9(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{36(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{84(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{126(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} + \frac{630(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5} - 1\right)}{315\,ad\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 32/315*(9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 36*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 + 84*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 126*(c
os(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 630*(cos(d*x + c) - 1)^5/(cos(d*x
 + c) + 1)^5 - 1)/(a*d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^9)
```

3.58 $\int \frac{\sin^7(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{\sin^6(c+dx)}{6ad}+\frac{\cos^7(c+dx)}{7ad}-\frac{2\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}$$

```
[Out] Cos[c + d*x]^3/(3*a*d) - (2*Cos[c + d*x]^5)/(5*a*d) + Cos[c + d*x]^7/(7*a*d
) + Sin[c + d*x]^6/(6*a*d)
```

Rubi [A] time = 0.154761, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3872, 2835, 2564, 30, 2565, 270}

$$\frac{\sin^6(c+dx)}{6ad}+\frac{\cos^7(c+dx)}{7ad}-\frac{2\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a + a*Sec[c + d*x]),x]
```


```
[Out] Cos[c + d*x]^3/(3*a*d) - (2*Cos[c + d*x]^5)/(5*a*d) + Cos[c + d*x]^7/(7*a*d
) + Sin[c + d*x]^6/(6*a*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
```

```
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^7(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin^7(c+dx)}{-a-a\cos(c+dx)}\,dx \\
&= \frac{\int \cos(c+dx)\sin^5(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^5(c+dx)\,dx}{a} \\
&= \frac{\operatorname{Subst}\left(\int x^5\,dx, x, \sin(c+dx)\right)}{ad} + \frac{\operatorname{Subst}\left(\int x^2\left(1-x^2\right)^2\,dx, x, \cos(c+dx)\right)}{ad} \\
&= \frac{\sin^6(c+dx)}{6ad} + \frac{\operatorname{Subst}\left(\int \left(x^2-2x^4+x^6\right)dx, x, \cos(c+dx)\right)}{ad} \\
&= \frac{\cos^3(c+dx)}{3ad} - \frac{2\cos^5(c+dx)}{5ad} + \frac{\cos^7(c+dx)}{7ad} + \frac{\sin^6(c+dx)}{6ad}
\end{aligned}
$$

Mathematica [A] time = 1.55637, size = 52, normalized size = 0.71

$$\frac{4\sin^8\left(\frac{1}{2}(c+dx)\right)(197\cos(c+dx)+85\cos(2(c+dx))+15\cos(3(c+dx))+123)}{105ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a + a*Sec[c + d*x]),x]
```

```
[Out] (4*(123 + 197*Cos[c + d*x] + 85*Cos[2*(c + d*x)] + 15*Cos[3*(c + d*x)])*Sin
[(c + d*x)/2]^8)/(105*a*d)
```

Maple [A] time = 0.085, size = 70, normalized size = 1.

$$-\frac{1}{da}\left(\frac{1}{6\left(\sec(dx+c)\right)^6} - \frac{1}{3\left(\sec(dx+c)\right)^3} + \frac{1}{2\left(\sec(dx+c)\right)^2} - \frac{1}{2\left(\sec(dx+c)\right)^4} + \frac{2}{5\left(\sec(dx+c)\right)^5} - \frac{1}{7\left(\sec(dx+\dots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/d/a*(1/6/sec(d*x+c)^6-1/3/sec(d*x+c)^3+1/2/sec(d*x+c)^2-1/2/sec(d*x+c)^4
+2/5/sec(d*x+c)^5-1/7/sec(d*x+c)^7)
```

Maxima [A] time = 1.01735, size = 93, normalized size = 1.27

$$\frac{30\cos(dx+c)^7 - 35\cos(dx+c)^6 - 84\cos(dx+c)^5 + 105\cos(dx+c)^4 + 70\cos(dx+c)^3 - 105\cos(dx+c)^2}{210\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/210*(30*cos(d*x + c)^7 - 35*cos(d*x + c)^6 - 84*cos(d*x + c)^5 + 105*cos(
d*x + c)^4 + 70*cos(d*x + c)^3 - 105*cos(d*x + c)^2)/(a*d)
```

Fricas [A] time = 1.71823, size = 182, normalized size = 2.49

$$\frac{30\cos(dx+c)^7 - 35\cos(dx+c)^6 - 84\cos(dx+c)^5 + 105\cos(dx+c)^4 + 70\cos(dx+c)^3 - 105\cos(dx+c)^2}{210\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/210*(30*cos(d*x + c)^7 - 35*cos(d*x + c)^6 - 84*cos(d*x + c)^5 + 105*cos(
d*x + c)^4 + 70*cos(d*x + c)^3 - 105*cos(d*x + c)^2)/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23589, size = 161, normalized size = 2.21

$$\frac{16\left(\frac{7(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{21(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{35(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{140(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} - 1\right)}{105\,ad\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 16/105*(7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 21*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 + 35*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 140*(c
os(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 1)/(a*d*((cos(d*x + c) - 1)/(cos(
d*x + c) + 1) - 1)^7)
```

3.59 $\int \frac{\sin^5(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{\sin^4(c+dx)}{4ad}-\frac{\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}$$

```
[Out] Cos[c + d*x]^3/(3*a*d) - Cos[c + d*x]^5/(5*a*d) + Sin[c + d*x]^4/(4*a*d)
```

Rubi [A] time = 0.148124, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3872, 2835, 2564, 30, 2565, 14}

$$\frac{\sin^4(c+dx)}{4ad}-\frac{\cos^5(c+dx)}{5ad}+\frac{\cos^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cos[c + d*x]^3/(3*a*d) - Cos[c + d*x]^5/(5*a*d) + Sin[c + d*x]^4/(4*a*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^5(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin^5(c+dx)}{-a-a\cos(c+dx)}\,dx\\ &= \frac{\int \cos(c+dx)\sin^3(c+dx)\,dx}{a}-\frac{\int \cos^2(c+dx)\sin^3(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}\left(\int x^3\,dx,x,\sin(c+dx)\right)}{ad}+\frac{\text{Subst}\left(\int x^2\left(1-x^2\right)\,dx,x,\cos(c+dx)\right)}{ad}\\ &= \frac{\sin^4(c+dx)}{4ad}+\frac{\text{Subst}\left(\int \left(x^2-x^4\right)\,dx,x,\cos(c+dx)\right)}{ad}\\ &= \frac{\cos^3(c+dx)}{3ad}-\frac{\cos^5(c+dx)}{5ad}+\frac{\sin^4(c+dx)}{4ad}\end{aligned}$$

Mathematica [A] time = 0.334041, size = 42, normalized size = 0.76

$$\frac{2\sin^6\left(\frac{1}{2}(c+dx)\right)(21\cos(c+dx)+6\cos(2(c+dx))+13)}{15ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + a*Sec[c + d*x]),x]

[Out] (2*(13 + 21*Cos[c + d*x] + 6*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]^6)/(15*a*d)
```

Maple [A] time = 0.072, size = 49, normalized size = 0.9

$$\frac{1}{da}\left(\frac{1}{3\left(\sec(dx+c)\right)^3}-\frac{1}{2\left(\sec(dx+c)\right)^2}+\frac{1}{4\left(\sec(dx+c)\right)^4}-\frac{1}{5\left(\sec(dx+c)\right)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+a*sec(d*x+c)),x)

[Out] 1/d/a*(1/3/sec(d*x+c)^3-1/2/sec(d*x+c)^2+1/4/sec(d*x+c)^4-1/5/sec(d*x+c)^5)
```

Maxima [A] time = 0.994888, size = 66, normalized size = 1.2

$$-\frac{12\cos(dx+c)^5-15\cos(dx+c)^4-20\cos(dx+c)^3+30\cos(dx+c)^2}{60\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*(12*cos(d*x + c)^5 - 15*cos(d*x + c)^4 - 20*cos(d*x + c)^3 + 30*cos(d
*x + c)^2)/(a*d)
```

Fricas [A] time = 1.66676, size = 126, normalized size = 2.29

$$-\frac{12 \cos\left(dx+c\right)^{5} - 15 \cos\left(dx+c\right)^{4} - 20 \cos\left(dx+c\right)^{3} + 30 \cos\left(dx+c\right)^{2}}{60 \, ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(12*cos(d*x + c)^5 - 15*cos(d*x + c)^4 - 20*cos(d*x + c)^3 + 30*cos(d
*x + c)^2)/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2043, size = 131, normalized size = 2.38

$$\frac{4 \left(\frac{5 (\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{10 (\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}} + \frac{30 (\cos(dx+c)-1)^{3}}{(\cos(dx+c)+1)^{3}} - 1\right)}{15 \, ad \left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 4/15*(5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 10*(cos(d*x + c) - 1)^2/(co
s(d*x + c) + 1)^2 + 30*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 1)/(a*d*
((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5)
```

3.60 $\int \frac{\sin^3(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\sin^2(c+dx)}{2ad}+\frac{\cos^3(c+dx)}{3ad}$$

```
[Out] Cos[c + d*x]^3/(3*a*d) + Sin[c + d*x]^2/(2*a*d)
```

Rubi [A] time = 0.126194, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2835, 2564, 30, 2565}

$$\frac{\sin^2(c+dx)}{2ad}+\frac{\cos^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cos[c + d*x]^3/(3*a*d) + Sin[c + d*x]^2/(2*a*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2565

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_.)*sin[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> -Dist[(a*f)^(-1), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x
, a*Cos[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&
!(IntegerQ[(m - 1)/2] && GtQ[m, 0] && LeQ[m, n])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^3(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin^3(c+dx)}{-a-a\cos(c+dx)}\,dx \\
&= \frac{\int \cos(c+dx)\sin(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin(c+dx)\,dx}{a} \\
&= \frac{\text{Subst}(\int x\,dx, x, \sin(c+dx))}{ad} + \frac{\text{Subst}\left(\int x^2\,dx, x, \cos(c+dx)\right)}{ad} \\
&= \frac{\cos^3(c+dx)}{3ad} + \frac{\sin^2(c+dx)}{2ad}
\end{aligned}
$$

Mathematica [A] time = 0.110726, size = 32, normalized size = 0.86

$$\frac{2\sin^4\left(\frac{1}{2}(c+dx)\right)(2\cos(c+dx)+1)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*(1 + 2*Cos[c + d*x])*Sin[(c + d*x)/2]^4)/(3*a*d)
```

Maple [A] time = 0.056, size = 30, normalized size = 0.8

$$-\frac{1}{da}\left(-\frac{1}{3\,(\sec(dx+c))^3}+\frac{1}{2\,(\sec(dx+c))^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/d/a*(-1/3/sec(d*x+c)^3+1/2/sec(d*x+c)^2)
```

Maxima [A] time = 0.991526, size = 39, normalized size = 1.05

$$\frac{2\,\cos(dx+c)^3-3\,\cos(dx+c)^2}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(2*cos(d*x + c)^3 - 3*cos(d*x + c)^2)/(a*d)
```

Fricas [A] time = 1.66242, size = 66, normalized size = 1.78

$$\frac{2\,\cos(dx+c)^3-3\,\cos(dx+c)^2}{6\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*cos(d*x + c)^3 - 3*cos(d*x + c)^2)/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22407, size = 43, normalized size = 1.16

$$\frac{\frac{2\cos(dx+c)^3}{d} - \frac{3\cos(dx+c)^2}{d}}{6a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(2*cos(d*x + c)^3/d - 3*cos(d*x + c)^2/d)/a
```

3.61 $\int \frac{\sin(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=31

$$\frac{\log(\cos(c+dx)+1)}{ad}-\frac{\cos(c+dx)}{ad}$$

```
[Out] -(Cos[c + d*x]/(a*d)) + Log[1 + Cos[c + d*x]]/(a*d)
```

Rubi [A] time = 0.0714901, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3872, 2833, 12, 43}

$$\frac{\log(\cos(c+dx)+1)}{ad}-\frac{\cos(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + a*Sec[c + d*x]),x]

[Out] -(Cos[c + d*x]/(a*d)) + Log[1 + Cos[c + d*x]]/(a*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin(c+dx)}{-a-a\cos(c+dx)}\,dx \\
&= \frac{\text{Subst}\left(\int \frac{x}{a(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{ad} \\
&= \frac{\text{Subst}\left(\int \frac{x}{-a+x}\,dx, x, -a\cos(c+dx)\right)}{a^2d} \\
&= \frac{\text{Subst}\left(\int \left(1-\frac{a}{a-x}\right)\,dx, x, -a\cos(c+dx)\right)}{a^2d} \\
&= -\frac{\cos(c+dx)}{ad} + \frac{\log(1+\cos(c+dx))}{ad}
\end{aligned}
$$

Mathematica [A] time = 0.0800689, size = 28, normalized size = 0.9

$$
-\frac{\cos(c+dx) - 2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{ad}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((Cos[c + d*x] - 2*Log[Cos[(c + d*x)/2]])/(a*d))
```

Maple [A] time = 0.023, size = 49, normalized size = 1.6

$$
\frac{\ln(1+\sec(dx+c))}{da} - \frac{1}{da\sec(dx+c)} - \frac{\ln(\sec(dx+c))}{da}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/d/a*ln(1+sec(d*x+c))-1/d/a/sec(d*x+c)-1/d/a*ln(sec(d*x+c))
```

Maxima [A] time = 1.00837, size = 41, normalized size = 1.32

$$
-\frac{\frac{\cos(dx+c)}{a} - \frac{\log(\cos(dx+c)+1)}{a}}{d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(cos(d*x + c)/a - log(cos(d*x + c) + 1)/a)/d
```

Fricas [A] time = 1.70961, size = 72, normalized size = 2.32

$$
-\frac{\cos(dx+c) - \log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)}{ad}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(cos(d*x + c) - log(1/2*cos(d*x + c) + 1/2))/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sin(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)/(sec(c + d*x) + 1), x)/a
```

Giac [A] time = 1.25922, size = 46, normalized size = 1.48

$$-\frac{\cos(dx+c)}{ad}+\frac{\log(|-\cos(dx+c)-1|)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -cos(d*x + c)/(a*d) + log(abs(-cos(d*x + c) - 1))/(a*d)
```

3.62 $\int \frac{\csc(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=58

$$-\frac{\csc^2(c+dx)}{2ad}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

```
[Out] -ArcTanh[Cos[c + d*x]]/(2*a*d) + (Cot[c + d*x]*Csc[c + d*x])/(2*a*d) - Csc[
c + d*x]^2/(2*a*d)
```

Rubi [A] time = 0.0970141, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3872, 2706, 2606, 30, 2611, 3770}

$$-\frac{\csc^2(c+dx)}{2ad}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] -ArcTanh[Cos[c + d*x]]/(2*a*d) + (Cot[c + d*x]*Csc[c + d*x])/(2*a*d) - Csc[
c + d*x]^2/(2*a*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2706

```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Dist[1/a, Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x]
 - Dist[1/(b*g), Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; FreeQ
[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cot(c+dx)}{-a-a\cos(c+dx)}\,dx \\
&= -\frac{\int \cot^2(c+dx)\csc(c+dx)\,dx}{a}+\frac{\int \cot(c+dx)\csc^2(c+dx)\,dx}{a} \\
&= \frac{\cot(c+dx)\csc(c+dx)}{2ad}+\frac{\int \csc(c+dx)\,dx}{2a}-\frac{\text{Subst}\left(\int x\,dx,x,\csc(c+dx)\right)}{ad} \\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{\cot(c+dx)\csc(c+dx)}{2ad}-\frac{\csc^2(c+dx)}{2ad}
\end{aligned}$$

Mathematica [A] time = 0.0958943, size = 67, normalized size = 1.16

$$-\frac{\sec(c+dx)\left(2\cos^2\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+1\right)}{2ad(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((1 + 2*Cos[(c + d*x)/2]^2*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])
)*Sec[c + d*x])/(2*a*d*(1 + Sec[c + d*x]))
```

Maple [A] time = 0.051, size = 54, normalized size = 0.9

$$-\frac{1}{2\,da\,(\cos(dx+c)+1)}-\frac{\ln(\cos(dx+c)+1)}{4\,da}+\frac{\ln(-1+\cos(dx+c))}{4\,da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/2/a/d/(cos(d*x+c)+1)-1/4*ln(cos(d*x+c)+1)/a/d+1/4/a/d*ln(-1+cos(d*x+c))
```

Maxima [A] time = 1.00968, size = 63, normalized size = 1.09

$$-\frac{\frac{\log(\cos(dx+c)+1)}{a}-\frac{\log(\cos(dx+c)-1)}{a}+\frac{2}{a\cos(dx+c)+a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*(log(cos(d*x + c) + 1)/a - log(cos(d*x + c) - 1)/a + 2/(a*cos(d*x + c)
 + a))/d
```

Fricas [A] time = 1.66203, size = 181, normalized size = 3.12

$$-\frac{(\cos(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-(\cos(dx+c)+1)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+2}{4\left(ad\cos(dx+c)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*((cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) - (cos(d*x + c) + 1)*l
og(-1/2*cos(d*x + c) + 1/2) + 2)/(a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\csc(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)/(sec(c + d*x) + 1), x)/a
```

Giac [A] time = 1.32623, size = 76, normalized size = 1.31

$$\frac{\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}+\frac{\cos(dx+c)-1}{a(\cos(dx+c)+1)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/4*(log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a + (cos(d*x + c) -
1)/(a*(cos(d*x + c) + 1)))/d
```

3.63 $\int \frac{\csc^3(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=82

$$-\frac{\csc^4(c+dx)}{4ad}-\frac{\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{\cot(c+dx)\csc(c+dx)}{8ad}$$

```
[Out] -ArcTanh[Cos[c + d*x]]/(8*a*d) - (Cot[c + d*x]*Csc[c + d*x])/(8*a*d) + (Cot
[c + d*x]*Csc[c + d*x]^3)/(4*a*d) - Csc[c + d*x]^4/(4*a*d)
```

Rubi [A] time = 0.158394, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2835, 2606, 30, 2611, 3768, 3770}

$$-\frac{\csc^4(c+dx)}{4ad}-\frac{\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{\cot(c+dx)\csc(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] -ArcTanh[Cos[c + d*x]]/(8*a*d) - (Cot[c + d*x]*Csc[c + d*x])/(8*a*d) + (Cot
[c + d*x]*Csc[c + d*x]^3)/(4*a*d) - Csc[c + d*x]^4/(4*a*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
```

```
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^3(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cot(c+dx)\csc^2(c+dx)}{-a-a\cos(c+dx)}\,dx\\ &= -\frac{\int \cot^2(c+dx)\csc^3(c+dx)\,dx}{a}+\frac{\int \cot(c+dx)\csc^4(c+dx)\,dx}{a}\\ &= \frac{\cot(c+dx)\csc^3(c+dx)}{4ad}+\frac{\int \csc^3(c+dx)\,dx}{4a}-\frac{\operatorname{Subst}\left(\int x^3\,dx,x,\csc(c+dx)\right)}{ad}\\ &= -\frac{\cot(c+dx)\csc(c+dx)}{8ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{\csc^4(c+dx)}{4ad}+\frac{\int \csc(c+dx)\,dx}{8a}\\ &= -\frac{\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\cot(c+dx)\csc(c+dx)}{8ad}+\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}-\frac{\csc^4(c+dx)}{4ad}\end{aligned}$$

Mathematica [A] time = 0.368848, size = 91, normalized size = 1.11

$$-\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(2\csc^2\left(\frac{1}{2}(c+dx)\right)+\sec^4\left(\frac{1}{2}(c+dx)\right)-4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)}{16ad(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(Cos[(c + d*x)/2]^2*(2*Csc[(c + d*x)/2]^2 + 4*Log[Cos[(c + d*x)/2]] - 4*Lo
g[Sin[(c + d*x)/2]] + Sec[(c + d*x)/2]^4)*Sec[c + d*x])/(16*a*d*(1 + Sec[c
+ d*x]))
```

Maple [A] time = 0.06, size = 72, normalized size = 0.9

$$-\frac{1}{8\,da\,(\cos(dx+c)+1)^2}-\frac{\ln(\cos(dx+c)+1)}{16\,da}+\frac{1}{8\,da\,(-1+\cos(dx+c))}+\frac{\ln(-1+\cos(dx+c))}{16\,da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/8/a/d/(cos(d*x+c)+1)^2-1/16*ln(cos(d*x+c)+1)/a/d+1/8/a/d/(-1+cos(d*x+c))
+1/16/a/d*ln(-1+cos(d*x+c))
```

Maxima [A] time = 1.02163, size = 116, normalized size = 1.41

$$\frac{\frac{2\left(\cos(dx+c)^2+\cos(dx+c)+2\right)}{a\cos(dx+c)^3+a\cos(dx+c)^2-a\cos(dx+c)-a}-\frac{\log(\cos(dx+c)+1)}{a}+\frac{\log(\cos(dx+c)-1)}{a}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(cos(d*x + c)^2 + cos(d*x + c) + 2)/(a*cos(d*x + c)^3 + a*cos(d*x +
 c)^2 - a*cos(d*x + c) - a) - log(cos(d*x + c) + 1)/a + log(cos(d*x + c) -
1)/a)/d
```

Fricas [A] time = 1.70858, size = 378, normalized size = 4.61

$$\frac{2\cos(dx+c)^2-\left(\cos(dx+c)^3+\cos(dx+c)^2-\cos(dx+c)-1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+\left(\cos(dx+c)^3+\cos(dx\right.}{16\left(ad\cos(dx+c)^3+ad\cos(dx+c)^2-ad\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(2*cos(d*x + c)^2 - (cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x + c) -
1)*log(1/2*cos(d*x + c) + 1/2) + (cos(d*x + c)^3 + cos(d*x + c)^2 - cos(d*x
 + c) - 1)*log(-1/2*cos(d*x + c) + 1/2) + 2*cos(d*x + c) + 4)/(a*d*cos(d*x
+ c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\csc^3(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**3/(sec(c + d*x) + 1), x)/a
```

Giac [A] time = 1.29878, size = 174, normalized size = 2.12

$$-\frac{\frac{2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)(\cos(dx+c)+1)}{a(\cos(dx+c)-1)}-\frac{2\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}-\frac{\frac{2a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^2}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/32*(2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)*(cos(d*x + c) + 1)/(a*
(cos(d*x + c) - 1)) - 2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a
 - (2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2)/a^2)/d
```

3.64 $\int \frac{\csc^5(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=106

$$-\frac{\csc^6(c+dx)}{6ad}-\frac{\tanh^{-1}(\cos(c+dx))}{16ad}+\frac{\cot(c+dx)\csc^5(c+dx)}{6ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{24ad}-\frac{\cot(c+dx)\csc(c+dx}{16ad}$$

```
[Out] -ArcTanh[Cos[c + d*x]]/(16*a*d) - (Cot[c + d*x]*Csc[c + d*x])/(16*a*d) - (C
ot[c + d*x]*Csc[c + d*x]^3)/(24*a*d) + (Cot[c + d*x]*Csc[c + d*x]^5)/(6*a*d
) - Csc[c + d*x]^6/(6*a*d)
```

Rubi [A] time = 0.172693, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2835, 2606, 30, 2611, 3768, 3770}

$$-\frac{\csc^6(c+dx)}{6ad}-\frac{\tanh^{-1}(\cos(c+dx))}{16ad}+\frac{\cot(c+dx)\csc^5(c+dx)}{6ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{24ad}-\frac{\cot(c+dx)\csc(c+dx}{16ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] -ArcTanh[Cos[c + d*x]]/(16*a*d) - (Cot[c + d*x]*Csc[c + d*x])/(16*a*d) - (C
ot[c + d*x]*Csc[c + d*x]^3)/(24*a*d) + (Cot[c + d*x]*Csc[c + d*x]^5)/(6*a*d
) - Csc[c + d*x]^6/(6*a*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2835

```
Int[(cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.))/((
a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[1/a, Int[Cos[e + f*
x]^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[1/(b*d), Int[Cos[e + f*x]^(p -
 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] &&
IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && IntegerQ[n] && (LtQ[0, n, (p +
1)/2] || (LeQ[p, -n] && LtQ[-n, 2*p - 3]) || (GtQ[n, 0] && LeQ[n, -p]))
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
```

```
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^5(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cot(c+dx)\csc^4(c+dx)}{-a-a\cos(c+dx)}\,dx\\
&= -\frac{\int \cot^2(c+dx)\csc^5(c+dx)\,dx}{a} + \frac{\int \cot(c+dx)\csc^6(c+dx)\,dx}{a}\\
&= \frac{\cot(c+dx)\csc^5(c+dx)}{6ad} + \frac{\int \csc^5(c+dx)\,dx}{6a} - \frac{\text{Subst}\left(\int x^5\,dx, x, \csc(c+dx)\right)}{ad}\\
&= -\frac{\cot(c+dx)\csc^3(c+dx)}{24ad} + \frac{\cot(c+dx)\csc^5(c+dx)}{6ad} - \frac{\csc^6(c+dx)}{6ad} + \frac{\int \csc^3(c+dx)\,dx}{8a}\\
&= -\frac{\cot(c+dx)\csc(c+dx)}{16ad} - \frac{\cot(c+dx)\csc^3(c+dx)}{24ad} + \frac{\cot(c+dx)\csc^5(c+dx)}{6ad} - \frac{\csc^6(c+dx}{6ad}\\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{16ad} - \frac{\cot(c+dx)\csc(c+dx)}{16ad} - \frac{\cot(c+dx)\csc^3(c+dx)}{24ad} + \frac{\cot(c+dx)\csc}{6ad}\end{aligned}$$

Mathematica [A] time = 0.474233, size = 122, normalized size = 1.15

$$-\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(3\csc^4\left(\frac{1}{2}(c+dx)\right)+12\csc^2\left(\frac{1}{2}(c+dx)\right)+2\sec^6\left(\frac{1}{2}(c+dx)\right)+3\sec^4\left(\frac{1}{2}(c+dx)\right)+24\left(\text{lo}\right.\right.}{192ad(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(Cos[(c + d*x)/2]^2*(12*Csc[(c + d*x)/2]^2 + 3*Csc[(c + d*x)/2]^4 + 24*(Lo
g[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]]) + 3*Sec[(c + d*x)/2]^4 + 2*Sec
[(c + d*x)/2]^6)*Sec[c + d*x])/(192*a*d*(1 + Sec[c + d*x]))
```

Maple [A] time = 0.067, size = 108, normalized size = 1.

$$-\frac{1}{24\,da\,(\cos(dx+c)+1)^3} - \frac{1}{32\,da\,(\cos(dx+c)+1)^2} - \frac{\ln(\cos(dx+c)+1)}{32\,da} - \frac{1}{32\,da\,(-1+\cos(dx+c))^2} + \frac{1}{16\,da\,(-1+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/24/a/d/(cos(d*x+c)+1)^3-1/32/a/d/(cos(d*x+c)+1)^2-1/32*ln(cos(d*x+c)+1)/
a/d-1/32/a/d/(-1+cos(d*x+c))^2+1/16/a/d/(-1+cos(d*x+c))+1/32/a/d*ln(-1+cos(
d*x+c))
```

Maxima [A] time = 1.01937, size = 176, normalized size = 1.66

$$\frac{\frac{2\left(3\cos(dx+c)^4+3\cos(dx+c)^3-5\cos(dx+c)^2-5\cos(dx+c)-8\right)}{a\cos(dx+c)^5+a\cos(dx+c)^4-2a\cos(dx+c)^3-2a\cos(dx+c)^2+a\cos(dx+c)+a}-\frac{3\log(\cos(dx+c)+1)}{a}+\frac{3\log(\cos(dx+c)-1)}{a}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/96*(2*(3*cos(d*x + c)^4 + 3*cos(d*x + c)^3 - 5*cos(d*x + c)^2 - 5*cos(d*x
 + c) - 8)/(a*cos(d*x + c)^5 + a*cos(d*x + c)^4 - 2*a*cos(d*x + c)^3 - 2*a*
cos(d*x + c)^2 + a*cos(d*x + c) + a) - 3*log(cos(d*x + c) + 1)/a + 3*log(co
s(d*x + c) - 1)/a)/d
```

Fricas [B] time = 1.78485, size = 603, normalized size = 5.69

$$\frac{6\cos(dx+c)^4+6\cos(dx+c)^3-10\cos(dx+c)^2-3\left(\cos(dx+c)^5+\cos(dx+c)^4-2\cos(dx+c)^3-2\cos(dx+\right.}{96\left(ad\cos(dx+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(6*cos(d*x + c)^4 + 6*cos(d*x + c)^3 - 10*cos(d*x + c)^2 - 3*(cos(d*x
+ c)^5 + cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c
) + 1)*log(1/2*cos(d*x + c) + 1/2) + 3*(cos(d*x + c)^5 + cos(d*x + c)^4 - 2
*cos(d*x + c)^3 - 2*cos(d*x + c)^2 + cos(d*x + c) + 1)*log(-1/2*cos(d*x + c
) + 1/2) - 10*cos(d*x + c) - 16)/(a*d*cos(d*x + c)^5 + a*d*cos(d*x + c)^4 -
 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c) + a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc^5(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**5/(sec(c + d*x) + 1), x)/a
```

Giac [A] time = 1.38418, size = 246, normalized size = 2.32

$$\frac{\frac{3\left(\frac{6(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{6(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^2}{a(\cos(dx+c)-1)^2}+\frac{12\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}+\frac{\frac{12a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{9a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{2a^2(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{a^3}}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/384*(3*(6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 6*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^2/(a*(cos(d*x + c) - 1)^2) + 1
2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a + (12*a^2*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 9*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^
2 + 2*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^3)/d
```

3.65 $\int \frac{\sin^8(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=125

$$\frac{\sin^7(c+dx)}{7ad}+\frac{\sin^5(c+dx)\cos^3(c+dx)}{8ad}+\frac{5\sin^3(c+dx)\cos^3(c+dx)}{48ad}+\frac{5\sin(c+dx)\cos^3(c+dx)}{64ad}-\frac{5\sin(c+dx)\cos(c+dx)}{128ad}$$

```
[Out] (-5*x)/(128*a) - (5*Cos[c + d*x]*Sin[c + d*x])/(128*a*d) + (5*Cos[c + d*x]^
3*Sin[c + d*x])/(64*a*d) + (5*Cos[c + d*x]^3*Sin[c + d*x]^3)/(48*a*d) + (Co
s[c + d*x]^3*Sin[c + d*x]^5)/(8*a*d) + Sin[c + d*x]^7/(7*a*d)
```

Rubi [A] time = 0.210315, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2839, 2564, 30, 2568, 2635, 8}

$$\frac{\sin^7(c+dx)}{7ad}+\frac{\sin^5(c+dx)\cos^3(c+dx)}{8ad}+\frac{5\sin^3(c+dx)\cos^3(c+dx)}{48ad}+\frac{5\sin(c+dx)\cos^3(c+dx)}{64ad}-\frac{5\sin(c+dx)\cos(c+dx)}{128ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-5*x)/(128*a) - (5*Cos[c + d*x]*Sin[c + d*x])/(128*a*d) + (5*Cos[c + d*x]^
3*Sin[c + d*x])/(64*a*d) + (5*Cos[c + d*x]^3*Sin[c + d*x]^3)/(48*a*d) + (Co
s[c + d*x]^3*Sin[c + d*x]^5)/(8*a*d) + Sin[c + d*x]^7/(7*a*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
```

```
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^8(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin^8(c+dx)}{-a-a\cos(c+dx)}\,dx\\ &= \frac{\int \cos(c+dx)\sin^6(c+dx)\,dx}{a}-\frac{\int \cos^2(c+dx)\sin^6(c+dx)\,dx}{a}\\ &= \frac{\cos^3(c+dx)\sin^5(c+dx)}{8ad}-\frac{5\int \cos^2(c+dx)\sin^4(c+dx)\,dx}{8a}+\frac{\operatorname{Subst}\left(\int x^6\,dx,x,\sin(c+dx)\right)}{ad}\\ &= \frac{5\cos^3(c+dx)\sin^3(c+dx)}{48ad}+\frac{\cos^3(c+dx)\sin^5(c+dx)}{8ad}+\frac{\sin^7(c+dx)}{7ad}-\frac{5\int \cos^2(c+dx)\sin^2}{16a}\\ &= \frac{5\cos^3(c+dx)\sin(c+dx)}{64ad}+\frac{5\cos^3(c+dx)\sin^3(c+dx)}{48ad}+\frac{\cos^3(c+dx)\sin^5(c+dx)}{8ad}+\frac{\sin^7(c}{7a}\\ &= -\frac{5\cos(c+dx)\sin(c+dx)}{128ad}+\frac{5\cos^3(c+dx)\sin(c+dx)}{64ad}+\frac{5\cos^3(c+dx)\sin^3(c+dx)}{48ad}+\frac{\cos^3(}{}\\ &= -\frac{5x}{128a}-\frac{5\cos(c+dx)\sin(c+dx)}{128ad}+\frac{5\cos^3(c+dx)\sin(c+dx)}{64ad}+\frac{5\cos^3(c+dx)\sin^3(c+dx)}{48ad}\end{aligned}$$

Mathematica [A] time = 1.19528, size = 132, normalized size = 1.06

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(1680\sin(c+dx)+336\sin(2(c+dx))-1008\sin(3(c+dx))+168\sin(4(c+dx))+336\sin(\right.}{10752ad(\sec(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(1176*c - 840*d*x + 1680*Sin[c + d*x] + 33
6*Sin[2*(c + d*x)] - 1008*Sin[3*(c + d*x)] + 168*Sin[4*(c + d*x)] + 336*Sin
[5*(c + d*x)] - 112*Sin[6*(c + d*x)] - 48*Sin[7*(c + d*x)] + 21*Sin[8*(c +
d*x)] - 1176*Tan[c/2]))/(10752*a*d*(1 + Sec[c + d*x]))
```

Maple [B] time = 0.099, size = 290, normalized size = 2.3

$$\frac{5}{64da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-8}+\frac{115}{192da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-8}+\frac{383}{192da}\left(\tan\left(\frac{dx}{2}+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^8/(a+a*sec(d*x+c)),x)
```

```
[Out] 5/64/a/d/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)+115/192/a/d/(1+tan(1
/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^3+383/192/a/d/(1+tan(1/2*d*x+1/2*c)^2
)^8*tan(1/2*d*x+1/2*c)^5+5053/1344/a/d/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d
*x+1/2*c)^7+44099/1344/a/d/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^9-
383/192/a/d/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^11-115/192/a/d/(1
+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^13-5/64/a/d/(1+tan(1/2*d*x+1/2*
c)^2)^8*tan(1/2*d*x+1/2*c)^15-5/64/d/a*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.52456, size = 486, normalized size = 3.89

$$\frac{\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{805\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{2681\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5053\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{44099\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{2681\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{805\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{105\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}}{a+\frac{8a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{56a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{70a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{56a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{28a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{8a\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{a\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}}-\frac{105\arctan\left(\frac{\sin(dx}{\cos(dx+}\right.}{a}}{1344\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/1344*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 + 2681*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5053*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7 + 44099*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 2681*s
in(d*x + c)^11/(cos(d*x + c) + 1)^11 - 805*sin(d*x + c)^13/(cos(d*x + c) +
1)^13 - 105*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)/(a + 8*a*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 28*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 56*a*sin(
d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8
+ 56*a*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a*sin(d*x + c)^12/(cos(d*
x + c) + 1)^12 + 8*a*sin(d*x + c)^14/(cos(d*x + c) + 1)^14 + a*sin(d*x + c)
^16/(cos(d*x + c) + 1)^16) - 105*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)
/d
```

Fricas [A] time = 1.79057, size = 261, normalized size = 2.09

$$-\frac{105\,dx-\left(336\,\cos\left(dx+c\right)^{7}-384\,\cos\left(dx+c\right)^{6}-952\,\cos\left(dx+c\right)^{5}+1152\,\cos\left(dx+c\right)^{4}+826\,\cos\left(dx+c\right)^{3}-1152}{2688\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2688*(105*d*x - (336*cos(d*x + c)^7 - 384*cos(d*x + c)^6 - 952*cos(d*x +
 c)^5 + 1152*cos(d*x + c)^4 + 826*cos(d*x + c)^3 - 1152*cos(d*x + c)^2 - 10
5*cos(d*x + c) + 384)*sin(d*x + c))/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**8/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31033, size = 188, normalized size = 1.5

$$-\frac{\frac{105\,(dx+c)}{a}+\frac{2\left(105\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+805\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+2681\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-44099\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-5053\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-2681\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{8}a}}{2688\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2688*(105*(d*x + c)/a + 2*(105*tan(1/2*d*x + 1/2*c)^15 + 805*tan(1/2*d*x
 + 1/2*c)^13 + 2681*tan(1/2*d*x + 1/2*c)^11 - 44099*tan(1/2*d*x + 1/2*c)^9
- 5053*tan(1/2*d*x + 1/2*c)^7 - 2681*tan(1/2*d*x + 1/2*c)^5 - 805*tan(1/2*d
*x + 1/2*c)^3 - 105*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^8*a
))/d
```

3.66 $\int \frac{\sin^6(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=99

$$\frac{\sin^5(c+dx)}{5ad}+\frac{\sin^3(c+dx)\cos^3(c+dx)}{6ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{8ad}-\frac{\sin(c+dx)\cos(c+dx)}{16ad}-\frac{x}{16a}$$

```
[Out] -x/(16*a) - (Cos[c + d*x]*Sin[c + d*x])/(16*a*d) + (Cos[c + d*x]^3*Sin[c +
d*x])/(8*a*d) + (Cos[c + d*x]^3*Sin[c + d*x]^3)/(6*a*d) + Sin[c + d*x]^5/(5
*a*d)
```

Rubi [A] time = 0.177497, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2839, 2564, 30, 2568, 2635, 8}

$$\frac{\sin^5(c+dx)}{5ad}+\frac{\sin^3(c+dx)\cos^3(c+dx)}{6ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{8ad}-\frac{\sin(c+dx)\cos(c+dx)}{16ad}-\frac{x}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] -x/(16*a) - (Cos[c + d*x]*Sin[c + d*x])/(16*a*d) + (Cos[c + d*x]^3*Sin[c +
d*x])/(8*a*d) + (Cos[c + d*x]^3*Sin[c + d*x]^3)/(6*a*d) + Sin[c + d*x]^5/(5
*a*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
```

```
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^6(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin^6(c+dx)}{-a-a\cos(c+dx)}\,dx\\
&= \frac{\int \cos(c+dx)\sin^4(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^4(c+dx)\,dx}{a}\\
&= \frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad} - \frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{2a} + \frac{\text{Subst}\left(\int x^4\,dx, x, \sin(c+dx)\right)}{ad}\\
&= \frac{\cos^3(c+dx)\sin(c+dx)}{8ad} + \frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad} + \frac{\sin^5(c+dx)}{5ad} - \frac{\int \cos^2(c+dx)\,dx}{8a}\\
&= -\frac{\cos(c+dx)\sin(c+dx)}{16ad} + \frac{\cos^3(c+dx)\sin(c+dx)}{8ad} + \frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad} + \frac{\sin^5(c+dx}{5ad}\\
&= -\frac{x}{16a} - \frac{\cos(c+dx)\sin(c+dx)}{16ad} + \frac{\cos^3(c+dx)\sin(c+dx)}{8ad} + \frac{\cos^3(c+dx)\sin^3(c+dx)}{6ad} + \frac{\sin^5}{}\end{aligned}$$

Mathematica [A] time = 0.695006, size = 112, normalized size = 1.13

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(120\sin(c+dx)+15\sin(2(c+dx))-60\sin(3(c+dx))+15\sin(4(c+dx))+12\sin(5(c+d\right.}{480ad(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(75*c - 60*d*x + 120*Sin[c + d*x] + 15*Sin
[2*(c + d*x)] - 60*Sin[3*(c + d*x)] + 15*Sin[4*(c + d*x)] + 12*Sin[5*(c + d
*x)] - 5*Sin[6*(c + d*x)] - 75*Tan[c/2]))/(480*a*d*(1 + Sec[c + d*x]))
```

Maple [B] time = 0.08, size = 222, normalized size = 2.2

$$-\frac{1}{8\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{11}\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-6} - \frac{17}{24\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{9}\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-6} + \frac{223}{20\,da}\left(\tan\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/8/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11-17/24/a/d/(1+tan(
1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9+223/20/a/d/(1+tan(1/2*d*x+1/2*c)^2
)^6*tan(1/2*d*x+1/2*c)^7+33/20/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1
/2*c)^5+17/24/a/d/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3+1/8/a/d/(
1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)-1/8/d/a*arctan(tan(1/2*d*x+1/2
*c))
```

Maxima [B] time = 1.51048, size = 375, normalized size = 3.79

$$\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{85\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{198\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{1338\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{85\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{15\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a+\frac{6a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}-\frac{15\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/120*((15*sin(d*x + c)/(cos(d*x + c) + 1) + 85*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 198*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1338*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7 - 85*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 15*sin(d*x +
c)^11/(cos(d*x + c) + 1)^11)/(a + 6*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 +
 15*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a*sin(d*x + c)^6/(cos(d*x +
c) + 1)^6 + 15*a*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a*sin(d*x + c)^10/
(cos(d*x + c) + 1)^10 + a*sin(d*x + c)^12/(cos(d*x + c) + 1)^12) - 15*arcta
n(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A] time = 1.70145, size = 190, normalized size = 1.92

$$-\frac{15\,dx+\left(40\,\cos\left(dx+c\right)^5-48\,\cos\left(dx+c\right)^4-70\,\cos\left(dx+c\right)^3+96\,\cos\left(dx+c\right)^2+15\,\cos\left(dx+c\right)-48\right)\sin\left(dx+\right.}{240\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/240*(15*d*x + (40*cos(d*x + c)^5 - 48*cos(d*x + c)^4 - 70*cos(d*x + c)^3
 + 96*cos(d*x + c)^2 + 15*cos(d*x + c) - 48)*sin(d*x + c))/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3087, size = 153, normalized size = 1.55

$$-\frac{\frac{15\,(dx+c)}{a}+\frac{2\left(15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+85\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-1338\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-198\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-85\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{6}a}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/240*(15*(d*x + c)/a + 2*(15*tan(1/2*d*x + 1/2*c)^11 + 85*tan(1/2*d*x + 1
/2*c)^9 - 1338*tan(1/2*d*x + 1/2*c)^7 - 198*tan(1/2*d*x + 1/2*c)^5 - 85*tan
(1/2*d*x + 1/2*c)^3 - 15*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1
)^6*a))/d
```

3.67 $\int \frac{\sin^4(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{\sin^3(c+dx)}{3ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\sin(c+dx)\cos(c+dx)}{8ad}-\frac{x}{8a}$$

```
[Out] -x/(8*a) - (Cos[c + d*x]*Sin[c + d*x])/(8*a*d) + (Cos[c + d*x]^3*Sin[c + d*
x])/(4*a*d) + Sin[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.15003, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2839, 2564, 30, 2568, 2635, 8}

$$\frac{\sin^3(c+dx)}{3ad}+\frac{\sin(c+dx)\cos^3(c+dx)}{4ad}-\frac{\sin(c+dx)\cos(c+dx)}{8ad}-\frac{x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] -x/(8*a) - (Cos[c + d*x]*Sin[c + d*x])/(8*a*d) + (Cos[c + d*x]^3*Sin[c + d*
x])/(4*a*d) + Sin[c + d*x]^3/(3*a*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
```

```
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^4(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin^4(c+dx)}{-a-a\cos(c+dx)}\,dx\\ &= \frac{\int \cos(c+dx)\sin^2(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\sin^2(c+dx)\,dx}{a}\\ &= \frac{\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{\int \cos^2(c+dx)\,dx}{4a} + \frac{\text{Subst}\left(\int x^2\,dx, x, \sin(c+dx)\right)}{ad}\\ &= -\frac{\cos(c+dx)\sin(c+dx)}{8ad} + \frac{\cos^3(c+dx)\sin(c+dx)}{4ad} + \frac{\sin^3(c+dx)}{3ad} - \frac{\int 1\,dx}{8a}\\ &= -\frac{x}{8a} - \frac{\cos(c+dx)\sin(c+dx)}{8ad} + \frac{\cos^3(c+dx)\sin(c+dx)}{4ad} + \frac{\sin^3(c+dx)}{3ad}\end{aligned}$$

Mathematica [A] time = 0.578878, size = 83, normalized size = 1.14

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(24\sin(c+dx)-8\sin(3(c+dx))+3\left(\sin(4(c+dx))+4c-4\tan\left(\frac{c}{2}\right)-4dx\right)\right)}{48ad(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*Sec[c + d*x]*(24*Sin[c + d*x] - 8*Sin[3*(c + d*x)] + 3*
(4*c - 4*d*x + Sin[4*(c + d*x)] - 4*Tan[c/2])))/(48*a*d*(1 + Sec[c + d*x]))
```

Maple [B] time = 0.072, size = 154, normalized size = 2.1

$$-\frac{1}{4\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-4}+\frac{53}{12\,da}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-4}+\frac{11}{12\,da}\left(\tan\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7+53/12/a/d/(1+tan(1
/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5+11/12/a/d/(1+tan(1/2*d*x+1/2*c)^2)^
4*tan(1/2*d*x+1/2*c)^3+1/4/a/d/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c
)-1/4/d/a*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.50441, size = 265, normalized size = 3.63

$$\frac{\frac{\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{11\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{53\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a+\frac{4a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*((3*sin(d*x + c)/(cos(d*x + c) + 1) + 11*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 53*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7)/(a + 4*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a*sin(d*x +
c)^4/(cos(d*x + c) + 1)^4 + 4*a*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + a*sin
(d*x + c)^8/(cos(d*x + c) + 1)^8) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1
))/a)/d
```

Fricas [A] time = 1.67449, size = 128, normalized size = 1.75

$$-\frac{3\,dx-\left(6\,\cos\left(dx+c\right)^3-8\,\cos\left(dx+c\right)^2-3\,\cos\left(dx+c\right)+8\right)\sin\left(dx+c\right)}{24\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/24*(3*d*x - (6*cos(d*x + c)^3 - 8*cos(d*x + c)^2 - 3*cos(d*x + c) + 8)*s
in(d*x + c))/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sin^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

Giac [A] time = 1.3091, size = 117, normalized size = 1.6

$$-\frac{\frac{3(dx+c)}{a}+\frac{2\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-53\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-11\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^4 a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/24*(3*(d*x + c)/a + 2*(3*tan(1/2*d*x + 1/2*c)^7 - 53*tan(1/2*d*x + 1/2*c
)^5 - 11*tan(1/2*d*x + 1/2*c)^3 - 3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1
/2*c)^2 + 1)^4*a))/d
```

3.68 $\int \frac{\sin^2(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=44

$$\frac{\sin(c+dx)}{ad}-\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{x}{2a}$$

```
[Out] -x/(2*a) + Sin[c + d*x]/(a*d) - (Cos[c + d*x]*Sin[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.108727, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2839, 2637, 2635, 8}

$$\frac{\sin(c+dx)}{ad}-\frac{\sin(c+dx)\cos(c+dx)}{2ad}-\frac{x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] -x/(2*a) + Sin[c + d*x]/(a*d) - (Cos[c + d*x]*Sin[c + d*x])/(2*a*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sin^2(c+dx)}{-a-a\cos(c+dx)}\,dx\\ &= \frac{\int \cos(c+dx)\,dx}{a} - \frac{\int \cos^2(c+dx)\,dx}{a}\\ &= \frac{\sin(c+dx)}{ad} - \frac{\cos(c+dx)\sin(c+dx)}{2ad} - \frac{\int 1\,dx}{2a}\\ &= -\frac{x}{2a} + \frac{\sin(c+dx)}{ad} - \frac{\cos(c+dx)\sin(c+dx)}{2ad}\end{aligned}$$

Mathematica [A] time = 0.2682, size = 68, normalized size = 1.55

$$-\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\left(-4\sin(c+dx)+\sin(2(c+dx))-c+\tan\left(\frac{c}{2}\right)+2dx\right)}{2ad(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(Cos[(c + d*x)/2]^2*Sec[c + d*x]*(-c + 2*d*x - 4*Sin[c + d*x] + Sin[2*(c +
 d*x)] + Tan[c/2]))/(2*a*d*(1 + Sec[c + d*x]))
```

Maple [B] time = 0.065, size = 85, normalized size = 1.9

$$3\,\frac{(\tan(1/2\,dx+c/2))^3}{da\left(1+(\tan(1/2\,dx+c/2))^2\right)^2}+\frac{1}{da}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-2}-\frac{1}{da}\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+a*sec(d*x+c)),x)
```

```
[Out] 3/a/d/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+1/a/d/(1+tan(1/2*d*x+
1/2*c)^2)^2*tan(1/2*d*x+1/2*c)-1/d/a*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.52014, size = 151, normalized size = 3.43

$$\frac{\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] ((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/
(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4) - arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a)/d
```

Fricas [A] time = 1.67787, size = 70, normalized size = 1.59

$$-\frac{dx + (\cos(dx + c) - 2)\sin(dx + c)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(d*x + (cos(d*x + c) - 2)*sin(d*x + c))/(a*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sin^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

Giac [A] time = 1.33594, size = 78, normalized size = 1.77

$$-\frac{\frac{dx+c}{a} - \frac{2\left(3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*((d*x + c)/a - 2*(3*tan(1/2*d*x + 1/2*c)^3 + tan(1/2*d*x + 1/2*c))/((t
an(1/2*d*x + 1/2*c)^2 + 1)^2*a))/d
```

3.69 $\int \frac{\csc^2(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^3(c+dx)}{3ad}$$

```
[Out] Cot[c + d*x]^3/(3*a*d) - Csc[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.1261, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2839, 2606, 30, 2607}

$$\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cot[c + d*x]^3/(3*a*d) - Csc[c + d*x]^3/(3*a*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{a+a\sec(c+dx)}\,dx = -\int \frac{\cot(c+dx)\csc(c+dx)}{-a-a\cos(c+dx)}\,dx$$
$$= -\frac{\int \cot^2(c+dx)\csc^2(c+dx)\,dx}{a} + \frac{\int \cot(c+dx)\csc^3(c+dx)\,dx}{a}$$
$$= -\frac{\text{Subst}\left(\int x^2\,dx, x, -\cot(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int x^2\,dx, x, \csc(c+dx)\right)}{ad}$$
$$= \frac{\cot^3(c+dx)}{3ad} - \frac{\csc^3(c+dx)}{3ad}$$

Mathematica [A] time = 0.209556, size = 66, normalized size = 1.78

$$\frac{\csc(c)(2\sin(c+dx)+\sin(2(c+dx))+2\sin(c+2dx)-6\sin(c)+4\sin(dx))\csc(2(c+dx))}{6ad(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Csc[c]*Csc[2*(c + d*x)]*(-6*Sin[c] + 4*Sin[d*x] + 2*Sin[c + d*x] + Sin[2*(
c + d*x)] + 2*Sin[c + 2*d*x]))/(6*a*d*(1 + Sec[c + d*x]))
```

Maple [A] time = 0.049, size = 36, normalized size = 1.

$$\frac{1}{4da}\left(-\frac{1}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/4/d/a*(-1/3*tan(1/2*d*x+1/2*c)^3-1/tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 0.996431, size = 66, normalized size = 1.78

$$-\frac{\frac{3(\cos(dx+c)+1)}{a\sin(dx+c)}+\frac{\sin(dx+c)^3}{a(\cos(dx+c)+1)^3}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/12*(3*(cos(d*x + c) + 1)/(a*sin(d*x + c)) + sin(d*x + c)^3/(a*(cos(d*x +
 c) + 1)^3))/d
```

Fricas [A] time = 1.5547, size = 111, normalized size = 3.

$$-\frac{\cos(dx+c)^2+\cos(dx+c)+1}{3\,(ad\cos(dx+c)+ad)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3*(cos(d*x + c)^2 + cos(d*x + c) + 1)/((a*d*cos(d*x + c) + a*d)*sin(d*x
+ c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**2/(sec(c + d*x) + 1), x)/a
```

Giac [A] time = 1.30797, size = 50, normalized size = 1.35

$$-\frac{\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3}{a}+\frac{3}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/12*(tan(1/2*d*x + 1/2*c)^3/a + 3/(a*tan(1/2*d*x + 1/2*c)))/d
```

3.70 $\int \frac{\csc^4(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^5(c+dx)}{5ad}$$

```
[Out] Cot[c + d*x]^3/(3*a*d) + Cot[c + d*x]^5/(5*a*d) - Csc[c + d*x]^5/(5*a*d)
```

Rubi [A] time = 0.14281, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3872, 2839, 2606, 30, 2607, 14}

$$\frac{\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cot[c + d*x]^3/(3*a*d) + Cot[c + d*x]^5/(5*a*d) - Csc[c + d*x]^5/(5*a*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^4(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cot(c+dx)\csc^3(c+dx)}{-a-a\cos(c+dx)}\,dx\\ &= -\frac{\int \cot^2(c+dx)\csc^4(c+dx)\,dx}{a}+\frac{\int \cot(c+dx)\csc^5(c+dx)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int x^4\,dx,x,\csc(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int x^2\left(1+x^2\right)\,dx,x,-\cot(c+dx)\right)}{ad}\\ &= -\frac{\csc^5(c+dx)}{5ad}-\frac{\text{Subst}\left(\int \left(x^2+x^4\right)\,dx,x,-\cot(c+dx)\right)}{ad}\\ &= \frac{\cot^3(c+dx)}{3ad}+\frac{\cot^5(c+dx)}{5ad}-\frac{\csc^5(c+dx)}{5ad}\end{aligned}$$

Mathematica [B] time = 0.500291, size = 116, normalized size = 2.11

$$-\frac{\csc(c)(-54\sin(c+dx)-18\sin(2(c+dx))+18\sin(3(c+dx))+9\sin(4(c+dx))-32\sin(c+2dx)+32\sin(2c+3dx)+}{960ad(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(Csc[c]*Csc[c + d*x]^3*Sec[c + d*x]*(240*Sin[c] - 96*Sin[d*x] - 54*Sin[c +
 d*x] - 18*Sin[2*(c + d*x)] + 18*Sin[3*(c + d*x)] + 9*Sin[4*(c + d*x)] - 32
*Sin[c + 2*d*x] + 32*Sin[2*c + 3*d*x] + 16*Sin[3*c + 4*d*x]))/(960*a*d*(1 +
 Sec[c + d*x]))
```

Maple [A] time = 0.057, size = 62, normalized size = 1.1

$$\frac{1}{16\,da}\left(-\frac{1}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{2}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{1}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-3}-2\,\left(\tan\left(1/2\,dx+c/2\right)\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/16/d/a*(-1/5*tan(1/2*d*x+1/2*c)^5-2/3*tan(1/2*d*x+1/2*c)^3-1/3/tan(1/2*d*
x+1/2*c)^3-2/tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 0.977802, size = 130, normalized size = 2.36

$$-\frac{\frac{\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a}+\frac{5\left(\frac{6\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)(\cos(dx+c)+1)^3}{a\sin(dx+c)^3}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/240*((10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x
 + c) + 1)^5)/a + 5*(6*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)*(cos(d*x +
c) + 1)^3/(a*sin(d*x + c)^3))/d
```

Fricas [A] time = 1.67212, size = 225, normalized size = 4.09

$$-\frac{2\cos\left(dx+c\right)^{4}+2\cos\left(dx+c\right)^{3}-3\cos\left(dx+c\right)^{2}-3\cos\left(dx+c\right)-3}{15\left(ad\cos\left(dx+c\right)^{3}+ad\cos\left(dx+c\right)^{2}-ad\cos\left(dx+c\right)-ad\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/15*(2*cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 3*cos(d*x + c)^2 - 3*cos(d*x +
 c) - 3)/((a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2 - a*d*cos(d*x + c) - a*d
)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\csc^{4}(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**4/(sec(c + d*x) + 1), x)/a
```

Giac [A] time = 1.32857, size = 100, normalized size = 1.82

$$-\frac{\frac{5\left(6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+1\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}}+\frac{3a^{4}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+10a^{4}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}}{a^{5}}}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/240*(5*(6*tan(1/2*d*x + 1/2*c)^2 + 1)/(a*tan(1/2*d*x + 1/2*c)^3) + (3*a^
4*tan(1/2*d*x + 1/2*c)^5 + 10*a^4*tan(1/2*d*x + 1/2*c)^3)/a^5)/d
```

3.71 $\int \frac{\csc^6(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{\cot^7(c+dx)}{7ad}+\frac{2\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^7(c+dx)}{7ad}$$

```
[Out] Cot[c + d*x]^3/(3*a*d) + (2*Cot[c + d*x]^5)/(5*a*d) + Cot[c + d*x]^7/(7*a*d
) - Csc[c + d*x]^7/(7*a*d)
```

Rubi [A] time = 0.147004, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3872, 2839, 2606, 30, 2607, 270}

$$\frac{\cot^7(c+dx)}{7ad}+\frac{2\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^7(c+dx)}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] Cot[c + d*x]^3/(3*a*d) + (2*Cot[c + d*x]^5)/(5*a*d) + Cot[c + d*x]^7/(7*a*d
) - Csc[c + d*x]^7/(7*a*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^6(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cot(c+dx)\csc^5(c+dx)}{-a-a\cos(c+dx)}\,dx\\ &= -\frac{\int \cot^2(c+dx)\csc^6(c+dx)\,dx}{a}+\frac{\int \cot(c+dx)\csc^7(c+dx)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int x^6\,dx,x,\csc(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int x^2\left(1+x^2\right)^2\,dx,x,-\cot(c+dx)\right)}{ad}\\ &= -\frac{\csc^7(c+dx)}{7ad}-\frac{\text{Subst}\left(\int \left(x^2+2x^4+x^6\right)\,dx,x,-\cot(c+dx)\right)}{ad}\\ &= \frac{\cot^3(c+dx)}{3ad}+\frac{2\cot^5(c+dx)}{5ad}+\frac{\cot^7(c+dx)}{7ad}-\frac{\csc^7(c+dx)}{7ad}\end{aligned}$$

Mathematica [B] time = 0.618358, size = 158, normalized size = 2.16

$$\csc(c)(1500\sin(c+dx)+375\sin(2(c+dx))-750\sin(3(c+dx))-300\sin(4(c+dx))+150\sin(5(c+dx))+75\sin(6$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Csc[c]*Csc[c + d*x]^5*Sec[c + d*x]*(-8960*Sin[c] + 2560*Sin[d*x] + 1500*Si
n[c + d*x] + 375*Sin[2*(c + d*x)] - 750*Sin[3*(c + d*x)] - 300*Sin[4*(c + d
*x)] + 150*Sin[5*(c + d*x)] + 75*Sin[6*(c + d*x)] + 640*Sin[c + 2*d*x] - 12
80*Sin[2*c + 3*d*x] - 512*Sin[3*c + 4*d*x] + 256*Sin[4*c + 5*d*x] + 128*Sin
[5*c + 6*d*x]))/(53760*a*d*(1 + Sec[c + d*x]))
```

Maple [A] time = 0.062, size = 88, normalized size = 1.2

$$\frac{1}{64\,da}\left(-\frac{1}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{4}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{5}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{4}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-3}-5\,(\tan(1/2\,dx+c/2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/64/d/a*(-1/7*tan(1/2*d*x+1/2*c)^7-4/5*tan(1/2*d*x+1/2*c)^5-5/3*tan(1/2*d*
x+1/2*c)^3-4/3/tan(1/2*d*x+1/2*c)^3-5/tan(1/2*d*x+1/2*c)-1/5/tan(1/2*d*x+1/
2*c)^5)
```

Maxima [B] time = 0.997052, size = 184, normalized size = 2.52

$$-\frac{\frac{\frac{175\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{84\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a}+\frac{7\left(\frac{20\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{75\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+3\right)(\cos(dx+c)+1)^5}{a\sin(dx+c)^5}}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6720*((175*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 84*sin(d*x + c)^5/(cos(
d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a + 7*(20*sin(d*x
 + c)^2/(cos(d*x + c) + 1)^2 + 75*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 3)*
(cos(d*x + c) + 1)^5/(a*sin(d*x + c)^5))/d
```

Fricas [B] time = 1.67308, size = 347, normalized size = 4.75

$$-\frac{8 \cos (dx+c)^6+8 \cos (dx+c)^5-20 \cos (dx+c)^4-20 \cos (dx+c)^3+15 \cos (dx+c)^2+15 \cos (dx+c)+15}{105 \left(ad \cos (dx+c)^5+ad \cos (dx+c)^4-2\, ad \cos (dx+c)^3-2\, ad \cos (dx+c)^2+ad \cos (dx+c)+ad\right) \sin (dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/105*(8*cos(d*x + c)^6 + 8*cos(d*x + c)^5 - 20*cos(d*x + c)^4 - 20*cos(d*
x + c)^3 + 15*cos(d*x + c)^2 + 15*cos(d*x + c) + 15)/((a*d*cos(d*x + c)^5 +
 a*d*cos(d*x + c)^4 - 2*a*d*cos(d*x + c)^3 - 2*a*d*cos(d*x + c)^2 + a*d*cos
(d*x + c) + a*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3124, size = 139, normalized size = 1.9

$$-\frac{\frac{7\left(75 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+20 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+3\right)}{a \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5}+\frac{15\, a^6 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+84\, a^6 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+175\, a^6 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3}{a^7}}{6720\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/6720*(7*(75*tan(1/2*d*x + 1/2*c)^4 + 20*tan(1/2*d*x + 1/2*c)^2 + 3)/(a*t
an(1/2*d*x + 1/2*c)^5) + (15*a^6*tan(1/2*d*x + 1/2*c)^7 + 84*a^6*tan(1/2*d*
x + 1/2*c)^5 + 175*a^6*tan(1/2*d*x + 1/2*c)^3)/a^7)/d
```

3.72 $\int \frac{\csc^8(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=91

$$\frac{\cot^9(c+dx)}{9ad}+\frac{3\cot^7(c+dx)}{7ad}+\frac{3\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^9(c+dx)}{9ad}$$

```
[Out] Cot[c + d*x]^3/(3*a*d) + (3*Cot[c + d*x]^5)/(5*a*d) + (3*Cot[c + d*x]^7)/(7
*a*d) + Cot[c + d*x]^9/(9*a*d) - Csc[c + d*x]^9/(9*a*d)
```

Rubi [A] time = 0.150765, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3872, 2839, 2606, 30, 2607, 270}

$$\frac{\cot^9(c+dx)}{9ad}+\frac{3\cot^7(c+dx)}{7ad}+\frac{3\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^9(c+dx)}{9ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```


```
[Out] Cot[c + d*x]^3/(3*a*d) + (3*Cot[c + d*x]^5)/(5*a*d) + (3*Cot[c + d*x]^7)/(7
*a*d) + Cot[c + d*x]^9/(9*a*d) - Csc[c + d*x]^9/(9*a*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^8(c+dx)}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cot(c+dx)\csc^7(c+dx)}{-a-a\cos(c+dx)}\,dx\\ &= -\frac{\int \cot^2(c+dx)\csc^8(c+dx)\,dx}{a}+\frac{\int \cot(c+dx)\csc^9(c+dx)\,dx}{a}\\ &= -\frac{\operatorname{Subst}\left(\int x^8\,dx, x, \csc(c+dx)\right)}{ad}-\frac{\operatorname{Subst}\left(\int x^2\left(1+x^2\right)^3\,dx, x, -\cot(c+dx)\right)}{ad}\\ &= -\frac{\csc^9(c+dx)}{9ad}-\frac{\operatorname{Subst}\left(\int \left(x^2+3x^4+3x^6+x^8\right)dx, x, -\cot(c+dx)\right)}{ad}\\ &= \frac{\cot^3(c+dx)}{3ad}+\frac{3\cot^5(c+dx)}{5ad}+\frac{3\cot^7(c+dx)}{7ad}+\frac{\cot^9(c+dx)}{9ad}-\frac{\csc^9(c+dx)}{9ad}\end{aligned}$$

Mathematica [B] time = 0.979606, size = 200, normalized size = 2.2

$$-\frac{\csc(c)(-85750\sin(c+dx)-17150\sin(2(c+dx))+51450\sin(3(c+dx))+17150\sin(4(c+dx))-17150\sin(5(c+dx))}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^8/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(Csc[c]*Csc[c + d*x]^7*Sec[c + d*x]*(645120*Sin[c] - 143360*Sin[d*x] - 857
50*Sin[c + d*x] - 17150*Sin[2*(c + d*x)] + 51450*Sin[3*(c + d*x)] + 17150*S
in[4*(c + d*x)] - 17150*Sin[5*(c + d*x)] - 7350*Sin[6*(c + d*x)] + 2450*Sin
[7*(c + d*x)] + 1225*Sin[8*(c + d*x)] - 28672*Sin[c + 2*d*x] + 86016*Sin[2*
c + 3*d*x] + 28672*Sin[3*c + 4*d*x] - 28672*Sin[4*c + 5*d*x] - 12288*Sin[5*
c + 6*d*x] + 4096*Sin[6*c + 7*d*x] + 2048*Sin[7*c + 8*d*x]))/(5160960*a*d*(
1 + Sec[c + d*x]))
```

Maple [A] time = 0.065, size = 114, normalized size = 1.3

$$\frac{1}{256\,da}\left(-\frac{1}{9}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9-\frac{6}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7-\frac{14}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{14}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{14}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^8/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/256/d/a*(-1/9*tan(1/2*d*x+1/2*c)^9-6/7*tan(1/2*d*x+1/2*c)^7-14/5*tan(1/2*
d*x+1/2*c)^5-14/3*tan(1/2*d*x+1/2*c)^3-14/3/tan(1/2*d*x+1/2*c)^3-14/tan(1/2
*d*x+1/2*c)-6/5/tan(1/2*d*x+1/2*c)^5-1/7/tan(1/2*d*x+1/2*c)^7)
```

Maxima [B] time = 1.00339, size = 238, normalized size = 2.62

$$-\frac{\frac{\frac{1470\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{882\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{270\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{35\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a}+\frac{3\left(\frac{126\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{490\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{1470\,\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+15\right)(\cos(dx+c)+1)^7}{a\sin(dx+c)^7}}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/80640*((1470*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 882*sin(d*x + c)^5/(c
os(d*x + c) + 1)^5 + 270*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 35*sin(d*x +
 c)^9/(cos(d*x + c) + 1)^9)/a + 3*(126*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
+ 490*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 1470*sin(d*x + c)^6/(cos(d*x +
c) + 1)^6 + 15)*(cos(d*x + c) + 1)^7/(a*sin(d*x + c)^7))/d
```

Fricas [B] time = 1.79155, size = 466, normalized size = 5.12

$$-\frac{16\,\cos(dx+c)^8+16\,\cos(dx+c)^7-56\,\cos(dx+c)^6-56\,\cos(dx+c)^5+70\,\cos(dx+c)^4+70\,\cos(dx+c)^3-3}{315\left(ad\cos(dx+c)^7+ad\cos(dx+c)^6-3\,ad\cos(dx+c)^5-3\,ad\cos(dx+c)^4+3\,ad\cos(dx+c)^3+3\,ad\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/315*(16*cos(d*x + c)^8 + 16*cos(d*x + c)^7 - 56*cos(d*x + c)^6 - 56*cos(
d*x + c)^5 + 70*cos(d*x + c)^4 + 70*cos(d*x + c)^3 - 35*cos(d*x + c)^2 - 35
*cos(d*x + c) - 35)/((a*d*cos(d*x + c)^7 + a*d*cos(d*x + c)^6 - 3*a*d*cos(d
*x + c)^5 - 3*a*d*cos(d*x + c)^4 + 3*a*d*cos(d*x + c)^3 + 3*a*d*cos(d*x + c
)^2 - a*d*cos(d*x + c) - a*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**8/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.2839, size = 178, normalized size = 1.96

$$-\frac{\frac{3\left(1470\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^6+490\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4+126\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+15\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7}+\frac{35\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+270\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+882\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+14}{a^9}}{80640\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/80640*(3*(1470*tan(1/2*d*x + 1/2*c)^6 + 490*tan(1/2*d*x + 1/2*c)^4 + 126
*tan(1/2*d*x + 1/2*c)^2 + 15)/(a*tan(1/2*d*x + 1/2*c)^7) + (35*a^8*tan(1/2*
d*x + 1/2*c)^9 + 270*a^8*tan(1/2*d*x + 1/2*c)^7 + 882*a^8*tan(1/2*d*x + 1/2
*c)^5 + 1470*a^8*tan(1/2*d*x + 1/2*c)^3)/a^9)/d
```

3.73 $\int \frac{\csc^{10}(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=109

$$\frac{\cot^{11}(c+dx)}{11ad}+\frac{4\cot^9(c+dx)}{9ad}+\frac{6\cot^7(c+dx)}{7ad}+\frac{4\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^{11}(c+dx)}{11ad}$$

```
[Out] Cot[c + d*x]^3/(3*a*d) + (4*Cot[c + d*x]^5)/(5*a*d) + (6*Cot[c + d*x]^7)/(7
*a*d) + (4*Cot[c + d*x]^9)/(9*a*d) + Cot[c + d*x]^11/(11*a*d) - Csc[c + d*x
]^11/(11*a*d)
```

Rubi [A] time = 0.155081, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3872, 2839, 2606, 30, 2607, 270}

$$\frac{\cot^{11}(c+dx)}{11ad}+\frac{4\cot^9(c+dx)}{9ad}+\frac{6\cot^7(c+dx)}{7ad}+\frac{4\cot^5(c+dx)}{5ad}+\frac{\cot^3(c+dx)}{3ad}-\frac{\csc^{11}(c+dx)}{11ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^10/(a + a*Sec[c + d*x]),x]
```


```
[Out] Cot[c + d*x]^3/(3*a*d) + (4*Cot[c + d*x]^5)/(5*a*d) + (6*Cot[c + d*x]^7)/(7
*a*d) + (4*Cot[c + d*x]^9)/(9*a*d) + Cot[c + d*x]^11/(11*a*d) - Csc[c + d*x
]^11/(11*a*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
```

```
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^{10}(c+dx)}{a+a\sec(c+dx)}dx &= -\int \frac{\cot(c+dx)\csc^9(c+dx)}{-a-a\cos(c+dx)}dx\\ &= -\frac{\int \cot^2(c+dx)\csc^{10}(c+dx)\,dx}{a}+\frac{\int \cot(c+dx)\csc^{11}(c+dx)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int x^{10}\,dx,x,\csc(c+dx)\right)}{ad}-\frac{\text{Subst}\left(\int x^2\left(1+x^2\right)^4\,dx,x,-\cot(c+dx)\right)}{ad}\\ &= -\frac{\csc^{11}(c+dx)}{11ad}-\frac{\text{Subst}\left(\int \left(x^2+4x^4+6x^6+4x^8+x^{10}\right)dx,x,-\cot(c+dx)\right)}{ad}\\ &= \frac{\cot^3(c+dx)}{3ad}+\frac{4\cot^5(c+dx)}{5ad}+\frac{6\cot^7(c+dx)}{7ad}+\frac{4\cot^9(c+dx)}{9ad}+\frac{\cot^{11}(c+dx)}{11ad}-\frac{\csc^{11}(c+}{11ad}\end{aligned}$$

Mathematica [B] time = 1.4677, size = 242, normalized size = 2.22

$$\csc(c)(5000940\sin(c+dx)+833490\sin(2(c+dx))-3333960\sin(3(c+dx))-952560\sin(4(c+dx))+1428840\sin(5($$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^10/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Csc[c]*Csc[c + d*x]^9*Sec[c + d*x]*(-45416448*Sin[c] + 8257536*Sin[d*x] +
5000940*Sin[c + d*x] + 833490*Sin[2*(c + d*x)] - 3333960*Sin[3*(c + d*x)] -
 952560*Sin[4*(c + d*x)] + 1428840*Sin[5*(c + d*x)] + 535815*Sin[6*(c + d*x
)] - 357210*Sin[7*(c + d*x)] - 158760*Sin[8*(c + d*x)] + 39690*Sin[9*(c + d
*x)] + 19845*Sin[10*(c + d*x)] + 1376256*Sin[c + 2*d*x] - 5505024*Sin[2*c +
 3*d*x] - 1572864*Sin[3*c + 4*d*x] + 2359296*Sin[4*c + 5*d*x] + 884736*Sin[
5*c + 6*d*x] - 589824*Sin[6*c + 7*d*x] - 262144*Sin[7*c + 8*d*x] + 65536*Si
n[8*c + 9*d*x] + 32768*Sin[9*c + 10*d*x]))/(454164480*a*d*(1 + Sec[c + d*x]
))
```

Maple [A] time = 0.065, size = 140, normalized size = 1.3

$$\frac{1}{1024\,da}\left(-\frac{1}{11}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{11}-\frac{8}{9}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{9}-\frac{27}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}-\frac{48}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}-14\,(\tan(1/2\,dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^10/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/1024/d/a*(-1/11*tan(1/2*d*x+1/2*c)^11-8/9*tan(1/2*d*x+1/2*c)^9-27/7*tan(1
/2*d*x+1/2*c)^7-48/5*tan(1/2*d*x+1/2*c)^5-14*tan(1/2*d*x+1/2*c)^3-16/tan(1/
```

```
2*d*x+1/2*c)^3-42/tan(1/2*d*x+1/2*c)-27/5/tan(1/2*d*x+1/2*c)^5-8/7/tan(1/2*
d*x+1/2*c)^7-1/9/tan(1/2*d*x+1/2*c)^9)
```

Maxima [B] time = 1.03037, size = 292, normalized size = 2.68

$$-\frac{\frac{\frac{48510 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{33264 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{13365 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{3080 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{315 \sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a}+\frac{11\left(\frac{360 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{1701 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{5040 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{13230 \sin(dx+c)}{(\cos(dx+c)+1}\right.}{a \sin(dx+c)^9}}{3548160\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/3548160*((48510*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 33264*sin(d*x + c)
^5/(cos(d*x + c) + 1)^5 + 13365*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 3080*
sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 315*sin(d*x + c)^11/(cos(d*x + c) + 1
)^11)/a + 11*(360*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1701*sin(d*x + c)^4
/(cos(d*x + c) + 1)^4 + 5040*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 13230*si
n(d*x + c)^8/(cos(d*x + c) + 1)^8 + 35)*(cos(d*x + c) + 1)^9/(a*sin(d*x + c
)^9))/d
```

Fricas [B] time = 1.71921, size = 602, normalized size = 5.52

$$-\frac{128 \cos(dx+c)^{10}+128 \cos(dx+c)^9-576 \cos(dx+c)^8-576 \cos(dx+c)^7+1008 \cos(dx+c)^6+1008 \cos(dx}{3465\left(ad\cos(dx+c)^9+ad\cos(dx+c)^8-4\,ad\cos(dx+c)^7-4\,ad\cos(dx+c)^6+6\,ad\cos(dx+c)^5+6\,ad\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/3465*(128*cos(d*x + c)^10 + 128*cos(d*x + c)^9 - 576*cos(d*x + c)^8 - 57
6*cos(d*x + c)^7 + 1008*cos(d*x + c)^6 + 1008*cos(d*x + c)^5 - 840*cos(d*x
+ c)^4 - 840*cos(d*x + c)^3 + 315*cos(d*x + c)^2 + 315*cos(d*x + c) + 315)/
((a*d*cos(d*x + c)^9 + a*d*cos(d*x + c)^8 - 4*a*d*cos(d*x + c)^7 - 4*a*d*co
s(d*x + c)^6 + 6*a*d*cos(d*x + c)^5 + 6*a*d*cos(d*x + c)^4 - 4*a*d*cos(d*x
+ c)^3 - 4*a*d*cos(d*x + c)^2 + a*d*cos(d*x + c) + a*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**10/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33764, size = 217, normalized size = 1.99

$$-\frac{\frac{11\left(13230\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{8}+5040\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{6}+1701\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{4}+360\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+35\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}}+\frac{315a^{10}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+3080a^{10}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}\cdots}{}}{3548160d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^10/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3548160*(11*(13230*tan(1/2*d*x + 1/2*c)^8 + 5040*tan(1/2*d*x + 1/2*c)^6
+ 1701*tan(1/2*d*x + 1/2*c)^4 + 360*tan(1/2*d*x + 1/2*c)^2 + 35)/(a*tan(1/2
*d*x + 1/2*c)^9) + (315*a^10*tan(1/2*d*x + 1/2*c)^11 + 3080*a^10*tan(1/2*d*
x + 1/2*c)^9 + 13365*a^10*tan(1/2*d*x + 1/2*c)^7 + 33264*a^10*tan(1/2*d*x +
 1/2*c)^5 + 48510*a^10*tan(1/2*d*x + 1/2*c)^3)/a^11)/d
```

3.74 $\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=137

$$-\frac{(a-a\cos(c+dx))^{11}}{11a^{13}d}+\frac{4(a-a\cos(c+dx))^{10}}{5a^{12}d}-\frac{25(a-a\cos(c+dx))^9}{9a^{11}d}+\frac{19(a-a\cos(c+dx))^8}{4a^{10}d}-\frac{4(a-a\cos(c+d}{a^9d}$$

```
[Out] (4*(a - a*Cos[c + d*x])^6)/(3*a^8*d) - (4*(a - a*Cos[c + d*x])^7)/(a^9*d) +
 (19*(a - a*Cos[c + d*x])^8)/(4*a^10*d) - (25*(a - a*Cos[c + d*x])^9)/(9*a^
11*d) + (4*(a - a*Cos[c + d*x])^10)/(5*a^12*d) - (a - a*Cos[c + d*x])^11/(1
1*a^13*d)
```

Rubi [A] time = 0.185802, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2836, 12, 88}

$$-\frac{(a-a\cos(c+dx))^{11}}{11a^{13}d}+\frac{4(a-a\cos(c+dx))^{10}}{5a^{12}d}-\frac{25(a-a\cos(c+dx))^9}{9a^{11}d}+\frac{19(a-a\cos(c+dx))^8}{4a^{10}d}-\frac{4(a-a\cos(c+d}{a^9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^11/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*(a - a*Cos[c + d*x])^6)/(3*a^8*d) - (4*(a - a*Cos[c + d*x])^7)/(a^9*d) +
 (19*(a - a*Cos[c + d*x])^8)/(4*a^10*d) - (25*(a - a*Cos[c + d*x])^9)/(9*a^
11*d) + (4*(a - a*Cos[c + d*x])^10)/(5*a^12*d) - (a - a*Cos[c + d*x])^11/(1
1*a^13*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin^{11}(c+dx)}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^5x^2(-a+x)^3}{a^2}\,dx, x, -a\cos(c+dx)\right)}{a^{11}d}$$
$$= \frac{\text{Subst}\left(\int (-a-x)^5x^2(-a+x)^3\,dx, x, -a\cos(c+dx)\right)}{a^{13}d}$$
$$= \frac{\text{Subst}\left(\int \left(-8a^5(-a-x)^5 - 28a^4(-a-x)^6 - 38a^3(-a-x)^7 - 25a^2(-a-x)^8 - 8a(-a-x)^9 - \right.\right.}{a^{13}d}$$
$$= \frac{4(a-a\cos(c+dx))^6}{3a^8d} - \frac{4(a-a\cos(c+dx))^7}{a^9d} + \frac{19(a-a\cos(c+dx))^8}{4a^{10}d} - \frac{25(a-a\cos(c+d}{9a^{11}d}$$

Mathematica [A] time = 4.82492, size = 72, normalized size = 0.53

$$\frac{4\sin^{12}\left(\frac{1}{2}(c+dx)\right)(4038\cos(c+dx)+2586\cos(2(c+dx))+1189\cos(3(c+dx))+342\cos(4(c+dx))+45\cos(5(c+d}{495a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^11/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*(2360 + 4038*Cos[c + d*x] + 2586*Cos[2*(c + d*x)] + 1189*Cos[3*(c + d*x)
] + 342*Cos[4*(c + d*x)] + 45*Cos[5*(c + d*x)])*Sin[(c + d*x)/2]^12)/(495*a
^2*d)
```

Maple [A] time = 0.112, size = 88, normalized size = 0.6

$$-\frac{1}{da^2}\left(\frac{1}{3(\sec(dx+c))^3} + \frac{1}{5(\sec(dx+c))^{10}} - \frac{3}{4(\sec(dx+c))^8} + (\sec(dx+c))^{-6} - \frac{1}{11(\sec(dx+c))^{11}} - \frac{1}{2(\sec(dx+c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^11/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/d/a^2*(1/3/sec(d*x+c)^3+1/5/sec(d*x+c)^10-3/4/sec(d*x+c)^8+1/sec(d*x+c)^
6-1/11/sec(d*x+c)^11-1/2/sec(d*x+c)^4-2/5/sec(d*x+c)^5+2/9/sec(d*x+c)^9)
```

Maxima [A] time = 1.01579, size = 120, normalized size = 0.88

$$\frac{180\cos(dx+c)^{11} - 396\cos(dx+c)^{10} - 440\cos(dx+c)^9 + 1485\cos(dx+c)^8 - 1980\cos(dx+c)^6 + 792\cos(dx+c}{1980a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/1980*(180*cos(d*x + c)^11 - 396*cos(d*x + c)^10 - 440*cos(d*x + c)^9 + 14
85*cos(d*x + c)^8 - 1980*cos(d*x + c)^6 + 792*cos(d*x + c)^5 + 990*cos(d*x
+ c)^4 - 660*cos(d*x + c)^3)/(a^2*d)
```

Fricas [A] time = 1.80994, size = 254, normalized size = 1.85

$$\frac{180 \cos(dx+c)^{11} - 396 \cos(dx+c)^{10} - 440 \cos(dx+c)^{9} + 1485 \cos(dx+c)^{8} - 1980 \cos(dx+c)^{6} + 792 \cos(dx+c)^{5} + 990 \cos(dx+c)^{4} - 660 \cos(dx+c)^{3}}{1980\, a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/1980*(180*cos(d*x + c)^11 - 396*cos(d*x + c)^10 - 440*cos(d*x + c)^9 + 14
85*cos(d*x + c)^8 - 1980*cos(d*x + c)^6 + 792*cos(d*x + c)^5 + 990*cos(d*x
+ c)^4 - 660*cos(d*x + c)^3)/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**11/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34112, size = 250, normalized size = 1.82

$$-\frac{64\left(\frac{11\,(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{55\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{165\,(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{330\,(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} + \frac{462\,(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5} + \frac{198\,(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6} + \frac{990\,(\cos(dx+c)-1)^7}{(\cos(dx+c)+1)^7} - 1\right)}{495\, a^2 d \left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -64/495*(11*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 55*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 + 165*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 330
*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 462*(cos(d*x + c) - 1)^5/(cos(
d*x + c) + 1)^5 + 198*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 990*(cos(
d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 1)/(a^2*d*((cos(d*x + c) - 1)/(cos(d
*x + c) + 1) - 1)^11)
```

3.75 $\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^2} dx$

Optimal. Leaf size=114

$$\frac{(a-a\cos(c+dx))^9}{9a^{11}d}-\frac{3(a-a\cos(c+dx))^8}{4a^{10}d}+\frac{13(a-a\cos(c+dx))^7}{7a^9d}-\frac{2(a-a\cos(c+dx))^6}{a^8d}+\frac{4(a-a\cos(c+dx))^5}{5a^7d}$$

```
[Out] (4*(a - a*Cos[c + d*x])^5)/(5*a^7*d) - (2*(a - a*Cos[c + d*x])^6)/(a^8*d) +
 (13*(a - a*Cos[c + d*x])^7)/(7*a^9*d) - (3*(a - a*Cos[c + d*x])^8)/(4*a^10
*d) + (a - a*Cos[c + d*x])^9/(9*a^11*d)
```

Rubi [A] time = 0.180202, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2836, 12, 88}

$$\frac{(a-a\cos(c+dx))^9}{9a^{11}d}-\frac{3(a-a\cos(c+dx))^8}{4a^{10}d}+\frac{13(a-a\cos(c+dx))^7}{7a^9d}-\frac{2(a-a\cos(c+dx))^6}{a^8d}+\frac{4(a-a\cos(c+dx))^5}{5a^7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^9/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*(a - a*Cos[c + d*x])^5)/(5*a^7*d) - (2*(a - a*Cos[c + d*x])^6)/(a^8*d) +
 (13*(a - a*Cos[c + d*x])^7)/(7*a^9*d) - (3*(a - a*Cos[c + d*x])^8)/(4*a^10
*d) + (a - a*Cos[c + d*x])^9/(9*a^11*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin^9(c+dx)}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^4x^2(-a+x)^2}{a^2}\,dx, x, -a\cos(c+dx)\right)}{a^9d}$$
$$= \frac{\text{Subst}\left(\int (-a-x)^4x^2(-a+x)^2\,dx, x, -a\cos(c+dx)\right)}{a^{11}d}$$
$$= \frac{\text{Subst}\left(\int \left(4a^4(-a-x)^4+12a^3(-a-x)^5+13a^2(-a-x)^6+6a(-a-x)^7+(-a-x)^8\right)dx,\right.}{a^{11}d}$$
$$= \frac{4(a-a\cos(c+dx))^5}{5a^7d} - \frac{2(a-a\cos(c+dx))^6}{a^8d} + \frac{13(a-a\cos(c+dx))^7}{7a^9d} - \frac{3(a-a\cos(c+}{4a^{10}d}$$

Mathematica [A] time = 3.44935, size = 62, normalized size = 0.54

$$\frac{2\sin^{10}\left(\frac{1}{2}(c+dx)\right)(1615\cos(c+dx)+970\cos(2(c+dx))+385\cos(3(c+dx))+70\cos(4(c+dx))+992)}{315a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^9/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*(992 + 1615*Cos[c + d*x] + 970*Cos[2*(c + d*x)] + 385*Cos[3*(c + d*x)] +
 70*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]^10)/(315*a^2*d)
```

Maple [A] time = 0.102, size = 79, normalized size = 0.7

$$\frac{1}{da^2}\left(-\frac{1}{3(\sec(dx+c))^3}+\frac{1}{2(\sec(dx+c))^4}+\frac{1}{4(\sec(dx+c))^8}-\frac{2}{3(\sec(dx+c))^6}+\frac{1}{5(\sec(dx+c))^5}+\frac{1}{7(\sec(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^9/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(-1/3/sec(d*x+c)^3+1/2/sec(d*x+c)^4+1/4/sec(d*x+c)^8-2/3/sec(d*x+c)
^6+1/5/sec(d*x+c)^5+1/7/sec(d*x+c)^7-1/9/sec(d*x+c)^9)
```

Maxima [A] time = 1.02519, size = 107, normalized size = 0.94

$$-\frac{140\cos(dx+c)^9-315\cos(dx+c)^8-180\cos(dx+c)^7+840\cos(dx+c)^6-252\cos(dx+c)^5-630\cos(dx+c}{1260\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/1260*(140*cos(d*x + c)^9 - 315*cos(d*x + c)^8 - 180*cos(d*x + c)^7 + 840
*cos(d*x + c)^6 - 252*cos(d*x + c)^5 - 630*cos(d*x + c)^4 + 420*cos(d*x + c
)^3)/(a^2*d)
```

Fricas [A] time = 1.7359, size = 221, normalized size = 1.94

$$-\frac{140 \cos(dx+c)^9 - 315 \cos(dx+c)^8 - 180 \cos(dx+c)^7 + 840 \cos(dx+c)^6 - 252 \cos(dx+c)^5 - 630 \cos(dx+c)^4 +}{1260\,a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/1260*(140*cos(d*x + c)^9 - 315*cos(d*x + c)^8 - 180*cos(d*x + c)^7 + 840
*cos(d*x + c)^6 - 252*cos(d*x + c)^5 - 630*cos(d*x + c)^4 + 420*cos(d*x + c
)^3)/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**9/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31472, size = 190, normalized size = 1.67

$$-\frac{64\left(\frac{9(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{36(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{84(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{126(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} - \frac{210(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6} - 1\right)}{315\,a^2 d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -64/315*(9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 36*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2 + 84*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 126*(
cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 210*(cos(d*x + c) - 1)^6/(cos(d*
x + c) + 1)^6 - 1)/(a^2*d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^9)
```

3.76 $\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=73

$$\frac{\cos^7(c+dx)}{7a^2d}-\frac{\cos^6(c+dx)}{3a^2d}+\frac{\cos^4(c+dx)}{2a^2d}-\frac{\cos^3(c+dx)}{3a^2d}$$

```
[Out] -Cos[c + d*x]^3/(3*a^2*d) + Cos[c + d*x]^4/(2*a^2*d) - Cos[c + d*x]^6/(3*a^
2*d) + Cos[c + d*x]^7/(7*a^2*d)
```

Rubi [A] time = 0.159336, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2836, 12, 75}

$$\frac{\cos^7(c+dx)}{7a^2d}-\frac{\cos^6(c+dx)}{3a^2d}+\frac{\cos^4(c+dx)}{2a^2d}-\frac{\cos^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -Cos[c + d*x]^3/(3*a^2*d) + Cos[c + d*x]^4/(2*a^2*d) - Cos[c + d*x]^6/(3*a^
2*d) + Cos[c + d*x]^7/(7*a^2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin^7(c+dx)}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^3x^2(-a+x)}{a^2}\,dx, x, -a\cos(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int (-a-x)^3x^2(-a+x)\,dx, x, -a\cos(c+dx)\right)}{a^9d}$$
$$= \frac{\text{Subst}\left(\int \left(a^4x^2+2a^3x^3-2ax^5-x^6\right)dx, x, -a\cos(c+dx)\right)}{a^9d}$$
$$= -\frac{\cos^3(c+dx)}{3a^2d}+\frac{\cos^4(c+dx)}{2a^2d}-\frac{\cos^6(c+dx)}{3a^2d}+\frac{\cos^7(c+dx)}{7a^2d}$$

Mathematica [A] time = 1.76868, size = 53, normalized size = 0.73

$$\frac{4\sin^8\left(\frac{1}{2}(c+dx)\right)(17\cos(c+dx)+10\cos(2(c+dx))+3(\cos(3(c+dx))+4))}{21a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*(17*Cos[c + d*x] + 10*Cos[2*(c + d*x)] + 3*(4 + Cos[3*(c + d*x)]))*Sin[(
c + d*x)/2]^8)/(21*a^2*d)
```

Maple [A] time = 0.086, size = 50, normalized size = 0.7

$$-\frac{1}{da^2}\left(\frac{1}{3(\sec(dx+c))^3}-\frac{1}{2(\sec(dx+c))^4}+\frac{1}{3(\sec(dx+c))^6}-\frac{1}{7(\sec(dx+c))^7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/d/a^2*(1/3/sec(d*x+c)^3-1/2/sec(d*x+c)^4+1/3/sec(d*x+c)^6-1/7/sec(d*x+c)
^7)
```

Maxima [A] time = 0.990104, size = 66, normalized size = 0.9

$$\frac{6\cos(dx+c)^7-14\cos(dx+c)^6+21\cos(dx+c)^4-14\cos(dx+c)^3}{42a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/42*(6*cos(d*x + c)^7 - 14*cos(d*x + c)^6 + 21*cos(d*x + c)^4 - 14*cos(d*x
 + c)^3)/(a^2*d)
```

Fricas [A] time = 1.77204, size = 126, normalized size = 1.73

$$\frac{6\cos\left(dx+c\right)^{7}-14\cos\left(dx+c\right)^{6}+21\cos\left(dx+c\right)^{4}-14\cos\left(dx+c\right)^{3}}{42\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/42*(6*cos(d*x + c)^7 - 14*cos(d*x + c)^6 + 21*cos(d*x + c)^4 - 14*cos(d*x
 + c)^3)/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34276, size = 190, normalized size = 2.6

$$-\frac{8\left(\frac{7\,(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{21\,(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{35\,(\cos(dx+c)-1)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{14\,(\cos(dx+c)-1)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{42\,(\cos(dx+c)-1)^{5}}{(\cos(dx+c)+1)^{5}}-1\right)}{21\,a^{2}d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -8/21*(7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 21*(cos(d*x + c) - 1)^2/(c
os(d*x + c) + 1)^2 + 35*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 14*(cos
(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 42*(cos(d*x + c) - 1)^5/(cos(d*x +
c) + 1)^5 - 1)/(a^2*d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^7)
```

3.77 $\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=55

$$-\frac{\cos^5(c+dx)}{5a^2d}+\frac{\cos^4(c+dx)}{2a^2d}-\frac{\cos^3(c+dx)}{3a^2d}$$

```
[Out] -Cos[c + d*x]^3/(3*a^2*d) + Cos[c + d*x]^4/(2*a^2*d) - Cos[c + d*x]^5/(5*a^
2*d)
```

Rubi [A] time = 0.154196, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2836, 12, 43}

$$-\frac{\cos^5(c+dx)}{5a^2d}+\frac{\cos^4(c+dx)}{2a^2d}-\frac{\cos^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -Cos[c + d*x]^3/(3*a^2*d) + Cos[c + d*x]^4/(2*a^2*d) - Cos[c + d*x]^5/(5*a^
2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin^5(c+dx)}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^2x^2}{a^2}\,dx, x, -a\cos(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int (-a-x)^2x^2\,dx, x, -a\cos(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2x^2+2ax^3+x^4\right)\,dx, x, -a\cos(c+dx)\right)}{a^7d}$$
$$= -\frac{\cos^3(c+dx)}{3a^2d} + \frac{\cos^4(c+dx)}{2a^2d} - \frac{\cos^5(c+dx)}{5a^2d}$$

Mathematica [A] time = 0.570548, size = 42, normalized size = 0.76

$$\frac{4\sin^6\left(\frac{1}{2}(c+dx)\right)(3\cos(c+dx)+3\cos(2(c+dx))+4)}{15a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*(4 + 3*Cos[c + d*x] + 3*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]^6)/(15*a^2*d)
```

Maple [A] time = 0.069, size = 39, normalized size = 0.7

$$\frac{1}{da^2}\left(-\frac{1}{3\left(\sec(dx+c)\right)^3}+\frac{1}{2\left(\sec(dx+c)\right)^4}-\frac{1}{5\left(\sec(dx+c)\right)^5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(-1/3/sec(d*x+c)^3+1/2/sec(d*x+c)^4-1/5/sec(d*x+c)^5)
```

Maxima [A] time = 0.979226, size = 53, normalized size = 0.96

$$-\frac{6\cos(dx+c)^5-15\cos(dx+c)^4+10\cos(dx+c)^3}{30\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/30*(6*cos(d*x + c)^5 - 15*cos(d*x + c)^4 + 10*cos(d*x + c)^3)/(a^2*d)
```

Fricas [A] time = 1.68046, size = 100, normalized size = 1.82

$$-\frac{6\cos(dx+c)^5-15\cos(dx+c)^4+10\cos(dx+c)^3}{30\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/30*(6*cos(d*x + c)^5 - 15*cos(d*x + c)^4 + 10*cos(d*x + c)^3)/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31988, size = 161, normalized size = 2.93

$$-\frac{8\left(\frac{10(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{20(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{15(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{15(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-2\right)}{15\,a^2d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -8/15*(10*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 20*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 + 15*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 15*(co
s(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 2)/(a^2*d*((cos(d*x + c) - 1)/(cos
(d*x + c) + 1) - 1)^5)
```

3.78 $\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=66

$$\frac{\cos^3(c+dx)}{3a^2d}-\frac{\cos^2(c+dx)}{a^2d}+\frac{2\cos(c+dx)}{a^2d}-\frac{2\log(\cos(c+dx)+1)}{a^2d}$$

```
[Out] (2*Cos[c + d*x])/(a^2*d) - Cos[c + d*x]^2/(a^2*d) + Cos[c + d*x]^3/(3*a^2*d
) - (2*Log[1 + Cos[c + d*x]])/(a^2*d)
```

Rubi [A] time = 0.163219, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2836, 12, 77}

$$\frac{\cos^3(c+dx)}{3a^2d}-\frac{\cos^2(c+dx)}{a^2d}+\frac{2\cos(c+dx)}{a^2d}-\frac{2\log(\cos(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*Cos[c + d*x])/(a^2*d) - Cos[c + d*x]^2/(a^2*d) + Cos[c + d*x]^3/(3*a^2*d
) - (2*Log[1 + Cos[c + d*x]])/(a^2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{\operatorname{Subst}\left(\int \frac{(-a-x)x^2}{a^2(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{a^3d}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{(-a-x)x^2}{-a+x}\,dx, x, -a\cos(c+dx)\right)}{a^5d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(-2a^2+\frac{2a^3}{a-x}-2ax-x^2\right)dx, x, -a\cos(c+dx)\right)}{a^5d}$$
$$= \frac{2\cos(c+dx)}{a^2d} - \frac{\cos^2(c+dx)}{a^2d} + \frac{\cos^3(c+dx)}{3a^2d} - \frac{2\log(1+\cos(c+dx))}{a^2d}$$

Mathematica [A] time = 0.205094, size = 51, normalized size = 0.77

$$\frac{27\cos(c+dx)-6\cos(2(c+dx))+\cos(3(c+dx))-48\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-22}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-22 + 27*Cos[c + d*x] - 6*Cos[2*(c + d*x)] + Cos[3*(c + d*x)] - 48*Log[Cos
[(c + d*x)/2]])/(12*a^2*d)
```

Maple [A] time = 0.083, size = 82, normalized size = 1.2

$$-2\,\frac{\ln(1+\sec(dx+c))}{da^2}+\frac{1}{3\,da^2(\sec(dx+c))^3}-\frac{1}{da^2(\sec(dx+c))^2}+2\,\frac{1}{da^2\sec(dx+c)}+2\,\frac{\ln(\sec(dx+c))}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -2/d/a^2*ln(1+sec(d*x+c))+1/3/d/a^2/sec(d*x+c)^3-1/d/a^2/sec(d*x+c)^2+2/d/a
^2/sec(d*x+c)+2/d/a^2*ln(sec(d*x+c))
```

Maxima [A] time = 1.00854, size = 69, normalized size = 1.05

$$\frac{\frac{\cos(dx+c)^3-3\cos(dx+c)^2+6\cos(dx+c)}{a^2}-\frac{6\log(\cos(dx+c)+1)}{a^2}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*((cos(d*x + c)^3 - 3*cos(d*x + c)^2 + 6*cos(d*x + c))/a^2 - 6*log(cos(d
*x + c) + 1)/a^2)/d
```

Fricas [A] time = 1.75959, size = 132, normalized size = 2.

$$\frac{\cos(dx+c)^3 - 3\cos(dx+c)^2 + 6\cos(dx+c) - 6\log\left(\frac{1}{2}\cos(dx+c) + \frac{1}{2}\right)}{3a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c)^3 - 3*cos(d*x + c)^2 + 6*cos(d*x + c) - 6*log(1/2*cos(d*x
 + c) + 1/2))/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32994, size = 101, normalized size = 1.53

$$-\frac{2\log\left(|-\cos(dx+c) - 1|\right)}{a^2d} + \frac{a^4d^2\cos(dx+c)^3 - 3a^4d^2\cos(dx+c)^2 + 6a^4d^2\cos(dx+c)}{3a^6d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -2*log(abs(-cos(d*x + c) - 1))/(a^2*d) + 1/3*(a^4*d^2*cos(d*x + c)^3 - 3*a^
4*d^2*cos(d*x + c)^2 + 6*a^4*d^2*cos(d*x + c))/(a^6*d^3)
```

3.79 $\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=52

$$-\frac{\cos(c+dx)}{a^2d}+\frac{1}{d\left(a^2\cos(c+dx)+a^2\right)}+\frac{2\log(\cos(c+dx)+1)}{a^2d}$$

```
[Out] -(Cos[c + d*x]/(a^2*d)) + 1/(d*(a^2 + a^2*Cos[c + d*x])) + (2*Log[1 + Cos[c
 + d*x]])/(a^2*d)
```

Rubi [A] time = 0.102093, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3872, 2833, 12, 43}

$$-\frac{\cos(c+dx)}{a^2d}+\frac{1}{d\left(a^2\cos(c+dx)+a^2\right)}+\frac{2\log(\cos(c+dx)+1)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(Cos[c + d*x]/(a^2*d)) + 1/(d*(a^2 + a^2*Cos[c + d*x])) + (2*Log[1 + Cos[c
 + d*x]])/(a^2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin(c+dx)}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{a^2(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{a^2}{(a-x)^2}-\frac{2a}{a-x}\right)dx, x, -a\cos(c+dx)\right)}{a^3d}$$
$$= -\frac{\cos(c+dx)}{a^2d} + \frac{1}{d\left(a^2+a^2\cos(c+dx)\right)} + \frac{2\log(1+\cos(c+dx))}{a^2d}$$

Mathematica [A] time = 0.184668, size = 64, normalized size = 1.23

$$-\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\cos(2(c+dx))-8\cos(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-8\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-3\right)}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -((-3 + Cos[2*(c + d*x)] - 8*Log[Cos[(c + d*x)/2]] - 8*Cos[c + d*x]*Log[Cos
[(c + d*x)/2]])*Sec[(c + d*x)/2]^2)/(4*a^2*d)
```

Maple [A] time = 0.026, size = 68, normalized size = 1.3

$$-\frac{1}{da^2(1+\sec(dx+c))}+2\frac{\ln(1+\sec(dx+c))}{da^2}-\frac{1}{da^2\sec(dx+c)}-2\frac{\ln(\sec(dx+c))}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/d/a^2/(1+sec(d*x+c))+2/d/a^2*ln(1+sec(d*x+c))-1/d/a^2/sec(d*x+c)-2/d/a^2
*ln(sec(d*x+c))
```

Maxima [A] time = 1.00639, size = 62, normalized size = 1.19

$$\frac{\frac{1}{a^2\cos(dx+c)+a^2}-\frac{\cos(dx+c)}{a^2}+\frac{2\log(\cos(dx+c)+1)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (1/(a^2*cos(d*x + c) + a^2) - cos(d*x + c)/a^2 + 2*log(cos(d*x + c) + 1)/a^
2)/d
```

Fricas [A] time = 1.72558, size = 159, normalized size = 3.06

$$-\frac{\cos(dx+c)^2 - 2(\cos(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+\cos(dx+c)-1}{a^2d\cos(dx+c)+a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(cos(d*x + c)^2 - 2*(cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + cos(d
*x + c) - 1)/(a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sin(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.30162, size = 70, normalized size = 1.35

$$-\frac{\cos(dx+c)}{a^2d}+\frac{2\log(|-\cos(dx+c)-1|)}{a^2d}+\frac{1}{a^2d(\cos(dx+c)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -cos(d*x + c)/(a^2*d) + 2*log(abs(-cos(d*x + c) - 1))/(a^2*d) + 1/(a^2*d*(c
os(d*x + c) + 1))
```

3.80 $\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=60

$$-\frac{3}{4d\left(a^2\cos(c+dx)+a^2\right)}-\frac{\tanh^{-1}(\cos(c+dx))}{4a^2d}+\frac{1}{4d(a\cos(c+dx)+a)^2}$$

```
[Out] -ArcTanh[Cos[c + d*x]]/(4*a^2*d) + 1/(4*d*(a + a*Cos[c + d*x])^2) - 3/(4*d*
(a^2 + a^2*Cos[c + d*x]))
```

Rubi [A] time = 0.126629, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3872, 2836, 12, 88, 206}

$$-\frac{3}{4d\left(a^2\cos(c+dx)+a^2\right)}-\frac{\tanh^{-1}(\cos(c+dx))}{4a^2d}+\frac{1}{4d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -ArcTanh[Cos[c + d*x]]/(4*a^2*d) + 1/(4*d*(a + a*Cos[c + d*x])^2) - 3/(4*d*
(a^2 + a^2*Cos[c + d*x]))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cos(c+dx)\cot(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{a\,\text{Subst}\left(\int \frac{x^2}{a^2(-a-x)(-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \frac{x^2}{(-a-x)(-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{ad} \\
&= \frac{\text{Subst}\left(\int \left(\frac{a}{2(a-x)^3}-\frac{3}{4(a-x)^2}+\frac{1}{4\left(a^2-x^2\right)}\right)dx, x, -a\cos(c+dx)\right)}{ad} \\
&= \frac{1}{4d(a+a\cos(c+dx))^2}-\frac{3}{4d\left(a^2+a^2\cos(c+dx)\right)}+\frac{\text{Subst}\left(\int \frac{1}{a^2-x^2}\,dx, x, -a\cos(c+dx)\right)}{4ad} \\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{4a^2d}+\frac{1}{4d(a+a\cos(c+dx))^2}-\frac{3}{4d\left(a^2+a^2\cos(c+dx)\right)}
\end{aligned}
$$

Mathematica [A] time = 0.174836, size = 83, normalized size = 1.38

$$
-\frac{\sec^2(c+dx)\left(6\cos^2\left(\frac{1}{2}(c+dx)\right)+4\cos^4\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-1\right)}{4a^2d(\sec(c+dx)+1)^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -((-1 + 6*Cos[(c + d*x)/2]^2 + 4*Cos[(c + d*x)/2]^4*(Log[Cos[(c + d*x)/2]]
- Log[Sin[(c + d*x)/2]]))*Sec[c + d*x]^2)/(4*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 0.06, size = 72, normalized size = 1.2

$$
\frac{1}{4\,da^2\,(\cos(dx+c)+1)^2}-\frac{3}{4\,da^2\,(\cos(dx+c)+1)}-\frac{\ln(\cos(dx+c)+1)}{8\,da^2}+\frac{\ln(-1+\cos(dx+c))}{8\,da^2}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/4/d/a^2/(cos(d*x+c)+1)^2-3/4/d/a^2/(cos(d*x+c)+1)-1/8*ln(cos(d*x+c)+1)/a^
2/d+1/8/d/a^2*ln(-1+cos(d*x+c))
```

Maxima [A] time = 1.00546, size = 100, normalized size = 1.67

$$
-\frac{\frac{2\,(3\,\cos(dx+c)+2)}{a^2\cos(dx+c)^2+2\,a^2\cos(dx+c)+a^2}+\frac{\log(\cos(dx+c)+1)}{a^2}-\frac{\log(\cos(dx+c)-1)}{a^2}}{8\,d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/8*(2*(3*cos(d*x + c) + 2)/(a^2*cos(d*x + c)^2 + 2*a^2*cos(d*x + c) + a^2
) + log(cos(d*x + c) + 1)/a^2 - log(cos(d*x + c) - 1)/a^2)/d
```

Fricas [A] time = 1.70344, size = 294, normalized size = 4.9

$$-\frac{\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1\right)\log\left(-\frac{1}{2}\,\cos\left(dx+\right.\right.}{8\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/8*((cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) - (
cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 6*cos(d
*x + c) + 4)/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.30401, size = 117, normalized size = 1.95

$$\frac{\frac{2\log\left(\frac{\left|-\cos\left(dx+c\right)+1\right|}{\left|\cos\left(dx+c\right)+1\right|}\right)}{a^{2}}+\frac{\frac{4\,a^{2}\left(\cos\left(dx+c\right)-1\right)}{\cos\left(dx+c\right)+1}+\frac{a^{2}\left(\cos\left(dx+c\right)-1\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}}{a^{4}}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/16*(2*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^2 + (4*a^2*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2)/a^4)/d
```

3.81 $\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^2} dx$

Optimal. Leaf size=42

$$-\frac{2a\cos(c+dx)+a}{6d(1-\cos(c+dx))(a\cos(c+dx)+a)^3}$$

```
[Out] -(a + 2*a*Cos[c + d*x])/(6*d*(1 - Cos[c + d*x])*(a + a*Cos[c + d*x])^3)
```

Rubi [A] time = 0.127259, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2836, 12, 81}

$$-\frac{2a\cos(c+dx)+a}{6d(1-\cos(c+dx))(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a + 2*a*Cos[c + d*x])/(6*d*(1 - Cos[c + d*x])*(a + a*Cos[c + d*x])^3)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[(b*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)*(2*a*d*f*(n
 + p + 3) - b*(d*e*(n + 2) + c*f*(p + 2)) + b*d*f*(n + p + 2)*x))/(d^2*f^2*
(n + p + 2)*(n + p + 3)), x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && NeQ[n
 + p + 2, 0] && NeQ[n + p + 3, 0] && EqQ[d*f*(n + p + 2)*(a^2*d*f*(n + p +
3) - b*(b*c*e + a*(d*e*(n + 1) + c*f*(p + 1)))) - b*(d*e*(n + 1) + c*f*(p +
 1))*(a*d*f*(n + p + 4) - b*(d*e*(n + 2) + c*f*(p + 2))), 0]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)\csc(c+dx)}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{x^2}{a^2(-a-x)^2(-a+x)^4}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{x^2}{(-a-x)^2(-a+x)^4}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= -\frac{a+2a\cos(c+dx)}{6d(1-\cos(c+dx))(a+a\cos(c+dx))^3}$$

Mathematica [A] time = 0.0893072, size = 38, normalized size = 0.9

$$-\frac{(2\cos(c+dx)+1)\csc^2(c+dx)}{6a^2d(\cos(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -((1 + 2*Cos[c + d*x])*Csc[c + d*x]^2)/(6*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A] time = 0.066, size = 57, normalized size = 1.4

$$\frac{1}{da^2}\left(\frac{1}{12\,(\cos(dx+c)+1)^3}-\frac{1}{8\,(\cos(dx+c)+1)^2}-\frac{1}{16\,\cos(dx+c)+16}+\frac{1}{-16+16\,\cos(dx+c)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/d/a^2*(1/12/(cos(d*x+c)+1)^3-1/8/(cos(d*x+c)+1)^2-1/16/(cos(d*x+c)+1)+1/1
6/(-1+cos(d*x+c)))
```

Maxima [A] time = 0.9793, size = 80, normalized size = 1.9

$$\frac{2\,\cos(dx+c)+1}{6\left(a^2\cos(dx+c)^4+2\,a^2\cos(dx+c)^3-2\,a^2\cos(dx+c)-a^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/6*(2*cos(d*x + c) + 1)/((a^2*cos(d*x + c)^4 + 2*a^2*cos(d*x + c)^3 - 2*a^
2*cos(d*x + c) - a^2)*d)
```

Fricas [A] time = 1.64883, size = 142, normalized size = 3.38

$$\frac{2\,\cos(dx+c)+1}{6\left(a^2d\cos(dx+c)^4+2\,a^2d\cos(dx+c)^3-2\,a^2d\cos(dx+c)-a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*cos(d*x + c) + 1)/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 - 2
*a^2*d*cos(d*x + c) - a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc^3(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [B] time = 1.33295, size = 111, normalized size = 2.64

$$\frac{\frac{3(\cos(dx+c)+1)}{a^2(\cos(dx+c)-1)}+\frac{\frac{6a^4(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a^4(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{a^6}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/96*(3*(cos(d*x + c) + 1)/(a^2*(cos(d*x + c) - 1)) + (6*a^4*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - a^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^
6)/d
```

3.82 $\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=146

$$\frac{a^2}{32d(a\cos(c+dx)+a)^4}-\frac{1}{64d\left(a^2-a^2\cos(c+dx)\right)}-\frac{1}{32d\left(a^2\cos(c+dx)+a^2\right)}+\frac{\tanh^{-1}(\cos(c+dx))}{64a^2d}-\frac{}{48d(a\cos(}$$

```
[Out] ArcTanh[Cos[c + d*x]]/(64*a^2*d) - 1/(64*d*(a - a*Cos[c + d*x])^2) + a^2/(3
2*d*(a + a*Cos[c + d*x])^4) - a/(48*d*(a + a*Cos[c + d*x])^3) - 1/(32*d*(a
+ a*Cos[c + d*x])^2) - 1/(64*d*(a^2 - a^2*Cos[c + d*x])) - 1/(32*d*(a^2 + a
^2*Cos[c + d*x]))
```

Rubi [A] time = 0.216907, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2836, 12, 88, 206}

$$\frac{a^2}{32d(a\cos(c+dx)+a)^4}-\frac{1}{64d\left(a^2-a^2\cos(c+dx)\right)}-\frac{1}{32d\left(a^2\cos(c+dx)+a^2\right)}+\frac{\tanh^{-1}(\cos(c+dx))}{64a^2d}-\frac{}{48d(a\cos(}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(64*a^2*d) - 1/(64*d*(a - a*Cos[c + d*x])^2) + a^2/(3
2*d*(a + a*Cos[c + d*x])^4) - a/(48*d*(a + a*Cos[c + d*x])^3) - 1/(32*d*(a
+ a*Cos[c + d*x])^2) - 1/(64*d*(a^2 - a^2*Cos[c + d*x])) - 1/(32*d*(a^2 + a
^2*Cos[c + d*x]))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)\csc^3(c+dx)}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{a^5\,\text{Subst}\left(\int \frac{x^2}{a^2(-a-x)^3(-a+x)^5}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{x^2}{(-a-x)^3(-a+x)^5}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{8a(a-x)^5} - \frac{1}{16a^2(a-x)^4} - \frac{1}{16a^3(a-x)^3} - \frac{1}{32a^4(a-x)^2} + \frac{1}{32a^3(a+x)^3} + \frac{1}{64a^4(a+x)^2} - \frac{1}{64a^4(a^2-x^2)}\right.}{d}$$
$$= -\frac{1}{64d(a-a\cos(c+dx))^2} + \frac{a^2}{32d(a+a\cos(c+dx))^4} - \frac{a}{48d(a+a\cos(c+dx))^3} - \frac{}{32d(a+a}$$
$$= \frac{\tanh^{-1}(\cos(c+dx))}{64a^2d} - \frac{1}{64d(a-a\cos(c+dx))^2} + \frac{a^2}{32d(a+a\cos(c+dx))^4} - \frac{a}{48d(a+a\cos(}$$

Mathematica [A] time = 0.745516, size = 152, normalized size = 1.04

$$-\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(6\csc^4\left(\frac{1}{2}(c+dx)\right)+12\csc^2\left(\frac{1}{2}(c+dx)\right)-3\sec^8\left(\frac{1}{2}(c+dx)\right)+4\sec^6\left(\frac{1}{2}(c+dx)\right)+12\,\text{se}\right.}{384a^2d(\sec(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(Cos[(c + d*x)/2]^4*(12*Csc[(c + d*x)/2]^2 + 6*Csc[(c + d*x)/2]^4 + 24*(-L
og[Cos[(c + d*x)/2]] + Log[Sin[(c + d*x)/2]]) + 24*Sec[(c + d*x)/2]^2 + 12*
Sec[(c + d*x)/2]^4 + 4*Sec[(c + d*x)/2]^6 - 3*Sec[(c + d*x)/2]^8)*Sec[c + d
*x]^2)/(384*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 0.072, size = 144, normalized size = 1.

$$\frac{1}{32\,da^2\,(\cos(dx+c)+1)^4} - \frac{1}{48\,da^2\,(\cos(dx+c)+1)^3} - \frac{1}{32\,da^2\,(\cos(dx+c)+1)^2} - \frac{1}{32\,da^2\,(\cos(dx+c)+1)} + \frac{\ln(\cos(}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/32/d/a^2/(cos(d*x+c)+1)^4-1/48/d/a^2/(cos(d*x+c)+1)^3-1/32/d/a^2/(cos(d*x
+c)+1)^2-1/32/d/a^2/(cos(d*x+c)+1)+1/128*ln(cos(d*x+c)+1)/a^2/d-1/64/d/a^2/
(-1+cos(d*x+c))^2+1/64/d/a^2/(-1+cos(d*x+c))-1/128/d/a^2*ln(-1+cos(d*x+c))
```

Maxima [A] time = 1.0289, size = 225, normalized size = 1.54

$$-\frac{\frac{2\left(3\cos(dx+c)^5+6\cos(dx+c)^4-2\cos(dx+c)^3-10\cos(dx+c)^2+35\cos(dx+c)+16\right)}{a^2\cos(dx+c)^6+2a^2\cos(dx+c)^5-a^2\cos(dx+c)^4-4a^2\cos(dx+c)^3-a^2\cos(dx+c)^2+2a^2\cos(dx+c)+a^2}-\frac{3\log(\cos(dx+c)+1)}{a^2}+\frac{3\log(\cos(dx+c)-1)}{a^2}}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/384*(2*(3*cos(d*x + c)^5 + 6*cos(d*x + c)^4 - 2*cos(d*x + c)^3 - 10*cos(
d*x + c)^2 + 35*cos(d*x + c) + 16)/(a^2*cos(d*x + c)^6 + 2*a^2*cos(d*x + c)
^5 - a^2*cos(d*x + c)^4 - 4*a^2*cos(d*x + c)^3 - a^2*cos(d*x + c)^2 + 2*a^2
*cos(d*x + c) + a^2) - 3*log(cos(d*x + c) + 1)/a^2 + 3*log(cos(d*x + c) - 1
)/a^2)/d
```

Fricas [B] time = 1.73821, size = 734, normalized size = 5.03

$$-\frac{6\cos(dx+c)^5+12\cos(dx+c)^4-4\cos(dx+c)^3-20\cos(dx+c)^2-3\left(\cos(dx+c)^6+2\cos(dx+c)^5-\cos(dx\right.}{38}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/384*(6*cos(d*x + c)^5 + 12*cos(d*x + c)^4 - 4*cos(d*x + c)^3 - 20*cos(d*
x + c)^2 - 3*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 - 4*cos(d*
x + c)^3 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2)
 + 3*(cos(d*x + c)^6 + 2*cos(d*x + c)^5 - cos(d*x + c)^4 - 4*cos(d*x + c)^3
 - cos(d*x + c)^2 + 2*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 70*c
os(d*x + c) + 32)/(a^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5 - a^2*d*co
s(d*x + c)^4 - 4*a^2*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(
d*x + c) + a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3645, size = 279, normalized size = 1.91

$$\frac{\frac{6\left(\frac{4(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^2}{a^2(\cos(dx+c)-1)^2}-\frac{12\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2}+\frac{\frac{48a^6(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{6a^6(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{8a^6(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{3a^6(\cos(dx+c)-}{(\cos(dx+c)+1)}}{a^8}}{1536d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/1536*(6*(4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^2/(a^2*(cos(d*x + c) - 1)^2)
- 12*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^2 + (48*a^6*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) - 6*a^6*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 - 8*a^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 3*a^6*(cos(d*x +
 c) - 1)^4/(cos(d*x + c) + 1)^4)/a^8)/d
```

3.83 $\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=167

$$\frac{2\sin^7(c+dx)}{7a^2d}-\frac{2\sin^5(c+dx)}{5a^2d}-\frac{\sin^3(c+dx)\cos^5(c+dx)}{8a^2d}-\frac{\sin^3(c+dx)\cos^3(c+dx)}{6a^2d}-\frac{\sin(c+dx)\cos^5(c+dx)}{16a^2d}-$$

```
[Out] (11*x)/(128*a^2) + (11*Cos[c + d*x]*Sin[c + d*x])/(128*a^2*d) - (7*Cos[c +
d*x]^3*Sin[c + d*x])/(64*a^2*d) - (Cos[c + d*x]^5*Sin[c + d*x])/(16*a^2*d)
- (Cos[c + d*x]^3*Sin[c + d*x]^3)/(6*a^2*d) - (Cos[c + d*x]^5*Sin[c + d*x]^
3)/(8*a^2*d) - (2*Sin[c + d*x]^5)/(5*a^2*d) + (2*Sin[c + d*x]^7)/(7*a^2*d)
```

Rubi [A] time = 0.440065, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2875, 2873, 2568, 2635, 8, 2564, 14}

$$\frac{2\sin^7(c+dx)}{7a^2d}-\frac{2\sin^5(c+dx)}{5a^2d}-\frac{\sin^3(c+dx)\cos^5(c+dx)}{8a^2d}-\frac{\sin^3(c+dx)\cos^3(c+dx)}{6a^2d}-\frac{\sin(c+dx)\cos^5(c+dx)}{16a^2d}-$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (11*x)/(128*a^2) + (11*Cos[c + d*x]*Sin[c + d*x])/(128*a^2*d) - (7*Cos[c +
d*x]^3*Sin[c + d*x])/(64*a^2*d) - (Cos[c + d*x]^5*Sin[c + d*x])/(16*a^2*d)
- (Cos[c + d*x]^3*Sin[c + d*x]^3)/(6*a^2*d) - (Cos[c + d*x]^5*Sin[c + d*x]^
3)/(8*a^2*d) - (2*Sin[c + d*x]^5)/(5*a^2*d) + (2*Sin[c + d*x]^7)/(7*a^2*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
```

```
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin^8(c+dx)}{(-a-a\cos(c+dx))^2}\,dx$$

$$= \frac{\int \cos^2(c+dx)(-a+a\cos(c+dx))^2\sin^4(c+dx)\,dx}{a^4}$$

$$= \frac{\int \left(a^2\cos^2(c+dx)\sin^4(c+dx) - 2a^2\cos^3(c+dx)\sin^4(c+dx) + a^2\cos^4(c+dx)\sin^4(c+dx\right.}{a^4}$$

$$= \frac{\int \cos^2(c+dx)\sin^4(c+dx)\,dx}{a^2} + \frac{\int \cos^4(c+dx)\sin^4(c+dx)\,dx}{a^2} - \frac{2\int \cos^3(c+dx)\sin^4(c+}{a^2}$$

$$= -\frac{\cos^3(c+dx)\sin^3(c+dx)}{6a^2d} - \frac{\cos^5(c+dx)\sin^3(c+dx)}{8a^2d} + \frac{3\int \cos^4(c+dx)\sin^2(c+dx)\,dx}{8a^2} +$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{8a^2d} - \frac{\cos^5(c+dx)\sin(c+dx)}{16a^2d} - \frac{\cos^3(c+dx)\sin^3(c+dx)}{6a^2d} - \frac{\cos^5(c-}{}$$

$$= \frac{\cos(c+dx)\sin(c+dx)}{16a^2d} - \frac{7\cos^3(c+dx)\sin(c+dx)}{64a^2d} - \frac{\cos^5(c+dx)\sin(c+dx)}{16a^2d} - \frac{\cos^3(c+}{}$$

$$= \frac{x}{16a^2} + \frac{11\cos(c+dx)\sin(c+dx)}{128a^2d} - \frac{7\cos^3(c+dx)\sin(c+dx)}{64a^2d} - \frac{\cos^5(c+dx)\sin(c+dx)}{16a^2d} -$$

$$= \frac{11x}{128a^2} + \frac{11\cos(c+dx)\sin(c+dx)}{128a^2d} - \frac{7\cos^3(c+dx)\sin(c+dx)}{64a^2d} - \frac{\cos^5(c+dx)\sin(c+dx)}{16a^2d}$$

Mathematica [A] time = 2.75518, size = 131, normalized size = 0.78

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(-10080\sin(c+dx) - 1680\sin(2(c+dx)) + 3360\sin(3(c+dx)) - 2520\sin(4(c+dx)) + 67\right.}{26880a^2d(\sec(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(9240*d*x - 10080*Sin[c + d*x] - 1680*Si
n[2*(c + d*x)] + 3360*Sin[3*(c + d*x)] - 2520*Sin[4*(c + d*x)] + 672*Sin[5*
(c + d*x)] + 560*Sin[6*(c + d*x)] - 480*Sin[7*(c + d*x)] + 105*Sin[8*(c + d
*x)] + 980*Tan[c/2]))/(26880*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 0.102, size = 290, normalized size = 1.7

$$-\frac{11}{64\,da^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-8}-\frac{253}{192\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-8}-\frac{4213}{960\,da^2}\left(\tan\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^8/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -11/64/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)-253/192/d/a^2/(1
+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^3-4213/960/d/a^2/(1+tan(1/2*d*x
+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^5-55583/6720/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)
^8*tan(1/2*d*x+1/2*c)^7+31007/6720/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2
*d*x+1/2*c)^9-20363/960/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)
^11+253/192/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^13+11/64/d/
a^2/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^15+11/64/d/a^2*arctan(tan
(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.53106, size = 510, normalized size = 3.05

$$-\frac{\frac{\frac{1155\sin(dx+c)}{\cos(dx+c)+1}+\frac{8855\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{29491\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{55583\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{31007\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{142541\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}-\frac{8855\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{1155\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}}{a^2+\frac{8a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{56a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{70a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{56a^2\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{28a^2\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{8a^2\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{a^2\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}}-\frac{1155\arctan}{}}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6720*((1155*sin(d*x + c)/(cos(d*x + c) + 1) + 8855*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 29491*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 55583*sin(d*x
+ c)^7/(cos(d*x + c) + 1)^7 - 31007*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 1
42541*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 - 8855*sin(d*x + c)^13/(cos(d*x
 + c) + 1)^13 - 1155*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)/(a^2 + 8*a^2*si
n(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1
)^4 + 56*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^2*sin(d*x + c)^8/(c
os(d*x + c) + 1)^8 + 56*a^2*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a^2*
sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 8*a^2*sin(d*x + c)^14/(cos(d*x + c)
 + 1)^14 + a^2*sin(d*x + c)^16/(cos(d*x + c) + 1)^16) - 1155*arctan(sin(d*x
 + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.74874, size = 270, normalized size = 1.62

$$\frac{1155\,dx+\left(1680\,\cos(dx+c)^7-3840\,\cos(dx+c)^6-280\,\cos(dx+c)^5+6144\,\cos(dx+c)^4-3710\,\cos(dx+c)^3-7\right.}{13440\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/13440*(1155*d*x + (1680*cos(d*x + c)^7 - 3840*cos(d*x + c)^6 - 280*cos(d*
x + c)^5 + 6144*cos(d*x + c)^4 - 3710*cos(d*x + c)^3 - 768*cos(d*x + c)^2 +
 1155*cos(d*x + c) - 1536)*sin(d*x + c))/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**8/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33326, size = 188, normalized size = 1.13

$$\frac{\frac{1155\,(dx+c)}{a^2}+\frac{2\left(1155\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15}+8855\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}-142541\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+31007\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}-55583\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-29491\,\tan\left(\frac{1}{2}\,dx \cdots\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^8 a^2}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/13440*(1155*(d*x + c)/a^2 + 2*(1155*tan(1/2*d*x + 1/2*c)^15 + 8855*tan(1/
2*d*x + 1/2*c)^13 - 142541*tan(1/2*d*x + 1/2*c)^11 + 31007*tan(1/2*d*x + 1/
2*c)^9 - 55583*tan(1/2*d*x + 1/2*c)^7 - 29491*tan(1/2*d*x + 1/2*c)^5 - 8855
*tan(1/2*d*x + 1/2*c)^3 - 1155*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)
^2 + 1)^8*a^2))/d
```

3.84 $\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=104

$$-\frac{\sin^5(c+dx)}{10a^2d}-\frac{\sin^3(c+dx)(a-a\cos(c+dx))^3}{6a^5d}-\frac{\sin^3(c+dx)\cos(c+dx)}{8a^2d}-\frac{3\sin(c+dx)\cos(c+dx)}{16a^2d}+\frac{3x}{16a^2}$$

```
[Out] (3*x)/(16*a^2) - (3*Cos[c + d*x]*Sin[c + d*x])/(16*a^2*d) - (Cos[c + d*x]*S
in[c + d*x]^3)/(8*a^2*d) - ((a - a*Cos[c + d*x])^3*Sin[c + d*x]^3)/(6*a^5*d
) - Sin[c + d*x]^5/(10*a^2*d)
```

Rubi [A] time = 0.311139, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3872, 2875, 2870, 2669, 2635, 8}

$$-\frac{\sin^5(c+dx)}{10a^2d}-\frac{\sin^3(c+dx)(a-a\cos(c+dx))^3}{6a^5d}-\frac{\sin^3(c+dx)\cos(c+dx)}{8a^2d}-\frac{3\sin(c+dx)\cos(c+dx)}{16a^2d}+\frac{3x}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (3*x)/(16*a^2) - (3*Cos[c + d*x]*Sin[c + d*x])/(16*a^2*d) - (Cos[c + d*x]*S
in[c + d*x]^3)/(8*a^2*d) - ((a - a*Cos[c + d*x])^3*Sin[c + d*x]^3)/(6*a^5*d
) - Sin[c + d*x]^5/(10*a^2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2870

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*sin[(e_.) + (f_.)*(x_)]^2*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> -Simp[((g*Cos[e + f*x])^(
p + 1)*(a + b*Sin[e + f*x])^(m + 1))/(2*b*f*g*(m + 1)), x] + Dist[a/(2*g^2)
, Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^(m - 1), x], x] /; Free
Q[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && EqQ[m - p, 0]
```

Rule 2669

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1))/(f*g*(p + 1)), x] + D
ist[a, Int[(g*Cos[e + f*x])^p, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && (I
ntegerQ[2*p] || NeQ[a^2 - b^2, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)\sin^6(c+dx)}{(-a-a\cos(c+dx))^2}\,dx\\ &= \frac{\int \cos^2(c+dx)(-a+a\cos(c+dx))^2\sin^2(c+dx)\,dx}{a^4}\\ &= -\frac{(a-a\cos(c+dx))^3\sin^3(c+dx)}{6a^5d} - \frac{\int(-a+a\cos(c+dx))\sin^4(c+dx)\,dx}{2a^3}\\ &= -\frac{(a-a\cos(c+dx))^3\sin^3(c+dx)}{6a^5d} - \frac{\sin^5(c+dx)}{10a^2d} + \frac{\int \sin^4(c+dx)\,dx}{2a^2}\\ &= -\frac{\cos(c+dx)\sin^3(c+dx)}{8a^2d} - \frac{(a-a\cos(c+dx))^3\sin^3(c+dx)}{6a^5d} - \frac{\sin^5(c+dx)}{10a^2d} + \frac{3\int \sin^2(c+\\ &= -\frac{3\cos(c+dx)\sin(c+dx)}{16a^2d} - \frac{\cos(c+dx)\sin^3(c+dx)}{8a^2d} - \frac{(a-a\cos(c+dx))^3\sin^3(c+dx)}{6a^5d} -\\ &= \frac{3x}{16a^2} - \frac{3\cos(c+dx)\sin(c+dx)}{16a^2d} - \frac{\cos(c+dx)\sin^3(c+dx)}{8a^2d} - \frac{(a-a\cos(c+dx))^3\sin^3(c+\\ \end{aligned}$$

Mathematica [A] time = 0.869438, size = 111, normalized size = 1.07

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(-480\sin(c+dx)+30\sin(2(c+dx))+80\sin(3(c+dx))-90\sin(4(c+dx))+48\sin(5(c\right.}{480a^2d(\sec(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(360*d*x - 480*Sin[c + d*x] + 30*Sin[2*(
c + d*x)] + 80*Sin[3*(c + d*x)] - 90*Sin[4*(c + d*x)] + 48*Sin[5*(c + d*x)]
 - 10*Sin[6*(c + d*x)] + 25*Tan[c/2]))/(480*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [B] time = 0.086, size = 222, normalized size = 2.1

$$\frac{3}{8\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{11}\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-6}-\frac{205}{24\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{9}\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-6}-\frac{29}{20\,da^2}\left(\tan\left(\frac{d}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 3/8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11-205/24/d/a^2/(1+
tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9-29/20/d/a^2/(1+tan(1/2*d*x+1/2
*c)^2)^6*tan(1/2*d*x+1/2*c)^7-99/20/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/
```

```
2*d*x+1/2*c)^5-17/8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3-3
/8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)+3/8/d/a^2*arctan(tan
(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.54564, size = 394, normalized size = 3.79

$$-\frac{\frac{\frac{45 \sin(dx+c)}{\cos(dx+c)+1}+\frac{255 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{594 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{174 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{1025 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{45 \sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a^2+\frac{6a^2 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^2 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a^2 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a^2 \sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a^2 \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^2 \sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}-\frac{45 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{120 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/120*((45*sin(d*x + c)/(cos(d*x + c) + 1) + 255*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 594*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 174*sin(d*x + c)^7/(
cos(d*x + c) + 1)^7 + 1025*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 - 45*sin(d*x
 + c)^11/(cos(d*x + c) + 1)^11)/(a^2 + 6*a^2*sin(d*x + c)^2/(cos(d*x + c) +
 1)^2 + 15*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a^2*sin(d*x + c)^6/
(cos(d*x + c) + 1)^6 + 15*a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*a^2*s
in(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^2*sin(d*x + c)^12/(cos(d*x + c) +
1)^12) - 45*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.7213, size = 192, normalized size = 1.85

$$\frac{45\, dx - \left(40 \cos(dx+c)^5 - 96 \cos(dx+c)^4 + 50 \cos(dx+c)^3 + 32 \cos(dx+c)^2 - 45 \cos(dx+c) + 64\right) \sin(dx+c)}{240\, a^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/240*(45*d*x - (40*cos(d*x + c)^5 - 96*cos(d*x + c)^4 + 50*cos(d*x + c)^3
+ 32*cos(d*x + c)^2 - 45*cos(d*x + c) + 64)*sin(d*x + c))/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29965, size = 153, normalized size = 1.47

$$\frac{\frac{45\,(dx+c)}{a^2}+\frac{2\left(45 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-1025 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-174 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-594 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-255 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-45 \tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^6 a^2}}{240\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(45*(d*x + c)/a^2 + 2*(45*tan(1/2*d*x + 1/2*c)^11 - 1025*tan(1/2*d*x
+ 1/2*c)^9 - 174*tan(1/2*d*x + 1/2*c)^7 - 594*tan(1/2*d*x + 1/2*c)^5 - 255*
tan(1/2*d*x + 1/2*c)^3 - 45*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2
+ 1)^6*a^2))/d
```

3.85 $\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=87

$$\frac{2\sin^3(c+dx)}{3a^2d}-\frac{2\sin(c+dx)}{a^2d}+\frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}+\frac{7\sin(c+dx)\cos(c+dx)}{8a^2d}+\frac{7x}{8a^2}$$

```
[Out] (7*x)/(8*a^2) - (2*Sin[c + d*x])/(a^2*d) + (7*Cos[c + d*x]*Sin[c + d*x])/(8
*a^2*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*a^2*d) + (2*Sin[c + d*x]^3)/(3*a
^2*d)
```

Rubi [A] time = 0.232528, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3872, 2869, 2757, 2635, 8, 2633}

$$\frac{2\sin^3(c+dx)}{3a^2d}-\frac{2\sin(c+dx)}{a^2d}+\frac{\sin(c+dx)\cos^3(c+dx)}{4a^2d}+\frac{7\sin(c+dx)\cos(c+dx)}{8a^2d}+\frac{7x}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (7*x)/(8*a^2) - (2*Sin[c + d*x])/(a^2*d) + (7*Cos[c + d*x]*Sin[c + d*x])/(8
*a^2*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*a^2*d) + (2*Sin[c + d*x]^3)/(3*a
^2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)\sin^4(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{\int \cos^2(c+dx)(-a+a\cos(c+dx))^2\,dx}{a^4} \\
&= \frac{\int \left(a^2\cos^2(c+dx)-2a^2\cos^3(c+dx)+a^2\cos^4(c+dx)\right)dx}{a^4} \\
&= \frac{\int \cos^2(c+dx)\,dx}{a^2}+\frac{\int \cos^4(c+dx)\,dx}{a^2}-\frac{2\int \cos^3(c+dx)\,dx}{a^2} \\
&= \frac{\cos(c+dx)\sin(c+dx)}{2a^2d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d}+\frac{\int 1\,dx}{2a^2}+\frac{3\int \cos^2(c+dx)\,dx}{4a^2}+\frac{2\,\text{Sub}}{}\\
&= \frac{x}{2a^2}-\frac{2\sin(c+dx)}{a^2d}+\frac{7\cos(c+dx)\sin(c+dx)}{8a^2d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d}+\frac{2\sin^3(c+dx)}{3a^2d} \\
&= \frac{7x}{8a^2}-\frac{2\sin(c+dx)}{a^2d}+\frac{7\cos(c+dx)\sin(c+dx)}{8a^2d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d}+\frac{2\sin^3(c+dx)}{3a^2d}
\end{aligned}
$$

Mathematica [A] time = 0.544488, size = 91, normalized size = 1.05

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\left(-144\sin(c+dx)+48\sin(2(c+dx))-16\sin(3(c+dx))+3\sin(4(c+dx))+2\tan\left(\frac{c}{2}\right)+8\right.}{24a^2d(\sec(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(84*d*x - 144*Sin[c + d*x] + 48*Sin[2*(c
 + d*x)] - 16*Sin[3*(c + d*x)] + 3*Sin[4*(c + d*x)] + 2*Tan[c/2]))/(24*a^2*
d*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 0.089, size = 154, normalized size = 1.8

$$-\frac{25}{4\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-4}-\frac{83}{12\,da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-4}-\frac{77}{12\,da^2}\left(\tan\left(\frac{d}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -25/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7-83/12/d/a^2/(1+
tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-77/12/d/a^2/(1+tan(1/2*d*x+1/2
*c)^2)^4*tan(1/2*d*x+1/2*c)^3-7/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*
d*x+1/2*c)+7/4/d/a^2*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.52328, size = 278, normalized size = 3.2

$$-\frac{\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{77\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{83\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{75\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^2+\frac{4a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^2\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{21\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/12*((21*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 83*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 75*sin(d*x + c)^7/(cos(
d*x + c) + 1)^7)/(a^2 + 4*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^2*s
in(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a^2*sin(d*x + c)^6/(cos(d*x + c) + 1
)^6 + a^2*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 21*arctan(sin(d*x + c)/(co
s(d*x + c) + 1))/a^2)/d
```

Fricas [A] time = 1.72595, size = 135, normalized size = 1.55

$$\frac{21\,dx+\left(6\,\cos\left(dx+c\right)^{3}-16\,\cos\left(dx+c\right)^{2}+21\,\cos\left(dx+c\right)-32\right)\sin\left(dx+c\right)}{24\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(21*d*x + (6*cos(d*x + c)^3 - 16*cos(d*x + c)^2 + 21*cos(d*x + c) - 32
)*sin(d*x + c))/(a^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36183, size = 117, normalized size = 1.34

$$\frac{\frac{21\,(dx+c)}{a^2}-\frac{2\left(75\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+83\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+77\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+21\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^4a^2}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(21*(d*x + c)/a^2 - 2*(75*tan(1/2*d*x + 1/2*c)^7 + 83*tan(1/2*d*x + 1/
2*c)^5 + 77*tan(1/2*d*x + 1/2*c)^3 + 21*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x
 + 1/2*c)^2 + 1)^4*a^2))/d
```

3.86 $\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=69

$$\frac{2\sin(c+dx)}{a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{2a^2d}+\frac{2\sin(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{5x}{2a^2}$$

```
[Out] (-5*x)/(2*a^2) + (2*Sin[c + d*x])/(a^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(2*
a^2*d) + (2*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x]))
```

Rubi [A] time = 0.316197, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2874, 2950, 2709, 2637, 2635, 8, 2648}

$$\frac{2\sin(c+dx)}{a^2d}-\frac{\sin(c+dx)\cos(c+dx)}{2a^2d}+\frac{2\sin(c+dx)}{a^2d(\cos(c+dx)+1)}-\frac{5x}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-5*x)/(2*a^2) + (2*Sin[c + d*x])/(a^2*d) - (Cos[c + d*x]*Sin[c + d*x])/(2*
a^2*d) + (2*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x]))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2950

```
Int[sin[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[a^n*c^n,
 Int[Tan[e + f*x]^p*(a + b*Sin[e + f*x])^(m - n), x], x] /; FreeQ[{a, b, c,
 d, e, f, m}, x] && EqQ[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && EqQ[p + 2*n,
0] && IntegerQ[n]
```

Rule 2709

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Dist[a^p, Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e
+ f*x])^(m - p/2))/(a - b*Sin[e + f*x])^(p/2), x], x], x] /; FreeQ[{a, b, e
, f}, x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p/2] && (LtQ[p, 0] || GtQ[m -
 p/2, 0])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))}{-a-a\cos(c+dx)}\,dx}{a^2} \\
&= \frac{\int (-a+a\cos(c+dx))^2\cot^2(c+dx)\,dx}{a^4} \\
&= \frac{\int \left(-2+2\cos(c+dx)-\cos^2(c+dx)+\frac{2}{1+\cos(c+dx)}\right)dx}{a^2} \\
&= -\frac{2x}{a^2}-\frac{\int \cos^2(c+dx)\,dx}{a^2}+\frac{2\int \cos(c+dx)\,dx}{a^2}+\frac{2\int \frac{1}{1+\cos(c+dx)}\,dx}{a^2} \\
&= -\frac{2x}{a^2}+\frac{2\sin(c+dx)}{a^2d}-\frac{\cos(c+dx)\sin(c+dx)}{2a^2d}+\frac{2\sin(c+dx)}{a^2d(1+\cos(c+dx))}-\frac{\int 1\,dx}{2a^2} \\
&= -\frac{5x}{2a^2}+\frac{2\sin(c+dx)}{a^2d}-\frac{\cos(c+dx)\sin(c+dx)}{2a^2d}+\frac{2\sin(c+dx)}{a^2d(1+\cos(c+dx))}
\end{aligned}
$$

Mathematica [A] time = 0.321016, size = 121, normalized size = 1.75

$$
-\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(-25\sin\left(c+\frac{dx}{2}\right)-21\sin\left(c+\frac{3dx}{2}\right)-21\sin\left(2c+\frac{3dx}{2}\right)+3\sin\left(2c+\frac{5dx}{2}\right)+3\sin\left(3c+\frac{5dx}{2}\right)+6\right.}{48a^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(Sec[c/2]*Sec[(c + d*x)/2]*(60*d*x*Cos[(d*x)/2] + 60*d*x*Cos[c + (d*x)/2]
- 119*Sin[(d*x)/2] - 25*Sin[c + (d*x)/2] - 21*Sin[c + (3*d*x)/2] - 21*Sin[2
*c + (3*d*x)/2] + 3*Sin[2*c + (5*d*x)/2] + 3*Sin[3*c + (5*d*x)/2]))/(48*a^2
*d)
```

Maple [A] time = 0.086, size = 103, normalized size = 1.5

$$2\,{\frac {\tan \left(1/2\,dx+c/2 \right) }{d{a}^{2}}}+5\,{\frac { \left(\tan \left(1/2\,dx+c/2 \right) \right) ^{3}}{d{a}^{2} \left(1+ \left(\tan \left(1/2\,dx+c/2 \right) \right) ^{2} \right) ^{2}}}+3\,{\frac {\tan \left(1/2\,dx+c/2 \right) }{d{a}^{2} \left(1+ \left(\tan \left(1/2\,dx+c/2 \right) \right) ^{2} \right) ^{2}}}-5\,{\frac {\arctan \left(\tan \left(1/2\,dx+
\right. }{d{a}^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 2/d/a^2*tan(1/2*d*x+1/2*c)+5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1
/2*c)^3+3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)-5/d/a^2*arcta
n(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.53877, size = 189, normalized size = 2.74

$$\frac{\frac{\frac{3 \, \sin\left(d x + c\right)}{\cos\left(d x + c\right) + 1} + \frac{5 \, \sin\left(d x + c\right)^{3}}{{\left(\cos\left(d x + c\right) + 1\right)}^{3}}}{a^{2} + \frac{2 \, a^{2} \sin\left(d x + c\right)^{2}}{{\left(\cos\left(d x + c\right) + 1\right)}^{2}} + \frac{a^{2} \sin\left(d x + c\right)^{4}}{{\left(\cos\left(d x + c\right) + 1\right)}^{4}}} - \frac{5 \, \arctan\left(\frac{\sin\left(d x + c\right)}{\cos\left(d x + c\right) + 1}\right)}{a^{2}} + \frac{2 \, \sin\left(d x + c\right)}{a^{2} {\left(\cos\left(d x + c\right) + 1\right)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] ((3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x + c) + 1)^3
)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(co
s(d*x + c) + 1)^4) - 5*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 2*sin(
d*x + c)/(a^2*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.70279, size = 158, normalized size = 2.29

$$-\frac{5 \, d x \cos\left(d x + c\right) + 5 \, d x + {\left(\cos\left(d x + c\right)^{2} - 3 \, \cos\left(d x + c\right) - 8\right)} \sin\left(d x + c\right)}{2 \, {\left(a^{2} d \cos\left(d x + c\right) + a^{2} d\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(5*d*x*cos(d*x + c) + 5*d*x + (cos(d*x + c)^2 - 3*cos(d*x + c) - 8)*si
n(d*x + c))/(a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sin^{2}{\left (c + d x \right)}}{\sec^{2}{\left (c + d x \right)} + 2 \sec{\left (c + d x \right)} + 1}\, dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.32468, size = 101, normalized size = 1.46

$$-\frac{\frac{5\,(dx+c)}{a^2}-\frac{4\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^2}-\frac{2\left(5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2 a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(5*(d*x + c)/a^2 - 4*tan(1/2*d*x + 1/2*c)/a^2 - 2*(5*tan(1/2*d*x + 1/2
*c)^3 + 3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2))/d
```

3.87 $\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^2} \, dx$

Optimal. Leaf size=73

$$-\frac{2\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}-\frac{2\csc^3(c+dx)}{3a^2d}$$

```
[Out] -Cot[c + d*x]^3/(3*a^2*d) - (2*Cot[c + d*x]^5)/(5*a^2*d) - (2*Csc[c + d*x]^
3)/(3*a^2*d) + (2*Csc[c + d*x]^5)/(5*a^2*d)
```

Rubi [A] time = 0.199552, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3872, 2711, 2607, 30, 2606, 14}

$$-\frac{2\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{2\csc^5(c+dx)}{5a^2d}-\frac{2\csc^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -Cot[c + d*x]^3/(3*a^2*d) - (2*Cot[c + d*x]^5)/(5*a^2*d) - (2*Csc[c + d*x]^
3)/(3*a^2*d) + (2*Csc[c + d*x]^5)/(5*a^2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2711

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((g_.)*tan[(e_.) + (f_.)*(x
_)])^(p_.), x_Symbol] :> Dist[a^(2*m), Int[ExpandIntegrand[(g*Tan[e + f*x])
^p/Sec[e + f*x]^m, (a*Sec[e + f*x] - b*Tan[e + f*x])^(-m), x], x], x] /; Fr
eeQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{\int \left(a^2\cot^4(c+dx)\csc^2(c+dx) - 2a^2\cot^3(c+dx)\csc^3(c+dx) + a^2\cot^2(c+dx)\csc^4(c+dx\right.}{a^4}$$
$$= \frac{\int \cot^4(c+dx)\csc^2(c+dx)\,dx}{a^2} + \frac{\int \cot^2(c+dx)\csc^4(c+dx)\,dx}{a^2} - \frac{2\int \cot^3(c+dx)\csc^3(c+}{a^2}$$
$$= \frac{\text{Subst}\left(\int x^4\,dx, x, -\cot(c+dx)\right)}{a^2d} + \frac{\text{Subst}\left(\int x^2\left(1+x^2\right)dx, x, -\cot(c+dx)\right)}{a^2d} + \frac{2\,\text{Subst}\left(\int\right.}{}$$
$$= -\frac{\cot^5(c+dx)}{5a^2d} + \frac{\text{Subst}\left(\int \left(x^2+x^4\right)dx, x, -\cot(c+dx)\right)}{a^2d} + \frac{2\,\text{Subst}\left(\int \left(-x^2+x^4\right)dx, x, \text{cs}\right.}{a^2d}$$
$$= -\frac{\cot^3(c+dx)}{3a^2d} - \frac{2\cot^5(c+dx)}{5a^2d} - \frac{2\csc^3(c+dx)}{3a^2d} + \frac{2\csc^5(c+dx)}{5a^2d}$$

Mathematica [A] time = 0.4256, size = 105, normalized size = 1.44

$$\frac{\csc(c)(55\sin(c+dx) + 44\sin(2(c+dx)) + 11\sin(3(c+dx)) - 60\sin(2c+dx) + 16\sin(c+2dx) + 4\sin(2c+3dx) - 80\,\text{s}}{240a^2d(\sec(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Csc[c]*Csc[c + d*x]*Sec[c + d*x]^2*(-80*Sin[c] + 80*Sin[d*x] + 55*Sin[c +
d*x] + 44*Sin[2*(c + d*x)] + 11*Sin[3*(c + d*x)] - 60*Sin[2*c + d*x] + 16*S
in[c + 2*d*x] + 4*Sin[2*c + 3*d*x]))/(240*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 0.058, size = 60, normalized size = 0.8

$$\frac{1}{8\,da^2}\left(\frac{1}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5 - \frac{1}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \tan\left(\frac{dx}{2}+\frac{c}{2}\right) - \left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/8/d/a^2*(1/5*tan(1/2*d*x+1/2*c)^5-1/3*tan(1/2*d*x+1/2*c)^3-tan(1/2*d*x+1/
2*c)-1/tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.01255, size = 122, normalized size = 1.67

$$-\frac{\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1} + \frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3} - \frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^2} + \frac{15(\cos(dx+c)+1)}{a^2\sin(dx+c)}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/120*((15*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 - 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^2 + 15*(cos(d*x + c) +
1)/(a^2*sin(d*x + c)))/d
```

Fricas [A] time = 1.60875, size = 180, normalized size = 2.47

$$-\frac{\cos(dx+c)^3+2\,\cos(dx+c)^2+8\,\cos(dx+c)+4}{15\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(cos(d*x + c)^3 + 2*cos(d*x + c)^2 + 8*cos(d*x + c) + 4)/((a^2*d*cos(
d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.29461, size = 100, normalized size = 1.37

$$-\frac{\frac{15}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\frac{3\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-5\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,a^8\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{10}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/120*(15/(a^2*tan(1/2*d*x + 1/2*c)) - (3*a^8*tan(1/2*d*x + 1/2*c)^5 - 5*a
^8*tan(1/2*d*x + 1/2*c)^3 - 15*a^8*tan(1/2*d*x + 1/2*c))/a^10)/d
```

3.88 $\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=91

$$-\frac{2\cot^7(c+dx)}{7a^2d}-\frac{3\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{2\csc^7(c+dx)}{7a^2d}-\frac{2\csc^5(c+dx)}{5a^2d}$$

```
[Out] -Cot[c + d*x]^3/(3*a^2*d) - (3*Cot[c + d*x]^5)/(5*a^2*d) - (2*Cot[c + d*x]^
7)/(7*a^2*d) - (2*Csc[c + d*x]^5)/(5*a^2*d) + (2*Csc[c + d*x]^7)/(7*a^2*d)
```

Rubi [A] time = 0.344943, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2875, 2873, 2607, 14, 2606, 270}

$$-\frac{2\cot^7(c+dx)}{7a^2d}-\frac{3\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{2\csc^7(c+dx)}{7a^2d}-\frac{2\csc^5(c+dx)}{5a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -Cot[c + d*x]^3/(3*a^2*d) - (3*Cot[c + d*x]^5)/(5*a^2*d) - (2*Cot[c + d*x]^
7)/(7*a^2*d) - (2*Csc[c + d*x]^5)/(5*a^2*d) + (2*Csc[c + d*x]^7)/(7*a^2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
```

```
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(-a-a\cos(c+dx))^2}\,dx \\
&= \frac{\int (-a+a\cos(c+dx))^2\cot^2(c+dx)\csc^6(c+dx)\,dx}{a^4} \\
&= \frac{\int \left(a^2\cot^4(c+dx)\csc^4(c+dx)-2a^2\cot^3(c+dx)\csc^5(c+dx)+a^2\cot^2(c+dx)\csc^6(c+\right.}{a^4} \\
&= \frac{\int \cot^4(c+dx)\csc^4(c+dx)\,dx}{a^2}+\frac{\int \cot^2(c+dx)\csc^6(c+dx)\,dx}{a^2}-\frac{2\int \cot^3(c+dx)\csc^5(}{a^2} \\
&= \frac{\operatorname{Subst}\left(\int x^4\left(1+x^2\right)\,dx,x,-\cot(c+dx)\right)}{a^2d}+\frac{\operatorname{Subst}\left(\int x^2\left(1+x^2\right)^2\,dx,x,-\cot(c+dx)\right)}{a^2d} \\
&= \frac{\operatorname{Subst}\left(\int \left(x^4+x^6\right)\,dx,x,-\cot(c+dx)\right)}{a^2d}+\frac{\operatorname{Subst}\left(\int \left(x^2+2x^4+x^6\right)\,dx,x,-\cot(c+dx\right.}{a^2d} \\
&= -\frac{\cot^3(c+dx)}{3a^2d}-\frac{3\cot^5(c+dx)}{5a^2d}-\frac{2\cot^7(c+dx)}{7a^2d}-\frac{2\csc^5(c+dx)}{5a^2d}+\frac{2\csc^7(c+dx)}{7a^2d}
\end{aligned}$$

Mathematica [A] time = 0.648676, size = 149, normalized size = 1.64

$$-\frac{\csc(c)(-714\sin(c+dx)-408\sin(2(c+dx))+153\sin(3(c+dx))+204\sin(4(c+dx))+51\sin(5(c+dx))+1680\sin}{1}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(Csc[c]*Csc[c + d*x]^3*Sec[c + d*x]^2*(1344*Sin[c] - 1456*Sin[d*x] - 714*S
in[c + d*x] - 408*Sin[2*(c + d*x)] + 153*Sin[3*(c + d*x)] + 204*Sin[4*(c +
d*x)] + 51*Sin[5*(c + d*x)] + 1680*Sin[2*c + d*x] + 128*Sin[c + 2*d*x] - 48
*Sin[2*c + 3*d*x] - 64*Sin[3*c + 4*d*x] - 16*Sin[4*c + 5*d*x]))/(13440*a^2*
d*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 0.066, size = 86, normalized size = 1.

$$\frac{1}{32\,da^2}\left(\frac{1}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{1}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{2}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-2\,\tan(1/2\,dx+c/2)-\frac{1}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/32/d/a^2*(1/7*tan(1/2*d*x+1/2*c)^7+1/5*tan(1/2*d*x+1/2*c)^5-2/3*tan(1/2*d
*x+1/2*c)^3-2*tan(1/2*d*x+1/2*c)-1/3/tan(1/2*d*x+1/2*c)^3-1/tan(1/2*d*x+1/2
*c))
```

Maxima [A] time = 1.00241, size = 181, normalized size = 1.99

$$-\frac{\frac{\frac{210\sin(dx+c)}{\cos(dx+c)+1}+\frac{70\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^2}+\frac{35\left(\frac{3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+1\right)(\cos(dx+c)+1)^3}{a^2\sin(dx+c)^3}}{3360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3360*((210*sin(d*x + c)/(cos(d*x + c) + 1) + 70*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^2 + 35*(3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 1)*(c
os(d*x + c) + 1)^3/(a^2*sin(d*x + c)^3))/d
```

Fricas [A] time = 1.66706, size = 267, normalized size = 2.93

$$\frac{2\,\cos(dx+c)^5+4\,\cos(dx+c)^4-\cos(dx+c)^3-6\,\cos(dx+c)^2+24\,\cos(dx+c)+12}{105\left(a^2d\cos(dx+c)^4+2\,a^2d\cos(dx+c)^3-2\,a^2d\cos(dx+c)-a^2d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/105*(2*cos(d*x + c)^5 + 4*cos(d*x + c)^4 - cos(d*x + c)^3 - 6*cos(d*x + c
)^2 + 24*cos(d*x + c) + 12)/((a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3
 - 2*a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\csc^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**2
```

Giac [A] time = 1.23884, size = 142, normalized size = 1.56

$$-\frac{\frac{35\left(3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+1\right)}{a^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}}-\frac{15a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+21a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-70a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-210a^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{14}}}{3360d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/3360*(35*(3*tan(1/2*d*x + 1/2*c)^2 + 1)/(a^2*tan(1/2*d*x + 1/2*c)^3) - (
15*a^12*tan(1/2*d*x + 1/2*c)^7 + 21*a^12*tan(1/2*d*x + 1/2*c)^5 - 70*a^12*t
an(1/2*d*x + 1/2*c)^3 - 210*a^12*tan(1/2*d*x + 1/2*c))/a^14)/d
```

3.89 $\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^2} dx$

Optimal. Leaf size=109

$$-\frac{2\cot^9(c+dx)}{9a^2d}-\frac{5\cot^7(c+dx)}{7a^2d}-\frac{4\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{2\csc^9(c+dx)}{9a^2d}-\frac{2\csc^7(c+dx)}{7a^2d}$$

```
[Out] -Cot[c + d*x]^3/(3*a^2*d) - (4*Cot[c + d*x]^5)/(5*a^2*d) - (5*Cot[c + d*x]^
7)/(7*a^2*d) - (2*Cot[c + d*x]^9)/(9*a^2*d) - (2*Csc[c + d*x]^7)/(7*a^2*d)
+ (2*Csc[c + d*x]^9)/(9*a^2*d)
```

Rubi [A] time = 0.35068, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2875, 2873, 2607, 270, 2606, 14}

$$-\frac{2\cot^9(c+dx)}{9a^2d}-\frac{5\cot^7(c+dx)}{7a^2d}-\frac{4\cot^5(c+dx)}{5a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{2\csc^9(c+dx)}{9a^2d}-\frac{2\csc^7(c+dx)}{7a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -Cot[c + d*x]^3/(3*a^2*d) - (4*Cot[c + d*x]^5)/(5*a^2*d) - (5*Cot[c + d*x]^
7)/(7*a^2*d) - (2*Cot[c + d*x]^9)/(9*a^2*d) - (2*Csc[c + d*x]^7)/(7*a^2*d)
+ (2*Csc[c + d*x]^9)/(9*a^2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)\csc^4(c+dx)}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{\int(-a+a\cos(c+dx))^2\cot^2(c+dx)\csc^8(c+dx)\,dx}{a^4}$$
$$= \frac{\int\left(a^2\cot^4(c+dx)\csc^6(c+dx)-2a^2\cot^3(c+dx)\csc^7(c+dx)+a^2\cot^2(c+dx)\csc^8(c+$$
$$= \frac{\int\cot^4(c+dx)\csc^6(c+dx)\,dx}{a^2}+\frac{\int\cot^2(c+dx)\csc^8(c+dx)\,dx}{a^2}-\frac{2\int\cot^3(c+dx)\csc^7(}{a^2}$$
$$= \frac{\mathrm{Subst}\left(\int x^4\left(1+x^2\right)^2\,dx,x,-\cot(c+dx)\right)}{a^2d}+\frac{\mathrm{Subst}\left(\int x^2\left(1+x^2\right)^3\,dx,x,-\cot(c+dx)\right)}{a^2d}$$
$$= \frac{\mathrm{Subst}\left(\int\left(x^4+2x^6+x^8\right)dx,x,-\cot(c+dx)\right)}{a^2d}+\frac{\mathrm{Subst}\left(\int\left(x^2+3x^4+3x^6+x^8\right)dx,x,}{a^2d}$$
$$= -\frac{\cot^3(c+dx)}{3a^2d}-\frac{4\cot^5(c+dx)}{5a^2d}-\frac{5\cot^7(c+dx)}{7a^2d}-\frac{2\cot^9(c+dx)}{9a^2d}-\frac{2\csc^7(c+dx)}{7a^2d}+\frac{2}{}$$

Mathematica [A] time = 0.97313, size = 191, normalized size = 1.75

$$\csc(c)(25875\sin(c+dx)+11500\sin(2(c+dx))-10925\sin(3(c+dx))-9200\sin(4(c+dx))+575\sin(5(c+dx))+$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Csc[c]*Csc[c + d*x]^5*Sec[c + d*x]^2*(-61440*Sin[c] + 84480*Sin[d*x] + 258
75*Sin[c + d*x] + 11500*Sin[2*(c + d*x)] - 10925*Sin[3*(c + d*x)] - 9200*Si
n[4*(c + d*x)] + 575*Sin[5*(c + d*x)] + 2300*Sin[6*(c + d*x)] + 575*Sin[7*(
c + d*x)] - 107520*Sin[2*c + d*x] - 10240*Sin[c + 2*d*x] + 9728*Sin[2*c + 3
*d*x] + 8192*Sin[3*c + 4*d*x] - 512*Sin[4*c + 5*d*x] - 2048*Sin[5*c + 6*d*x
] - 512*Sin[6*c + 7*d*x]))/(1290240*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 0.07, size = 112, normalized size = 1.

$$\frac{1}{128\,da^2}\left(\frac{1}{9}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9+\frac{3}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{1}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{5}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-5\tan\left(1/2\,dx+c/2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/128/d/a^2*(1/9*tan(1/2*d*x+1/2*c)^9+3/7*tan(1/2*d*x+1/2*c)^7+1/5*tan(1/2*
d*x+1/2*c)^5-5/3*tan(1/2*d*x+1/2*c)^3-5*tan(1/2*d*x+1/2*c)-1/tan(1/2*d*x+1/
2*c)^3-1/tan(1/2*d*x+1/2*c)-1/5/tan(1/2*d*x+1/2*c)^5)
```

Maxima [A] time = 0.99652, size = 235, normalized size = 2.16

$$-\frac{\frac{\frac{1575\sin(dx+c)}{\cos(dx+c)+1}+\frac{525\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{135\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{35\sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^2}+\frac{63\left(\frac{5\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{5\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+1\right)(\cos(dx+c)+1)^5}{a^2\sin(dx+c)^5}}{40320\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/40320*((1575*sin(d*x + c)/(cos(d*x + c) + 1) + 525*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 - 63*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 135*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7 - 35*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^2 + 63*(
5*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 5*sin(d*x + c)^4/(cos(d*x + c) + 1)
^4 + 1)*(cos(d*x + c) + 1)^5/(a^2*sin(d*x + c)^5))/d
```

Fricas [A] time = 1.73779, size = 423, normalized size = 3.88

$$\frac{8\cos(dx+c)^7+16\cos(dx+c)^6-12\cos(dx+c)^5-40\cos(dx+c)^4-5\cos(dx+c)^3+30\cos(dx+c)^2-40\,\mathrm{c}}{315\left(a^2d\cos(dx+c)^6+2\,a^2d\cos(dx+c)^5-a^2d\cos(dx+c)^4-4\,a^2d\cos(dx+c)^3-a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/315*(8*cos(d*x + c)^7 + 16*cos(d*x + c)^6 - 12*cos(d*x + c)^5 - 40*cos(d*
x + c)^4 - 5*cos(d*x + c)^3 + 30*cos(d*x + c)^2 - 40*cos(d*x + c) - 20)/((a
^2*d*cos(d*x + c)^6 + 2*a^2*d*cos(d*x + c)^5 - a^2*d*cos(d*x + c)^4 - 4*a^2
*d*cos(d*x + c)^3 - a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)*si
n(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38236, size = 181, normalized size = 1.66

$$-\frac{\frac{63\left(5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4+5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5}-\frac{35a^{16}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+135a^{16}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+63a^{16}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-525a^{16}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-1575}{a^{18}}}{40320d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/40320*(63*(5*tan(1/2*d*x + 1/2*c)^4 + 5*tan(1/2*d*x + 1/2*c)^2 + 1)/(a^2
*tan(1/2*d*x + 1/2*c)^5) - (35*a^16*tan(1/2*d*x + 1/2*c)^9 + 135*a^16*tan(1
/2*d*x + 1/2*c)^7 + 63*a^16*tan(1/2*d*x + 1/2*c)^5 - 525*a^16*tan(1/2*d*x +
 1/2*c)^3 - 1575*a^16*tan(1/2*d*x + 1/2*c))/a^18)/d
```

3.90 $\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=125

$$-\frac{2\cot^{11}(c+dx)}{11a^2d}-\frac{7\cot^9(c+dx)}{9a^2d}-\frac{9\cot^7(c+dx)}{7a^2d}-\frac{\cot^5(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{2\csc^{11}(c+dx)}{11a^2d}-\frac{2\csc^9(c+dx)}{9a^2d}$$

```
[Out] -Cot[c + d*x]^3/(3*a^2*d) - Cot[c + d*x]^5/(a^2*d) - (9*Cot[c + d*x]^7)/(7*
a^2*d) - (7*Cot[c + d*x]^9)/(9*a^2*d) - (2*Cot[c + d*x]^11)/(11*a^2*d) - (2
*Csc[c + d*x]^9)/(9*a^2*d) + (2*Csc[c + d*x]^11)/(11*a^2*d)
```

Rubi [A] time = 0.367212, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2875, 2873, 2607, 270, 2606, 14}

$$-\frac{2\cot^{11}(c+dx)}{11a^2d}-\frac{7\cot^9(c+dx)}{9a^2d}-\frac{9\cot^7(c+dx)}{7a^2d}-\frac{\cot^5(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{2\csc^{11}(c+dx)}{11a^2d}-\frac{2\csc^9(c+dx)}{9a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] -Cot[c + d*x]^3/(3*a^2*d) - Cot[c + d*x]^5/(a^2*d) - (9*Cot[c + d*x]^7)/(7*
a^2*d) - (7*Cot[c + d*x]^9)/(9*a^2*d) - (2*Cot[c + d*x]^11)/(11*a^2*d) - (2
*Csc[c + d*x]^9)/(9*a^2*d) + (2*Csc[c + d*x]^11)/(11*a^2*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)\csc^6(c+dx)}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{\int (-a+a\cos(c+dx))^2\cot^2(c+dx)\csc^{10}(c+dx)\,dx}{a^4}$$
$$= \frac{\int \left(a^2\cot^4(c+dx)\csc^8(c+dx)-2a^2\cot^3(c+dx)\csc^9(c+dx)+a^2\cot^2(c+dx)\csc^{10}(c\right.}{a^4}$$
$$= \frac{\int \cot^4(c+dx)\csc^8(c+dx)\,dx}{a^2}+\frac{\int \cot^2(c+dx)\csc^{10}(c+dx)\,dx}{a^2}-\frac{2\int \cot^3(c+dx)\csc^9}{a^2}$$
$$= \frac{\operatorname{Subst}\left(\int x^4\left(1+x^2\right)^3 dx, x, -\cot(c+dx)\right)}{a^2d}+\frac{\operatorname{Subst}\left(\int x^2\left(1+x^2\right)^4 dx, x, -\cot(c+dx)\right)}{a^2d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(x^4+3x^6+3x^8+x^{10}\right) dx, x, -\cot(c+dx)\right)}{a^2d}+\frac{\operatorname{Subst}\left(\int \left(x^2+4x^4+6x^6+4x^8\right.\right.}{a^2d}$$
$$= -\frac{\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{a^2d}-\frac{9\cot^7(c+dx)}{7a^2d}-\frac{7\cot^9(c+dx)}{9a^2d}-\frac{2\cot^{11}(c+dx)}{11a^2d}-\frac{2\,c}{}$$

Mathematica [A] time = 1.40975, size = 233, normalized size = 1.86

$$-\frac{\csc(c)(-218834\sin(c+dx)-79576\sin(2(c+dx))+119364\sin(3(c+dx))+79576\sin(4(c+dx))-28420\sin(5(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^8/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(Csc[c]*Csc[c + d*x]^7*Sec[c + d*x]^2*(630784*Sin[c] - 1103872*Sin[d*x] -
218834*Sin[c + d*x] - 79576*Sin[2*(c + d*x)] + 119364*Sin[3*(c + d*x)] + 79
576*Sin[4*(c + d*x)] - 28420*Sin[5*(c + d*x)] - 34104*Sin[6*(c + d*x)] - 14
21*Sin[7*(c + d*x)] + 5684*Sin[8*(c + d*x)] + 1421*Sin[9*(c + d*x)] + 14192
64*Sin[2*c + d*x] + 114688*Sin[c + 2*d*x] - 172032*Sin[2*c + 3*d*x] - 11468
8*Sin[3*c + 4*d*x] + 40960*Sin[4*c + 5*d*x] + 49152*Sin[5*c + 6*d*x] + 2048
*Sin[6*c + 7*d*x] - 8192*Sin[7*c + 8*d*x] - 2048*Sin[8*c + 9*d*x]))/(227082
24*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 0.078, size = 112, normalized size = 0.9

$$\frac{1}{512\,da^2}\left(\frac{1}{11}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{11}+\frac{5}{9}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{9}+\frac{8}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}-\frac{14}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}-14\tan(1/2\,dx+c/2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^8/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/512/d/a^2*(1/11*tan(1/2*d*x+1/2*c)^11+5/9*tan(1/2*d*x+1/2*c)^9+8/7*tan(1/
2*d*x+1/2*c)^7-14/3*tan(1/2*d*x+1/2*c)^3-14*tan(1/2*d*x+1/2*c)-8/3/tan(1/2*
d*x+1/2*c)^3-1/tan(1/2*d*x+1/2*c)^5-1/7/tan(1/2*d*x+1/2*c)^7)
```

Maxima [A] time = 1.00744, size = 235, normalized size = 1.88

$$-\frac{\frac{\frac{9702\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{3234\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{792\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}-\frac{385\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}-\frac{63\,\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a^2}+\frac{33\left(\frac{21\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{56\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+3\right)(\cos(dx+c)+1)^7}{a^2\sin(dx+c)^7}}{354816\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/354816*((9702*sin(d*x + c)/(cos(d*x + c) + 1) + 3234*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3 - 792*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 - 385*sin(d*x +
c)^9/(cos(d*x + c) + 1)^9 - 63*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)/a^2 +
 33*(21*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 56*sin(d*x + c)^4/(cos(d*x +
c) + 1)^4 + 3)*(cos(d*x + c) + 1)^7/(a^2*sin(d*x + c)^7))/d
```

Fricas [A] time = 1.85061, size = 521, normalized size = 4.17

$$\frac{16\cos(dx+c)^9+32\cos(dx+c)^8-40\cos(dx+c)^7-112\cos(dx+c)^6+14\cos(dx+c)^5+140\cos(dx+c)^4+35\,\mathrm{cc}}{693\left(a^2d\cos(dx+c)^8+2\,a^2d\cos(dx+c)^7-2\,a^2d\cos(dx+c)^6-6\,a^2d\cos(dx+c)^5+6\,a^2d\cos(dx+c)^3+2\,a^2d\,\mathrm{co}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/693*(16*cos(d*x + c)^9 + 32*cos(d*x + c)^8 - 40*cos(d*x + c)^7 - 112*cos(
d*x + c)^6 + 14*cos(d*x + c)^5 + 140*cos(d*x + c)^4 + 35*cos(d*x + c)^3 - 7
0*cos(d*x + c)^2 + 56*cos(d*x + c) + 28)/((a^2*d*cos(d*x + c)^8 + 2*a^2*d*c
os(d*x + c)^7 - 2*a^2*d*cos(d*x + c)^6 - 6*a^2*d*cos(d*x + c)^5 + 6*a^2*d*c
os(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 - 2*a^2*d*cos(d*x + c) - a^2*d)*sin(
d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**8/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.39539, size = 181, normalized size = 1.45

$$-\frac{\frac{33\left(56\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{4}+21\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+3\right)}{a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}}-\frac{63\,a^{20}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+385\,a^{20}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+792\,a^{20}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-3234\,a^{20}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}}{a^{22}}}{354816\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/354816*(33*(56*tan(1/2*d*x + 1/2*c)^4 + 21*tan(1/2*d*x + 1/2*c)^2 + 3)/(
a^2*tan(1/2*d*x + 1/2*c)^7) - (63*a^20*tan(1/2*d*x + 1/2*c)^11 + 385*a^20*t
an(1/2*d*x + 1/2*c)^9 + 792*a^20*tan(1/2*d*x + 1/2*c)^7 - 3234*a^20*tan(1/2
*d*x + 1/2*c)^3 - 9702*a^20*tan(1/2*d*x + 1/2*c))/a^22)/d
```

3.91 $\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=139

$$-\frac{(a-a\cos(c+dx))^{11}}{11a^{14}d}+\frac{7(a-a\cos(c+dx))^{10}}{10a^{13}d}-\frac{19(a-a\cos(c+dx))^9}{9a^{12}d}+\frac{25(a-a\cos(c+dx))^8}{8a^{11}d}-\frac{16(a-a\cos(c+dx)}{7a^{10}d}$$

```
[Out] (2*(a - a*Cos[c + d*x])^6)/(3*a^9*d) - (16*(a - a*Cos[c + d*x])^7)/(7*a^10*
d) + (25*(a - a*Cos[c + d*x])^8)/(8*a^11*d) - (19*(a - a*Cos[c + d*x])^9)/(
9*a^12*d) + (7*(a - a*Cos[c + d*x])^10)/(10*a^13*d) - (a - a*Cos[c + d*x])^
11/(11*a^14*d)
```

Rubi [A] time = 0.194717, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2836, 12, 88}

$$-\frac{(a-a\cos(c+dx))^{11}}{11a^{14}d}+\frac{7(a-a\cos(c+dx))^{10}}{10a^{13}d}-\frac{19(a-a\cos(c+dx))^9}{9a^{12}d}+\frac{25(a-a\cos(c+dx))^8}{8a^{11}d}-\frac{16(a-a\cos(c+dx)}{7a^{10}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^11/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (2*(a - a*Cos[c + d*x])^6)/(3*a^9*d) - (16*(a - a*Cos[c + d*x])^7)/(7*a^10*
d) + (25*(a - a*Cos[c + d*x])^8)/(8*a^11*d) - (19*(a - a*Cos[c + d*x])^9)/(
9*a^12*d) + (7*(a - a*Cos[c + d*x])^10)/(10*a^13*d) - (a - a*Cos[c + d*x])^
11/(11*a^14*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\sin^{11}(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^{11}(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^5x^3(-a+x)^2}{a^3}\,dx, x, -a\cos(c+dx)\right)}{a^{11}d}$$
$$= \frac{\text{Subst}\left(\int (-a-x)^5x^3(-a+x)^2\,dx, x, -a\cos(c+dx)\right)}{a^{14}d}$$
$$= \frac{\text{Subst}\left(\int \left(-4a^5(-a-x)^5 - 16a^4(-a-x)^6 - 25a^3(-a-x)^7 - 19a^2(-a-x)^8 - 7a(-a-x)^9\right.\right.}{a^{14}d}$$
$$= \frac{2(a-a\cos(c+dx))^6}{3a^9d} - \frac{16(a-a\cos(c+dx))^7}{7a^{10}d} + \frac{25(a-a\cos(c+dx))^8}{8a^{11}d} - \frac{19(a-a\cos(c}{9a^{12}d}$$

Mathematica [A] time = 4.27029, size = 120, normalized size = 0.86

$$\frac{2273040\cos(c+dx) - 1496880\cos(2(c+dx)) + 535920\cos(3(c+dx)) + 110880\cos(4(c+dx)) - 293832\cos(5(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^11/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-1615571 + 2273040*Cos[c + d*x] - 1496880*Cos[2*(c + d*x)] + 535920*Cos[3*
(c + d*x)] + 110880*Cos[4*(c + d*x)] - 293832*Cos[5*(c + d*x)] + 212520*Cos
[6*(c + d*x)] - 67320*Cos[7*(c + d*x)] - 27720*Cos[8*(c + d*x)] + 40040*Cos
[9*(c + d*x)] - 16632*Cos[10*(c + d*x)] + 2520*Cos[11*(c + d*x)])/(28385280
*a^3*d)
```

Maple [A] time = 0.12, size = 90, normalized size = 0.7

$$-\frac{1}{da^3}\left(-\frac{5}{8(\sec(dx+c))^8} + \frac{1}{6(\sec(dx+c))^6} - \frac{1}{11(\sec(dx+c))^{11}} + \frac{1}{4(\sec(dx+c))^4} + \frac{3}{10(\sec(dx+c))^{10}} - \frac{}{5(\sec}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^11/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/d/a^3*(-5/8/sec(d*x+c)^8+1/6/sec(d*x+c)^6-1/11/sec(d*x+c)^11+1/4/sec(d*x
+c)^4+3/10/sec(d*x+c)^10-3/5/sec(d*x+c)^5+5/7/sec(d*x+c)^7-1/9/sec(d*x+c)^9
)
```

Maxima [A] time = 0.998433, size = 120, normalized size = 0.86

$$\frac{2520\cos(dx+c)^{11} - 8316\cos(dx+c)^{10} + 3080\cos(dx+c)^9 + 17325\cos(dx+c)^8 - 19800\cos(dx+c)^7 - 4620\,c}{27720\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/27720*(2520*cos(d*x + c)^11 - 8316*cos(d*x + c)^10 + 3080*cos(d*x + c)^9
+ 17325*cos(d*x + c)^8 - 19800*cos(d*x + c)^7 - 4620*cos(d*x + c)^6 + 16632
```

```
*cos(d*x + c)^5 - 6930*cos(d*x + c)^4)/(a^3*d)
```

Fricas [A] time = 1.80858, size = 267, normalized size = 1.92

$$\frac{2520 \cos(dx+c)^{11} - 8316 \cos(dx+c)^{10} + 3080 \cos(dx+c)^{9} + 17325 \cos(dx+c)^{8} - 19800 \cos(dx+c)^{7} - 4620 \cos}{27720 a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/27720*(2520*cos(d*x + c)^11 - 8316*cos(d*x + c)^10 + 3080*cos(d*x + c)^9
+ 17325*cos(d*x + c)^8 - 19800*cos(d*x + c)^7 - 4620*cos(d*x + c)^6 + 16632
*cos(d*x + c)^5 - 6930*cos(d*x + c)^4)/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**11/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3818, size = 279, normalized size = 2.01

$$\frac{32\left(\frac{209(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{1045(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{3135(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{6270(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} + \frac{8778(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5} - \frac{13398(\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6} - \frac{2310}{(\mathrm{co}}\right.}{3465 a^3 d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right)^{11}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^11/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 32/3465*(209*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1045*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2 + 3135*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 -
 6270*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 8778*(cos(d*x + c) - 1)^5
/(cos(d*x + c) + 1)^5 - 13398*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 2
310*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 9240*(cos(d*x + c) - 1)^8/(
cos(d*x + c) + 1)^8 - 19)/(a^3*d*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1
)^11)
```

3.92 $\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=109

$$-\frac{\cos^9(c+dx)}{9a^3d}+\frac{3\cos^8(c+dx)}{8a^3d}-\frac{2\cos^7(c+dx)}{7a^3d}-\frac{\cos^6(c+dx)}{3a^3d}+\frac{3\cos^5(c+dx)}{5a^3d}-\frac{\cos^4(c+dx)}{4a^3d}$$

```
[Out] -Cos[c + d*x]^4/(4*a^3*d) + (3*Cos[c + d*x]^5)/(5*a^3*d) - Cos[c + d*x]^6/(
3*a^3*d) - (2*Cos[c + d*x]^7)/(7*a^3*d) + (3*Cos[c + d*x]^8)/(8*a^3*d) - Co
s[c + d*x]^9/(9*a^3*d)
```

Rubi [A] time = 0.178612, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2836, 12, 75}

$$-\frac{\cos^9(c+dx)}{9a^3d}+\frac{3\cos^8(c+dx)}{8a^3d}-\frac{2\cos^7(c+dx)}{7a^3d}-\frac{\cos^6(c+dx)}{3a^3d}+\frac{3\cos^5(c+dx)}{5a^3d}-\frac{\cos^4(c+dx)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^9/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -Cos[c + d*x]^4/(4*a^3*d) + (3*Cos[c + d*x]^5)/(5*a^3*d) - Cos[c + d*x]^6/(
3*a^3*d) - (2*Cos[c + d*x]^7)/(7*a^3*d) + (3*Cos[c + d*x]^8)/(8*a^3*d) - Co
s[c + d*x]^9/(9*a^3*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 75

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && EqQ[b*e + a*f, 0] &&  !(ILtQ[n + p
+ 2, 0] && GtQ[n + 2*p, 0])
```

Rubi steps

$$\int \frac{\sin^9(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^9(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^4x^3(-a+x)}{a^3}\,dx, x, -a\cos(c+dx)\right)}{a^9d}$$
$$= \frac{\text{Subst}\left(\int (-a-x)^4x^3(-a+x)\,dx, x, -a\cos(c+dx)\right)}{a^{12}d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^5x^3-3a^4x^4-2a^3x^5+2a^2x^6+3ax^7+x^8\right)dx, x, -a\cos(c+dx)\right)}{a^{12}d}$$
$$= -\frac{\cos^4(c+dx)}{4a^3d}+\frac{3\cos^5(c+dx)}{5a^3d}-\frac{\cos^6(c+dx)}{3a^3d}-\frac{2\cos^7(c+dx)}{7a^3d}+\frac{3\cos^8(c+dx)}{8a^3d}-\frac{\cos^9(c}{9a^3}$$

Mathematica [A] time = 2.86371, size = 100, normalized size = 0.92

$$-\frac{-52920\cos(c+dx)+37800\cos(2(c+dx))-18480\cos(3(c+dx))+3780\cos(4(c+dx))+3024\cos(5(c+dx))-4200}{322560a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^9/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(34771 - 52920*Cos[c + d*x] + 37800*Cos[2*(c + d*x)] - 18480*Cos[3*(c + d*
x)] + 3780*Cos[4*(c + d*x)] + 3024*Cos[5*(c + d*x)] - 4200*Cos[6*(c + d*x)]
 + 2700*Cos[7*(c + d*x)] - 945*Cos[8*(c + d*x)] + 140*Cos[9*(c + d*x)])/(32
2560*a^3*d)
```

Maple [A] time = 0.104, size = 69, normalized size = 0.6

$$\frac{1}{da^3}\left(-\frac{1}{4\,(\sec(dx+c))^4}+\frac{3}{8\,(\sec(dx+c))^8}-\frac{1}{3\,(\sec(dx+c))^6}+\frac{3}{5\,(\sec(dx+c))^5}-\frac{2}{7\,(\sec(dx+c))^7}-\frac{1}{9\,(\sec(dx+c))}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^9/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/d/a^3*(-1/4/sec(d*x+c)^4+3/8/sec(d*x+c)^8-1/3/sec(d*x+c)^6+3/5/sec(d*x+c)
^5-2/7/sec(d*x+c)^7-1/9/sec(d*x+c)^9)
```

Maxima [A] time = 0.985532, size = 93, normalized size = 0.85

$$-\frac{280\cos(dx+c)^9-945\cos(dx+c)^8+720\cos(dx+c)^7+840\cos(dx+c)^6-1512\cos(dx+c)^5+630\cos(dx+c)^4}{2520\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2520*(280*cos(d*x + c)^9 - 945*cos(d*x + c)^8 + 720*cos(d*x + c)^7 + 840
*cos(d*x + c)^6 - 1512*cos(d*x + c)^5 + 630*cos(d*x + c)^4)/(a^3*d)
```

Fricas [A] time = 1.75617, size = 194, normalized size = 1.78

$$-\frac{280\,\cos\left(dx+c\right)^{9}-945\,\cos\left(dx+c\right)^{8}+720\,\cos\left(dx+c\right)^{7}+840\,\cos\left(dx+c\right)^{6}-1512\,\cos\left(dx+c\right)^{5}+630\,\cos\left(dx+\right.}{2520\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2520*(280*cos(d*x + c)^9 - 945*cos(d*x + c)^8 + 720*cos(d*x + c)^7 + 840
*cos(d*x + c)^6 - 1512*cos(d*x + c)^5 + 630*cos(d*x + c)^4)/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**9/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34183, size = 250, normalized size = 2.29

$$\frac{32\left(\frac{36\,(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{144\,(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{336\,(\cos(dx+c)-1)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{504\,(\cos(dx+c)-1)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{630\,(\cos(dx+c)-1)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{105\,(\cos(dx+c)-1)^{6}}{(\cos(dx+c)+1)^{6}}+\frac{315\,(\cos(}{(\cos(dx}\right.}{315\,a^{3}d\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 32/315*(36*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 144*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2 + 336*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 504
*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 630*(cos(d*x + c) - 1)^5/(cos(
d*x + c) + 1)^5 - 105*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 315*(cos(
d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 4)/(a^3*d*((cos(d*x + c) - 1)/(cos(d
*x + c) + 1) - 1)^9)
```

3.93 $\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^3} dx$

Optimal. Leaf size=73

$$\frac{\cos^7(c+dx)}{7a^3d} - \frac{\cos^6(c+dx)}{2a^3d} + \frac{3\cos^5(c+dx)}{5a^3d} - \frac{\cos^4(c+dx)}{4a^3d}$$

```
[Out] -Cos[c + d*x]^4/(4*a^3*d) + (3*Cos[c + d*x]^5)/(5*a^3*d) - Cos[c + d*x]^6/(
2*a^3*d) + Cos[c + d*x]^7/(7*a^3*d)
```

Rubi [A] time = 0.164819, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2836, 12, 43}

$$\frac{\cos^7(c+dx)}{7a^3d} - \frac{\cos^6(c+dx)}{2a^3d} + \frac{3\cos^5(c+dx)}{5a^3d} - \frac{\cos^4(c+dx)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -Cos[c + d*x]^4/(4*a^3*d) + (3*Cos[c + d*x]^5)/(5*a^3*d) - Cos[c + d*x]^6/(
2*a^3*d) + Cos[c + d*x]^7/(7*a^3*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^7(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^3x^3}{a^3}\,dx, x, -a\cos(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int (-a-x)^3x^3\,dx, x, -a\cos(c+dx)\right)}{a^{10}d}$$
$$= \frac{\text{Subst}\left(\int \left(-a^3x^3-3a^2x^4-3ax^5-x^6\right)dx, x, -a\cos(c+dx)\right)}{a^{10}d}$$
$$= -\frac{\cos^4(c+dx)}{4a^3d} + \frac{3\cos^5(c+dx)}{5a^3d} - \frac{\cos^6(c+dx)}{2a^3d} + \frac{\cos^7(c+dx)}{7a^3d}$$

Mathematica [A] time = 1.66604, size = 80, normalized size = 1.1

$$\frac{4060\cos(c+dx) - 3220\cos(2(c+dx)) + 2100\cos(3(c+dx)) - 1120\cos(4(c+dx)) + 476\cos(5(c+dx)) - 140\cos(6}{8960a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-2421 + 4060*Cos[c + d*x] - 3220*Cos[2*(c + d*x)] + 2100*Cos[3*(c + d*x)]
- 1120*Cos[4*(c + d*x)] + 476*Cos[5*(c + d*x)] - 140*Cos[6*(c + d*x)] + 20*
Cos[7*(c + d*x)])/(8960*a^3*d)
```

Maple [A] time = 0.087, size = 50, normalized size = 0.7

$$-\frac{1}{da^3}\left(\frac{1}{4\,(\sec(dx+c))^4} + \frac{1}{2\,(\sec(dx+c))^6} - \frac{3}{5\,(\sec(dx+c))^5} - \frac{1}{7\,(\sec(dx+c))^7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/d/a^3*(1/4/sec(d*x+c)^4+1/2/sec(d*x+c)^6-3/5/sec(d*x+c)^5-1/7/sec(d*x+c)
^7)
```

Maxima [A] time = 0.985246, size = 66, normalized size = 0.9

$$\frac{20\cos(dx+c)^7 - 70\cos(dx+c)^6 + 84\cos(dx+c)^5 - 35\cos(dx+c)^4}{140\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/140*(20*cos(d*x + c)^7 - 70*cos(d*x + c)^6 + 84*cos(d*x + c)^5 - 35*cos(d
*x + c)^4)/(a^3*d)
```

Fricas [A] time = 1.7563, size = 128, normalized size = 1.75

$$\frac{20 \cos(dx+c)^7 - 70 \cos(dx+c)^6 + 84 \cos(dx+c)^5 - 35 \cos(dx+c)^4}{140 a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/140*(20*cos(d*x + c)^7 - 70*cos(d*x + c)^6 + 84*cos(d*x + c)^5 - 35*cos(d
*x + c)^4)/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.38915, size = 220, normalized size = 3.01

$$\frac{4 \left(\frac{91 (\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{273 (\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{455 (\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{490 (\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} + \frac{210 (\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5} - \frac{140 (\cos(dx+c)-1)^6}{(\cos(dx+c)+1)^6} - 13\right)}{35 a^3 d \left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 4/35*(91*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 273*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 + 455*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 490*(
cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 210*(cos(d*x + c) - 1)^5/(cos(d*
x + c) + 1)^5 - 140*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 13)/(a^3*d*
((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^7)
```

3.94 $\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=102

$$-\frac{\cos^5(c+dx)}{5a^3d}+\frac{3\cos^4(c+dx)}{4a^3d}-\frac{4\cos^3(c+dx)}{3a^3d}+\frac{2\cos^2(c+dx)}{a^3d}-\frac{4\cos(c+dx)}{a^3d}+\frac{4\log(\cos(c+dx)+1)}{a^3d}$$

```
[Out] (-4*Cos[c + d*x])/(a^3*d) + (2*Cos[c + d*x]^2)/(a^3*d) - (4*Cos[c + d*x]^3)
/(3*a^3*d) + (3*Cos[c + d*x]^4)/(4*a^3*d) - Cos[c + d*x]^5/(5*a^3*d) + (4*L
og[1 + Cos[c + d*x]])/(a^3*d)
```

Rubi [A] time = 0.181705, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2836, 12, 88}

$$-\frac{\cos^5(c+dx)}{5a^3d}+\frac{3\cos^4(c+dx)}{4a^3d}-\frac{4\cos^3(c+dx)}{3a^3d}+\frac{2\cos^2(c+dx)}{a^3d}-\frac{4\cos(c+dx)}{a^3d}+\frac{4\log(\cos(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-4*Cos[c + d*x])/(a^3*d) + (2*Cos[c + d*x]^2)/(a^3*d) - (4*Cos[c + d*x]^3)
/(3*a^3*d) + (3*Cos[c + d*x]^4)/(4*a^3*d) - Cos[c + d*x]^5/(5*a^3*d) + (4*L
og[1 + Cos[c + d*x]])/(a^3*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^5(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^2x^3}{a^3(-a+x)}\,dx, x, -a\cos(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)^2x^3}{-a+x}\,dx, x, -a\cos(c+dx)\right)}{a^8d}$$
$$= \frac{\text{Subst}\left(\int \left(4a^4 - \frac{4a^5}{a-x} + 4a^3x + 4a^2x^2 + 3ax^3 + x^4\right)dx, x, -a\cos(c+dx)\right)}{a^8d}$$
$$= -\frac{4\cos(c+dx)}{a^3d} + \frac{2\cos^2(c+dx)}{a^3d} - \frac{4\cos^3(c+dx)}{3a^3d} + \frac{3\cos^4(c+dx)}{4a^3d} - \frac{\cos^5(c+dx)}{5a^3d} + \frac{4\log(\cdots}{}$$

Mathematica [A] time = 1.01721, size = 73, normalized size = 0.72

$$\frac{-4920\cos(c+dx) + 1320\cos(2(c+dx)) - 380\cos(3(c+dx)) + 90\cos(4(c+dx)) - 12\cos(5(c+dx)) + 7680\log\left(\cos\left(\frac{1}{2}\cdots\right.\right.}{960a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3857 - 4920*Cos[c + d*x] + 1320*Cos[2*(c + d*x)] - 380*Cos[3*(c + d*x)] +
90*Cos[4*(c + d*x)] - 12*Cos[5*(c + d*x)] + 7680*Log[Cos[(c + d*x)/2]])/(96
0*a^3*d)
```

Maple [A] time = 0.108, size = 114, normalized size = 1.1

$$4\,\frac{\ln(1+\sec(dx+c))}{da^3} - \frac{1}{5\,da^3\,(\sec(dx+c))^5} + \frac{3}{4\,da^3\,(\sec(dx+c))^4} - \frac{4}{3\,da^3\,(\sec(dx+c))^3} + 2\,\frac{1}{da^3\,(\sec(dx+c))^2} - 4\,\frac{\cdots}{d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 4/d/a^3*ln(1+sec(d*x+c))-1/5/d/a^3/sec(d*x+c)^5+3/4/d/a^3/sec(d*x+c)^4-4/3/
d/a^3/sec(d*x+c)^3+2/d/a^3/sec(d*x+c)^2-4/d/a^3/sec(d*x+c)-4/d/a^3*ln(sec(d
*x+c))
```

Maxima [A] time = 1.01158, size = 99, normalized size = 0.97

$$-\frac{\frac{12\cos(dx+c)^5 - 45\cos(dx+c)^4 + 80\cos(dx+c)^3 - 120\cos(dx+c)^2 + 240\cos(dx+c)}{a^3} - \frac{240\log(\cos(dx+c)+1)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*((12*cos(d*x + c)^5 - 45*cos(d*x + c)^4 + 80*cos(d*x + c)^3 - 120*cos
(d*x + c)^2 + 240*cos(d*x + c))/a^3 - 240*log(cos(d*x + c) + 1)/a^3)/d
```

Fricas [A] time = 1.72607, size = 201, normalized size = 1.97

$$-\frac{12\,\cos\left(dx+c\right)^5-45\,\cos\left(dx+c\right)^4+80\,\cos\left(dx+c\right)^3-120\,\cos\left(dx+c\right)^2+240\,\cos\left(dx+c\right)-240\,\log\left(\frac{1}{2}\,\cos\left(dx+\right.\right.}{60\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(12*cos(d*x + c)^5 - 45*cos(d*x + c)^4 + 80*cos(d*x + c)^3 - 120*cos(
d*x + c)^2 + 240*cos(d*x + c) - 240*log(1/2*cos(d*x + c) + 1/2))/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38199, size = 232, normalized size = 2.27

$$-\frac{\frac{60\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)}{a^3}+\frac{\frac{85\,(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{20\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{200\,(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}+\frac{205\,(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4}-\frac{137\,(\cos(dx+c)-1)^5}{(\cos(dx+c)+1)^5}-29}{a^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/15*(60*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^3 + (85*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) - 20*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 - 200*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 205*(cos(d*x + c)
 - 1)^4/(cos(d*x + c) + 1)^4 - 137*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^
5 - 29)/(a^3*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5))/d
```

3.95 $\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=89

$$\frac{\cos^3(c+dx)}{3a^3d}-\frac{3\cos^2(c+dx)}{2a^3d}+\frac{5\cos(c+dx)}{a^3d}-\frac{2}{d\left(a^3\cos(c+dx)+a^3\right)}-\frac{7\log(\cos(c+dx)+1)}{a^3d}$$

```
[Out] (5*Cos[c + d*x])/(a^3*d) - (3*Cos[c + d*x]^2)/(2*a^3*d) + Cos[c + d*x]^3/(3
*a^3*d) - 2/(d*(a^3 + a^3*Cos[c + d*x])) - (7*Log[1 + Cos[c + d*x]])/(a^3*d
)
```

Rubi [A] time = 0.183736, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2836, 12, 77}

$$\frac{\cos^3(c+dx)}{3a^3d}-\frac{3\cos^2(c+dx)}{2a^3d}+\frac{5\cos(c+dx)}{a^3d}-\frac{2}{d\left(a^3\cos(c+dx)+a^3\right)}-\frac{7\log(\cos(c+dx)+1)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (5*Cos[c + d*x])/(a^3*d) - (3*Cos[c + d*x]^2)/(2*a^3*d) + Cos[c + d*x]^3/(3
*a^3*d) - 2/(d*(a^3 + a^3*Cos[c + d*x])) - (7*Log[1 + Cos[c + d*x]])/(a^3*d
)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^3(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)x^3}{a^3(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-a-x)x^3}{(-a+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^6d}$$
$$= \frac{\text{Subst}\left(\int \left(-5a^2 - \frac{2a^4}{(a-x)^2} + \frac{7a^3}{a-x} - 3ax - x^2\right)dx, x, -a\cos(c+dx)\right)}{a^6d}$$
$$= \frac{5\cos(c+dx)}{a^3d} - \frac{3\cos^2(c+dx)}{2a^3d} + \frac{\cos^3(c+dx)}{3a^3d} - \frac{2}{d\left(a^3+a^3\cos(c+dx)\right)} - \frac{7\log(1+\cos}{a^3d}$$

Mathematica [A] time = 0.411911, size = 99, normalized size = 1.11

$$-\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\left(-184\cos(2(c+dx)) + 28\cos(3(c+dx)) - 4\cos(4(c+dx)) + 1344\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + \cos(c+\right.}{24a^3d(\cos(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(Cos[(c + d*x)/2]^4*(389 - 184*Cos[2*(c + d*x)] + 28*Cos[3*(c + d*x)] - 4*
Cos[4*(c + d*x)] + 1344*Log[Cos[(c + d*x)/2]] + Cos[c + d*x]*(-19 + 1344*Lo
g[Cos[(c + d*x)/2]])))/(24*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.097, size = 100, normalized size = 1.1

$$2\frac{1}{da^3(1+\sec(dx+c))} - 7\frac{\ln(1+\sec(dx+c))}{da^3} + \frac{1}{3da^3(\sec(dx+c))^3} - \frac{3}{2da^3(\sec(dx+c))^2} + 5\frac{1}{da^3\sec(dx+c)} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 2/d/a^3/(1+sec(d*x+c))-7/d/a^3*ln(1+sec(d*x+c))+1/3/d/a^3/sec(d*x+c)^3-3/2/
d/a^3/sec(d*x+c)^2+5/d/a^3/sec(d*x+c)+7/d/a^3*ln(sec(d*x+c))
```

Maxima [A] time = 0.976867, size = 97, normalized size = 1.09

$$-\frac{\frac{12}{a^3\cos(dx+c)+a^3} - \frac{2\cos(dx+c)^3 - 9\cos(dx+c)^2 + 30\cos(dx+c)}{a^3} + \frac{42\log(\cos(dx+c)+1)}{a^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/6*(12/(a^3*cos(d*x + c) + a^3) - (2*cos(d*x + c)^3 - 9*cos(d*x + c)^2 +
30*cos(d*x + c))/a^3 + 42*log(cos(d*x + c) + 1)/a^3)/d
```

Fricas [A] time = 1.7707, size = 228, normalized size = 2.56

$$\frac{4\cos(dx+c)^4 - 14\cos(dx+c)^3 + 42\cos(dx+c)^2 - 84(\cos(dx+c)+1)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) + 69\cos(dx+c) -}{12\left(a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*cos(d*x + c)^4 - 14*cos(d*x + c)^3 + 42*cos(d*x + c)^2 - 84*(cos(d*
x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 69*cos(d*x + c) - 15)/(a^3*d*cos(
d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29883, size = 127, normalized size = 1.43

$$-\frac{7\log(|-\cos(dx+c)-1|)}{a^3d} - \frac{2}{a^3d(\cos(dx+c)+1)} + \frac{2a^6d^5\cos(dx+c)^3 - 9a^6d^5\cos(dx+c)^2 + 30a^6d^5\cos(dx+c)}{6a^9d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -7*log(abs(-cos(d*x + c) - 1))/(a^3*d) - 2/(a^3*d*(cos(d*x + c) + 1)) + 1/6
*(2*a^6*d^5*cos(d*x + c)^3 - 9*a^6*d^5*cos(d*x + c)^2 + 30*a^6*d^5*cos(d*x
+ c))/(a^9*d^6)
```

3.96 $\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=75

$$-\frac{\cos(c+dx)}{a^3d}+\frac{3}{d\left(a^3\cos(c+dx)+a^3\right)}+\frac{3\log(\cos(c+dx)+1)}{a^3d}-\frac{1}{2ad(a\cos(c+dx)+a)^2}$$

```
[Out] -(Cos[c + d*x]/(a^3*d)) - 1/(2*a*d*(a + a*Cos[c + d*x])^2) + 3/(d*(a^3 + a^
3*Cos[c + d*x])) + (3*Log[1 + Cos[c + d*x]])/(a^3*d)
```

Rubi [A] time = 0.116747, antiderivative size = 75, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3872, 2833, 12, 43}

$$-\frac{\cos(c+dx)}{a^3d}+\frac{3}{d\left(a^3\cos(c+dx)+a^3\right)}+\frac{3\log(\cos(c+dx)+1)}{a^3d}-\frac{1}{2ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(Cos[c + d*x]/(a^3*d)) - 1/(2*a*d*(a + a*Cos[c + d*x])^2) + 3/(d*(a^3 + a^
3*Cos[c + d*x])) + (3*Log[1 + Cos[c + d*x]])/(a^3*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\sin(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{a^3(-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{(-a+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^4d}$$
$$= \frac{\text{Subst}\left(\int \left(1-\frac{a^3}{(a-x)^3}+\frac{3a^2}{(a-x)^2}-\frac{3a}{a-x}\right)dx, x, -a\cos(c+dx)\right)}{a^4d}$$
$$= -\frac{\cos(c+dx)}{a^3d}-\frac{1}{2ad(a+a\cos(c+dx))^2}+\frac{3}{d\left(a^3+a^3\cos(c+dx)\right)}+\frac{3\log(1+\cos(c+dx))}{a^3d}$$

Mathematica [A] time = 0.322926, size = 103, normalized size = 1.37

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-2\cos(3(c+dx))+72\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+\cos(2(c+dx))\left(24\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-5\right)+\cos(c+\right.}{4a^3d(\cos(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(21 - 2*Cos[3*(c + d*x)] + 72*Log[Cos[(c + d*x)/2]] + C
os[2*(c + d*x)]*(-5 + 24*Log[Cos[(c + d*x)/2]]) + Cos[c + d*x]*(22 + 96*Log
[Cos[(c + d*x)/2]])))/(4*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A] time = 0.029, size = 86, normalized size = 1.2

$$-\frac{1}{2\,da^3\,(1+\sec(dx+c))^2}-2\,\frac{1}{da^3\,(1+\sec(dx+c))}+3\,\frac{\ln(1+\sec(dx+c))}{da^3}-\frac{1}{da^3\sec(dx+c)}-3\,\frac{\ln(\sec(dx+c))}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/2/d/a^3/(1+sec(d*x+c))^2-2/d/a^3/(1+sec(d*x+c))+3/d/a^3*ln(1+sec(d*x+c))
-1/d/a^3/sec(d*x+c)-3/d/a^3*ln(sec(d*x+c))
```

Maxima [A] time = 1.0238, size = 96, normalized size = 1.28

$$\frac{\frac{6\,\cos(dx+c)+5}{a^3\cos(dx+c)^2+2\,a^3\cos(dx+c)+a^3}-\frac{2\,\cos(dx+c)}{a^3}+\frac{6\,\log(\cos(dx+c)+1)}{a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((6*cos(d*x + c) + 5)/(a^3*cos(d*x + c)^2 + 2*a^3*cos(d*x + c) + a^3) -
 2*cos(d*x + c)/a^3 + 6*log(cos(d*x + c) + 1)/a^3)/d
```

Fricas [A] time = 1.73379, size = 255, normalized size = 3.4

$$-\frac{2 \cos (dx + c)^3 + 4 \cos (dx + c)^2 - 6 \left(\cos (dx + c)^2 + 2 \cos (dx + c) + 1\right) \log \left(\frac{1}{2} \cos (dx + c) + \frac{1}{2}\right) - 4 \cos (dx + c) -}{2 \left(a^3 d \cos (dx + c)^2 + 2 a^3 d \cos (dx + c) + a^3 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*cos(d*x + c)^3 + 4*cos(d*x + c)^2 - 6*(cos(d*x + c)^2 + 2*cos(d*x +
 c) + 1)*log(1/2*cos(d*x + c) + 1/2) - 4*cos(d*x + c) - 5)/(a^3*d*cos(d*x +
 c)^2 + 2*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sin (c+dx)}{\sec^3 (c+dx)+3 \sec^2 (c+dx)+3 \sec (c+dx)+1}\, dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x)
 + 1), x)/a**3
```

Giac [A] time = 1.34489, size = 85, normalized size = 1.13

$$-\frac{\cos (dx + c)}{a^3 d} + \frac{3 \log (|-\cos (dx + c) - 1|)}{a^3 d} + \frac{6 \cos (dx + c) + 5}{2 a^3 d(\cos (dx + c) + 1)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -cos(d*x + c)/(a^3*d) + 3*log(abs(-cos(d*x + c) - 1))/(a^3*d) + 1/2*(6*cos(
d*x + c) + 5)/(a^3*d*(cos(d*x + c) + 1)^2)
```

3.97 $\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=82

$$-\frac{7}{8d\left(a^3\cos(c+dx)+a^3\right)}-\frac{\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{5}{8ad(a\cos(c+dx)+a)^2}-\frac{1}{6d(a\cos(c+dx)+a)^3}$$

```
[Out] -ArcTanh[Cos[c + d*x]]/(8*a^3*d) - 1/(6*d*(a + a*Cos[c + d*x])^3) + 5/(8*a*
d*(a + a*Cos[c + d*x])^2) - 7/(8*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.151134, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3872, 2836, 12, 88, 206}

$$-\frac{7}{8d\left(a^3\cos(c+dx)+a^3\right)}-\frac{\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{5}{8ad(a\cos(c+dx)+a)^2}-\frac{1}{6d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -ArcTanh[Cos[c + d*x]]/(8*a^3*d) - 1/(6*d*(a + a*Cos[c + d*x])^3) + 5/(8*a*
d*(a + a*Cos[c + d*x])^2) - 7/(8*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cos^2(c+dx)\cot(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$
$$= \frac{a\,\text{Subst}\left(\int \frac{x^3}{a^3(-a-x)(-a+x)^4}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{(-a-x)(-a+x)^4}\,dx, x, -a\cos(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a^2}{2(a-x)^4}+\frac{5a}{4(a-x)^3}-\frac{7}{8(a-x)^2}+\frac{1}{8(a^2-x^2)}\right)dx, x, -a\cos(c+dx)\right)}{a^2d}$$
$$= -\frac{1}{6d(a+a\cos(c+dx))^3}+\frac{5}{8ad(a+a\cos(c+dx))^2}-\frac{7}{8d\left(a^3+a^3\cos(c+dx)\right)}+\frac{\text{Subst}\left(\right.}{}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{1}{6d(a+a\cos(c+dx))^3}+\frac{5}{8ad(a+a\cos(c+dx))^2}-\frac{}{8d\left(a^3+a^3\right.}$$

Mathematica [A] time = 0.325845, size = 97, normalized size = 1.18

$$-\frac{\sec^3(c+dx)\left(42\cos^4\left(\frac{1}{2}(c+dx)\right)-15\cos^2\left(\frac{1}{2}(c+dx)\right)+12\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\sin\left(\frac{1}{2}(c\right.\right.\right.\right.}{12a^3d(\sec(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -((2 - 15*Cos[(c + d*x)/2]^2 + 42*Cos[(c + d*x)/2]^4 + 12*Cos[(c + d*x)/2]^
6*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]]))*Sec[c + d*x]^3)/(12*a^3*
d*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 0.066, size = 90, normalized size = 1.1

$$-\frac{1}{6\,da^3\left(\cos(dx+c)+1\right)^3}+\frac{5}{8\,da^3\left(\cos(dx+c)+1\right)^2}-\frac{7}{8\,da^3\left(\cos(dx+c)+1\right)}-\frac{\ln(\cos(dx+c)+1)}{16\,da^3}+\frac{\ln(-1+\cos}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/6/d/a^3/(cos(d*x+c)+1)^3+5/8/d/a^3/(cos(d*x+c)+1)^2-7/8/d/a^3/(cos(d*x+c
)+1)-1/16*ln(cos(d*x+c)+1)/a^3/d+1/16/d/a^3*ln(-1+cos(d*x+c))
```

Maxima [A] time = 1.00691, size = 132, normalized size = 1.61

$$-\frac{\frac{2\left(21\cos(dx+c)^2+27\cos(dx+c)+10\right)}{a^3\cos(dx+c)^3+3a^3\cos(dx+c)^2+3a^3\cos(dx+c)+a^3}+\frac{3\log(\cos(dx+c)+1)}{a^3}-\frac{3\log(\cos(dx+c)-1)}{a^3}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/48*(2*(21*cos(d*x + c)^2 + 27*cos(d*x + c) + 10)/(a^3*cos(d*x + c)^3 + 3
*a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3) + 3*log(cos(d*x + c) + 1)/a
^3 - 3*log(cos(d*x + c) - 1)/a^3)/d
```

Fricas [B] time = 1.75499, size = 416, normalized size = 5.07

$$-\frac{42\cos(dx+c)^2+3\left(\cos(dx+c)^3+3\cos(dx+c)^2+3\cos(dx+c)+1\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)-3\left(\cos(dx+c)^3+\right.}{48\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\,c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/48*(42*cos(d*x + c)^2 + 3*(cos(d*x + c)^3 + 3*cos(d*x + c)^2 + 3*cos(d*x
 + c) + 1)*log(1/2*cos(d*x + c) + 1/2) - 3*(cos(d*x + c)^3 + 3*cos(d*x + c)
^2 + 3*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 54*cos(d*x + c) + 2
0)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) +
a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x)
 + 1), x)/a**3
```

Giac [A] time = 1.34449, size = 153, normalized size = 1.87

$$\frac{\frac{6\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3}+\frac{\frac{18a^6(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9a^6(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{2a^6(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}}{a^9}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/96*(6*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^3 + (18*a^6*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^6*(cos(d*x + c) - 1)^2/(cos(d*x +
c) + 1)^2 + 2*a^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/a^9)/d
```

3.98 $\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=126

$$-\frac{1}{32d\left(a^3-a^3\cos(c+dx)\right)}-\frac{1}{16d\left(a^3\cos(c+dx)+a^3\right)}+\frac{\tanh^{-1}(\cos(c+dx))}{32a^3d}-\frac{a}{16d(a\cos(c+dx)+a)^4}+\frac{1}{6d(a\cos(}$$

```
[Out] ArcTanh[Cos[c + d*x]]/(32*a^3*d) - a/(16*d*(a + a*Cos[c + d*x])^4) + 1/(6*d
*(a + a*Cos[c + d*x])^3) - 3/(32*a*d*(a + a*Cos[c + d*x])^2) - 1/(32*d*(a^3
 - a^3*Cos[c + d*x])) - 1/(16*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.134334, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2707, 88, 206}

$$-\frac{1}{32d\left(a^3-a^3\cos(c+dx)\right)}-\frac{1}{16d\left(a^3\cos(c+dx)+a^3\right)}+\frac{\tanh^{-1}(\cos(c+dx))}{32a^3d}-\frac{a}{16d(a\cos(c+dx)+a)^4}+\frac{1}{6d(a\cos(}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ArcTanh[Cos[c + d*x]]/(32*a^3*d) - a/(16*d*(a + a*Cos[c + d*x])^4) + 1/(6*d
*(a + a*Cos[c + d*x])^3) - 3/(32*a*d*(a + a*Cos[c + d*x])^2) - 1/(32*d*(a^3
 - a^3*Cos[c + d*x])) - 1/(16*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2707

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^(m - (p + 1)/2))/(a - x)
^((p + 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && Eq
Q[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cot^3(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{(-a-x)^2(-a+x)^5}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a}{4(a-x)^5} + \frac{1}{2(a-x)^4} - \frac{3}{16a(a-x)^3} - \frac{1}{16a^2(a-x)^2} + \frac{1}{32a^2(a+x)^2} - \frac{1}{32a^2(a^2-x^2)}\right) dx, x, -a\cos(c\right.}{d}$$
$$= -\frac{a}{16d(a+a\cos(c+dx))^4} + \frac{1}{6d(a+a\cos(c+dx))^3} - \frac{3}{32ad(a+a\cos(c+dx))^2} - \frac{}{32d\left(a^3-\right.}$$
$$= \frac{\tanh^{-1}(\cos(c+dx))}{32a^3d} - \frac{a}{16d(a+a\cos(c+dx))^4} + \frac{1}{6d(a+a\cos(c+dx))^3} - \frac{3}{32ad(a+a\cos(c}$$

Mathematica [A] time = 0.576518, size = 138, normalized size = 1.1

$$-\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(12\csc^2\left(\frac{1}{2}(c+dx)\right)+3\sec^8\left(\frac{1}{2}(c+dx)\right)-16\sec^6\left(\frac{1}{2}(c+dx)\right)+18\sec^4\left(\frac{1}{2}(c+dx)\right)+24\right.}{96a^3d(\sec(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(Cos[(c + d*x)/2]^6*(12*Csc[(c + d*x)/2]^2 + 24*(-Log[Cos[(c + d*x)/2]] +
Log[Sin[(c + d*x)/2]]) + 24*Sec[(c + d*x)/2]^2 + 18*Sec[(c + d*x)/2]^4 - 16
*Sec[(c + d*x)/2]^6 + 3*Sec[(c + d*x)/2]^8)*Sec[c + d*x]^3)/(96*a^3*d*(1 +
Sec[c + d*x])^3)
```

Maple [A] time = 0.079, size = 126, normalized size = 1.

$$-\frac{1}{16\,da^3\left(\cos(dx+c)+1\right)^4}+\frac{1}{6\,da^3\left(\cos(dx+c)+1\right)^3}-\frac{3}{32\,da^3\left(\cos(dx+c)+1\right)^2}-\frac{1}{16\,da^3\left(\cos(dx+c)+1\right)}+\frac{\ln(\cos(}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/16/d/a^3/(cos(d*x+c)+1)^4+1/6/d/a^3/(cos(d*x+c)+1)^3-3/32/d/a^3/(cos(d*x
+c)+1)^2-1/16/d/a^3/(cos(d*x+c)+1)+1/64*ln(cos(d*x+c)+1)/a^3/d+1/32/d/a^3/(
-1+cos(d*x+c))-1/64/d/a^3*ln(-1+cos(d*x+c))
```

Maxima [A] time = 1.00209, size = 197, normalized size = 1.56

$$-\frac{\frac{2\left(3\cos(dx+c)^4+9\cos(dx+c)^3-25\cos(dx+c)^2-27\cos(dx+c)-8\right)}{a^3\cos(dx+c)^5+3a^3\cos(dx+c)^4+2a^3\cos(dx+c)^3-2a^3\cos(dx+c)^2-3a^3\cos(dx+c)-a^3}-\frac{3\log(\cos(dx+c)+1)}{a^3}+\frac{3\log(\cos(dx+c)-1)}{a^3}}{192\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/192*(2*(3*cos(d*x + c)^4 + 9*cos(d*x + c)^3 - 25*cos(d*x + c)^2 - 27*cos
(d*x + c) - 8)/(a^3*cos(d*x + c)^5 + 3*a^3*cos(d*x + c)^4 + 2*a^3*cos(d*x +
 c)^3 - 2*a^3*cos(d*x + c)^2 - 3*a^3*cos(d*x + c) - a^3) - 3*log(cos(d*x +
c) + 1)/a^3 + 3*log(cos(d*x + c) - 1)/a^3)/d
```

Fricas [B] time = 1.77896, size = 640, normalized size = 5.08

$$-\frac{6\,\cos\left(dx+c\right)^{4}+18\,\cos\left(dx+c\right)^{3}-50\,\cos\left(dx+c\right)^{2}-3\left(\cos\left(dx+c\right)^{5}+3\,\cos\left(dx+c\right)^{4}+2\,\cos\left(dx+c\right)^{3}-2\,\cos\left(\right.\right.}{192\left(a^{3}d\cos\left(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/192*(6*cos(d*x + c)^4 + 18*cos(d*x + c)^3 - 50*cos(d*x + c)^2 - 3*(cos(d
*x + c)^5 + 3*cos(d*x + c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(
d*x + c) - 1)*log(1/2*cos(d*x + c) + 1/2) + 3*(cos(d*x + c)^5 + 3*cos(d*x +
 c)^4 + 2*cos(d*x + c)^3 - 2*cos(d*x + c)^2 - 3*cos(d*x + c) - 1)*log(-1/2*
cos(d*x + c) + 1/2) - 54*cos(d*x + c) - 16)/(a^3*d*cos(d*x + c)^5 + 3*a^3*d
*cos(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 - 3*a^3*d
*cos(d*x + c) - a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc^{3}\left(c+dx\right)}{\sec^{3}\left(c+dx\right)+3\sec^{2}\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**3/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A] time = 1.34542, size = 246, normalized size = 1.95

$$\frac{\frac{12\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)(\cos(dx+c)+1)}{a^{3}(\cos(dx+c)-1)}-\frac{12\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^{3}}+\frac{\frac{24a^{9}(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{12a^{9}(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}-\frac{4a^{9}(\cos(dx+c)-1)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{3a^{9}(\cos(dx+c)-1)^{4}}{(\cos(dx+c)+1)^{4}}}{a^{12}}}{768\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/768*(12*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)*(cos(d*x + c) + 1)/(a
^3*(cos(d*x + c) - 1)) - 12*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1
))/a^3 + (24*a^9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12*a^9*(cos(d*x +
c) - 1)^2/(cos(d*x + c) + 1)^2 - 4*a^9*(cos(d*x + c) - 1)^3/(cos(d*x + c) +
 1)^3 - 3*a^9*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/a^12)/d
```

3.99 $\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=128

$$-\frac{a^2}{40d(a\cos(c+dx)+a)^5}-\frac{3}{128d\left(a^3\cos(c+dx)+a^3\right)}+\frac{3\tanh^{-1}(\cos(c+dx))}{128a^3d}+\frac{3a}{64d(a\cos(c+dx)+a)^4}-\frac{}{128ad(a-}$$

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(128*a^3*d) - 1/(128*a*d*(a - a*Cos[c + d*x])^2)
- a^2/(40*d*(a + a*Cos[c + d*x])^5) + (3*a)/(64*d*(a + a*Cos[c + d*x])^4) -
 1/(64*a*d*(a + a*Cos[c + d*x])^2) - 3/(128*d*(a^3 + a^3*Cos[c + d*x]))
```

Rubi [A] time = 0.205667, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2836, 12, 88, 206}

$$-\frac{a^2}{40d(a\cos(c+dx)+a)^5}-\frac{3}{128d\left(a^3\cos(c+dx)+a^3\right)}+\frac{3\tanh^{-1}(\cos(c+dx))}{128a^3d}+\frac{3a}{64d(a\cos(c+dx)+a)^4}-\frac{}{128ad(a-}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3*ArcTanh[Cos[c + d*x]])/(128*a^3*d) - 1/(128*a*d*(a - a*Cos[c + d*x])^2)
- a^2/(40*d*(a + a*Cos[c + d*x])^5) + (3*a)/(64*d*(a + a*Cos[c + d*x])^4) -
 1/(64*a*d*(a + a*Cos[c + d*x])^2) - 3/(128*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2836

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x)^((p - 1)/2)*(c + (d*x)/b)^n,
x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, c, d, m, n}, x] && Integer
Q[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\csc^5(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cot^3(c+dx)\csc^2(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$
$$= \frac{a^5\,\text{Subst}\left(\int \frac{x^3}{a^3(-a-x)^3(-a+x)^6}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{x^3}{(-a-x)^3(-a+x)^6}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(-\frac{1}{8(a-x)^6}+\frac{3}{16a(a-x)^5}-\frac{1}{32a^3(a-x)^3}-\frac{3}{128a^4(a-x)^2}+\frac{1}{64a^3(a+x)^3}-\frac{3}{128a^4(a^2-x^2)}\right)dx, x\right.}{d}$$
$$= -\frac{1}{128ad(a-a\cos(c+dx))^2}-\frac{a^2}{40d(a+a\cos(c+dx))^5}+\frac{3a}{64d(a+a\cos(c+dx))^4}-\frac{}{64ad}$$
$$= \frac{3\tanh^{-1}(\cos(c+dx))}{128a^3d}-\frac{1}{128ad(a-a\cos(c+dx))^2}-\frac{a^2}{40d(a+a\cos(c+dx))^5}+\frac{}{64d(a+}$$

Mathematica [A] time = 5.16965, size = 137, normalized size = 1.07

$$-\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(60\cos^8\left(\frac{1}{2}(c+dx)\right)-15\cos^2\left(\frac{1}{2}(c+dx)\right)+10\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\cot^4\left(\frac{1}{2}(c+dx)\right)+2\right.\right.}{640a^3d(\sec(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -((4 - 15*Cos[(c + d*x)/2]^2 + 60*Cos[(c + d*x)/2]^8 + 10*Cos[(c + d*x)/2]^
6*(2 + Cot[(c + d*x)/2]^4) - 120*Cos[(c + d*x)/2]^10*(Log[Cos[(c + d*x)/2]]
 - Log[Sin[(c + d*x)/2]]))*Sec[(c + d*x)/2]^4*Sec[c + d*x]^3)/(640*a^3*d*(1
 + Sec[c + d*x])^3)
```

Maple [A] time = 0.081, size = 126, normalized size = 1.

$$-\frac{1}{40\,da^3\,(\cos(dx+c)+1)^5}+\frac{3}{64\,da^3\,(\cos(dx+c)+1)^4}-\frac{1}{64\,da^3\,(\cos(dx+c)+1)^2}-\frac{3}{128\,da^3\,(\cos(dx+c)+1)}+\frac{3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/40/d/a^3/(cos(d*x+c)+1)^5+3/64/d/a^3/(cos(d*x+c)+1)^4-1/64/d/a^3/(cos(d*
x+c)+1)^2-3/128/d/a^3/(cos(d*x+c)+1)+3/256*ln(cos(d*x+c)+1)/a^3/d-1/128/d/a
^3/(-1+cos(d*x+c))^2-3/256/d/a^3*ln(-1+cos(d*x+c))
```

Maxima [A] time = 1.01282, size = 254, normalized size = 1.98

$$-\frac{\frac{2\left(15\cos(dx+c)^6+45\cos(dx+c)^5+20\cos(dx+c)^4-60\cos(dx+c)^3+61\cos(dx+c)^2+63\cos(dx+c)+16\right)}{a^3\cos(dx+c)^7+3a^3\cos(dx+c)^6+a^3\cos(dx+c)^5-5a^3\cos(dx+c)^4-5a^3\cos(dx+c)^3+a^3\cos(dx+c)^2+3a^3\cos(dx+c)+a^3}-\frac{15\log(\cos(dx+c)+1)}{a^3}+\frac{15\log(}{}}{1280\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/1280*(2*(15*cos(d*x + c)^6 + 45*cos(d*x + c)^5 + 20*cos(d*x + c)^4 - 60*
cos(d*x + c)^3 + 61*cos(d*x + c)^2 + 63*cos(d*x + c) + 16)/(a^3*cos(d*x + c
)^7 + 3*a^3*cos(d*x + c)^6 + a^3*cos(d*x + c)^5 - 5*a^3*cos(d*x + c)^4 - 5*
a^3*cos(d*x + c)^3 + a^3*cos(d*x + c)^2 + 3*a^3*cos(d*x + c) + a^3) - 15*lo
g(cos(d*x + c) + 1)/a^3 + 15*log(cos(d*x + c) - 1)/a^3)/d
```

Fricas [B] time = 1.8624, size = 857, normalized size = 6.7

$$-\frac{30\cos(dx+c)^6+90\cos(dx+c)^5+40\cos(dx+c)^4-120\cos(dx+c)^3+122\cos(dx+c)^2-15\left(\cos(dx+c)^7+3\cos\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/1280*(30*cos(d*x + c)^6 + 90*cos(d*x + c)^5 + 40*cos(d*x + c)^4 - 120*co
s(d*x + c)^3 + 122*cos(d*x + c)^2 - 15*(cos(d*x + c)^7 + 3*cos(d*x + c)^6 +
 cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x + c)^2 + 3*
cos(d*x + c) + 1)*log(1/2*cos(d*x + c) + 1/2) + 15*(cos(d*x + c)^7 + 3*cos(
d*x + c)^6 + cos(d*x + c)^5 - 5*cos(d*x + c)^4 - 5*cos(d*x + c)^3 + cos(d*x
 + c)^2 + 3*cos(d*x + c) + 1)*log(-1/2*cos(d*x + c) + 1/2) + 126*cos(d*x +
c) + 32)/(a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 + a^3*d*cos(d*x + c
)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a^3*d*cos(d*x + c)^
2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40154, size = 313, normalized size = 2.45

$$\frac{\frac{10\left(\frac{2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right)(\cos(dx+c)+1)^2}{a^3(\cos(dx+c)-1)^2}-\frac{60\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3}+\frac{\frac{60a^{12}(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{30a^{12}(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{20a^{12}(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3}-\frac{5a^{12}(\cos(dx+\ldots}{(\cos(dx+c)\ldots}}{a^{15}}}{5120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/5120*(10*(2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^2/(a^3*(cos(d*x + c) - 1)^2)
```

```
 - 60*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^3 + (60*a^12*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + 30*a^12*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2 - 20*a^12*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 5*a^12*(co
s(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 4*a^12*(cos(d*x + c) - 1)^5/(cos(d
*x + c) + 1)^5)/a^15)/d
```

3.100 $\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=157

$$\frac{3\sin^7(c+dx)}{7a^3d}-\frac{7\sin^5(c+dx)}{5a^3d}+\frac{4\sin^3(c+dx)}{3a^3d}+\frac{\sin(c+dx)\cos^7(c+dx)}{8a^3d}+\frac{23\sin(c+dx)\cos^5(c+dx)}{48a^3d}-\frac{29\sin(c+d}{19}$$

```
[Out] (-29*x)/(128*a^3) - (29*Cos[c + d*x]*Sin[c + d*x])/(128*a^3*d) - (29*Cos[c
+ d*x]^3*Sin[c + d*x])/(192*a^3*d) + (23*Cos[c + d*x]^5*Sin[c + d*x])/(48*a
^3*d) + (Cos[c + d*x]^7*Sin[c + d*x])/(8*a^3*d) + (4*Sin[c + d*x]^3)/(3*a^3
*d) - (7*Sin[c + d*x]^5)/(5*a^3*d) + (3*Sin[c + d*x]^7)/(7*a^3*d)
```

Rubi [A] time = 0.460523, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3872, 2875, 2873, 2564, 14, 2568, 2635, 8, 270}

$$\frac{3\sin^7(c+dx)}{7a^3d}-\frac{7\sin^5(c+dx)}{5a^3d}+\frac{4\sin^3(c+dx)}{3a^3d}+\frac{\sin(c+dx)\cos^7(c+dx)}{8a^3d}+\frac{23\sin(c+dx)\cos^5(c+dx)}{48a^3d}-\frac{29\sin(c+d}{19}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-29*x)/(128*a^3) - (29*Cos[c + d*x]*Sin[c + d*x])/(128*a^3*d) - (29*Cos[c
+ d*x]^3*Sin[c + d*x])/(192*a^3*d) + (23*Cos[c + d*x]^5*Sin[c + d*x])/(48*a
^3*d) + (Cos[c + d*x]^7*Sin[c + d*x])/(8*a^3*d) + (4*Sin[c + d*x]^3)/(3*a^3
*d) - (7*Sin[c + d*x]^5)/(5*a^3*d) + (3*Sin[c + d*x]^7)/(7*a^3*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sin^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^8(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$

$$= -\frac{\int \cos^3(c+dx)(-a+a\cos(c+dx))^3\sin^2(c+dx)\,dx}{a^6}$$

$$= -\frac{\int \left(-a^3\cos^3(c+dx)\sin^2(c+dx)+3a^3\cos^4(c+dx)\sin^2(c+dx)-3a^3\cos^5(c+dx)\sin^2\right.}{a^6}$$

$$= \frac{\int \cos^3(c+dx)\sin^2(c+dx)\,dx}{a^3} - \frac{\int \cos^6(c+dx)\sin^2(c+dx)\,dx}{a^3} - \frac{3\int \cos^4(c+dx)\sin^2(}{a^3}$$

$$= \frac{\cos^5(c+dx)\sin(c+dx)}{2a^3d} + \frac{\cos^7(c+dx)\sin(c+dx)}{8a^3d} - \frac{\int \cos^6(c+dx)\,dx}{8a^3} - \frac{\int \cos^4(c+dx}{2a^3}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{8a^3d} + \frac{23\cos^5(c+dx)\sin(c+dx)}{48a^3d} + \frac{\cos^7(c+dx)\sin(c+dx)}{8a^3d} - \frac{5}{}$$

$$= -\frac{3\cos(c+dx)\sin(c+dx)}{16a^3d} - \frac{29\cos^3(c+dx)\sin(c+dx)}{192a^3d} + \frac{23\cos^5(c+dx)\sin(c+dx)}{48a^3d} +$$

$$= -\frac{3x}{16a^3} - \frac{29\cos(c+dx)\sin(c+dx)}{128a^3d} - \frac{29\cos^3(c+dx)\sin(c+dx)}{192a^3d} + \frac{23\cos^5(c+dx)\sin(}{48a^3d}$$

$$= -\frac{29x}{128a^3} - \frac{29\cos(c+dx)\sin(c+dx)}{128a^3d} - \frac{29\cos^3(c+dx)\sin(c+dx)}{192a^3d} + \frac{23\cos^5(c+dx)\sin}{48a^3d}$$

Mathematica [A] time = 4.53154, size = 131, normalized size = 0.83

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(38640\sin(c+dx)-6720\sin(2(c+dx))-3920\sin(3(c+dx))+5880\sin(4(c+dx))-436\right.}{13440a^3d(\sec(c+dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(-24360*d*x + 38640*Sin[c + d*x] - 6720*
Sin[2*(c + d*x)] - 3920*Sin[3*(c + d*x)] + 5880*Sin[4*(c + d*x)] - 4368*Sin
[5*(c + d*x)] + 2240*Sin[6*(c + d*x)] - 720*Sin[7*(c + d*x)] + 105*Sin[8*(c
 + d*x)] + 294*Tan[c/2]))/(13440*a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [B] time = 0.105, size = 290, normalized size = 1.9

$$\frac{29}{64\,da^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-8}+\frac{667}{192\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-8}+\frac{11107}{960\,da^3}\left(\tan\left(\frac{d}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^8/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 29/64/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)+667/192/d/a^3/(1+
tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^3+11107/960/d/a^3/(1+tan(1/2*d*x
+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^5+146537/6720/d/a^3/(1+tan(1/2*d*x+1/2*c)^2
)^8*tan(1/2*d*x+1/2*c)^7+72669/2240/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/
2*d*x+1/2*c)^9+1759/320/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)
^11+1143/64/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^13-29/64/d/
a^3/(1+tan(1/2*d*x+1/2*c)^2)^8*tan(1/2*d*x+1/2*c)^15-29/64/d/a^3*arctan(tan
(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.53776, size = 510, normalized size = 3.25

$$\frac{\dfrac{\frac{3045\sin(dx+c)}{\cos(dx+c)+1}+\frac{23345\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{77749\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{146537\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{218007\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{36939\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{120015\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}-\frac{3045\sin(dx+c)^{15}}{(\cos(dx+c)+1)^{15}}}{a^3+\frac{8a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{28a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{56a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{70a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{56a^3\sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{28a^3\sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}+\frac{8a^3\sin(dx+c)^{14}}{(\cos(dx+c)+1)^{14}}+\frac{a^3\sin(dx+c)^{16}}{(\cos(dx+c)+1)^{16}}}-\frac{3045\arctan(}{a^3}}{6720\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6720*((3045*sin(d*x + c)/(cos(d*x + c) + 1) + 23345*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 77749*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 146537*sin(d*x
 + c)^7/(cos(d*x + c) + 1)^7 + 218007*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 +
 36939*sin(d*x + c)^11/(cos(d*x + c) + 1)^11 + 120015*sin(d*x + c)^13/(cos(
d*x + c) + 1)^13 - 3045*sin(d*x + c)^15/(cos(d*x + c) + 1)^15)/(a^3 + 8*a^3
*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 28*a^3*sin(d*x + c)^4/(cos(d*x + c)
+ 1)^4 + 56*a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6 + 70*a^3*sin(d*x + c)^8
/(cos(d*x + c) + 1)^8 + 56*a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + 28*a
^3*sin(d*x + c)^12/(cos(d*x + c) + 1)^12 + 8*a^3*sin(d*x + c)^14/(cos(d*x +
 c) + 1)^14 + a^3*sin(d*x + c)^16/(cos(d*x + c) + 1)^16) - 3045*arctan(sin(
```

```
d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A] time = 1.80264, size = 274, normalized size = 1.75

$$-\frac{3045\,dx - \left(1680\,\cos\left(dx+c\right)^{7} - 5760\,\cos\left(dx+c\right)^{6} + 6440\,\cos\left(dx+c\right)^{5} - 1536\,\cos\left(dx+c\right)^{4} - 2030\,\cos\left(dx+c\right)^{3}}{13440\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/13440*(3045*d*x - (1680*cos(d*x + c)^7 - 5760*cos(d*x + c)^6 + 6440*cos(
d*x + c)^5 - 1536*cos(d*x + c)^4 - 2030*cos(d*x + c)^3 + 2432*cos(d*x + c)^
2 - 3045*cos(d*x + c) + 4864)*sin(d*x + c))/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**8/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28195, size = 188, normalized size = 1.2

$$-\frac{\frac{3045\,(dx+c)}{a^{3}} + \frac{2\left(3045\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{15} - 120015\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13} - 36939\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11} - 218007\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9} - 146537\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7} - 77749}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{8}a^{3}}}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/13440*(3045*(d*x + c)/a^3 + 2*(3045*tan(1/2*d*x + 1/2*c)^15 - 120015*tan
(1/2*d*x + 1/2*c)^13 - 36939*tan(1/2*d*x + 1/2*c)^11 - 218007*tan(1/2*d*x +
 1/2*c)^9 - 146537*tan(1/2*d*x + 1/2*c)^7 - 77749*tan(1/2*d*x + 1/2*c)^5 -
23345*tan(1/2*d*x + 1/2*c)^3 - 3045*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1
/2*c)^2 + 1)^8*a^3))/d
```

3.101 $\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^3} dx$

Optimal. Leaf size=129

$$\frac{3\sin^5(c+dx)}{5a^3d} - \frac{7\sin^3(c+dx)}{3a^3d} + \frac{4\sin(c+dx)}{a^3d} - \frac{\sin(c+dx)\cos^5(c+dx)}{6a^3d} - \frac{23\sin(c+dx)\cos^3(c+dx)}{24a^3d} - \frac{23\sin(c+dx}{16a$$

```
[Out] (-23*x)/(16*a^3) + (4*Sin[c + d*x])/(a^3*d) - (23*Cos[c + d*x]*Sin[c + d*x]
)/(16*a^3*d) - (23*Cos[c + d*x]^3*Sin[c + d*x])/(24*a^3*d) - (Cos[c + d*x]^
5*Sin[c + d*x])/(6*a^3*d) - (7*Sin[c + d*x]^3)/(3*a^3*d) + (3*Sin[c + d*x]^
5)/(5*a^3*d)
```

Rubi [A] time = 0.291331, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3872, 2869, 2757, 2633, 2635, 8}

$$\frac{3\sin^5(c+dx)}{5a^3d} - \frac{7\sin^3(c+dx)}{3a^3d} + \frac{4\sin(c+dx)}{a^3d} - \frac{\sin(c+dx)\cos^5(c+dx)}{6a^3d} - \frac{23\sin(c+dx)\cos^3(c+dx)}{24a^3d} - \frac{23\sin(c+dx}{16a$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (-23*x)/(16*a^3) + (4*Sin[c + d*x])/(a^3*d) - (23*Cos[c + d*x]*Sin[c + d*x]
)/(16*a^3*d) - (23*Cos[c + d*x]^3*Sin[c + d*x])/(24*a^3*d) - (Cos[c + d*x]^
5*Sin[c + d*x])/(6*a^3*d) - (7*Sin[c + d*x]^3)/(3*a^3*d) + (3*Sin[c + d*x]^
5)/(5*a^3*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2869

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[a^(2*m), Int[(d*S
in[e + f*x])^n/(a - b*Sin[e + f*x])^m, x], x] /; FreeQ[{a, b, d, e, f, n},
x] && EqQ[a^2 - b^2, 0] && IntegersQ[m, p] && EqQ[2*m + p, 0]
```

Rule 2757

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(a + b*sin[e + f*x])^m*(d*sin[e +
f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, n}, x] && EqQ[a^2 - b^2, 0] && IGt
Q[m, 0] && RationalQ[n]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^6(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$
$$= -\frac{\int \cos^3(c+dx)(-a+a\cos(c+dx))^3\,dx}{a^6}$$
$$= -\frac{\int \left(-a^3\cos^3(c+dx)+3a^3\cos^4(c+dx)-3a^3\cos^5(c+dx)+a^3\cos^6(c+dx)\right)dx}{a^6}$$
$$= \frac{\int \cos^3(c+dx)\,dx}{a^3} - \frac{\int \cos^6(c+dx)\,dx}{a^3} - \frac{3\int \cos^4(c+dx)\,dx}{a^3} + \frac{3\int \cos^5(c+dx)\,dx}{a^3}$$
$$= -\frac{3\cos^3(c+dx)\sin(c+dx)}{4a^3d} - \frac{\cos^5(c+dx)\sin(c+dx)}{6a^3d} - \frac{5\int \cos^4(c+dx)\,dx}{6a^3} - \frac{9\int \cos^2}{4}$$
$$= \frac{4\sin(c+dx)}{a^3d} - \frac{9\cos(c+dx)\sin(c+dx)}{8a^3d} - \frac{23\cos^3(c+dx)\sin(c+dx)}{24a^3d} - \frac{\cos^5(c+dx)\sin}{6a^3d}$$
$$= -\frac{9x}{8a^3} + \frac{4\sin(c+dx)}{a^3d} - \frac{23\cos(c+dx)\sin(c+dx)}{16a^3d} - \frac{23\cos^3(c+dx)\sin(c+dx)}{24a^3d} - \frac{\cos^5(}{}$$
$$= -\frac{23x}{16a^3} + \frac{4\sin(c+dx)}{a^3d} - \frac{23\cos(c+dx)\sin(c+dx)}{16a^3d} - \frac{23\cos^3(c+dx)\sin(c+dx)}{24a^3d} - \frac{\cos^5}{}$$

Mathematica [A] time = 1.83668, size = 111, normalized size = 0.86

$$\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\left(5040\sin(c+dx)-1890\sin(2(c+dx))+760\sin(3(c+dx))-270\sin(4(c+dx))+72\,\mathrm{s}\right.}{240a^3d(\sec(c+dx)+1)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]^6*Sec[c + d*x]^3*(-2760*d*x + 5040*Sin[c + d*x] - 1890*Si
n[2*(c + d*x)] + 760*Sin[3*(c + d*x)] - 270*Sin[4*(c + d*x)] + 72*Sin[5*(c
+ d*x)] - 10*Sin[6*(c + d*x)] + 9*Tan[c/2]))/(240*a^3*d*(1 + Sec[c + d*x])^
3)
```

Maple [A] time = 0.113, size = 222, normalized size = 1.7

$$\frac{105}{8\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{11}\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-6}+\frac{211}{8\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{9}\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-6}+\frac{969}{20\,da^3}\left(\tan\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 105/8/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11+211/8/d/a^3/(1
+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9+969/20/d/a^3/(1+tan(1/2*d*x+1
/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7+759/20/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan
(1/2*d*x+1/2*c)^5+391/24/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c
)^3+23/8/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)-23/8/d/a^3*arc
tan(tan(1/2*d*x+1/2*c))
```

Maxima [B] time = 1.53634, size = 394, normalized size = 3.05

$$\frac{\frac{\frac{345 \sin(dx+c)}{\cos(dx+c)+1}+\frac{1955 \sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{4554 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5814 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{3165 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{1575 \sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a^3+\frac{6a^3 \sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{15a^3 \sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{20a^3 \sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{15a^3 \sin(dx+c)^8}{(\cos(dx+c)+1)^8}+\frac{6a^3 \sin(dx+c)^{10}}{(\cos(dx+c)+1)^{10}}+\frac{a^3 \sin(dx+c)^{12}}{(\cos(dx+c)+1)^{12}}}-\frac{345 \arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{120\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/120*((345*sin(d*x + c)/(cos(d*x + c) + 1) + 1955*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 4554*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 5814*sin(d*x + c)^
7/(cos(d*x + c) + 1)^7 + 3165*sin(d*x + c)^9/(cos(d*x + c) + 1)^9 + 1575*si
n(d*x + c)^11/(cos(d*x + c) + 1)^11)/(a^3 + 6*a^3*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2 + 15*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 20*a^3*sin(d*x +
c)^6/(cos(d*x + c) + 1)^6 + 15*a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8 + 6*
a^3*sin(d*x + c)^10/(cos(d*x + c) + 1)^10 + a^3*sin(d*x + c)^12/(cos(d*x +
c) + 1)^12) - 345*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3)/d
```

Fricas [A] time = 1.76307, size = 201, normalized size = 1.56

$$-\frac{345\, dx + \left(40 \cos(dx+c)^5 - 144 \cos(dx+c)^4 + 230 \cos(dx+c)^3 - 272 \cos(dx+c)^2 + 345 \cos(dx+c) - 544\right) \sin(d}{240\, a^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/240*(345*d*x + (40*cos(d*x + c)^5 - 144*cos(d*x + c)^4 + 230*cos(d*x + c
)^3 - 272*cos(d*x + c)^2 + 345*cos(d*x + c) - 544)*sin(d*x + c))/(a^3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34619, size = 153, normalized size = 1.19

$$-\frac{\frac{345\,(dx+c)}{a^3} - \frac{2\left(1575\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+3165\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+5814\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+4554\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+1955\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+345\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{6}a^3}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/240*(345*(d*x + c)/a^3 - 2*(1575*tan(1/2*d*x + 1/2*c)^11 + 3165*tan(1/2*
d*x + 1/2*c)^9 + 5814*tan(1/2*d*x + 1/2*c)^7 + 4554*tan(1/2*d*x + 1/2*c)^5
+ 1955*tan(1/2*d*x + 1/2*c)^3 + 345*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1
/2*c)^2 + 1)^6*a^3))/d
```

3.102 $\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=108

$$\frac{\sin^3(c+dx)}{a^3d}-\frac{7\sin(c+dx)}{a^3d}+\frac{\sin(c+dx)\cos^3(c+dx)}{4a^3d}+\frac{19\sin(c+dx)\cos(c+dx)}{8a^3d}-\frac{4\sin(c+dx)}{a^3d(\cos(c+dx)+1)}+\frac{51x}{8a^3}$$

```
[Out] (51*x)/(8*a^3) - (7*Sin[c + d*x])/(a^3*d) + (19*Cos[c + d*x]*Sin[c + d*x])/
(8*a^3*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*a^3*d) - (4*Sin[c + d*x])/(a^3
*d*(1 + Cos[c + d*x])) + Sin[c + d*x]^3/(a^3*d)
```

Rubi [A] time = 0.318477, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2875, 2872, 2648, 2637, 2635, 8, 2633}

$$\frac{\sin^3(c+dx)}{a^3d}-\frac{7\sin(c+dx)}{a^3d}+\frac{\sin(c+dx)\cos^3(c+dx)}{4a^3d}+\frac{19\sin(c+dx)\cos(c+dx)}{8a^3d}-\frac{4\sin(c+dx)}{a^3d(\cos(c+dx)+1)}+\frac{51x}{8a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (51*x)/(8*a^3) - (7*Sin[c + d*x])/(a^3*d) + (19*Cos[c + d*x]*Sin[c + d*x])/
(8*a^3*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*a^3*d) - (4*Sin[c + d*x])/(a^3
*d*(1 + Cos[c + d*x])) + Sin[c + d*x]^3/(a^3*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2872

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/a^p, Int[Expand
Trig[(d*sin[e + f*x])^n*(a - b*sin[e + f*x])^(p/2)*(a + b*sin[e + f*x])^(m
+ p/2), x], x], x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && Int
egersQ[m, n, p/2] && ((GtQ[m, 0] && GtQ[p, 0] && LtQ[-m - p, n, -1]) || (Gt
Q[m, 2] && LtQ[p, 0] && GtQ[m + p/2, 0]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\int \frac{\cos^3(c+dx)\sin^4(c+dx)}{(-a-a\cos(c+dx))^3}\,dx \\
&= -\frac{\int \cos(c+dx)(-a+a\cos(c+dx))^3\cot^2(c+dx)\,dx}{a^6} \\
&= \frac{\int \left(4a+\frac{4a}{-1-\cos(c+dx)}-4a\cos(c+dx)+4a\cos^2(c+dx)-3a\cos^3(c+dx)+a\cos^4(c+dx\right.}{a^4} \\
&= \frac{4x}{a^3}+\frac{\int \cos^4(c+dx)\,dx}{a^3}-\frac{3\int \cos^3(c+dx)\,dx}{a^3}+\frac{4\int \frac{1}{-1-\cos(c+dx)}\,dx}{a^3}-\frac{4\int \cos(c+dx)\,dx}{a^3} \\
&= \frac{4x}{a^3}-\frac{4\sin(c+dx)}{a^3d}+\frac{2\cos(c+dx)\sin(c+dx)}{a^3d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4a^3d}-\frac{4\sin(c}{a^3d(1+\cos} \\
&= \frac{6x}{a^3}-\frac{7\sin(c+dx)}{a^3d}+\frac{19\cos(c+dx)\sin(c+dx)}{8a^3d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4a^3d}-\frac{4\sin(c}{a^3d(1+\text{co}} \\
&= \frac{51x}{8a^3}-\frac{7\sin(c+dx)}{a^3d}+\frac{19\cos(c+dx)\sin(c+dx)}{8a^3d}+\frac{\cos^3(c+dx)\sin(c+dx)}{4a^3d}-\frac{4\sin(}{a^3d(1+\text{c}}
\end{aligned}$$

Mathematica [A] time = 0.645708, size = 173, normalized size = 1.6

$$\frac{\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(-997\sin\left(c+\frac{dx}{2}\right)-800\sin\left(c+\frac{3dx}{2}\right)-800\sin\left(2c+\frac{3dx}{2}\right)+160\sin\left(2c+\frac{5dx}{2}\right)+160\sin\left(3c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]*(2040*d*x*Cos[(d*x)/2] + 2040*d*x*Cos[c + (d*x)/
2] - 3563*Sin[(d*x)/2] - 997*Sin[c + (d*x)/2] - 800*Sin[c + (3*d*x)/2] - 80
0*Sin[2*c + (3*d*x)/2] + 160*Sin[2*c + (5*d*x)/2] + 160*Sin[3*c + (5*d*x)/2
] - 35*Sin[3*c + (7*d*x)/2] - 35*Sin[4*c + (7*d*x)/2] + 5*Sin[4*c + (9*d*x)
/2] + 5*Sin[5*c + (9*d*x)/2]))/(640*a^3*d)
```

Maple [A] time = 0.107, size = 171, normalized size = 1.6

$$-4\,{\frac{\tan \left(1/2\,dx+c/2 \right) }{d{a}^{3}}}-{\frac{77}{4\,d{a}^{3}}\left(\tan \left({\frac{dx}{2}}+{\frac{c}{2}} \right) \right) ^{7} \left(1+ \left(\tan \left({\frac{dx}{2}}+{\frac{c}{2}} \right) \right) ^{2} \right) ^{-4}}-{\frac{149}{4\,d{a}^{3}}\left(\tan \left({\frac{dx}{2}}+{\frac{c}{2}} \right) \right) ^{5} \left(1+ \left(\tan \left({\frac{dx}{2}}+{\frac{c}{2}} \right) \right) ^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -4/d/a^3*tan(1/2*d*x+1/2*c)-77/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d
*x+1/2*c)^7-149/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-123
/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3-35/4/d/a^3/(1+tan(
1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)+51/4/d/a^3*arctan(tan(1/2*d*x+1/2*c)
)
```

Maxima [B] time = 1.51965, size = 306, normalized size = 2.83

$$-\frac{\frac{\frac{35\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{123\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{149\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{77\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^3+\frac{4\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{6\,a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{4\,a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\frac{a^3\sin(dx+c)^8}{(\cos(dx+c)+1)^8}}-\frac{51\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}+\frac{16\,\sin(dx+c)}{a^3(\cos(dx+c)+1)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*((35*sin(d*x + c)/(cos(d*x + c) + 1) + 123*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 149*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 77*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7)/(a^3 + 4*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + 6*a^3*
sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + 4*a^3*sin(d*x + c)^6/(cos(d*x + c) +
1)^6 + a^3*sin(d*x + c)^8/(cos(d*x + c) + 1)^8) - 51*arctan(sin(d*x + c)/(c
os(d*x + c) + 1))/a^3 + 16*sin(d*x + c)/(a^3*(cos(d*x + c) + 1)))/d
```

Fricas [A] time = 1.70069, size = 217, normalized size = 2.01

$$\frac{51\,dx\cos(dx+c)+51\,dx+\left(2\,\cos(dx+c)^4-6\,\cos(dx+c)^3+11\,\cos(dx+c)^2-29\,\cos(dx+c)-80\right)\sin(dx+c)}{8\left(a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/8*(51*d*x*cos(d*x + c) + 51*d*x + (2*cos(d*x + c)^4 - 6*cos(d*x + c)^3 +
11*cos(d*x + c)^2 - 29*cos(d*x + c) - 80)*sin(d*x + c))/(a^3*d*cos(d*x + c)
 + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sin^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A] time = 1.35869, size = 136, normalized size = 1.26

$$\frac{\frac{51\,(dx+c)}{a^3}-\frac{32\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}-\frac{2\left(77\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+149\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+123\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+35\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4 a^3}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(51*(d*x + c)/a^3 - 32*tan(1/2*d*x + 1/2*c)/a^3 - 2*(77*tan(1/2*d*x + 1
/2*c)^7 + 149*tan(1/2*d*x + 1/2*c)^5 + 123*tan(1/2*d*x + 1/2*c)^3 + 35*tan(
1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^3))/d
```

3.103 $\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=97

$$\frac{3\sin(c+dx)}{a^3d}-\frac{\sin(c+dx)\cos(c+dx)}{2a^3d}+\frac{19\sin(c+dx)}{3a^3d(\cos(c+dx)+1)}-\frac{2\sin(c+dx)}{3a^3d(\cos(c+dx)+1)^2}-\frac{11x}{2a^3}$$

```
[Out] (-11*x)/(2*a^3) + (3*Sin[c + d*x])/(a^3*d) - (Cos[c + d*x]*Sin[c + d*x])/(2
*a^3*d) - (2*Sin[c + d*x])/(3*a^3*d*(1 + Cos[c + d*x])^2) + (19*Sin[c + d*x
])/(3*a^3*d*(1 + Cos[c + d*x]))
```

Rubi [A] time = 0.309781, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2874, 2966, 2637, 2635, 8, 2650, 2648}

$$\frac{3\sin(c+dx)}{a^3d}-\frac{\sin(c+dx)\cos(c+dx)}{2a^3d}+\frac{19\sin(c+dx)}{3a^3d(\cos(c+dx)+1)}-\frac{2\sin(c+dx)}{3a^3d(\cos(c+dx)+1)^2}-\frac{11x}{2a^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-11*x)/(2*a^3) + (3*Sin[c + d*x])/(a^3*d) - (Cos[c + d*x]*Sin[c + d*x])/(2
*a^3*d) - (2*Sin[c + d*x])/(3*a^3*d*(1 + Cos[c + d*x])^2) + (19*Sin[c + d*x
])/(3*a^3*d*(1 + Cos[c + d*x]))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 2966

```
Int[sin[(e_.) + (f_.)*(x_)]^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[ExpandTrig[si
n[e + f*x]^n*(a + b*sin[e + f*x])^m*(A + B*sin[e + f*x]), x], x] /; FreeQ[{
a, b, e, f, A, B}, x] && EqQ[A*b + a*B, 0] && EqQ[a^2 - b^2, 0] && IntegerQ
[m] && IntegerQ[n]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^2(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$

$$= -\frac{\int \frac{\cos^3(c+dx)(-a+a\cos(c+dx))}{(-a-a\cos(c+dx))^2}\,dx}{a^2}$$

$$= -\frac{\int \left(\frac{5}{a}-\frac{3\cos(c+dx)}{a}+\frac{\cos^2(c+dx)}{a}+\frac{2}{a(1+\cos(c+dx))^2}-\frac{7}{a(1+\cos(c+dx))}\right)dx}{a^2}$$

$$= -\frac{5x}{a^3}-\frac{\int \cos^2(c+dx)\,dx}{a^3}-\frac{2\int \frac{1}{(1+\cos(c+dx))^2}\,dx}{a^3}+\frac{3\int \cos(c+dx)\,dx}{a^3}+\frac{7\int \frac{1}{1+\cos(c+dx)}\,dx}{a^3}$$

$$= -\frac{5x}{a^3}+\frac{3\sin(c+dx)}{a^3d}-\frac{\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{2\sin(c+dx)}{3a^3d(1+\cos(c+dx))^2}+\frac{7\sin(c+}{a^3d(1+\cos(}$$

$$= -\frac{11x}{2a^3}+\frac{3\sin(c+dx)}{a^3d}-\frac{\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{2\sin(c+dx)}{3a^3d(1+\cos(c+dx))^2}+\frac{19\sin(c}{3a^3d(1+\mathrm{co}}$$

Mathematica [A] time = 0.417204, size = 177, normalized size = 1.82

$$-\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(1326\sin\left(c+\frac{dx}{2}\right)-2012\sin\left(c+\frac{3dx}{2}\right)-498\sin\left(2c+\frac{3dx}{2}\right)-135\sin\left(2c+\frac{5dx}{2}\right)-135\sin\left(\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(Sec[c/2]*Sec[(c + d*x)/2]^3*(1980*d*x*Cos[(d*x)/2] + 1980*d*x*Cos[c + (d*
x)/2] + 660*d*x*Cos[c + (3*d*x)/2] + 660*d*x*Cos[2*c + (3*d*x)/2] - 3216*Si
n[(d*x)/2] + 1326*Sin[c + (d*x)/2] - 2012*Sin[c + (3*d*x)/2] - 498*Sin[2*c
+ (3*d*x)/2] - 135*Sin[2*c + (5*d*x)/2] - 135*Sin[3*c + (5*d*x)/2] + 15*Sin
[3*c + (7*d*x)/2] + 15*Sin[4*c + (7*d*x)/2]))/(960*a^3*d)
```

Maple [A] time = 0.092, size = 122, normalized size = 1.3

$$-\frac{1}{3\,da^3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+6\,\frac{\tan(1/2\,dx+c/2)}{da^3}+7\,\frac{(\tan(1/2\,dx+c/2))^3}{da^3\left(1+(\tan(1/2\,dx+c/2))^2\right)^2}+5\,\frac{\tan(1/2\,dx+c/2)}{da^3\left(1+(\tan(1/2\,dx+c/2))^2\right)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+a*sec(d*x+c))^3,x)
```

```
[Out] -1/3/d/a^3*tan(1/2*d*x+1/2*c)^3+6/d/a^3*tan(1/2*d*x+1/2*c)+7/d/a^3/(1+tan(1
/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+5/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*
tan(1/2*d*x+1/2*c)-11/d/a^3*arctan(tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.79722, size = 221, normalized size = 2.28

$$\frac{\frac{3\left(\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3+\frac{2\,a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{18\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^3}-\frac{33\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c)
^4/(cos(d*x + c) + 1)^4) + (18*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x +
c)^3/(cos(d*x + c) + 1)^3)/a^3 - 33*arctan(sin(d*x + c)/(cos(d*x + c) + 1))
/a^3)/d
```

Fricas [A] time = 1.81664, size = 259, normalized size = 2.67

$$-\frac{33\,dx\cos(dx+c)^2+66\,dx\cos(dx+c)+33\,dx+\left(3\,\cos(dx+c)^3-12\,\cos(dx+c)^2-71\,\cos(dx+c)-52\right)\sin(dx+c)}{6\left(a^3d\cos(dx+c)^2+2\,a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/6*(33*d*x*cos(d*x + c)^2 + 66*d*x*cos(d*x + c) + 33*d*x + (3*cos(d*x + c
)^3 - 12*cos(d*x + c)^2 - 71*cos(d*x + c) - 52)*sin(d*x + c))/(a^3*d*cos(d*
x + c)^2 + 2*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\sin^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A] time = 1.30563, size = 130, normalized size = 1.34

$$-\frac{\frac{33\,(dx+c)}{a^3}-\frac{6\left(7\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^3}+\frac{2\left(a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-18\,a^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{a^9}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/6*(33*(d*x + c)/a^3 - 6*(7*tan(1/2*d*x + 1/2*c)^3 + 5*tan(1/2*d*x + 1/2*
c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) + 2*(a^6*tan(1/2*d*x + 1/2*c)^3 -
18*a^6*tan(1/2*d*x + 1/2*c))/a^9)/d
```

3.104 $\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=89

$$\frac{4\cot^7(c+dx)}{7a^3d}+\frac{3\cot^5(c+dx)}{5a^3d}-\frac{4\csc^7(c+dx)}{7a^3d}+\frac{7\csc^5(c+dx)}{5a^3d}-\frac{\csc^3(c+dx)}{a^3d}$$

```
[Out] (3*Cot[c + d*x]^5)/(5*a^3*d) + (4*Cot[c + d*x]^7)/(7*a^3*d) - Csc[c + d*x]^
3/(a^3*d) + (7*Csc[c + d*x]^5)/(5*a^3*d) - (4*Csc[c + d*x]^7)/(7*a^3*d)
```

Rubi [A] time = 0.366936, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2875, 2873, 2607, 30, 2606, 270, 14}

$$\frac{4\cot^7(c+dx)}{7a^3d}+\frac{3\cot^5(c+dx)}{5a^3d}-\frac{4\csc^7(c+dx)}{7a^3d}+\frac{7\csc^5(c+dx)}{5a^3d}-\frac{\csc^3(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (3*Cot[c + d*x]^5)/(5*a^3*d) + (4*Cot[c + d*x]^7)/(7*a^3*d) - Csc[c + d*x]^
3/(a^3*d) + (7*Csc[c + d*x]^5)/(5*a^3*d) - (4*Csc[c + d*x]^7)/(7*a^3*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^2(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\int \frac{\cos(c+dx)\cot^2(c+dx)}{(-a-a\cos(c+dx))^3}\,dx\\ &= -\frac{\int (-a+a\cos(c+dx))^3\cot^3(c+dx)\csc^5(c+dx)\,dx}{a^6}\\ &= \frac{\int \left(-a^3\cot^6(c+dx)\csc^2(c+dx)+3a^3\cot^5(c+dx)\csc^3(c+dx)-3a^3\cot^4(c+dx)\csc^4(\right.}{a^6}\\ &= -\frac{\int \cot^6(c+dx)\csc^2(c+dx)\,dx}{a^3}+\frac{\int \cot^3(c+dx)\csc^5(c+dx)\,dx}{a^3}+\frac{3\int \cot^5(c+dx)\csc}{a^3}\\ &= -\frac{\text{Subst}\left(\int x^6\,dx,x,-\cot(c+dx)\right)}{a^3d}-\frac{\text{Subst}\left(\int x^4\left(-1+x^2\right)dx,x,\csc(c+dx)\right)}{a^3d}-\frac{3\,\text{Subs}}{}\\ &= \frac{\cot^7(c+dx)}{7a^3d}-\frac{\text{Subst}\left(\int \left(-x^4+x^6\right)dx,x,\csc(c+dx)\right)}{a^3d}-\frac{3\,\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)dx\right.}{a^3d}\\ &= \frac{3\cot^5(c+dx)}{5a^3d}+\frac{4\cot^7(c+dx)}{7a^3d}-\frac{\csc^3(c+dx)}{a^3d}+\frac{7\csc^5(c+dx)}{5a^3d}-\frac{4\csc^7(c+dx)}{7a^3d}\end{aligned}$$

Mathematica [A] time = 0.563895, size = 137, normalized size = 1.54

$$\frac{\csc(c)(602\sin(c+dx)+602\sin(2(c+dx))+258\sin(3(c+dx))+43\sin(4(c+dx))-560\sin(2c+dx)+168\sin(c+}{2240a^3d(\sec(c}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Csc[c]*Csc[c + d*x]*Sec[c + d*x]^3*(-840*Sin[c] + 448*Sin[d*x] + 602*Sin[c
 + d*x] + 602*Sin[2*(c + d*x)] + 258*Sin[3*(c + d*x)] + 43*Sin[4*(c + d*x)]
 - 560*Sin[2*c + d*x] + 168*Sin[c + 2*d*x] - 280*Sin[3*c + 2*d*x] - 48*Sin[
2*c + 3*d*x] - 8*Sin[3*c + 4*d*x]))/(2240*a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 0.066, size = 60, normalized size = 0.7

$$\frac{1}{16\,da^3}\left(-\frac{1}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^7+\frac{2}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-2\,\tan\left(1/2\,dx+c/2\right)-\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/16/d/a^3*(-1/7*tan(1/2*d*x+1/2*c)^7+2/5*tan(1/2*d*x+1/2*c)^5-2*tan(1/2*d*
x+1/2*c)-1/tan(1/2*d*x+1/2*c))
```

Maxima [A] time = 1.03617, size = 122, normalized size = 1.37

$$-\frac{\frac{\frac{70 \sin(dx+c)}{\cos(dx+c)+1}-\frac{14 \sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5 \sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^3}+\frac{35(\cos(dx+c)+1)}{a^3 \sin(dx+c)}}{560 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/560*((70*sin(d*x + c)/(cos(d*x + c) + 1) - 14*sin(d*x + c)^5/(cos(d*x +
c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^3 + 35*(cos(d*x + c) +
 1)/(a^3*sin(d*x + c)))/d
```

Fricas [A] time = 1.83632, size = 240, normalized size = 2.7

$$\frac{\cos(dx+c)^4+3\cos(dx+c)^3-15\cos(dx+c)^2-18\cos(dx+c)-6}{35\left(a^3 d\cos(dx+c)^3+3a^3 d\cos(dx+c)^2+3a^3 d\cos(dx+c)+a^3 d\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/35*(cos(d*x + c)^4 + 3*cos(d*x + c)^3 - 15*cos(d*x + c)^2 - 18*cos(d*x +
c) - 6)/((a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
 c) + a^3*d)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\csc^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x)/a**3
```

Giac [A] time = 1.31231, size = 99, normalized size = 1.11

$$-\frac{\frac{35}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}+\frac{5a^{18}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-14a^{18}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+70a^{18}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{21}}}{560d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/560*(35/(a^3*tan(1/2*d*x + 1/2*c)) + (5*a^18*tan(1/2*d*x + 1/2*c)^7 - 14
*a^18*tan(1/2*d*x + 1/2*c)^5 + 70*a^18*tan(1/2*d*x + 1/2*c))/a^21)/d
```

3.105 $\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^3} dx$

Optimal. Leaf size=103

$$\frac{4\cot^9(c+dx)}{9a^3d}+\frac{\cot^7(c+dx)}{a^3d}+\frac{3\cot^5(c+dx)}{5a^3d}-\frac{4\csc^9(c+dx)}{9a^3d}+\frac{\csc^7(c+dx)}{a^3d}-\frac{3\csc^5(c+dx)}{5a^3d}$$

```
[Out] (3*Cot[c + d*x]^5)/(5*a^3*d) + Cot[c + d*x]^7/(a^3*d) + (4*Cot[c + d*x]^9)/
(9*a^3*d) - (3*Csc[c + d*x]^5)/(5*a^3*d) + Csc[c + d*x]^7/(a^3*d) - (4*Csc[
c + d*x]^9)/(9*a^3*d)
```

Rubi [A] time = 0.378706, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2875, 2873, 2607, 14, 2606, 270}

$$\frac{4\cot^9(c+dx)}{9a^3d}+\frac{\cot^7(c+dx)}{a^3d}+\frac{3\cot^5(c+dx)}{5a^3d}-\frac{4\csc^9(c+dx)}{9a^3d}+\frac{\csc^7(c+dx)}{a^3d}-\frac{3\csc^5(c+dx)}{5a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (3*Cot[c + d*x]^5)/(5*a^3*d) + Cot[c + d*x]^7/(a^3*d) + (4*Cot[c + d*x]^9)/
(9*a^3*d) - (3*Csc[c + d*x]^5)/(5*a^3*d) + Csc[c + d*x]^7/(a^3*d) - (4*Csc[
c + d*x]^9)/(9*a^3*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cot^3(c+dx)\csc(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$
$$= -\frac{\int(-a+a\cos(c+dx))^3\cot^3(c+dx)\csc^7(c+dx)\,dx}{a^6}$$
$$= \frac{\int\left(-a^3\cot^6(c+dx)\csc^4(c+dx)+3a^3\cot^5(c+dx)\csc^5(c+dx)-3a^3\cot^4(c+dx)\csc^6(\right.}{a^6}$$
$$= -\frac{\int\cot^6(c+dx)\csc^4(c+dx)\,dx}{a^3}+\frac{\int\cot^3(c+dx)\csc^7(c+dx)\,dx}{a^3}+\frac{3\int\cot^5(c+dx)\csc}{a^3}$$
$$= -\frac{\text{Subst}\left(\int x^6\left(-1+x^2\right)dx,x,\csc(c+dx)\right)}{a^3d}-\frac{\text{Subst}\left(\int x^6\left(1+x^2\right)dx,x,-\cot(c+dx)\right)}{a^3d}$$
$$= -\frac{\text{Subst}\left(\int\left(-x^6+x^8\right)dx,x,\csc(c+dx)\right)}{a^3d}-\frac{\text{Subst}\left(\int\left(x^6+x^8\right)dx,x,-\cot(c+dx)\right)}{a^3d}-$$
$$= \frac{3\cot^5(c+dx)}{5a^3d}+\frac{\cot^7(c+dx)}{a^3d}+\frac{4\cot^9(c+dx)}{9a^3d}-\frac{3\csc^5(c+dx)}{5a^3d}+\frac{\csc^7(c+dx)}{a^3d}-\frac{4\csc}{9}$$

Mathematica [A] time = 0.713802, size = 175, normalized size = 1.7

$$-\frac{\csc(c)(-1764\sin(c+dx)-1323\sin(2(c+dx))+98\sin(3(c+dx))+588\sin(4(c+dx))+294\sin(5(c+dx))+49\sin}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(Csc[c]*Csc[2*(c + d*x)]^3*(5376*Sin[c] - 1152*Sin[d*x] - 1764*Sin[c + d*x
] - 1323*Sin[2*(c + d*x)] + 98*Sin[3*(c + d*x)] + 588*Sin[4*(c + d*x)] + 29
4*Sin[5*(c + d*x)] + 49*Sin[6*(c + d*x)] + 3456*Sin[2*c + d*x] - 1152*Sin[c
 + 2*d*x] + 2880*Sin[3*c + 2*d*x] - 128*Sin[2*c + 3*d*x] - 768*Sin[3*c + 4*
d*x] - 384*Sin[4*c + 5*d*x] - 64*Sin[5*c + 6*d*x]))/(5760*a^3*d*(1 + Sec[c
+ d*x])^3)
```

Maple [A] time = 0.074, size = 60, normalized size = 0.6

$$\frac{1}{64\,da^3}\left(-\frac{1}{9}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^9+\frac{3}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-3\,\tan(1/2\,dx+c/2)-\frac{1}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/64/d/a^3*(-1/9*tan(1/2*d*x+1/2*c)^9+3/5*tan(1/2*d*x+1/2*c)^5-3*tan(1/2*d*
x+1/2*c)-1/3/tan(1/2*d*x+1/2*c)^3)
```

Maxima [A] time = 1.12351, size = 124, normalized size = 1.2

$$-\frac{\frac{\frac{135 \sin(dx+c)}{\cos(dx+c)+1} - \frac{27 \sin(dx+c)^5}{(\cos(dx+c)+1)^5} + \frac{5 \sin(dx+c)^9}{(\cos(dx+c)+1)^9}}{a^3} + \frac{15 (\cos(dx+c)+1)^3}{a^3 \sin(dx+c)^3}}{2880\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2880*((135*sin(d*x + c)/(cos(d*x + c) + 1) - 27*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5 + 5*sin(d*x + c)^9/(cos(d*x + c) + 1)^9)/a^3 + 15*(cos(d*x + c)
 + 1)^3/(a^3*sin(d*x + c)^3))/d
```

Fricas [A] time = 1.89216, size = 359, normalized size = 3.49

$$\frac{2 \cos(dx+c)^6 + 6 \cos(dx+c)^5 + 3 \cos(dx+c)^4 - 7 \cos(dx+c)^3 + 3 \cos(dx+c)^2 + 6 \cos(dx+c) + 2}{45 \left(a^3 d \cos(dx+c)^5 + 3 a^3 d \cos(dx+c)^4 + 2 a^3 d \cos(dx+c)^3 - 2 a^3 d \cos(dx+c)^2 - 3 a^3 d \cos(dx+c) - a^3 d\right) \sin(dx+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/45*(2*cos(d*x + c)^6 + 6*cos(d*x + c)^5 + 3*cos(d*x + c)^4 - 7*cos(d*x +
c)^3 + 3*cos(d*x + c)^2 + 6*cos(d*x + c) + 2)/((a^3*d*cos(d*x + c)^5 + 3*a^
3*d*cos(d*x + c)^4 + 2*a^3*d*cos(d*x + c)^3 - 2*a^3*d*cos(d*x + c)^2 - 3*a^
3*d*cos(d*x + c) - a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28664, size = 99, normalized size = 0.96

$$-\frac{\frac{15}{a^3 \tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)^3} + \frac{5 a^{24} \tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)^9 - 27 a^{24} \tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)^5 + 135 a^{24} \tan\left(\frac{1}{2}\, dx + \frac{1}{2}\, c\right)}{a^{27}}}{2880\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2880*(15/(a^3*tan(1/2*d*x + 1/2*c)^3) + (5*a^24*tan(1/2*d*x + 1/2*c)^9 -
 27*a^24*tan(1/2*d*x + 1/2*c)^5 + 135*a^24*tan(1/2*d*x + 1/2*c))/a^27)/d
```

3.106 $\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=127

$$\frac{4\cot^{11}(c+dx)}{11a^3d}+\frac{11\cot^9(c+dx)}{9a^3d}+\frac{10\cot^7(c+dx)}{7a^3d}+\frac{3\cot^5(c+dx)}{5a^3d}-\frac{4\csc^{11}(c+dx)}{11a^3d}+\frac{7\csc^9(c+dx)}{9a^3d}-\frac{3\csc^7(c+dx)}{7a^3d}$$

```
[Out] (3*Cot[c + d*x]^5)/(5*a^3*d) + (10*Cot[c + d*x]^7)/(7*a^3*d) + (11*Cot[c +
d*x]^9)/(9*a^3*d) + (4*Cot[c + d*x]^11)/(11*a^3*d) - (3*Csc[c + d*x]^7)/(7*
a^3*d) + (7*Csc[c + d*x]^9)/(9*a^3*d) - (4*Csc[c + d*x]^11)/(11*a^3*d)
```

Rubi [A] time = 0.408049, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2875, 2873, 2607, 270, 2606, 14}

$$\frac{4\cot^{11}(c+dx)}{11a^3d}+\frac{11\cot^9(c+dx)}{9a^3d}+\frac{10\cot^7(c+dx)}{7a^3d}+\frac{3\cot^5(c+dx)}{5a^3d}-\frac{4\csc^{11}(c+dx)}{11a^3d}+\frac{7\csc^9(c+dx)}{9a^3d}-\frac{3\csc^7(c+dx)}{7a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (3*Cot[c + d*x]^5)/(5*a^3*d) + (10*Cot[c + d*x]^7)/(7*a^3*d) + (11*Cot[c +
d*x]^9)/(9*a^3*d) + (4*Cot[c + d*x]^11)/(11*a^3*d) - (3*Csc[c + d*x]^7)/(7*
a^3*d) + (7*Csc[c + d*x]^9)/(9*a^3*d) - (4*Csc[c + d*x]^11)/(11*a^3*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\csc^6(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\int \frac{\cot^3(c+dx)\csc^3(c+dx)}{(-a-a\cos(c+dx))^3}\,dx$$
$$= -\frac{\int(-a+a\cos(c+dx))^3\cot^3(c+dx)\csc^9(c+dx)\,dx}{a^6}$$
$$= \frac{\int\left(-a^3\cot^6(c+dx)\csc^6(c+dx)+3a^3\cot^5(c+dx)\csc^7(c+dx)-3a^3\cot^4(c+dx)\csc^8(\right.}{a^6}$$
$$= -\frac{\int\cot^6(c+dx)\csc^6(c+dx)\,dx}{a^3}+\frac{\int\cot^3(c+dx)\csc^9(c+dx)\,dx}{a^3}+\frac{3\int\cot^5(c+dx)\csc}{a^3}$$
$$= -\frac{\text{Subst}\left(\int x^8\left(-1+x^2\right)dx,x,\csc(c+dx)\right)}{a^3d}-\frac{\text{Subst}\left(\int x^6\left(1+x^2\right)^2dx,x,-\cot(c+dx)\right)}{a^3d}$$
$$= -\frac{\text{Subst}\left(\int\left(-x^8+x^{10}\right)dx,x,\csc(c+dx)\right)}{a^3d}-\frac{\text{Subst}\left(\int\left(x^6+2x^8+x^{10}\right)dx,x,-\cot(c+\right.}{a^3d}$$
$$= \frac{3\cot^5(c+dx)}{5a^3d}+\frac{10\cot^7(c+dx)}{7a^3d}+\frac{11\cot^9(c+dx)}{9a^3d}+\frac{4\cot^{11}(c+dx)}{11a^3d}-\frac{3\csc^7(c+dx)}{7a^3d}+$$

Mathematica [A] time = 1.21646, size = 223, normalized size = 1.76

$$\csc(c)(524150\sin(c+dx)+314490\sin(2(c+dx))-162010\sin(3(c+dx))-238250\sin(4(c+dx))-47650\sin(5(c+$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Csc[c]*Csc[c + d*x]^5*Sec[c + d*x]^3*(-3886080*Sin[c] + 563200*Sin[d*x] +
524150*Sin[c + d*x] + 314490*Sin[2*(c + d*x)] - 162010*Sin[3*(c + d*x)] - 2
38250*Sin[4*(c + d*x)] - 47650*Sin[5*(c + d*x)] + 47650*Sin[6*(c + d*x)] +
28590*Sin[7*(c + d*x)] + 4765*Sin[8*(c + d*x)] - 2027520*Sin[2*c + d*x] + 1
486848*Sin[c + 2*d*x] - 2365440*Sin[3*c + 2*d*x] + 452608*Sin[2*c + 3*d*x]
+ 665600*Sin[3*c + 4*d*x] + 133120*Sin[4*c + 5*d*x] - 133120*Sin[5*c + 6*d*
x] - 79872*Sin[6*c + 7*d*x] - 13312*Sin[7*c + 8*d*x]))/(56770560*a^3*d*(1 +
 Sec[c + d*x])^3)
```

Maple [A] time = 0.079, size = 112, normalized size = 0.9

$$\frac{1}{256\,da^3}\left(-\frac{1}{11}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{11}-\frac{2}{9}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{9}+\frac{2}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}+\frac{6}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{5}-6\,\tan\left(1/2\,dx+c/2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/256/d/a^3*(-1/11*tan(1/2*d*x+1/2*c)^11-2/9*tan(1/2*d*x+1/2*c)^9+2/7*tan(1
/2*d*x+1/2*c)^7+6/5*tan(1/2*d*x+1/2*c)^5-6*tan(1/2*d*x+1/2*c)-2/3/tan(1/2*d
*x+1/2*c)^3+2/tan(1/2*d*x+1/2*c)-1/5/tan(1/2*d*x+1/2*c)^5)
```

Maxima [A] time = 1.12955, size = 235, normalized size = 1.85

$$-\frac{\frac{\frac{20790\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{4158\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{990\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{770\,\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{315\,\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}}{a^3}+\frac{231\left(\frac{10\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}-\frac{30\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+3\right)(\cos(dx+c)+1)^5}{a^3\sin(dx+c)^5}}{887040\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/887040*((20790*sin(d*x + c)/(cos(d*x + c) + 1) - 4158*sin(d*x + c)^5/(co
s(d*x + c) + 1)^5 - 990*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 770*sin(d*x +
 c)^9/(cos(d*x + c) + 1)^9 + 315*sin(d*x + c)^11/(cos(d*x + c) + 1)^11)/a^3
 + 231*(10*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 - 30*sin(d*x + c)^4/(cos(d*x
 + c) + 1)^4 + 3)*(cos(d*x + c) + 1)^5/(a^3*sin(d*x + c)^5))/d
```

Fricas [A] time = 1.98898, size = 495, normalized size = 3.9

$$\frac{104\,\cos(dx+c)^8+312\,\cos(dx+c)^7+52\,\cos(dx+c)^6-676\,\cos(dx+c)^5-585\,\cos(dx+c)^4+325\,\cos(dx+c)^3-}{3465\left(a^3d\cos(dx+c)^7+3\,a^3d\cos(dx+c)^6+a^3d\cos(dx+c)^5-5\,a^3d\cos(dx+c)^4-5\,a^3d\cos(dx+c)^3+a^3d\cos(dx\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/3465*(104*cos(d*x + c)^8 + 312*cos(d*x + c)^7 + 52*cos(d*x + c)^6 - 676*c
os(d*x + c)^5 - 585*cos(d*x + c)^4 + 325*cos(d*x + c)^3 - 25*cos(d*x + c)^2
 - 150*cos(d*x + c) - 50)/((a^3*d*cos(d*x + c)^7 + 3*a^3*d*cos(d*x + c)^6 +
 a^3*d*cos(d*x + c)^5 - 5*a^3*d*cos(d*x + c)^4 - 5*a^3*d*cos(d*x + c)^3 + a
^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31689, size = 181, normalized size = 1.43

$$\frac{\frac{231\left(30\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-10\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3\right)}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5}-\frac{315a^{30}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+770a^{30}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-990a^{30}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-4158a^{30}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5}{a^{33}}}{887040d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/887040*(231*(30*tan(1/2*d*x + 1/2*c)^4 - 10*tan(1/2*d*x + 1/2*c)^2 - 3)/(
a^3*tan(1/2*d*x + 1/2*c)^5) - (315*a^30*tan(1/2*d*x + 1/2*c)^11 + 770*a^30*
tan(1/2*d*x + 1/2*c)^9 - 990*a^30*tan(1/2*d*x + 1/2*c)^7 - 4158*a^30*tan(1/
2*d*x + 1/2*c)^5 + 20790*a^30*tan(1/2*d*x + 1/2*c))/a^33)/d
```

3.107 $\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=145

$$\frac{4\cot^{13}(c+dx)}{13a^3d}+\frac{15\cot^{11}(c+dx)}{11a^3d}+\frac{7\cot^9(c+dx)}{3a^3d}+\frac{13\cot^7(c+dx)}{7a^3d}+\frac{3\cot^5(c+dx)}{5a^3d}-\frac{4\csc^{13}(c+dx)}{13a^3d}+\frac{7\csc^{11}(c+}{11a^3d}$$

```
[Out] (3*Cot[c + d*x]^5)/(5*a^3*d) + (13*Cot[c + d*x]^7)/(7*a^3*d) + (7*Cot[c + d
*x]^9)/(3*a^3*d) + (15*Cot[c + d*x]^11)/(11*a^3*d) + (4*Cot[c + d*x]^13)/(1
3*a^3*d) - Csc[c + d*x]^9/(3*a^3*d) + (7*Csc[c + d*x]^11)/(11*a^3*d) - (4*C
sc[c + d*x]^13)/(13*a^3*d)
```

Rubi [A] time = 0.419321, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2875, 2873, 2607, 270, 2606, 14}

$$\frac{4\cot^{13}(c+dx)}{13a^3d}+\frac{15\cot^{11}(c+dx)}{11a^3d}+\frac{7\cot^9(c+dx)}{3a^3d}+\frac{13\cot^7(c+dx)}{7a^3d}+\frac{3\cot^5(c+dx)}{5a^3d}-\frac{4\csc^{13}(c+dx)}{13a^3d}+\frac{7\csc^{11}(c+}{11a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (3*Cot[c + d*x]^5)/(5*a^3*d) + (13*Cot[c + d*x]^7)/(7*a^3*d) + (7*Cot[c + d
*x]^9)/(3*a^3*d) + (15*Cot[c + d*x]^11)/(11*a^3*d) + (4*Cot[c + d*x]^13)/(1
3*a^3*d) - Csc[c + d*x]^9/(3*a^3*d) + (7*Csc[c + d*x]^11)/(11*a^3*d) - (4*C
sc[c + d*x]^13)/(13*a^3*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] && !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \frac{\csc^8(c+dx)}{(a+a\sec(c+dx))^3}\,dx &= -\int \frac{\cot^3(c+dx)\csc^5(c+dx)}{(-a-a\cos(c+dx))^3}\,dx\\
&= -\frac{\int (-a+a\cos(c+dx))^3\cot^3(c+dx)\csc^{11}(c+dx)\,dx}{a^6}\\
&= \frac{\int \left(-a^3\cot^6(c+dx)\csc^8(c+dx)+3a^3\cot^5(c+dx)\csc^9(c+dx)-3a^3\cot^4(c+dx)\csc^{10}\right.}{a^6}\\
&= -\frac{\int \cot^6(c+dx)\csc^8(c+dx)\,dx}{a^3}+\frac{\int \cot^3(c+dx)\csc^{11}(c+dx)\,dx}{a^3}+\frac{3\int \cot^5(c+dx)\,\mathrm{cs}}{a^3}\\
&= -\frac{\mathrm{Subst}\left(\int x^{10}\left(-1+x^2\right)dx,x,\csc(c+dx)\right)}{a^3d}-\frac{\mathrm{Subst}\left(\int x^6\left(1+x^2\right)^3dx,x,-\cot(c+dx)\right.}{a^3d}\\
&= -\frac{\mathrm{Subst}\left(\int \left(-x^{10}+x^{12}\right)dx,x,\csc(c+dx)\right)}{a^3d}-\frac{\mathrm{Subst}\left(\int \left(x^6+3x^8+3x^{10}+x^{12}\right)dx,x,-\right.}{a^3d}\\
&= \frac{3\cot^5(c+dx)}{5a^3d}+\frac{13\cot^7(c+dx)}{7a^3d}+\frac{7\cot^9(c+dx)}{3a^3d}+\frac{15\cot^{11}(c+dx)}{11a^3d}+\frac{4\cot^{13}(c+dx)}{13a^3d}\end{aligned}$$

Mathematica [A] time = 1.76125, size = 265, normalized size = 1.83

$$-\frac{\csc(c)(-2764580\sin(c+dx)-1382290\sin(2(c+dx))+1275960\sin(3(c+dx))+1336720\sin(4(c+dx))-60760\,\mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^8/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(Csc[c]*Csc[c + d*x]^7*Sec[c + d*x]^3*(49201152*Sin[c] - 6336512*Sin[d*x]
- 2764580*Sin[c + d*x] - 1382290*Sin[2*(c + d*x)] + 1275960*Sin[3*(c + d*x)
] + 1336720*Sin[4*(c + d*x)] - 60760*Sin[5*(c + d*x)] - 524055*Sin[6*(c + d
*x)] - 167090*Sin[7*(c + d*x)] + 60760*Sin[8*(c + d*x)] + 45570*Sin[9*(c +
d*x)] + 7595*Sin[10*(c + d*x)] + 20500480*Sin[2*c + d*x] - 23668736*Sin[c +
 2*d*x] + 30750720*Sin[3*c + 2*d*x] - 6537216*Sin[2*c + 3*d*x] - 6848512*Si
n[3*c + 4*d*x] + 311296*Sin[4*c + 5*d*x] + 2684928*Sin[5*c + 6*d*x] + 85606
4*Sin[6*c + 7*d*x] - 311296*Sin[7*c + 8*d*x] - 233472*Sin[8*c + 9*d*x] - 38
912*Sin[9*c + 10*d*x]))/(984023040*a^3*d*(1 + Sec[c + d*x])^3)
```

Maple [A] time = 0.083, size = 138, normalized size = 1.

$$\frac{1}{1024\,da^3}\left(-\frac{1}{13}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{13}-\frac{4}{11}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{11}-\frac{1}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{9}+\frac{8}{7}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{7}+\frac{14}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^8/(a+a*sec(d*x+c))^3,x)
```

```
[Out] 1/1024/d/a^3*(-1/13*tan(1/2*d*x+1/2*c)^13-4/11*tan(1/2*d*x+1/2*c)^11-1/3*ta
n(1/2*d*x+1/2*c)^9+8/7*tan(1/2*d*x+1/2*c)^7+14/5*tan(1/2*d*x+1/2*c)^5-14*ta
n(1/2*d*x+1/2*c)-1/tan(1/2*d*x+1/2*c)^3+8/tan(1/2*d*x+1/2*c)-4/5/tan(1/2*d*
x+1/2*c)^5-1/7/tan(1/2*d*x+1/2*c)^7)
```

Maxima [A] time = 1.15545, size = 289, normalized size = 1.99

$$-\frac{\frac{\frac{210210\sin(dx+c)}{\cos(dx+c)+1}-\frac{42042\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{17160\sin(dx+c)^7}{(\cos(dx+c)+1)^7}+\frac{5005\sin(dx+c)^9}{(\cos(dx+c)+1)^9}+\frac{5460\sin(dx+c)^{11}}{(\cos(dx+c)+1)^{11}}+\frac{1155\sin(dx+c)^{13}}{(\cos(dx+c)+1)^{13}}}{a^3}+\frac{429\left(\frac{28\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{35\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{280\sin(dx+c)^6}{(\cos(dx+c)+1)^6}+\cdots\right.}{a^3\sin(dx+c)^7}}{15375360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/15375360*((210210*sin(d*x + c)/(cos(d*x + c) + 1) - 42042*sin(d*x + c)^5
/(cos(d*x + c) + 1)^5 - 17160*sin(d*x + c)^7/(cos(d*x + c) + 1)^7 + 5005*si
n(d*x + c)^9/(cos(d*x + c) + 1)^9 + 5460*sin(d*x + c)^11/(cos(d*x + c) + 1)
^11 + 1155*sin(d*x + c)^13/(cos(d*x + c) + 1)^13)/a^3 + 429*(28*sin(d*x + c
)^2/(cos(d*x + c) + 1)^2 + 35*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - 280*sin
(d*x + c)^6/(cos(d*x + c) + 1)^6 + 5)*(cos(d*x + c) + 1)^7/(a^3*sin(d*x + c
)^7))/d
```

Fricas [A] time = 2.05892, size = 567, normalized size = 3.91

$$\frac{304\cos(dx+c)^{10}+912\cos(dx+c)^{9}-152\cos(dx+c)^{8}-2888\cos(dx+c)^{7}-1862\cos(dx+c)^{6}+2926\cos(dx+c\cdots}{15015\left(a^3d\cos(dx+c)^9+3\,a^3d\cos(dx+c)^8-8\,a^3d\cos(dx+c)^6-6\,a^3d\cos(dx+c)^5+6\,a^3d\cos(d\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/15015*(304*cos(d*x + c)^10 + 912*cos(d*x + c)^9 - 152*cos(d*x + c)^8 - 28
88*cos(d*x + c)^7 - 1862*cos(d*x + c)^6 + 2926*cos(d*x + c)^5 + 3325*cos(d*
x + c)^4 - 665*cos(d*x + c)^3 - 35*cos(d*x + c)^2 + 210*cos(d*x + c) + 70)/
((a^3*d*cos(d*x + c)^9 + 3*a^3*d*cos(d*x + c)^8 - 8*a^3*d*cos(d*x + c)^6 -
6*a^3*d*cos(d*x + c)^5 + 6*a^3*d*cos(d*x + c)^4 + 8*a^3*d*cos(d*x + c)^3 -
3*a^3*d*cos(d*x + c) - a^3*d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**8/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35844, size = 220, normalized size = 1.52

$$\frac{\frac{429\left(280\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^6-35\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^4-28\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-5\right)}{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7}-\frac{1155a^{36}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{13}+5460a^{36}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+5005a^{36}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}}{}}{15375360\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/15375360*(429*(280*tan(1/2*d*x + 1/2*c)^6 - 35*tan(1/2*d*x + 1/2*c)^4 - 2
8*tan(1/2*d*x + 1/2*c)^2 - 5)/(a^3*tan(1/2*d*x + 1/2*c)^7) - (1155*a^36*tan
(1/2*d*x + 1/2*c)^13 + 5460*a^36*tan(1/2*d*x + 1/2*c)^11 + 5005*a^36*tan(1/
2*d*x + 1/2*c)^9 - 17160*a^36*tan(1/2*d*x + 1/2*c)^7 - 42042*a^36*tan(1/2*d
*x + 1/2*c)^5 + 210210*a^36*tan(1/2*d*x + 1/2*c))/a^39)/d
```

3.108 $\int (a + a\sec(c + dx))(e\sin(c + dx))^{5/2}\,dx$

Optimal. Leaf size=157

$$-\frac{ae^{5/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{ae^{5/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{6ae^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{2ae(e\sin(c+dx))^{3/2}}{3d}$$

```
[Out] -((a*e^(5/2)*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d) + (a*e^(5/2)*ArcTanh[
Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (6*a*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2
]*Sqrt[e*Sin[c + d*x]])/(5*d*Sqrt[Sin[c + d*x]]) - (2*a*e*(e*Sin[c + d*x])^
(3/2))/(3*d) - (2*a*e*Cos[c + d*x]*(e*Sin[c + d*x])^(3/2))/(5*d)
```

Rubi [A] time = 0.201328, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3872, 2838, 2564, 321, 329, 298, 203, 206, 2635, 2640, 2639}

$$-\frac{ae^{5/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{ae^{5/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{6ae^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{2ae(e\sin(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] -((a*e^(5/2)*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d) + (a*e^(5/2)*ArcTanh[
Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (6*a*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2
]*Sqrt[e*Sin[c + d*x]])/(5*d*Sqrt[Sin[c + d*x]]) - (2*a*e*(e*Sin[c + d*x])^
(3/2))/(3*d) - (2*a*e*Cos[c + d*x]*(e*Sin[c + d*x])^(3/2))/(5*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\sec(c+dx))(e\sin(c+dx))^{5/2}\,dx = -\int (-a-a\cos(c+dx))\sec(c+dx)(e\sin(c+dx))^{5/2}\,dx$$
$$= a\int (e\sin(c+dx))^{5/2}\,dx + a\int \sec(c+dx)(e\sin(c+dx))^{5/2}\,dx$$
$$= -\frac{2ae\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d} + \frac{a\,\text{Subst}\left(\int \frac{x^{5/2}}{1-\frac{x^2}{e^2}}dx, x, e\sin(c+dx)\right)}{de} + \frac{1}{5}$$
$$= -\frac{2ae(e\sin(c+dx))^{3/2}}{3d} - \frac{2ae\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d} + \frac{(ae)\,\text{Subst}\left(\int \frac{\sqrt{}}{1-}\right.}{}$$
$$= \frac{6ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} - \frac{2ae(e\sin(c+dx))^{3/2}}{3d} - \frac{2ae\cos(c}{}$$
$$= \frac{6ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} - \frac{2ae(e\sin(c+dx))^{3/2}}{3d} - \frac{2ae\cos(c}{}$$
$$= -\frac{ae^{5/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{ae^{5/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{6ae^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+d\right.\right.}{5d\sqrt{\sin}}$$

Mathematica [A] time = 0.301539, size = 106, normalized size = 0.68

$$-\frac{a(e\sin(c+dx))^{5/2}\left(10\sin^{\frac{3}{2}}(c+dx)+3\sin(2(c+dx))\sqrt{\sin(c+dx)}+18E\left(\frac{1}{4}(-2c-2dx+\pi)\middle|2\right)+15\tan^{-1}\left(\sqrt{\sin(c+d}\right.\right.}{15d\sin^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] -(a*(e*Sin[c + d*x])^(5/2)*(15*ArcTan[Sqrt[Sin[c + d*x]]] - 15*ArcTanh[Sqrt
[Sin[c + d*x]]] + 18*EllipticE[(-2*c + Pi - 2*d*x)/4, 2] + 10*Sin[c + d*x]^
(3/2) + 3*Sqrt[Sin[c + d*x]]*Sin[2*(c + d*x)]))/(15*d*Sin[c + d*x]^(5/2))
```

Maple [A] time = 1.403, size = 290, normalized size = 1.9

$$-\frac{2\,ae}{3\,d}\left(e\sin\left(dx+c\right)\right)^{\frac{3}{2}}+\frac{a}{d}e^{\frac{5}{2}}\text{Artanh}\left(\sqrt{e\sin\left(dx+c\right)}\frac{1}{\sqrt{e}}\right)-\frac{a}{d}e^{\frac{5}{2}}\arctan\left(\sqrt{e\sin\left(dx+c\right)}\frac{1}{\sqrt{e}}\right)+\frac{3\,ae^3}{5\,d\cos\left(dx+c\right)}\sqrt{-\sin\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/3*a*e*(e*sin(d*x+c))^(3/2)/d+a*e^(5/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1
/2))/d-a*e^(5/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d+3/5/d*a*e^3/cos(d*x
+c)/(e*sin(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d
*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-6/5/d*a*e^3/cos(d*
x+c)/(e*sin(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(
d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+2/5/d*a*e^3/cos(d
*x+c)/(e*sin(d*x+c))^(1/2)*sin(d*x+c)^4-2/5/d*a*e^3/cos(d*x+c)/(e*sin(d*x+c
```

```
))^(1/2)*sin(d*x+c)^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(ae^2\cos(dx+c)^2 - ae^2 + \left(ae^2\cos(dx+c)^2 - ae^2\right)\sec(dx+c)\right)\sqrt{e\sin(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*e^2*cos(d*x + c)^2 - a*e^2 + (a*e^2*cos(d*x + c)^2 - a*e^2)*se
c(d*x + c))*sqrt(e*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c) + a)(e\sin(dx+c))^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*(e*sin(d*x + c))^(5/2), x)
```

3.109 $\int (a + a\sec(c + dx))(e\sin(c + dx))^{3/2}\,dx$

Optimal. Leaf size=154

$$\frac{2ae^2\sqrt{\sin(c+dx)}\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3d\sqrt{e\sin(c+dx)}}+\frac{ae^{3/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{ae^{3/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}-\frac{2ae\sqrt{e\sin(c+}}{d}$$

```
[Out] (a*e^(3/2)*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (a*e^(3/2)*ArcTanh[Sqr
t[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*S
qrt[Sin[c + d*x]])/(3*d*Sqrt[e*Sin[c + d*x]]) - (2*a*e*Sqrt[e*Sin[c + d*x]]
)/d - (2*a*e*Cos[c + d*x]*Sqrt[e*Sin[c + d*x]])/(3*d)
```

Rubi [A] time = 0.199885, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3872, 2838, 2564, 321, 329, 212, 206, 203, 2635, 2642, 2641}

$$\frac{ae^{3/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{ae^{3/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2ae^2\sqrt{\sin(c+dx)}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3d\sqrt{e\sin(c+dx)}}-\frac{2ae\sqrt{e\sin(c+dx)}}{d}-2a$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a*e^(3/2)*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (a*e^(3/2)*ArcTanh[Sqr
t[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*S
qrt[Sin[c + d*x]])/(3*d*Sqrt[e*Sin[c + d*x]]) - (2*a*e*Sqrt[e*Sin[c + d*x]]
)/d - (2*a*e*Cos[c + d*x]*Sqrt[e*Sin[c + d*x]])/(3*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+a\sec(c+dx))(e\sin(c+dx))^{3/2}\,dx = -\int (-a-a\cos(c+dx))\sec(c+dx)(e\sin(c+dx))^{3/2}\,dx$$
$$= a\int (e\sin(c+dx))^{3/2}\,dx + a\int \sec(c+dx)(e\sin(c+dx))^{3/2}\,dx$$
$$= -\frac{2ae\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d} + \frac{a\,\text{Subst}\left(\int \frac{x^{3/2}}{1-\frac{x^2}{e^2}}\,dx, x, e\sin(c+dx)\right)}{de} + \frac{1}{3}\Big($$
$$= -\frac{2ae\sqrt{e\sin(c+dx)}}{d} - \frac{2ae\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d} + \frac{(ae)\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{e^2}\right.}\right.}{a}$$
$$= \frac{2ae^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}} - \frac{2ae\sqrt{e\sin(c+dx)}}{d} - \frac{2ae\cos(c+dx}{}$$
$$= \frac{2ae^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}} - \frac{2ae\sqrt{e\sin(c+dx)}}{d} - \frac{2ae\cos(c+dx}{}$$
$$= \frac{ae^{3/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{ae^{3/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{2ae^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right.\right.}{3d\sqrt{e\sin(}}$$

Mathematica [A] time = 0.574349, size = 170, normalized size = 1.1

$$\frac{a(e\sin(c+dx))^{3/2}\left(-8\text{EllipticF}\left(\frac{1}{4}(-2c-2dx+\pi),2\right)-24\sqrt{\sin(c+dx)}-3\log\left(1-\sqrt{\sin(c+dx)}\right)+3\log\left(\sqrt{\sin(c+dx}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (a*(e*Sin[c + d*x])^(3/2)*(12*ArcTan[Sqrt[Sin[c + d*x]]] + 6*ArcTanh[Sqrt[S
in[c + d*x]]] - 8*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] - 3*Log[1 - Sqrt[Sin[
c + d*x]]] + 3*Log[1 + Sqrt[Sin[c + d*x]]] - 24*Sqrt[Sin[c + d*x]] - 8*Cos[
c + d*x]*Sec[2*(c + d*x)]*Sqrt[Sin[c + d*x]] + 16*Cos[c + d*x]*Sec[2*(c + d
*x)]*Sin[c + d*x]^(5/2)))/(12*d*Sin[c + d*x]^(3/2))
```

Maple [A] time = 1.127, size = 210, normalized size = 1.4

$$\frac{a}{d}e^{\frac{3}{2}}\text{Artanh}\left(\sqrt{e\sin(dx+c)}\frac{1}{\sqrt{e}}\right)+\frac{a}{d}e^{\frac{3}{2}}\arctan\left(\sqrt{e\sin(dx+c)}\frac{1}{\sqrt{e}}\right)-2\,\frac{ae\sqrt{e\sin(dx+c)}}{d}-\frac{ae^2}{3\,d\cos(dx+c)}\sqrt{-\sin(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*sin(d*x+c))^(3/2),x)
```

```
[Out] a*e^(3/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))/d+a*e^(3/2)*arctan((e*sin(d
*x+c))^(1/2)/e^(1/2))/d-2*a*e*(e*sin(d*x+c))^(1/2)/d-1/3/d*a*e^2/cos(d*x+c)
/(e*sin(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+
c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+2/3/d*a*e^2/cos(d*x+c
)/(e*sin(d*x+c))^(1/2)*sin(d*x+c)^3-2/3/d*a*e^2/cos(d*x+c)/(e*sin(d*x+c))^(
```

```
1/2)*sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec (dx+c)+a)(e\sin (dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*(e*sin(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ae\sec (dx+c)+ae)\sqrt{e\sin (dx+c)}\sin (dx+c)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*e*sec(d*x + c) + a*e)*sqrt(e*sin(d*x + c))*sin(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec (dx+c)+a)(e\sin (dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*(e*sin(d*x + c))^(3/2), x)
```

3.110 $\int (a+a\sec(c+dx))\sqrt{e\sin(c+dx)}\,dx$

Optimal. Leaf size=104

$$-\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{a\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}$$

```
[Out] -((a*Sqrt[e]*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d) + (a*Sqrt[e]*ArcTanh[
Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sq
rt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]])
```

Rubi [A] time = 0.150244, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3872, 2838, 2564, 329, 298, 203, 206, 2640, 2639}

$$-\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{a\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] -((a*Sqrt[e]*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d) + (a*Sqrt[e]*ArcTanh[
Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sq
rt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int (a+a\sec(c+dx))\sqrt{e\sin(c+dx)}\,dx &= -\int (-a-a\cos(c+dx))\sec(c+dx)\sqrt{e\sin(c+dx)}\,dx \\
&= a\int \sqrt{e\sin(c+dx)}\,dx + a\int \sec(c+dx)\sqrt{e\sin(c+dx)}\,dx \\
&= \frac{a\,\mathrm{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{e^2}}\,dx, x, e\sin(c+dx)\right)}{de} + \frac{\left(a\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{\sqrt{\sin(c+dx)}} \\
&= \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{(2a)\,\mathrm{Subst}\left(\int \frac{x^2}{1-\frac{x^4}{e^2}}\,dx, x, \sqrt{e\sin(c
\end{aligned}$$

$$\begin{aligned}
&= \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{(ae)\,\mathrm{Subst}\left(\int \frac{1}{e-x^2}\,dx, x, \sqrt{e\sin(c
\end{aligned}$$

$$\begin{aligned}
&= -\frac{a\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{a\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right.\right.}{d\sqrt{\sin(}
\end{aligned}$$

Mathematica [A] time = 0.108921, size = 69, normalized size = 0.66

$$\frac{a\sqrt{e\sin(c+dx)}\left(-2E\left(\frac{1}{4}(-2c-2dx+\pi)\middle|2\right)-\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)+\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)\right)}{d\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (a*(-ArcTan[Sqrt[Sin[c + d*x]]] + ArcTanh[Sqrt[Sin[c + d*x]]] - 2*EllipticE
[(-2*c + Pi - 2*d*x)/4, 2])*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]])
```

Maple [A] time = 1.14, size = 198, normalized size = 1.9

$$\frac{a}{d}\mathrm{Artanh}\left(\sqrt{e\sin(dx+c)}\frac{1}{\sqrt{e}}\right)\sqrt{e}-\frac{a}{d}\arctan\left(\sqrt{e\sin(dx+c)}\frac{1}{\sqrt{e}}\right)\sqrt{e}-2\,\frac{ae\sqrt{-\sin(dx+c)+1}\sqrt{2+2\,\sin(dx+c)}\sqrt{\sin}}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*sin(d*x+c))^(1/2),x)
```

```
[Out] a*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))*e^(1/2)/d-a*arctan((e*sin(d*x+c))^(
1/2)/e^(1/2))*e^(1/2)/d-2/d*a*e*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2
)*sin(d*x+c)^(1/2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*EllipticE((-sin(d*x+c)+1
)^(1/2),1/2*2^(1/2))+1/d*a*e*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*s
in(d*x+c)^(1/2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(
1/2),1/2*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)\sqrt{e\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sqrt(e*sin(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((a\sec(dx+c)+a)\sqrt{e\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)*sqrt(e*sin(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\sqrt{e\sin(c+dx)}\,dx+\int\sqrt{e\sin(c+dx)}\sec(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(sqrt(e*sin(c + d*x)), x) + Integral(sqrt(e*sin(c + d*x))*sec(c
+ d*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sec (dx + c) + a)\sqrt{e \sin (dx + c)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*sqrt(e*sin(d*x + c)), x)
```

3.111 $\int \frac{a+a\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx$

Optimal. Leaf size=103

$$\frac{2a\sqrt{\sin(c+dx)}\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}+\frac{a\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}$$

```
[Out] (a*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*Sqrt[e]) + (a*ArcTanh[Sqrt[e*Si
n[c + d*x]]/Sqrt[e]])/(d*Sqrt[e]) + (2*a*EllipticF[(c - Pi/2 + d*x)/2, 2]*S
qrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c + d*x]])
```

Rubi [A] time = 0.151885, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3872, 2838, 2564, 329, 212, 206, 203, 2642, 2641}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}+\frac{2a\sqrt{\sin(c+dx)}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (a*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*Sqrt[e]) + (a*ArcTanh[Sqrt[e*Si
n[c + d*x]]/Sqrt[e]])/(d*Sqrt[e]) + (2*a*EllipticF[(c - Pi/2 + d*x)/2, 2]*S
qrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c + d*x]])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+a\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx &= -\int \frac{(-a-a\cos(c+dx))\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx\\
&= a\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx + a\int \frac{\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx\\
&= \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{e^2}\right)}\,dx, x, e\sin(c+dx)\right)}{de} + \frac{\left(a\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{\sqrt{e\sin(c+dx)}}\\
&= \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{1-\frac{x^4}{e^2}}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{de}\\
&= \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}} + \frac{a\,\text{Subst}\left(\int \frac{1}{e-x^2}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{d} + \frac{a\,\text{Subst}\left(\right.}{}\\
&= \frac{a\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{a\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}}
\end{aligned}
$$

Mathematica [A] time = 3.20554, size = 201, normalized size = 1.95

$$
\frac{4a\cos\left(\frac{1}{2}(c+dx)\right)\left(4\text{EllipticF}\left(\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{4}(c+dx)\right)}}\right),-1\right)+\sqrt{2}\left(-\Pi\left(-1-\sqrt{2};-\sin^{-1}\left(\frac{1}{\sqrt{\tan\left(\frac{1}{4}(c+dx)\right)}}\right)\middle|-1\right)+\Pi\left(1-\right.\right.\right.}{d\sqrt{\tan\left(\frac{1}{4}(c+dx)\right)}\sqrt{1-}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (4*a*Cos[(c + d*x)/2]*(4*EllipticF[ArcSin[1/Sqrt[Tan[(c + d*x)/4]]], -1] +
Sqrt[2]*(-EllipticPi[-1 - Sqrt[2], -ArcSin[1/Sqrt[Tan[(c + d*x)/4]]], -1] +
 EllipticPi[1 - Sqrt[2], -ArcSin[1/Sqrt[Tan[(c + d*x)/4]]], -1] + EllipticP
i[-1 + Sqrt[2], -ArcSin[1/Sqrt[Tan[(c + d*x)/4]]], -1] - EllipticPi[1 + Sqr
t[2], -ArcSin[1/Sqrt[Tan[(c + d*x)/4]]], -1])))/(d*Sqrt[1 - Cot[(c + d*x)/4
]^2]*Sqrt[e*Sin[c + d*x]]*Sqrt[Tan[(c + d*x)/4]])
```

Maple [A] time = 0.998, size = 122, normalized size = 1.2

$$\frac{a}{d}\mathrm{Artanh}\left(\sqrt{e\sin(dx+c)}\frac{1}{\sqrt{e}}\right)\frac{1}{\sqrt{e}}+\frac{a}{d}\arctan\left(\sqrt{e\sin(dx+c)}\frac{1}{\sqrt{e}}\right)\frac{1}{\sqrt{e}}-\frac{a}{d\cos(dx+c)}\sqrt{-\sin(dx+c)+1}\sqrt{2+2\sin(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x)
```

```
[Out] a*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(1/2)+a*arctan((e*sin(d*x+c))^(
1/2)/e^(1/2))/d/e^(1/2)-1/d*a*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*
sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))/cos(d*x+c)/(e
*sin(d*x+c))^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sec(dx+c)+a}{\sqrt{e\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(e*sin(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(a\sec(dx+c)+a)\sqrt{e\sin(dx+c)}}{e\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)*sqrt(e*sin(d*x + c))/(e*sin(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \frac{1}{\sqrt{e\sin\left(c+dx\right)}}\,dx+\int \frac{\sec\left(c+dx\right)}{\sqrt{e\sin\left(c+dx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(1/sqrt(e*sin(c + d*x)), x) + Integral(sec(c + d*x)/sqrt(e*sin(c
 + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sec\left(dx+c\right)+a}{\sqrt{e\sin\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(e*sin(d*x + c)), x)
```

3.112 $\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=155

$$-\frac{a\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}-\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}-\frac{2a}{de\sqrt{e\sin(c+dx)}}-\frac{2a\cos(c}{de\sqrt{e\sin}}$$

```
[Out] -((a*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(3/2))) + (a*ArcTanh[Sqrt[e
*Sin[c + d*x]]/Sqrt[e]])/(d*e^(3/2)) - (2*a)/(d*e*Sqrt[e*Sin[c + d*x]]) - (
2*a*Cos[c + d*x])/(d*e*Sqrt[e*Sin[c + d*x]]) - (2*a*EllipticE[(c - Pi/2 + d
*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*e^2*Sqrt[Sin[c + d*x]])
```

Rubi [A] time = 0.199393, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3872, 2838, 2564, 325, 329, 298, 203, 206, 2636, 2640, 2639}

$$-\frac{a\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}-\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}-\frac{2a}{de\sqrt{e\sin(c+dx)}}-\frac{2a\cos(c}{de\sqrt{e\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] -((a*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(3/2))) + (a*ArcTanh[Sqrt[e
*Sin[c + d*x]]/Sqrt[e]])/(d*e^(3/2)) - (2*a)/(d*e*Sqrt[e*Sin[c + d*x]]) - (
2*a*Cos[c + d*x])/(d*e*Sqrt[e*Sin[c + d*x]]) - (2*a*EllipticE[(c - Pi/2 + d
*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(d*e^2*Sqrt[Sin[c + d*x]])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx = -\int \frac{(-a-a\cos(c+dx))\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx$$

$$= a\int \frac{1}{(e\sin(c+dx))^{3/2}}\,dx + a\int \frac{\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx$$

$$= -\frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{a\int\sqrt{e\sin(c+dx)}\,dx}{e^2} + \frac{a\,\text{Subst}\left(\int \frac{1}{x^{3/2}\left(1-\frac{x^2}{e^2}\right)}dx, x, e\sin(c+dx)\right)}{de}$$

$$= -\frac{2a}{de\sqrt{e\sin(c+dx)}} - \frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{x}}{1-\frac{x^2}{e^2}}dx, x, e\sin(c+dx)\right)}{de^3} - \frac{\left(a\sqrt{e\sin(c+\right.}{e^2\sqrt{}}$$

$$= -\frac{2a}{de\sqrt{e\sin(c+dx)}} - \frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}} + \frac{(2a)\,\text{Subst}}{}$$

$$= -\frac{2a}{de\sqrt{e\sin(c+dx)}} - \frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}} + \frac{a\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{a\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} + \frac{a\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} - \frac{2a}{de\sqrt{e\sin(c+dx)}} - \frac{2a\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2aE}{}$$

Mathematica [A] time = 0.362867, size = 143, normalized size = 0.92

$$-\frac{a\sin^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)\sec\left(\frac{1}{2}(c+dx)\right)\left(2\sqrt{\sin(c+dx)}\csc\left(\frac{1}{2}(c+dx)\right)-2\sec\left(\frac{1}{2}(c+dx)\right)E\left(\frac{1}{4}(-2c-2dx+\pi)\right.\right.}{2d(e\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] -(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]*(ArcTan[Sqrt[Sin[c + d*x]]]*Sec[(c
+ d*x)/2] - ArcTanh[Sqrt[Sin[c + d*x]]]*Sec[(c + d*x)/2] - 2*EllipticE[(-2*
c + Pi - 2*d*x)/4, 2]*Sec[(c + d*x)/2] + 2*Csc[(c + d*x)/2]*Sqrt[Sin[c + d*
x]])*Sin[c + d*x]^(3/2))/(2*d*(e*Sin[c + d*x])^(3/2))
```

Maple [A] time = 1.326, size = 247, normalized size = 1.6

$$\frac{a}{d}\text{Artanh}\left(\sqrt{e\sin(dx+c)}\frac{1}{\sqrt{e}}\right)e^{-\frac{3}{2}} - \frac{a}{d}\arctan\left(\sqrt{e\sin(dx+c)}\frac{1}{\sqrt{e}}\right)e^{-\frac{3}{2}} - 2\frac{a}{ed\sqrt{e\sin(dx+c)}} + 2\frac{a\sqrt{-\sin(dx+c)+1}\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x)
```

```
[Out] a*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(3/2)-a*arctan((e*sin(d*x+c))^(
1/2)/e^(1/2))/d/e^(3/2)-2*a/d/e/(e*sin(d*x+c))^(1/2)+2/d*a/e/cos(d*x+c)/(e*
sin(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(
1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/d*a/e/cos(d*x+c)/(e*sin
(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2
)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*a*cos(d*x+c)/d/e/(e*sin(d*
```

```
x+c))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(a\sec(dx+c)+a)\sqrt{e\sin(dx+c)}}{e^2\cos(dx+c)^2-e^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*sec(d*x + c) + a)*sqrt(e*sin(d*x + c))/(e^2*cos(d*x + c)^2 - e
^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{a\sec(dx+c)+a}{(e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/(e*sin(d*x + c))^(3/2), x)
```

3.113 $\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=160

$$\frac{2a\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3de^2\sqrt{e\sin(c+dx)}}+\frac{a\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}-\frac{2a}{3de(e\sin(c+dx))^{3/2}}-$$

```
[Out] (a*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(5/2)) + (a*ArcTanh[Sqrt[e*Si
n[c + d*x]]/Sqrt[e]])/(d*e^(5/2)) - (2*a)/(3*d*e*(e*Sin[c + d*x])^(3/2)) -
(2*a*Cos[c + d*x])/(3*d*e*(e*Sin[c + d*x])^(3/2)) + (2*a*EllipticF[(c - Pi/
2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Rubi [A] time = 0.200585, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3872, 2838, 2564, 325, 329, 212, 206, 203, 2636, 2642, 2641}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}+\frac{a\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}+\frac{2a\sqrt{\sin(c+dx)}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3de^2\sqrt{e\sin(c+dx)}}-\frac{2a}{3de(e\sin(c+dx))^{3/2}}-\frac{2a\cos}{3de(e\sin}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (a*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(5/2)) + (a*ArcTanh[Sqrt[e*Si
n[c + d*x]]/Sqrt[e]])/(d*e^(5/2)) - (2*a)/(3*d*e*(e*Sin[c + d*x])^(3/2)) -
(2*a*Cos[c + d*x])/(3*d*e*(e*Sin[c + d*x])^(3/2)) + (2*a*EllipticF[(c - Pi/
2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
```

```
x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx = -\int \frac{(-a-a\cos(c+dx))\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx$$
$$= a\int \frac{1}{(e\sin(c+dx))^{5/2}}\,dx + a\int \frac{\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx$$
$$= -\frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3e^2} + \frac{a\,\text{Subst}\left(\int \frac{1}{x^{5/2}\left(1-\frac{x^2}{e^2}\right)}\,dx, x, e\sin(c+dx)\right)}{de}$$
$$= -\frac{2a}{3de(e\sin(c+dx))^{3/2}} - \frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{e^2}\right)}\,dx, x, e\sin(c+dx)\right)}{de^3} + \frac{(a\cdot$$
$$= -\frac{2a}{3de(e\sin(c+dx))^{3/2}} - \frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}} + \frac{(2a)\,\text{S}}{}$$
$$= -\frac{2a}{3de(e\sin(c+dx))^{3/2}} - \frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3de^2\sqrt{e\sin(c+dx)}} + \frac{a\,\text{Sub}}{}$$
$$= \frac{a\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} + \frac{a\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} - \frac{2a}{3de(e\sin(c+dx))^{3/2}} - \frac{2a\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} +$$

Mathematica [A] time = 0.347795, size = 120, normalized size = 0.75

$$-\frac{a\sqrt{\sin(c+dx)}(\cos(c+dx)+1)\sec^2\left(\frac{1}{2}(c+dx)\right)\left(2\text{EllipticF}\left(\frac{1}{4}(-2c-2dx+\pi),2\right)-3\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)-3\tanh^{-}\right.}{6de^2\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] -(a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(-3*ArcTan[Sqrt[Sin[c + d*x]]] -
3*ArcTanh[Sqrt[Sin[c + d*x]]] + 2*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] + Csc
[(c + d*x)/2]^2*Sqrt[Sin[c + d*x]])*Sqrt[Sin[c + d*x]])/(6*d*e^2*Sqrt[e*Sin
[c + d*x]])
```

Maple [A] time = 1.561, size = 212, normalized size = 1.3

$$-\frac{2a}{3ed}(e\sin(dx+c))^{-\frac{3}{2}} + \frac{a}{d}\text{Artanh}\left(\sqrt{e\sin(dx+c)}\frac{1}{\sqrt{e}}\right)e^{-\frac{5}{2}} + \frac{a}{d}\arctan\left(\sqrt{e\sin(dx+c)}\frac{1}{\sqrt{e}}\right)e^{-\frac{5}{2}} - \frac{a}{3de^2\cos(dx+c)}\sqrt{s}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x)
```

```
[Out] -2/3*a/d/e/(e*sin(d*x+c))^(3/2)+a*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e
^(5/2)+a*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))/d/e^(5/2)-1/3/d*a/e^2*sin(d*x
+c)^(1/2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*
x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+2/3/d*a/e^2*sin(d*
x+c)/cos(d*x+c)/(e*sin(d*x+c))^(1/2)-2/3/d*a/e^2/sin(d*x+c)/cos(d*x+c)/(e*s
```

```
in(d*x+c))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(a\sec(dx+c)+a)\sqrt{e\sin(dx+c)}}{\left(e^3\cos(dx+c)^2-e^3\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*sec(d*x + c) + a)*sqrt(e*sin(d*x + c))/((e^3*cos(d*x + c)^2 -
e^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sec(dx+c)+a}{(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/(e*sin(d*x + c))^(5/2), x)
```

3.114 $\int (a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}\,dx$

Optimal. Leaf size=194

$$-\frac{2a^2e^{5/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2a^2e^{5/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}-\frac{9a^2e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{4a^2e(e\sin(c+}{3d}$$

```
[Out] (-2*a^2*e^(5/2)*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a^2*e^(5/2)*Ar
cTanh[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d - (9*a^2*e^2*EllipticE[(c - Pi/2 + d
*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*d*Sqrt[Sin[c + d*x]]) - (4*a^2*e*(e*Sin[
c + d*x])^(3/2))/(3*d) - (2*a^2*e*Cos[c + d*x]*(e*Sin[c + d*x])^(3/2))/(5*d
) + (a^2*e*Sec[c + d*x]*(e*Sin[c + d*x])^(3/2))/d
```

Rubi [A] time = 0.382437, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {3872, 2873, 2635, 2640, 2639, 2564, 321, 329, 298, 203, 206, 2566}

$$-\frac{2a^2e^{5/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2a^2e^{5/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}-\frac{9a^2e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}}-\frac{4a^2e(e\sin(c+}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (-2*a^2*e^(5/2)*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a^2*e^(5/2)*Ar
cTanh[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d - (9*a^2*e^2*EllipticE[(c - Pi/2 + d
*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*d*Sqrt[Sin[c + d*x]]) - (4*a^2*e*(e*Sin[
c + d*x])^(3/2))/(3*d) - (2*a^2*e*Cos[c + d*x]*(e*Sin[c + d*x])^(3/2))/(5*d
) + (a^2*e*Sec[c + d*x]*(e*Sin[c + d*x])^(3/2))/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}\,dx = \int (-a-a\cos(c+dx))^2\sec^2(c+dx)(e\sin(c+dx))^{5/2}\,dx$$

$$= \int \left(a^2(e\sin(c+dx))^{5/2} + 2a^2\sec(c+dx)(e\sin(c+dx))^{5/2} + a^2\sec^2(c+dx)(\right.$$

$$= a^2\int (e\sin(c+dx))^{5/2}\,dx + a^2\int \sec^2(c+dx)(e\sin(c+dx))^{5/2}\,dx + \left(2a^2\right)\int$$

$$= -\frac{2a^2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d} + \frac{a^2e\sec(c+dx)(e\sin(c+dx))^{3/2}}{d} + \frac{(2a}{}$$

$$= -\frac{4a^2e(e\sin(c+dx))^{3/2}}{3d} - \frac{2a^2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5d} + \frac{a^2e\sec(c+dx}{}$$

$$= -\frac{9a^2e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} - \frac{4a^2e(e\sin(c+dx))^{3/2}}{3d} - \frac{2a^2e\,c}{}$$

$$= -\frac{9a^2e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5d\sqrt{\sin(c+dx)}} - \frac{4a^2e(e\sin(c+dx))^{3/2}}{3d} - \frac{2a^2e\,c}{}$$

$$= -\frac{2a^2e^{5/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{2a^2e^{5/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} - \frac{9a^2e^2E\left(\frac{1}{2}\left(c-\right.\right.}{5}$$

Mathematica [C] time = 16.6173, size = 205, normalized size = 1.06

$$\frac{2a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)(e\sin(c+dx))^{5/2}\sec^4\left(\frac{1}{2}\sin^{-1}(\sin(c+dx))\right)\left(9\sin^{\frac{3}{2}}(c+dx)\sqrt{\cos^2(c+dx)}\text{Hypergeome}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*Cos[(c + d*x)/2]^4*Sec[c + d*x]*Sec[ArcSin[Sin[c + d*x]]/2]^4*(e*Sin
[c + d*x])^(5/2)*(-15*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] + 15*
ArcTanh[Sqrt[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] - 9*Sin[c + d*x]^(3/2) - 1
0*Sqrt[Cos[c + d*x]^2]*Sin[c + d*x]^(3/2) + 9*Sqrt[Cos[c + d*x]^2]*Hypergeo
metric2F1[3/4, 3/2, 7/4, Sin[c + d*x]^2]*Sin[c + d*x]^(3/2) + 3*Sin[c + d*x
]^(7/2)))/(15*d*Sin[c + d*x]^(5/2))
```

Maple [A] time = 2.388, size = 265, normalized size = 1.4

$$\frac{a^2}{30\,d\cos(dx+c)}\left(60\,\mathrm{Artanh}\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)\sqrt{e\sin(dx+c)}e^{5/2}\cos(dx+c) - 60\sqrt{e\sin(dx+c)}\arctan\left(\frac{\sqrt{e\sin(dx+}}{\sqrt{e}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/30/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*a^2*(60*arctanh((e*sin(d*x+c))^(1/2)/e
^(1/2))*(e*sin(d*x+c))^(1/2)*e^(5/2)*cos(d*x+c)-60*(e*sin(d*x+c))^(1/2)*arc
tan((e*sin(d*x+c))^(1/2)/e^(1/2))*e^(5/2)*cos(d*x+c)+54*(-sin(d*x+c)+1)^(1/
```

```
2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),
1/2*2^(1/2))*e^3-27*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)
^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*e^3+12*e^3*cos(d*x+c)^4
+40*e^3*cos(d*x+c)^3-42*e^3*cos(d*x+c)^2-40*e^3*cos(d*x+c)+30*e^3)/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$\mathrm{integral}\left(-\left(a^2e^2\cos\left(dx+c\right)^2-a^2e^2+\left(a^2e^2\cos\left(dx+c\right)^2-a^2e^2\right)\sec\left(dx+c\right)^2+2\left(a^2e^2\cos\left(dx+c\right)^2-a^2e^2\right)\sec\left(dx\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*e^2*cos(d*x + c)^2 - a^2*e^2 + (a^2*e^2*cos(d*x + c)^2 - a^2
*e^2)*sec(d*x + c)^2 + 2*(a^2*e^2*cos(d*x + c)^2 - a^2*e^2)*sec(d*x + c))*s
qrt(e*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(dx+c\right)+a\right)^2\left(e\sin\left(dx+c\right)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(5/2), x)
```

3.115 $\int (a+a\sec(c+dx))^2 (e\sin(c+dx))^{3/2}\,dx$

Optimal. Leaf size=192

$$-\frac{a^2e^2\sqrt{\sin(c+dx)}\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3d\sqrt{e\sin(c+dx)}}+\frac{2a^2e^{3/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2a^2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}-\frac{4a^2e\sqrt{e\sin}}{}$$

```
[Out] (2*a^2*e^(3/2)*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a^2*e^(3/2)*Arc
Tanh[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d - (a^2*e^2*EllipticF[(c - Pi/2 + d*x)
/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*Sqrt[e*Sin[c + d*x]]) - (4*a^2*e*Sqrt[e*Sin
[c + d*x]])/d - (2*a^2*e*Cos[c + d*x]*Sqrt[e*Sin[c + d*x]])/(3*d) + (a^2*e*
Sec[c + d*x]*Sqrt[e*Sin[c + d*x]])/d
```

Rubi [A] time = 0.380084, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {3872, 2873, 2635, 2642, 2641, 2564, 321, 329, 212, 206, 203, 2566}

$$\frac{2a^2e^{3/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2a^2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}-\frac{a^2e^2\sqrt{\sin(c+dx)}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3d\sqrt{e\sin(c+dx)}}-\frac{4a^2e\sqrt{e\sin(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2),x]
```


```
[Out] (2*a^2*e^(3/2)*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a^2*e^(3/2)*Arc
Tanh[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d - (a^2*e^2*EllipticF[(c - Pi/2 + d*x)
/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*Sqrt[e*Sin[c + d*x]]) - (4*a^2*e*Sqrt[e*Sin
[c + d*x]])/d - (2*a^2*e*Cos[c + d*x]*Sqrt[e*Sin[c + d*x]])/(3*d) + (a^2*e*
Sec[c + d*x]*Sqrt[e*Sin[c + d*x]])/d
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}\,dx = \int (-a-a\cos(c+dx))^2\sec^2(c+dx)(e\sin(c+dx))^{3/2}\,dx$$
$$= \int \left(a^2(e\sin(c+dx))^{3/2}+2a^2\sec(c+dx)(e\sin(c+dx))^{3/2}+a^2\sec^2(c+dx)(e$$
$$= a^2\int (e\sin(c+dx))^{3/2}\,dx + a^2\int \sec^2(c+dx)(e\sin(c+dx))^{3/2}\,dx + \left(2a^2\right)\int$$
$$= -\frac{2a^2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d} + \frac{a^2e\sec(c+dx)\sqrt{e\sin(c+dx)}}{d} + \frac{\left(2a^2\right)\mathrm{Su}}{}$$
$$= -\frac{4a^2e\sqrt{e\sin(c+dx)}}{d} - \frac{2a^2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3d} + \frac{a^2e\sec(c+dx)\sqrt{e\mathrm{s}}}{d}$$
$$= -\frac{a^2e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}} - \frac{4a^2e\sqrt{e\sin(c+dx)}}{d} - \frac{2a^2e\cos(c}{}$$
$$= -\frac{a^2e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3d\sqrt{e\sin(c+dx)}} - \frac{4a^2e\sqrt{e\sin(c+dx)}}{d} - \frac{2a^2e\cos(c}{}$$
$$= \frac{2a^2e^{3/2}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{2a^2e^{3/2}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} - \frac{a^2e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}\right.\right.}{3d\sqrt{e}}$$

Mathematica [C] time = 14.6744, size = 204, normalized size = 1.06

$$\frac{16a^2e\sin^4\left(\frac{1}{2}\sin^{-1}(\sin(c+dx))\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{e\sin(c+dx)}\left(-\sqrt{\sin(c+dx)}\sqrt{\cos^2(c+dx)}\mathrm{Hypergeome}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (16*a^2*e*Cos[(c + d*x)/2]^4*Sec[c + d*x]*Sqrt[e*Sin[c + d*x]]*(6*ArcTan[Sq
rt[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] + 6*ArcTanh[Sqrt[Sin[c + d*x]]]*Sqrt
[Cos[c + d*x]^2] + Sqrt[Sin[c + d*x]] - 12*Sqrt[Cos[c + d*x]^2]*Sqrt[Sin[c
+ d*x]] - Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/4, 1/2, 5/4, Sin[c + d*x
]^2]*Sqrt[Sin[c + d*x]] + 2*Sin[c + d*x]^(5/2))*Sin[ArcSin[Sin[c + d*x]]/2]
^4)/(3*d*Sin[c + d*x]^(9/2))
```

Maple [A] time = 2.076, size = 201, normalized size = 1.1

$$\frac{a^2}{6d\cos(dx+c)}\left(12\cos(dx+c)e^{3/2}\sqrt{e\sin(dx+c)}\arctan\left(\frac{\sqrt{e\sin(dx+c)}}{\sqrt{e}}\right)+12\cos(dx+c)e^{3/2}\sqrt{e\sin(dx+c)}\mathrm{Artanh}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/6/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*a^2*(12*cos(d*x+c)*e^(3/2)*(e*sin(d*x+c
))^(1/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))+12*cos(d*x+c)*e^(3/2)*(e*sin(
```

```
d*x+c))^(1/2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))+(-sin(d*x+c)+1)^(1/2)*(
2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*
2^(1/2))*e^2-4*e^2*sin(d*x+c)*cos(d*x+c)^2-24*e^2*sin(d*x+c)*cos(d*x+c)+6*e
^2*sin(d*x+c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^2 (e\sin(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^2e\sec(dx+c)^2+2\,a^2e\sec(dx+c)+a^2e\right)\sqrt{e\sin(dx+c)}\sin(dx+c)\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*e*sec(d*x + c)^2 + 2*a^2*e*sec(d*x + c) + a^2*e)*sqrt(e*sin(d
*x + c))*sin(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^2 (e\sin(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2), x)
```

3.116 $\int (a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}\,dx$

Optimal. Leaf size=138

$$-\frac{2a^2\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2a^2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de}+\frac{a^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e}}{d\sqrt{\sin(c+dx)}}$$

```
[Out] (-2*a^2*Sqrt[e]*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a^2*Sqrt[e]*Ar
cTanh[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (a^2*EllipticE[(c - Pi/2 + d*x)/2,
 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]]) + (a^2*Sec[c + d*x]*(e*Sin
[c + d*x])^(3/2))/(d*e)
```

Rubi [A] time = 0.307368, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3872, 2873, 2640, 2639, 2564, 329, 298, 203, 206, 2571}

$$-\frac{2a^2\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{2a^2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d}+\frac{a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de}+\frac{a^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e}}{d\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (-2*a^2*Sqrt[e]*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (2*a^2*Sqrt[e]*Ar
cTanh[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/d + (a^2*EllipticE[(c - Pi/2 + d*x)/2,
 2]*Sqrt[e*Sin[c + d*x]])/(d*Sqrt[Sin[c + d*x]]) + (a^2*Sec[c + d*x]*(e*Sin
[c + d*x])^(3/2))/(d*e)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
```

```
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}\,dx = \int (-a-a\cos(c+dx))^2\sec^2(c+dx)\sqrt{e\sin(c+dx)}\,dx$$
$$= \int \left(a^2\sqrt{e\sin(c+dx)}+2a^2\sec(c+dx)\sqrt{e\sin(c+dx)}+a^2\sec^2(c+dx)\sqrt{e\sin(c+dx)}\right.$$
$$= a^2\int \sqrt{e\sin(c+dx)}\,dx + a^2\int \sec^2(c+dx)\sqrt{e\sin(c+dx)}\,dx + \left(2a^2\right)\int \sec(c$$
$$= \frac{a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de} - \frac{1}{2}a^2\int \sqrt{e\sin(c+dx)}\,dx + \frac{\left(2a^2\right)\text{Subst}\left(\int \right.}{}$$
$$= \frac{2a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de} + $$
$$= \frac{a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{d\sqrt{\sin(c+dx)}} + \frac{a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de} + $$
$$= -\frac{2a^2\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{2a^2\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d} + \frac{a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+d\right.\right.}{d\sqrt{\sin}}$$

Mathematica [C] time = 1.93746, size = 168, normalized size = 1.22

$$-\frac{2a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{e\sin(c+dx)}\sec^4\left(\frac{1}{2}\sin^{-1}(\sin(c+dx))\right)\left(\sin^{\frac{3}{2}}(c+dx)\sqrt{\cos^2(c+dx)}\text{Hypergeometri}\right.}{3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (-2*a^2*Cos[(c + d*x)/2]^4*Sec[c + d*x]*Sec[ArcSin[Sin[c + d*x]]/2]^4*Sqrt[
e*Sin[c + d*x]]*(3*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] - 3*ArcT
anh[Sqrt[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] - 3*Sin[c + d*x]^(3/2) + Sqrt[
Cos[c + d*x]^2]*Hypergeometric2F1[3/4, 3/2, 7/4, Sin[c + d*x]^2]*Sin[c + d*
x]^(3/2)))/(3*d*Sqrt[Sin[c + d*x]])
```

Maple [A] time = 2.126, size = 219, normalized size = 1.6

$$\frac{a^2}{2d\cos(dx+c)}\left(\sqrt{-\sin(dx+c)+1}\sqrt{2+2\sin(dx+c)}\sqrt{\sin(dx+c)}\text{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)e-2\sqrt{-\sin(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*a^2*((-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x
+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*e-
2*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((
-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*e+4*cos(d*x+c)*(e*sin(d*x+c))^(1/2)*e^(1/
2)*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))-4*cos(d*x+c)*(e*sin(d*x+c))^(1/2)*
e^(1/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))-2*e*cos(d*x+c)^2+2*e)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^2\sqrt{e\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sqrt(e*sin(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2\sec(dx+c)^2+2a^2\sec(dx+c)+a^2\right)\sqrt{e\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*sqrt(e*sin(d*x + c
)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^2\sqrt{e\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*sqrt(e*sin(d*x + c)), x)
```

3.117 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx$

Optimal. Leaf size=139

$$\frac{3a^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{d\sqrt{e\sin(c+dx)}}+\frac{2a^2\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}+\frac{a^2\sec(c+dx)\sqrt{e\,\text{s}}}{de}$$

```
[Out] (2*a^2*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*Sqrt[e]) + (2*a^2*ArcTanh[S
qrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*Sqrt[e]) + (3*a^2*EllipticF[(c - Pi/2 + d*
x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c + d*x]]) + (a^2*Sec[c + d*x]*S
qrt[e*Sin[c + d*x]])/(d*e)
```

Rubi [A] time = 0.307262, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3872, 2873, 2642, 2641, 2564, 329, 212, 206, 203, 2571}

$$\frac{2a^2\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}}+\frac{a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{de}+\frac{3a^2\sqrt{\sin(c+dx)}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\right|}{d\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (2*a^2*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*Sqrt[e]) + (2*a^2*ArcTanh[S
qrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*Sqrt[e]) + (3*a^2*EllipticF[(c - Pi/2 + d*
x)/2, 2]*Sqrt[Sin[c + d*x]])/(d*Sqrt[e*Sin[c + d*x]]) + (a^2*Sec[c + d*x]*S
qrt[e*Sin[c + d*x]])/(d*e)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx = \int \frac{(-a-a\cos(c+dx))^2\sec^2(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx$$
$$= \int \left(\frac{a^2}{\sqrt{e\sin(c+dx)}} + \frac{2a^2\sec(c+dx)}{\sqrt{e\sin(c+dx)}} + \frac{a^2\sec^2(c+dx)}{\sqrt{e\sin(c+dx)}}\right)dx$$
$$= a^2\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx + a^2\int \frac{\sec^2(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx + \left(2a^2\right)\int \frac{\sec(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx$$
$$= \frac{a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{de} + \frac{1}{2}a^2\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx + \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{\sqrt{x}\left(1-\frac{x^2}{e^2}\right)}dx, x, e\right.}{de}$$
$$= \frac{2a^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}} + \frac{a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{de} + \frac{\left(4a^2\right)\text{Subst}\left(\int \frac{1}{1-\ldots}\right.}{}$$
$$= \frac{3a^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}} + \frac{a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{de} + \frac{\left(2a^2\right)\text{Subst}\left(\int \frac{1}{e-x\ldots}\right.}{}$$
$$= \frac{2a^2\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{2a^2\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{d\sqrt{e}} + \frac{3a^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d\sqrt{e\sin(c+dx)}}$$

Mathematica [C] time = 64.3412, size = 164, normalized size = 1.18

$$\frac{a^2\sqrt{\sin(c+dx)}\cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sec^4\left(\frac{1}{2}\sin^{-1}(\sin(c+dx))\right)\left(3\sqrt{\sin(c+dx)}\sqrt{\cos^2(c+dx)}\text{Hypergeometric2}\ldots\right.}{d\sqrt{e\,\text{si}\ldots}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sqrt[e*Sin[c + d*x]],x]
```

```
[Out] (a^2*Cos[(c + d*x)/2]^4*Sec[c + d*x]*Sec[ArcSin[Sin[c + d*x]]/2]^4*(2*ArcTa
n[Sqrt[Sin[c + d*x]]]*Sqrt[Cos[c + d*x]^2] + 2*ArcTanh[Sqrt[Sin[c + d*x]]]*
Sqrt[Cos[c + d*x]^2] + Sqrt[Sin[c + d*x]] + 3*Sqrt[Cos[c + d*x]^2]*Hypergeo
metric2F1[1/4, 1/2, 5/4, Sin[c + d*x]^2]*Sqrt[Sin[c + d*x]])*Sqrt[Sin[c + d
*x]])/(d*Sqrt[e*Sin[c + d*x]])
```

Maple [A] time = 2.059, size = 163, normalized size = 1.2

$$-\frac{a^2}{2d\cos(dx+c)}\left(3\sqrt{e}\sqrt{-\sin(dx+c)+1}\sqrt{2+2\sin(dx+c)}\sqrt{\sin(dx+c)}\text{EllipticF}\left(\sqrt{-\sin(dx+c)+1},1/2\sqrt{2}\right)-4\,\text{c}\ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x)
```

```
[Out] -1/2/e^(1/2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*a^2*(3*e^(1/2)*(-sin(d*x+c)+1)
^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1
/2),1/2*2^(1/2))-4*cos(d*x+c)*(e*sin(d*x+c))^(1/2)*arctanh((e*sin(d*x+c))^(
1/2)/e^(1/2))-4*cos(d*x+c)*(e*sin(d*x+c))^(1/2)*arctan((e*sin(d*x+c))^(1/2)
/e^(1/2))-2*e^(1/2)*sin(d*x+c))/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\left(a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2\right)\sqrt{e\sin\left(dx+c\right)}}{e\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*sqrt(e*sin(d*x + c
))/(e*sin(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int\frac{1}{\sqrt{e\sin\left(c+dx\right)}}\,dx+\int\frac{2\sec\left(c+dx\right)}{\sqrt{e\sin\left(c+dx\right)}}\,dx+\int\frac{\sec^2\left(c+dx\right)}{\sqrt{e\sin\left(c+dx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(1/sqrt(e*sin(c + d*x)), x) + Integral(2*sec(c + d*x)/sqrt(e*
sin(c + d*x)), x) + Integral(sec(c + d*x)**2/sqrt(e*sin(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\left(a\sec\left(dx+c\right)+a\right)^2}{\sqrt{e\sin\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(e*sin(d*x + c)), x)
```

3.118 $\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=224

$$-\frac{2a^2\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}+\frac{3a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de^3}-\frac{5a^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}$$

```
[Out] (-2*a^2*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(3/2)) + (2*a^2*ArcTanh[
Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(3/2)) - (4*a^2)/(d*e*Sqrt[e*Sin[c + d*
x]]) - (2*a^2*Cos[c + d*x])/(d*e*Sqrt[e*Sin[c + d*x]]) - (2*a^2*Sec[c + d*x
])/(d*e*Sqrt[e*Sin[c + d*x]]) - (5*a^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqr
t[e*Sin[c + d*x]])/(d*e^2*Sqrt[Sin[c + d*x]]) + (3*a^2*Sec[c + d*x]*(e*Sin[
c + d*x])^(3/2))/(d*e^3)
```

Rubi [A] time = 0.424021, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {3872, 2873, 2636, 2640, 2639, 2564, 325, 329, 298, 203, 206, 2570, 2571}

$$-\frac{2a^2\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}}+\frac{3a^2\sec(c+dx)(e\sin(c+dx))^{3/2}}{de^3}-\frac{5a^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^2*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(3/2)) + (2*a^2*ArcTanh[
Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(3/2)) - (4*a^2)/(d*e*Sqrt[e*Sin[c + d*
x]]) - (2*a^2*Cos[c + d*x])/(d*e*Sqrt[e*Sin[c + d*x]]) - (2*a^2*Sec[c + d*x
])/(d*e*Sqrt[e*Sin[c + d*x]]) - (5*a^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqr
t[e*Sin[c + d*x]])/(d*e^2*Sqrt[Sin[c + d*x]]) + (3*a^2*Sec[c + d*x]*(e*Sin[
c + d*x])^(3/2))/(d*e^3)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx = \int \frac{(-a-a\cos(c+dx))^2\sec^2(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx$$

$$= \int \left(\frac{a^2}{(e\sin(c+dx))^{3/2}} + \frac{2a^2\sec(c+dx)}{(e\sin(c+dx))^{3/2}} + \frac{a^2\sec^2(c+dx)}{(e\sin(c+dx))^{3/2}}\right)dx$$

$$= a^2\int \frac{1}{(e\sin(c+dx))^{3/2}}\,dx + a^2\int \frac{\sec^2(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx + \left(2a^2\right)\int \frac{\sec(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx$$

$$= -\frac{2a^2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\sec(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{a^2\int\sqrt{e\sin(c+dx)}\,dx}{e^2} + \frac{\left(3a^2\right)\int\sec^2(c+dx)\sqrt{e\sin}}{e^2}$$

$$= -\frac{4a^2}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\sec(c+dx)}{de\sqrt{e\sin(c+dx)}} + \frac{3a^2\sec(c+dx)(e\sin(c+dx))^{3/}}{de^3}$$

$$= -\frac{4a^2}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\sec(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin}}{de^2\sqrt{\sin(c+dx)}}$$

$$= -\frac{4a^2}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\sec(c+dx)}{de\sqrt{e\sin(c+dx)}} - \frac{5a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin}}{de^2\sqrt{\sin(c+dx)}}$$

$$= -\frac{2a^2\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} + \frac{2a^2\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{3/2}} - \frac{4a^2}{de\sqrt{e\sin(c+dx)}} - \frac{2a^2\cos(c+dx)}{de\sqrt{e\sin(c+dx)}}$$

Mathematica [C] time = 10.6163, size = 135, normalized size = 0.6

$$-\frac{2a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\cot(c+dx)\sqrt{e\sin(c+dx)}\sec^4\left(\frac{1}{2}\sin^{-1}(\sin(c+dx))\right)\left(\sin^2(c+dx)\text{Hypergeometric2F1}\left(\frac{3}{4},\frac{3}{2},\frac{7}{4},\right.\right.}{3de^2\sqrt{\cos^2(}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Sin[c + d*x])^(3/2),x]
```

```
[Out] (-2*a^2*Cos[(c + d*x)/2]^4*Cot[c + d*x]*Sec[ArcSin[Sin[c + d*x]]/2]^4*Sqrt[
e*Sin[c + d*x]]*(6*Hypergeometric2F1[-1/4, 1, 3/4, Sin[c + d*x]^2] + 6*Hype
rgeometric2F1[-1/4, 3/2, 3/4, Sin[c + d*x]^2] + Hypergeometric2F1[3/4, 3/2,
 7/4, Sin[c + d*x]^2]*Sin[c + d*x]^2))/(3*d*e^2*Sqrt[Cos[c + d*x]^2])
```

Maple [A] time = 2.436, size = 238, normalized size = 1.1

$$\frac{a^2}{2d\cos(dx+c)}\left(10e^{3/2}\sqrt{-\sin(dx+c)+1}\sqrt{2+2\sin(dx+c)}\sqrt{\sin(dx+c)}\text{EllipticE}\left(\sqrt{-\sin(dx+c)+1},1/2\sqrt{2}\right)-5e\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/2/e^(5/2)/(e*sin(d*x+c))^(1/2)/cos(d*x+c)*a^2*(10*e^(3/2)*(-sin(d*x+c)+1)
^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1
/2),1/2*2^(1/2))-5*e^(3/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin
(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-10*e^(3/2)*cos(d
*x+c)^2-8*e^(3/2)*cos(d*x+c)+4*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))*(e*sin
(d*x+c))^(1/2)*e*cos(d*x+c)-4*(e*sin(d*x+c))^(1/2)*arctan((e*sin(d*x+c))^(1
/2)/e^(1/2))*e*cos(d*x+c)+2*e^(3/2))/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a^2 \sec(dx+c)^2 + 2\,a^2 \sec(dx+c) + a^2\right)\sqrt{e \sin(dx+c)}}{e^2 \cos(dx+c)^2 - e^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*sqrt(e*sin(d*x +
c))/(e^2*cos(d*x + c)^2 - e^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a \sec(dx+c) + a)^2}{(e \sin(dx+c))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="giac")

[Out] integrate((a*sec(d*x + c) + a)^2/(e*sin(d*x + c))^(3/2), x)
```

3.119 $\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=234

$$\frac{7a^2\sqrt{\sin(c+dx)}\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3de^2\sqrt{e\sin(c+dx)}}+\frac{2a^2\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}+\frac{5a^2\sec(c+dx)}{3a}$$

```
[Out] (2*a^2*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(5/2)) + (2*a^2*ArcTanh[S
qrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(5/2)) - (4*a^2)/(3*d*e*(e*Sin[c + d*x])
^(3/2)) - (2*a^2*Cos[c + d*x])/(3*d*e*(e*Sin[c + d*x])^(3/2)) - (2*a^2*Sec[
c + d*x])/(3*d*e*(e*Sin[c + d*x])^(3/2)) + (7*a^2*EllipticF[(c - Pi/2 + d*x
)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*e^2*Sqrt[e*Sin[c + d*x]]) + (5*a^2*Sec[c +
 d*x]*Sqrt[e*Sin[c + d*x]])/(3*d*e^3)
```

Rubi [A] time = 0.419121, antiderivative size = 234, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {3872, 2873, 2636, 2642, 2641, 2564, 325, 329, 212, 206, 203, 2570, 2571}

$$\frac{2a^2\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}+\frac{2a^2\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}}+\frac{5a^2\sec(c+dx)\sqrt{e\sin(c+dx)}}{3de^3}+\frac{7a^2\sqrt{\sin(c+dx)}F\left(\frac{1}{2}\left(c+dx-\right.\right.}{3de^2\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*ArcTan[Sqrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(5/2)) + (2*a^2*ArcTanh[S
qrt[e*Sin[c + d*x]]/Sqrt[e]])/(d*e^(5/2)) - (4*a^2)/(3*d*e*(e*Sin[c + d*x])
^(3/2)) - (2*a^2*Cos[c + d*x])/(3*d*e*(e*Sin[c + d*x])^(3/2)) - (2*a^2*Sec[
c + d*x])/(3*d*e*(e*Sin[c + d*x])^(3/2)) + (7*a^2*EllipticF[(c - Pi/2 + d*x
)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d*e^2*Sqrt[e*Sin[c + d*x]]) + (5*a^2*Sec[c +
 d*x]*Sqrt[e*Sin[c + d*x]])/(3*d*e^3)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx = \int \frac{(-a-a\cos(c+dx))^2\sec^2(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx$$
$$= \int \left(\frac{a^2}{(e\sin(c+dx))^{5/2}}+\frac{2a^2\sec(c+dx)}{(e\sin(c+dx))^{5/2}}+\frac{a^2\sec^2(c+dx)}{(e\sin(c+dx))^{5/2}}\right)dx$$
$$= a^2\int \frac{1}{(e\sin(c+dx))^{5/2}}\,dx + a^2\int \frac{\sec^2(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx + \left(2a^2\right)\int \frac{\sec(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx$$
$$= -\frac{2a^2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2\sec(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{a^2\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3e^2} + \frac{\left(5a^2\right)\int \frac{\sec^2(c+dx)}{\sqrt{e\sin(c+dx)}}}{3e^2}$$
$$= -\frac{4a^2}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2\sec(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{5a^2\sec(c+dx)\sqrt{}}{3de^3}$$
$$= -\frac{4a^2}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2\sec(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{2a^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+\right.\right.}{3de^2\sqrt{e}}$$
$$= -\frac{4a^2}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2\cos(c+dx)}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2\sec(c+dx)}{3de(e\sin(c+dx))^{3/2}} + \frac{7a^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+\right.\right.}{3de^2\sqrt{e}}$$
$$= \frac{2a^2\tan^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} + \frac{2a^2\tanh^{-1}\left(\frac{\sqrt{e\sin(c+dx)}}{\sqrt{e}}\right)}{de^{5/2}} - \frac{4a^2}{3de(e\sin(c+dx))^{3/2}} - \frac{2a^2\cos(c}{3de(e\sin(c}$$

Mathematica [C] time = 46.6692, size = 169, normalized size = 0.72

$$-\frac{a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{e\sin(c+dx)}\sec^4\left(\frac{1}{2}\sin^{-1}(\sin(c+dx))\right)\left(3\sqrt{\cos^2(c+dx)}\text{Hypergeometric2F1}\left(\frac{1}{4},\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Sin[c + d*x])^(5/2),x]
```

```
[Out] -(a^2*Cos[(c + d*x)/2]^4*(3 + 4*Sqrt[Cos[c + d*x]^2]*Csc[c + d*x]^2*Hyperge
ometric2F1[-3/4, 1, 1/4, Sin[c + d*x]^2] + 4*Sqrt[Cos[c + d*x]^2]*Csc[c + d
*x]^2*Hypergeometric2F1[-3/4, 3/2, 1/4, Sin[c + d*x]^2] + 3*Sqrt[Cos[c + d*
x]^2]*Hypergeometric2F1[1/4, 1/2, 5/4, Sin[c + d*x]^2])*Sec[c + d*x]*Sec[Ar
cSin[Sin[c + d*x]]/2]^4*Sqrt[e*Sin[c + d*x]])/(3*d*e^3)
```

Maple [A] time = 2.245, size = 301, normalized size = 1.3

$$\frac{a^2}{6\cos(dx+c)\left((\cos(dx+c))^2-1\right)d}\left(7\sqrt{-\sin(dx+c)+1}\sqrt{2+2\sin(dx+c)}(\sin(dx+c))^{7/2}\text{EllipticF}\left(\sqrt{-\sin(dx}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/6/e^(9/2)/(e*sin(d*x+c))^(3/2)/cos(d*x+c)/(cos(d*x+c)^2-1)*a^2*(7*(-sin(d
*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(7/2)*EllipticF((-sin(d*x+
c)+1)^(1/2),1/2*2^(1/2))*e^(7/2)-14*e^(7/2)*cos(d*x+c)^4-8*e^(7/2)*cos(d*x+
c)^3+12*arctanh((e*sin(d*x+c))^(1/2)/e^(1/2))*(e*sin(d*x+c))^(3/2)*e^2*cos(
d*x+c)^3+12*(e*sin(d*x+c))^(3/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))*e^2*c
os(d*x+c)^3+20*e^(7/2)*cos(d*x+c)^2+8*e^(7/2)*cos(d*x+c)-12*arctanh((e*sin(
d*x+c))^(1/2)/e^(1/2))*(e*sin(d*x+c))^(3/2)*e^2*cos(d*x+c)-12*(e*sin(d*x+c)
)^(3/2)*arctan((e*sin(d*x+c))^(1/2)/e^(1/2))*e^2*cos(d*x+c)-6*e^(7/2))/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(a^{2} \sec\left(dx+c\right)^{2}+2\,a^{2} \sec\left(dx+c\right)+a^{2}\right)\sqrt{e \sin\left(dx+c\right)}}{\left(e^{3} \cos\left(dx+c\right)^{2}-e^{3}\right)\sin\left(dx+c\right)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*sqrt(e*sin(d*x +
c))/((e^3*cos(d*x + c)^2 - e^3)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(a \sec\left(dx+c\right)+a\right)^{2}}{\left(e \sin\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/(e*sin(d*x + c))^(5/2), x)
```

3.120 $\int \frac{(e\sin(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=139

$$-\frac{4e^4\sqrt{\sin(c+dx)}\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{21ad\sqrt{e\sin(c+dx)}}+\frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7ad}-\frac{2e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21ad}+\frac{2e(e}{}$$

```
[Out] (-4*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*a*d*Sqrt[e
*Sin[c + d*x]]) - (2*e^3*Cos[c + d*x]*Sqrt[e*Sin[c + d*x]])/(21*a*d) + (2*e
^3*Cos[c + d*x]^3*Sqrt[e*Sin[c + d*x]])/(7*a*d) + (2*e*(e*Sin[c + d*x])^(5/
2))/(5*a*d)
```

Rubi [A] time = 0.279631, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3872, 2839, 2564, 30, 2568, 2569, 2642, 2641}

$$\frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7ad}-\frac{2e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21ad}-\frac{4e^4\sqrt{\sin(c+dx)}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{21ad\sqrt{e\sin(c+dx)}}+\frac{2e(e\sin(c+d}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(7/2)/(a + a*Sec[c + d*x]),x]
```


```
[Out] (-4*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*a*d*Sqrt[e
*Sin[c + d*x]]) - (2*e^3*Cos[c + d*x]*Sqrt[e*Sin[c + d*x]])/(21*a*d) + (2*e
^3*Cos[c + d*x]^3*Sqrt[e*Sin[c + d*x]])/(7*a*d) + (2*e*(e*Sin[c + d*x])^(5/
2))/(5*a*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+a\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)(e\sin(c+dx))^{7/2}}{-a-a\cos(c+dx)}\,dx$$
$$= \frac{e^2\int \cos(c+dx)(e\sin(c+dx))^{3/2}\,dx}{a} - \frac{e^2\int \cos^2(c+dx)(e\sin(c+dx))^{3/2}\,dx}{a}$$
$$= \frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7ad} + \frac{e\,\mathrm{Subst}\left(\int x^{3/2}\,dx, x, e\sin(c+dx)\right)}{ad} - \frac{e^4\int \frac{\cos^2(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{7a}$$
$$= -\frac{2e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21ad} + \frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7ad} + \frac{2e(e\sin(c+dx))^{5/2}}{5ad} - $$
$$= -\frac{2e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21ad} + \frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7ad} + \frac{2e(e\sin(c+dx))^{5/2}}{5ad} - $$
$$= -\frac{4e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21ad\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21ad} + \frac{2e^3\cos^3(c+dx)}{7a}$$

Mathematica [A] time = 0.677895, size = 122, normalized size = 0.88

$$\frac{e^3\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{e\sin(c+dx)}\left(40\mathrm{EllipticF}\left(\frac{1}{4}(-2c-2dx+\pi),2\right)+\sqrt{\sin(c+dx)}(25\cos(c+dx)-42\right.}{105ad\sqrt{\sin(c+dx)}(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sin[c + d*x])^(7/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (e^3*Cos[(c + d*x)/2]^2*Sec[c + d*x]*(40*EllipticF[(-2*c + Pi - 2*d*x)/4, 2
] + (42 + 25*Cos[c + d*x] - 42*Cos[2*(c + d*x)] + 15*Cos[3*(c + d*x)])*Sqrt
```

```
[Sin[c + d*x]])*Sqrt[e*Sin[c + d*x]])/(105*a*d*(1 + Sec[c + d*x])*Sqrt[Sin[
c + d*x]])
```

Maple [A] time = 1.288, size = 128, normalized size = 0.9

$$\frac{1}{d}\left(\frac{2e}{5a}(e\sin(dx+c))^{\frac{5}{2}}+\frac{2e^4}{21a\cos(dx+c)}\left(3(\sin(dx+c))^5+\sqrt{-\sin(dx+c)+1}\sqrt{2+2\sin(dx+c)}\sqrt{\sin(dx+c)}\mathrm{Ellip}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (2/5/a*e*(e*sin(d*x+c))^(5/2)+2/21*e^4*(3*sin(d*x+c)^5+(-sin(d*x+c)+1)^(1/2
)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1
/2*2^(1/2))-5*sin(d*x+c)^3+2*sin(d*x+c))/a/cos(d*x+c)/(e*sin(d*x+c))^(1/2))
/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{(e\sin(dx+c))^{\frac{7}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^(7/2)/(a*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(e^3\cos(dx+c)^2-e^3\right)\sqrt{e\sin(dx+c)}\sin(dx+c)}{a\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(-(e^3*cos(d*x + c)^2 - e^3)*sqrt(e*sin(d*x + c))*sin(d*x + c)/(a*s
ec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(7/2)/(a*sec(d*x + c) + a), x)
```

3.121 $\int \frac{(e\sin(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=104

$$-\frac{4e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5ad\sqrt{\sin(c+dx)}}+\frac{2e(e\sin(c+dx))^{3/2}}{3ad}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5ad}$$

```
[Out] (-4*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*a*d*Sqrt[
Sin[c + d*x]]) + (2*e*(e*Sin[c + d*x])^(3/2))/(3*a*d) - (2*e*Cos[c + d*x]*(
e*Sin[c + d*x])^(3/2))/(5*a*d)
```

Rubi [A] time = 0.219942, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3872, 2839, 2564, 30, 2569, 2640, 2639}

$$-\frac{4e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5ad\sqrt{\sin(c+dx)}}+\frac{2e(e\sin(c+dx))^{3/2}}{3ad}-\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-4*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*a*d*Sqrt[
Sin[c + d*x]]) + (2*e*(e*Sin[c + d*x])^(3/2))/(3*a*d) - (2*e*Cos[c + d*x]*(
e*Sin[c + d*x])^(3/2))/(5*a*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)(e\sin(c+dx))^{5/2}}{-a-a\cos(c+dx)}\,dx$$
$$= \frac{e^2\int \cos(c+dx)\sqrt{e\sin(c+dx)}\,dx}{a} - \frac{e^2\int \cos^2(c+dx)\sqrt{e\sin(c+dx)}\,dx}{a}$$
$$= -\frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5ad} + \frac{e\,\mathrm{Subst}\left(\int \sqrt{x}\,dx, x, e\sin(c+dx)\right)}{ad} - \frac{\left(2e^2\right)\int \sqrt{e\sin(c+}}{5a}$$
$$= \frac{2e(e\sin(c+dx))^{3/2}}{3ad} - \frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5ad} - \frac{\left(2e^2\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}}{5a\sqrt{\sin(c+dx)}}$$
$$= -\frac{4e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5ad\sqrt{\sin(c+dx)}} + \frac{2e(e\sin(c+dx))^{3/2}}{3ad} - \frac{2e\cos(c+dx)(e\sin(c+d}{5ad}$$

Mathematica [C] time = 4.71814, size = 232, normalized size = 2.23

$$\frac{2\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)(e\sin(c+dx))^{5/2}\left(\sqrt{\sin(c+dx)}(10\sin(c)\cos(dx)-3\sin(2c)\cos(2dx)+10\cos(c)\sin(dx}{15ad\sin^{\frac{5}{2}}(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sin[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]^2*Sec[c + d*x]*(e*Sin[c + d*x])^(5/2)*((2*Sqrt[2 - 2*E^
((2*I)*(c + d*x))]*(3*Hypergeometric2F1[-1/4, 1/2, 3/4, E^((2*I)*(c + d*x))
] + E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))])*Se
c[c])/(E^(I*d*x)*Sqrt[((-I)*(-1 + E^((2*I)*(c + d*x))))/E^(I*(c + d*x))]) +
 Sqrt[Sin[c + d*x]]*(10*Cos[d*x]*Sin[c] - 3*Cos[2*d*x]*Sin[2*c] + 10*Cos[c]
*Sin[d*x] - 3*Cos[2*c]*Sin[2*d*x] - 12*Tan[c])))/(15*a*d*(1 + Sec[c + d*x])
*Sin[c + d*x]^(5/2))
```

Maple [A] time = 1.322, size = 173, normalized size = 1.7

$$\frac{2e^3}{15a\cos(dx+c)d}\left(6\sqrt{-\sin(dx+c)+1}\sqrt{2+2\sin(dx+c)}\sqrt{\sin(dx+c)}\mathrm{EllipticE}\left(\sqrt{-\sin(dx+c)+1},1/2\sqrt{2}\right)-3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 2/15/a/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e^3*(6*(-sin(d*x+c)+1)^(1/2)*(2+2*si
n(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2
))-3*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*Elliptic
F((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+3*cos(d*x+c)^4-5*cos(d*x+c)^3-3*cos(d*
x+c)^2+5*cos(d*x+c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^(5/2)/(a*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(e^2\cos(dx+c)^2-e^2\right)\sqrt{e\sin(dx+c)}}{a\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(-(e^2*cos(d*x + c)^2 - e^2)*sqrt(e*sin(d*x + c))/(a*sec(d*x + c) +
 a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(5/2)/(a*sec(d*x + c) + a), x)
```

3.122 $\int \frac{(e\sin(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=102

$$-\frac{4e^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3ad\sqrt{e\sin(c+dx)}}+\frac{2e\sqrt{e\sin(c+dx)}}{ad}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3ad}$$

```
[Out] (-4*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*a*d*Sqrt[e*
Sin[c + d*x]]) + (2*e*Sqrt[e*Sin[c + d*x]])/(a*d) - (2*e*Cos[c + d*x]*Sqrt[
e*Sin[c + d*x]])/(3*a*d)
```

Rubi [A] time = 0.222465, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3872, 2839, 2564, 30, 2569, 2642, 2641}

$$-\frac{4e^2\sqrt{\sin(c+dx)}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3ad\sqrt{e\sin(c+dx)}}+\frac{2e\sqrt{e\sin(c+dx)}}{ad}-\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-4*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*a*d*Sqrt[e*
Sin[c + d*x]]) + (2*e*Sqrt[e*Sin[c + d*x]])/(a*d) - (2*e*Cos[c + d*x]*Sqrt[
e*Sin[c + d*x]])/(3*a*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\sin(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)(e\sin(c+dx))^{3/2}}{-a-a\cos(c+dx)}\,dx \\
&= \frac{e^2\int \frac{\cos(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{a} - \frac{e^2\int \frac{\cos^2(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{a} \\
&= -\frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3ad} + \frac{e\,\mathrm{Subst}\left(\int \frac{1}{\sqrt{x}}\,dx, x, e\sin(c+dx)\right)}{ad} - \frac{(2e^2)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3a} \\
&= \frac{2e\sqrt{e\sin(c+dx)}}{ad} - \frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3ad} - \frac{\left(2e^2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3a\sqrt{e\sin(c+dx)}} \\
&= -\frac{4e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3ad\sqrt{e\sin(c+dx)}} + \frac{2e\sqrt{e\sin(c+dx)}}{ad} - \frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3ad}
\end{aligned}$$

Mathematica [A] time = 19.8295, size = 69, normalized size = 0.68

$$-\frac{2(e\sin(c+dx))^{3/2}\left(\sqrt{\sin(c+dx)}(\cos(c+dx)-3)-2\,\mathrm{EllipticF}\left(\frac{1}{4}(-2c-2dx+\pi),2\right)\right)}{3ad\sin^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sin[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*(-2*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] + (-3 + Cos[c + d*x])*Sqrt[Sin[
c + d*x]])*(e*Sin[c + d*x])^(3/2))/(3*a*d*Sin[c + d*x]^(3/2))
```

Maple [A] time = 1.348, size = 112, normalized size = 1.1

$$\frac{2\,e^2}{3\,a\cos(dx+c)\,d}\left(\sqrt{-\sin(dx+c)+1}\sqrt{2+2\,\sin(dx+c)}\sqrt{\sin(dx+c)}\mathrm{EllipticF}\left(\sqrt{-\sin(dx+c)+1},\frac{\sqrt{2}}{2}\right)-(\cos(d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 2/3/a/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e^2*((-sin(d*x+c)+1)^(1/2)*(2+2*sin(d
*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-
cos(d*x+c)^2*sin(d*x+c)+3*cos(d*x+c)*sin(d*x+c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\sin(dx+c)}e\sin(dx+c)}{a\sec(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*sin(d*x + c))*e*sin(d*x + c)/(a*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)
```

3.123 $\int \frac{\sqrt{e\sin(c+dx)}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=95

$$-\frac{2e}{ad\sqrt{e\sin(c+dx)}}+\frac{2e\cos(c+dx)}{ad\sqrt{e\sin(c+dx)}}+\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}}$$

```
[Out] (-2*e)/(a*d*Sqrt[e*Sin[c + d*x]]) + (2*e*Cos[c + d*x])/(a*d*Sqrt[e*Sin[c +
d*x]]) + (4*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(a*d*Sqr
t[Sin[c + d*x]])
```

Rubi [A] time = 0.207713, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3872, 2839, 2564, 30, 2567, 2640, 2639}

$$-\frac{2e}{ad\sqrt{e\sin(c+dx)}}+\frac{2e\cos(c+dx)}{ad\sqrt{e\sin(c+dx)}}+\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sin[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-2*e)/(a*d*Sqrt[e*Sin[c + d*x]]) + (2*e*Cos[c + d*x])/(a*d*Sqrt[e*Sin[c +
d*x]]) + (4*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(a*d*Sqr
t[Sin[c + d*x]])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{e\sin(c+dx)}}{a+a\sec(c+dx)}\,dx &= -\int \frac{\cos(c+dx)\sqrt{e\sin(c+dx)}}{-a-a\cos(c+dx)}\,dx \\
&= \frac{e^2\int \frac{\cos(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx}{a} - \frac{e^2\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx}{a} \\
&= \frac{2e\cos(c+dx)}{ad\sqrt{e\sin(c+dx)}} + \frac{2\int \sqrt{e\sin(c+dx)}\,dx}{a} + \frac{e\,\mathrm{Subst}\left(\int \frac{1}{x^{3/2}}\,dx, x, e\sin(c+dx)\right)}{ad} \\
&= -\frac{2e}{ad\sqrt{e\sin(c+dx)}} + \frac{2e\cos(c+dx)}{ad\sqrt{e\sin(c+dx)}} + \frac{\left(2\sqrt{e\sin(c+dx)}\right)\int \sqrt{\sin(c+dx)}\,dx}{a\sqrt{\sin(c+dx)}} \\
&= -\frac{2e}{ad\sqrt{e\sin(c+dx)}} + \frac{2e\cos(c+dx)}{ad\sqrt{e\sin(c+dx)}} + \frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}}
\end{aligned}$$

Mathematica [C] time = 0.587984, size = 249, normalized size = 2.62

$$\frac{2\left(12e^{2ic}\sqrt{1-e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},e^{2i(c+dx)}\right)+4e^{2i(c+dx)}\sqrt{1-e^{2i(c+dx)}}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},e^{2\cdots}\right.\right.}{3a\left(1+ie^{ic}\right)\left(e^{ic}+i\right)d\left(-1+e^{i(c+dx)}\right)\left(1+e^{i(c+\cdots}\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sin[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*(3 - 9*E^((2*I)*c) + 6*E^(I*(c + d*x)) - 9*E^((2*I)*(c + d*x)) + 3*E^((2
*I)*(2*c + d*x)) + 6*E^(I*(3*c + d*x)) + 12*E^((2*I)*c)*Sqrt[1 - E^((2*I)*(
c + d*x))]*Hypergeometric2F1[-1/4, 1/2, 3/4, E^((2*I)*(c + d*x))] + 4*E^((2
*I)*(c + d*x))*Sqrt[1 - E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/
4, E^((2*I)*(c + d*x))])*Sqrt[e*Sin[c + d*x]])/(3*a*d*(1 + I*E^(I*c))*(I +
E^(I*c))*(-1 + E^(I*(c + d*x)))*(1 + E^(I*(c + d*x))))
```

Maple [A] time = 1.421, size = 149, normalized size = 1.6

$$-2\,\frac{e\left(2\sqrt{-\sin(dx+c)+1}\sqrt{2+2\sin(dx+c)}\sqrt{\sin(dx+c)}\text{EllipticE}\left(\sqrt{-\sin(dx+c)+1},1/2\sqrt{2}\right)-\sqrt{-\sin(dx+c)+1}\cdots\right.}{a\cos(dx+c)\sqrt{e\sin\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -2/a/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e*(2*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*
x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-(
-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-si
n(d*x+c)+1)^(1/2),1/2*2^(1/2))-cos(d*x+c)^2+cos(d*x+c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\sin(dx+c)}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*sin(d*x + c))/(a*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\sin(dx+c)}}{a\sec(dx+c)+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*sin(d*x + c))/(a*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{e\sin(c+dx)}}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(e*sin(c + d*x))/(sec(c + d*x) + 1), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\sin(dx+c)}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*sin(d*x + c))/(a*sec(d*x + c) + a), x)
```

3.124 $\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx$

Optimal. Leaf size=101

$$\frac{4\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3ad\sqrt{e\sin(c+dx)}}-\frac{2e}{3ad(e\sin(c+dx))^{3/2}}+\frac{2e\cos(c+dx)}{3ad(e\sin(c+dx))^{3/2}}$$

```
[Out] (-2*e)/(3*a*d*(e*Sin[c + d*x])^(3/2)) + (2*e*Cos[c + d*x])/(3*a*d*(e*Sin[c
+ d*x])^(3/2)) + (4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3
*a*d*Sqrt[e*Sin[c + d*x]])
```

Rubi [A] time = 0.211353, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3872, 2839, 2564, 30, 2567, 2642, 2641}

$$-\frac{2e}{3ad(e\sin(c+dx))^{3/2}}+\frac{2e\cos(c+dx)}{3ad(e\sin(c+dx))^{3/2}}+\frac{4\sqrt{\sin(c+dx)}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3ad\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] (-2*e)/(3*a*d*(e*Sin[c + d*x])^(3/2)) + (2*e*Cos[c + d*x])/(3*a*d*(e*Sin[c
+ d*x])^(3/2)) + (4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3
*a*d*Sqrt[e*Sin[c + d*x]])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(a+a\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx &= -\int \frac{\cos(c+dx)}{(-a-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx \\
&= \frac{e^2\int \frac{\cos(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx}{a} - \frac{e^2\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx}{a} \\
&= \frac{2e\cos(c+dx)}{3ad(e\sin(c+dx))^{3/2}} + \frac{2\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3a} + \frac{e\,\text{Subst}\left(\int \frac{1}{x^{5/2}}\,dx, x, e\sin(c+dx)\right)}{ad} \\
&= -\frac{2e}{3ad(e\sin(c+dx))^{3/2}} + \frac{2e\cos(c+dx)}{3ad(e\sin(c+dx))^{3/2}} + \frac{\left(2\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,d}{3a\sqrt{e\sin(c+dx)}} \\
&= -\frac{2e}{3ad(e\sin(c+dx))^{3/2}} + \frac{2e\cos(c+dx)}{3ad(e\sin(c+dx))^{3/2}} + \frac{4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c}}{3ad\sqrt{e\sin(c+dx)}}
\end{aligned}$$

Mathematica [A] time = 0.528417, size = 77, normalized size = 0.76

$$\frac{2\cot\left(\frac{1}{2}(c+dx)\right)\left(-2\sin^{\frac{3}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{4}(-2c-2dx+\pi),2\right)+\cos(c+dx)-1\right)}{3ad(\cos(c+dx)+1)\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] (2*Cot[(c + d*x)/2]*(-1 + Cos[c + d*x] - 2*EllipticF[(-2*c + Pi - 2*d*x)/4,
 2]*Sin[c + d*x]^(3/2)))/(3*a*d*(1 + Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])
```

Maple [A] time = 1.324, size = 121, normalized size = 1.2

$$\frac{1}{d}\left(-\frac{2e}{3a}(e\sin(dx+c))^{-\frac{3}{2}} - \frac{2}{3a(\sin(dx+c))^2\cos(dx+c)}\left(\sqrt{-\sin(dx+c)+1}\sqrt{2+2\sin(dx+c)}(\sin(dx+c))^{\frac{5}{2}}\text{Ellipt}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x)
```

```
[Out] (-2/3/a*e/(e*sin(d*x+c))^(3/2)-2/3*((-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^
(1/2)*sin(d*x+c)^(5/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+sin(d*x
+c)^3-sin(d*x+c))/a/sin(d*x+c)^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\sin\left(dx+c\right)}}{\left(ae\sec\left(dx+c\right)+ae\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*sin(d*x + c))/((a*e*sec(d*x + c) + a*e)*sin(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sqrt{e\sin\left(c+dx\right)}\sec\left(c+dx\right)+\sqrt{e\sin\left(c+dx\right)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*sin(c + d*x))*sec(c + d*x) + sqrt(e*sin(c + d*x))), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)\sqrt{e\sin\left(dx+c\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*sqrt(e*sin(d*x + c))), x)
```

3.125 $\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=135

$$-\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5ade^2\sqrt{\sin(c+dx)}}-\frac{2e}{5ad(e\sin(c+dx))^{5/2}}+\frac{2e\cos(c+dx)}{5ad(e\sin(c+dx))^{5/2}}-\frac{4\cos(c+dx)}{5ade\sqrt{e\sin(c+dx)}}$$

```
[Out] (-2*e)/(5*a*d*(e*Sin[c + d*x])^(5/2)) + (2*e*Cos[c + d*x])/(5*a*d*(e*Sin[c
+ d*x])^(5/2)) - (4*Cos[c + d*x])/(5*a*d*e*Sqrt[e*Sin[c + d*x]]) - (4*Ellip
ticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*a*d*e^2*Sqrt[Sin[c + d
*x]])
```

Rubi [A] time = 0.248337, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3872, 2839, 2564, 30, 2567, 2636, 2640, 2639}

$$-\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5ade^2\sqrt{\sin(c+dx)}}-\frac{2e}{5ad(e\sin(c+dx))^{5/2}}+\frac{2e\cos(c+dx)}{5ad(e\sin(c+dx))^{5/2}}-\frac{4\cos(c+dx)}{5ade\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (-2*e)/(5*a*d*(e*Sin[c + d*x])^(5/2)) + (2*e*Cos[c + d*x])/(5*a*d*(e*Sin[c
+ d*x])^(5/2)) - (4*Cos[c + d*x])/(5*a*d*e*Sqrt[e*Sin[c + d*x]]) - (4*Ellip
ticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*a*d*e^2*Sqrt[Sin[c + d
*x]])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = -\int \frac{\cos(c+dx)}{(-a-a\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx$$

$$= \frac{e^2\int \frac{\cos(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx}{a} - \frac{e^2\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx}{a}$$

$$= \frac{2e\cos(c+dx)}{5ad(e\sin(c+dx))^{5/2}} + \frac{2\int \frac{1}{(e\sin(c+dx))^{3/2}}\,dx}{5a} + \frac{e\,\text{Subst}\left(\int \frac{1}{x^{7/2}}\,dx, x, e\sin(\cdots\right.}{ad}$$

$$= -\frac{2e}{5ad(e\sin(c+dx))^{5/2}} + \frac{2e\cos(c+dx)}{5ad(e\sin(c+dx))^{5/2}} - \frac{4\cos(c+dx)}{5ade\sqrt{e\sin(c+dx)}} - \frac{2\cdots}{\cdots}$$

$$= -\frac{2e}{5ad(e\sin(c+dx))^{5/2}} + \frac{2e\cos(c+dx)}{5ad(e\sin(c+dx))^{5/2}} - \frac{4\cos(c+dx)}{5ade\sqrt{e\sin(c+dx)}} - \frac{(2\cdots}{\cdots}$$

$$= -\frac{2e}{5ad(e\sin(c+dx))^{5/2}} + \frac{2e\cos(c+dx)}{5ad(e\sin(c+dx))^{5/2}} - \frac{4\cos(c+dx)}{5ade\sqrt{e\sin(c+dx)}} - \frac{4\cdots}{\cdots}$$

Mathematica [C] time = 1.06398, size = 124, normalized size = 0.92

$$\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(\cos(c+dx)+i\sin(c+dx))\left(2\sqrt{1-e^{2i(c+dx)}}(\cos(c+dx)+1)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},e^{2i(c+dx)}\right)\cdots\right.}{15ade\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (Sec[(c + d*x)/2]^2*(Cos[c + d*x] + I*Sin[c + d*x])*(-6 - 9*Cos[c + d*x] +
2*Sqrt[1 - E^((2*I)*(c + d*x))]*(1 + Cos[c + d*x])*Hypergeometric2F1[1/2, 3
```

```
/4, 7/4, E^((2*I)*(c + d*x))] + (3*I)*Sin[c + d*x]))/(15*a*d*e*Sqrt[e*Sin[c
 + d*x]])
```

Maple [A] time = 1.458, size = 187, normalized size = 1.4

$$\frac{1}{d}\left(-\frac{2e}{5a}(e\sin(dx+c))^{-\frac{5}{2}}+\frac{2}{5ae(\sin(dx+c))^{3}\cos(dx+c)}\left(2\sqrt{-\sin(dx+c)+1}\sqrt{2+2\sin(dx+c)}(\sin(dx+c))^{7/2}\,\mathrm{El}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x)
```

```
[Out] (-2/5/a*e/(e*sin(d*x+c))^(5/2)+2/5/e*(2*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+
c))^(1/2)*sin(d*x+c)^(7/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-(-s
in(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(7/2)*EllipticF((-sin(
d*x+c)+1)^(1/2),1/2*2^(1/2))+2*sin(d*x+c)^5-3*sin(d*x+c)^3+sin(d*x+c))/a/si
n(d*x+c)^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{e\sin(dx+c)}}{ae^{2}\cos(dx+c)^{2}-ae^{2}+\left(ae^{2}\cos(dx+c)^{2}-ae^{2}\right)\sec(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*sin(d*x + c))/(a*e^2*cos(d*x + c)^2 - a*e^2 + (a*e^2*cos(d
*x + c)^2 - a*e^2)*sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)\left(e\sin\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*(e*sin(d*x + c))^(3/2)), x)
```

3.126 $\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=135

$$\frac{4\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{21ade^2\sqrt{e\sin(c+dx)}}-\frac{2e}{7ad(e\sin(c+dx))^{7/2}}+\frac{2e\cos(c+dx)}{7ad(e\sin(c+dx))^{7/2}}-\frac{4\cos(c+dx)}{21ade(e\sin(c+dx))^{3/2}}$$

```
[Out] (-2*e)/(7*a*d*(e*Sin[c + d*x])^(7/2)) + (2*e*Cos[c + d*x])/(7*a*d*(e*Sin[c
+ d*x])^(7/2)) - (4*Cos[c + d*x])/(21*a*d*e*(e*Sin[c + d*x])^(3/2)) + (4*El
lipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*a*d*e^2*Sqrt[e*Sin[c
 + d*x]])
```

Rubi [A] time = 0.250353, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3872, 2839, 2564, 30, 2567, 2636, 2642, 2641}

$$\frac{4\sqrt{\sin(c+dx)}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{21ade^2\sqrt{e\sin(c+dx)}}-\frac{2e}{7ad(e\sin(c+dx))^{7/2}}+\frac{2e\cos(c+dx)}{7ad(e\sin(c+dx))^{7/2}}-\frac{4\cos(c+dx)}{21ade(e\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (-2*e)/(7*a*d*(e*Sin[c + d*x])^(7/2)) + (2*e*Cos[c + d*x])/(7*a*d*(e*Sin[c
+ d*x])^(7/2)) - (4*Cos[c + d*x])/(21*a*d*e*(e*Sin[c + d*x])^(3/2)) + (4*El
lipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*a*d*e^2*Sqrt[e*Sin[c
 + d*x]])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx = -\int \frac{\cos(c+dx)}{(-a-a\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx$$
$$= \frac{e^2\int \frac{\cos(c+dx)}{(e\sin(c+dx))^{9/2}}\,dx}{a} - \frac{e^2\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{9/2}}\,dx}{a}$$
$$= \frac{2e\cos(c+dx)}{7ad(e\sin(c+dx))^{7/2}} + \frac{2\int \frac{1}{(e\sin(c+dx))^{5/2}}\,dx}{7a} + \frac{e\,\text{Subst}\left(\int \frac{1}{x^{9/2}}\,dx, x, e\sin(\ldots\right.}{ad}$$
$$= -\frac{2e}{7ad(e\sin(c+dx))^{7/2}} + \frac{2e\cos(c+dx)}{7ad(e\sin(c+dx))^{7/2}} - \frac{4\cos(c+dx)}{21ade(e\sin(c+dx))^{3/2}} + \ldots$$
$$= -\frac{2e}{7ad(e\sin(c+dx))^{7/2}} + \frac{2e\cos(c+dx)}{7ad(e\sin(c+dx))^{7/2}} - \frac{4\cos(c+dx)}{21ade(e\sin(c+dx))^{3/2}} + \ldots$$
$$= -\frac{2e}{7ad(e\sin(c+dx))^{7/2}} + \frac{2e\cos(c+dx)}{7ad(e\sin(c+dx))^{7/2}} - \frac{4\cos(c+dx)}{21ade(e\sin(c+dx))^{3/2}} + \ldots$$

Mathematica [A] time = 1.25271, size = 91, normalized size = 0.67

$$-\frac{2\left(\sin^{\frac{7}{2}}(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)\text{EllipticF}\left(\frac{1}{4}(-2c-2dx+\pi),2\right)+2\cos(c+dx)+\cos(2(c+dx))+4\right)}{21ade(\cos(c+dx)+1)(e\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] (-2*(4 + 2*Cos[c + d*x] + Cos[2*(c + d*x)] + Csc[(c + d*x)/2]^2*EllipticF[(
-2*c + Pi - 2*d*x)/4, 2]*Sin[c + d*x]^(7/2)))/(21*a*d*e*(1 + Cos[c + d*x])*
```

```
(e*Sin[c + d*x])^(3/2))
```

Maple [A] time = 1.51, size = 136, normalized size = 1.

$$\frac{1}{d}\left(-\frac{2e}{7a}\left(e\sin\left(dx+c\right)\right)^{-\frac{7}{2}}-\frac{2}{21\,ae^{2}\left(\sin\left(dx+c\right)\right)^{4}\cos\left(dx+c\right)}\left(\sqrt{-\sin\left(dx+c\right)+1}\sqrt{2+2\,\sin\left(dx+c\right)}\left(\sin\left(dx+c\right)\right)^{\frac{9}{2}}\mathrm{Ell}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x)
```

```
[Out] (-2/7/a*e/(e*sin(d*x+c))^(7/2)-2/21/e^2*((-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x
+c))^(1/2)*sin(d*x+c)^(9/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2*
sin(d*x+c)^5+5*sin(d*x+c)^3-3*sin(d*x+c))/a/sin(d*x+c)^4/cos(d*x+c)/(e*sin(
d*x+c))^(1/2))/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{e\sin\left(dx+c\right)}}{\left(ae^{3}\cos\left(dx+c\right)^{2}-ae^{3}+\left(ae^{3}\cos\left(dx+c\right)^{2}-ae^{3}\right)\sec\left(dx+c\right)\right)\sin\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*sin(d*x + c))/((a*e^3*cos(d*x + c)^2 - a*e^3 + (a*e^3*cos(
d*x + c)^2 - a*e^3)*sec(d*x + c))*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)\left(e\sin\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)*(e*sin(d*x + c))^(5/2)), x)
```

3.127 $\int \frac{(e\sin(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=162

$$\frac{52e^4\sqrt{\sin(c+dx)}\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{21a^2d\sqrt{e\sin(c+dx)}}-\frac{4e^3\sqrt{e\sin(c+dx)}}{a^2d}+\frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7a^2d}+\frac{26e^3\cos(c+dx)\ldots}{21a\ldots}$$

```
[Out] (52*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*a^2*d*Sqrt
[e*Sin[c + d*x]]) - (4*e^3*Sqrt[e*Sin[c + d*x]])/(a^2*d) + (26*e^3*Cos[c +
d*x]*Sqrt[e*Sin[c + d*x]])/(21*a^2*d) + (2*e^3*Cos[c + d*x]^3*Sqrt[e*Sin[c
+ d*x]])/(7*a^2*d) + (4*e*(e*Sin[c + d*x])^(5/2))/(5*a^2*d)
```

Rubi [A] time = 0.551093, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3872, 2875, 2873, 2569, 2642, 2641, 2564, 14}

$$-\frac{4e^3\sqrt{e\sin(c+dx)}}{a^2d}+\frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7a^2d}+\frac{26e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d}+\frac{52e^4\sqrt{\sin(c+dx)}F\left(\frac{1}{2}\left(c+dx\ldots\right.\right.}{21a^2d\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(7/2)/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (52*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*a^2*d*Sqrt
[e*Sin[c + d*x]]) - (4*e^3*Sqrt[e*Sin[c + d*x]])/(a^2*d) + (26*e^3*Cos[c +
d*x]*Sqrt[e*Sin[c + d*x]])/(21*a^2*d) + (2*e^3*Cos[c + d*x]^3*Sqrt[e*Sin[c
+ d*x]])/(7*a^2*d) + (4*e*(e*Sin[c + d*x])^(5/2))/(5*a^2*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
```

```
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{7/2}}{(-a-a\cos(c+dx))^2}\,dx$$

$$= \frac{e^4\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sqrt{e\sin(c+dx)}}\,dx}{a^4}$$

$$= \frac{e^4\int \left(\frac{a^2\cos^2(c+dx)}{\sqrt{e\sin(c+dx)}}-\frac{2a^2\cos^3(c+dx)}{\sqrt{e\sin(c+dx)}}+\frac{a^2\cos^4(c+dx)}{\sqrt{e\sin(c+dx)}}\right)dx}{a^4}$$

$$= \frac{e^4\int \frac{\cos^2(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{a^2}+\frac{e^4\int \frac{\cos^4(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{a^2}-\frac{\left(2e^4\right)\int \frac{\cos^3(c+dx)}{\sqrt{e\sin(c+dx)}}\,dx}{a^2}$$

$$= \frac{2e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d}+\frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7a^2d}-\frac{\left(2e^3\right)\mathrm{Subst}\left(\int \frac{1-\frac{x^2}{e^2}}{\sqrt{x}}\,d\right.}{a^2d}$$

$$= \frac{26e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d}+\frac{2e^3\cos^3(c+dx)\sqrt{e\sin(c+dx)}}{7a^2d}-\frac{\left(2e^3\right)\mathrm{Subst}\left(\int\left(\frac{1}{\sqrt{x}}\right.\right.}{}$$

$$= \frac{4e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3a^2d\sqrt{e\sin(c+dx)}}-\frac{4e^3\sqrt{e\sin(c+dx)}}{a^2d}+\frac{26e^3\cos(c+dx)\sqrt{e\sin(c+}}{21a^2d}$$

$$= \frac{52e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21a^2d\sqrt{e\sin(c+dx)}}-\frac{4e^3\sqrt{e\sin(c+dx)}}{a^2d}+\frac{26e^3\cos(c+dx)\sqrt{e\sin(c}}{21a^2d}$$

Mathematica [A] time = 1.55736, size = 94, normalized size = 0.58

$$-\frac{e^3\sqrt{e\sin(c+dx)}\left(520\mathrm{EllipticF}\left(\frac{1}{4}(-2c-2dx+\pi),2\right)+\sqrt{\sin(c+dx)}(-305\cos(c+dx)+84\cos(2(c+dx))-15\cos\right.}{210a^2d\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sin[c + d*x])^(7/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(e^3*(520*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] + (756 - 305*Cos[c + d*x] +
84*Cos[2*(c + d*x)] - 15*Cos[3*(c + d*x)])*Sqrt[Sin[c + d*x]])*Sqrt[e*Sin[c
 + d*x]])/(210*a^2*d*Sqrt[Sin[c + d*x]])
```

Maple [A] time = 1.74, size = 145, normalized size = 0.9

$$-\frac{2\,e^4}{105\,a^2\cos\left(dx+c\right)d}\left(-15\,\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)^4+65\,\sqrt{-\sin\left(dx+c\right)+1}\sqrt{2+2\,\sin\left(dx+c\right)}\sqrt{\sin\left(dx+c\right)}\mathrm{Elliptic}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -2/105/a^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e^4*(-15*sin(d*x+c)*cos(d*x+c)^4
+65*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF
((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+42*cos(d*x+c)^3*sin(d*x+c)-65*cos(d*x+c
)^2*sin(d*x+c)+168*cos(d*x+c)*sin(d*x+c))/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\left(e^3\cos\left(dx+c\right)^2-e^3\right)\sqrt{e\sin\left(dx+c\right)}\sin\left(dx+c\right)}{a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(e^3*cos(d*x + c)^2 - e^3)*sqrt(e*sin(d*x + c))*sin(d*x + c)/(a^2
*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(7/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(7/2)/(a*sec(d*x + c) + a)^2, x)
```

3.128 $\int \frac{(e\sin(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=187

$$\frac{4e^3}{a^2d\sqrt{e\sin(c+dx)}}-\frac{2e^3\cos^3(c+dx)}{a^2d\sqrt{e\sin(c+dx)}}-\frac{2e^3\cos(c+dx)}{a^2d\sqrt{e\sin(c+dx)}}-\frac{44e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}}+\frac{4e(e\sin(c+d\ldots}{3a^2d}$$

```
[Out] (4*e^3)/(a^2*d*Sqrt[e*Sin[c + d*x]]) - (2*e^3*Cos[c + d*x])/(a^2*d*Sqrt[e*S
in[c + d*x]]) - (2*e^3*Cos[c + d*x]^3)/(a^2*d*Sqrt[e*Sin[c + d*x]]) - (44*e
^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*a^2*d*Sqrt[Sin
[c + d*x]]) + (4*e*(e*Sin[c + d*x])^(3/2))/(3*a^2*d) - (12*e*Cos[c + d*x]*(
e*Sin[c + d*x])^(3/2))/(5*a^2*d)
```

Rubi [A] time = 0.596152, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3872, 2875, 2873, 2567, 2640, 2639, 2564, 14, 2569}

$$\frac{4e^3}{a^2d\sqrt{e\sin(c+dx)}}-\frac{2e^3\cos^3(c+dx)}{a^2d\sqrt{e\sin(c+dx)}}-\frac{2e^3\cos(c+dx)}{a^2d\sqrt{e\sin(c+dx)}}-\frac{44e^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}}+\frac{4e(e\sin(c+d\ldots}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*e^3)/(a^2*d*Sqrt[e*Sin[c + d*x]]) - (2*e^3*Cos[c + d*x])/(a^2*d*Sqrt[e*S
in[c + d*x]]) - (2*e^3*Cos[c + d*x]^3)/(a^2*d*Sqrt[e*Sin[c + d*x]]) - (44*e
^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*a^2*d*Sqrt[Sin
[c + d*x]]) + (4*e*(e*Sin[c + d*x])^(3/2))/(3*a^2*d) - (12*e*Cos[c + d*x]*(
e*Sin[c + d*x])^(3/2))/(5*a^2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
```

```
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{5/2}}{(-a-a\cos(c+dx))^2}\,dx$$

$$= \frac{e^4\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{(e\sin(c+dx))^{3/2}}\,dx}{a^4}$$

$$= \frac{e^4\int \left(\frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{3/2}} - \frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{3/2}} + \frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{3/2}}\right)dx}{a^4}$$

$$= \frac{e^4\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx}{a^2} + \frac{e^4\int \frac{\cos^4(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx}{a^2} - \frac{\left(2e^4\right)\int \frac{\cos^3(c+dx)}{(e\sin(c+dx))^{3/2}}\,dx}{a^2}$$

$$= -\frac{2e^3\cos(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos^3(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{\left(2e^2\right)\int \sqrt{e\sin(c+dx)}\,dx}{a^2} - \frac{\left(6e^2\right)\int \cos^2(c+dx}{a^2}$$

$$= -\frac{2e^3\cos(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos^3(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{12e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5a^2d} - \frac{\left(12e^2\right)\int \sqrt{e\,s}}{5a}$$

$$= \frac{4e^3}{a^2d\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos^3(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{4e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\,s}}{a^2d\sqrt{\sin(c+dx)}}$$

$$= \frac{4e^3}{a^2d\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{2e^3\cos^3(c+dx)}{a^2d\sqrt{e\sin(c+dx)}} - \frac{44e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e}}{5a^2d\sqrt{\sin(c+dx)}}$$

Mathematica [C] time = 3.08798, size = 249, normalized size = 1.33

$$\frac{4\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)(e\sin(c+dx))^{5/2}\left(\csc^2(c+dx)\left(20\sin(c)\cos(dx) - 3\sin(2c)\cos(2dx) + 20\cos(c)\sin(dx)\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sin[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*(e*Sin[c + d*x])^(5/2)*(((352*I)*E^((2
*I)*(2*c + d*x))*(3*Hypergeometric2F1[-1/4, 1/2, 3/4, E^((2*I)*(c + d*x))]
+ E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))]))/((1
 + E^((2*I)*c))*(1 - E^((2*I)*(c + d*x)))^(5/2)) + Csc[c + d*x]^2*(20*Cos[d
*x]*Sin[c] - 3*Cos[2*d*x]*Sin[2*c] + Sec[c/2]*(-36*Sec[c]*Sin[(3*c)/2] + 60
*Sec[(c + d*x)/2]*Sin[(d*x)/2]) + 20*Cos[c]*Sin[d*x] - 3*Cos[2*c]*Sin[2*d*x
] - 96*Sec[c]*Tan[c/2])))/(15*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 1.766, size = 173, normalized size = 0.9

$$\frac{2\,e^3}{15\,a^2\cos(dx+c)\,d}\left(66\sqrt{-\sin(dx+c)+1}\sqrt{2+2\sin(dx+c)}\sqrt{\sin(dx+c)}\mathrm{EllipticE}\left(\sqrt{-\sin(dx+c)+1},1/2\sqrt{2}\right) - 33\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 2/15/a^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*e^3*(66*(-sin(d*x+c)+1)^(1/2)*(2+2
*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(
```

```
1/2))-33*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*Elli
pticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+3*cos(d*x+c)^4-10*cos(d*x+c)^3-33*
cos(d*x+c)^2+40*cos(d*x+c))/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\left(e^2\cos\left(dx+c\right)^2-e^2\right)\sqrt{e\sin\left(dx+c\right)}}{a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(-(e^2*cos(d*x + c)^2 - e^2)*sqrt(e*sin(d*x + c))/(a^2*sec(d*x + c)
^2 + 2*a^2*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(5/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e\sin\left(dx+c\right)\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

3.129 $\int \frac{(e\sin(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=189

$$-\frac{4e^2\sqrt{\sin(c+dx)}\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{a^2d\sqrt{e\sin(c+dx)}}+\frac{4e^3}{3a^2d(e\sin(c+dx))^{3/2}}-\frac{2e^3\cos^3(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}}-\frac{2e^3\cos(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}}$$

```
[Out] (4*e^3)/(3*a^2*d*(e*Sin[c + d*x])^(3/2)) - (2*e^3*Cos[c + d*x])/(3*a^2*d*(e
*Sin[c + d*x])^(3/2)) - (2*e^3*Cos[c + d*x]^3)/(3*a^2*d*(e*Sin[c + d*x])^(3
/2)) - (4*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^2*d*S
qrt[e*Sin[c + d*x]]) + (4*e*Sqrt[e*Sin[c + d*x]])/(a^2*d) - (4*e*Cos[c + d*
x]*Sqrt[e*Sin[c + d*x]])/(3*a^2*d)
```

Rubi [A] time = 0.593564, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3872, 2875, 2873, 2567, 2642, 2641, 2564, 14, 2569}

$$\frac{4e^3}{3a^2d(e\sin(c+dx))^{3/2}}-\frac{2e^3\cos^3(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}}-\frac{2e^3\cos(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}}-\frac{4e^2\sqrt{\sin(c+dx)}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{a^2d\sqrt{e\sin(c+dx)}}+\frac{4e\sqrt{e\sin}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (4*e^3)/(3*a^2*d*(e*Sin[c + d*x])^(3/2)) - (2*e^3*Cos[c + d*x])/(3*a^2*d*(e
*Sin[c + d*x])^(3/2)) - (2*e^3*Cos[c + d*x]^3)/(3*a^2*d*(e*Sin[c + d*x])^(3
/2)) - (4*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^2*d*S
qrt[e*Sin[c + d*x]]) + (4*e*Sqrt[e*Sin[c + d*x]])/(a^2*d) - (4*e*Cos[c + d*
x]*Sqrt[e*Sin[c + d*x]])/(3*a^2*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
```

```
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{3/2}}{(-a-a\cos(c+dx))^2}\,dx$$

$$= \frac{e^4\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{(e\sin(c+dx))^{5/2}}\,dx}{a^4}$$

$$= \frac{e^4\int \left(\frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{5/2}} - \frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{5/2}} + \frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{5/2}}\right)dx}{a^4}$$

$$= \frac{e^4\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx}{a^2} + \frac{e^4\int \frac{\cos^4(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx}{a^2} - \frac{(2e^4)\int \frac{\cos^3(c+dx)}{(e\sin(c+dx))^{5/2}}\,dx}{a^2}$$

$$= -\frac{2e^3\cos(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{2e^3\cos^3(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{(2e^2)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3a^2} - \frac{(2e^2)\int \frac{\cos^2(c+dx)}{\sqrt{e\sin(c+dx)}}}{a^2}$$

$$= -\frac{2e^3\cos(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{2e^3\cos^3(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{4e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d} - \frac{(4e^2)\int}{3}$$

$$= \frac{4e^3}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{2e^3\cos(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{2e^3\cos^3(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{4e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+d\right.\right.}{3a^2d\sqrt{e\,s}}$$

$$= \frac{4e^3}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{2e^3\cos(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{2e^3\cos^3(c+dx)}{3a^2d(e\sin(c+dx))^{3/2}} - \frac{4e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+d\right.\right.}{a^2d\sqrt{e\,si}}$$

Mathematica [A] time = 1.83444, size = 119, normalized size = 0.63

$$\frac{2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)(e\sin(c+dx))^{3/2}\left(\frac{24\text{EllipticF}\left(\frac{1}{4}(-2c-2dx+\pi),2\right)}{\sin^{\frac{3}{2}}(c+dx)} + (10\cos(c+dx)-\cos(2(c+dx))+15)\csc(c+\right.}{3a^2d(\sec(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Sin[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*((15 + 10*Cos[c + d*x] - Cos[2*(c + d*
x)])*Csc[c + d*x]*Sec[(c + d*x)/2]^2 + (24*EllipticF[(-2*c + Pi - 2*d*x)/4,
 2])/Sin[c + d*x]^(3/2))*(e*Sin[c + d*x])^(3/2))/(3*a^2*d*(1 + Sec[c + d*x]
)^2)
```

Maple [A] time = 1.671, size = 153, normalized size = 0.8

$$-\frac{2e^3}{3a^2\cos(dx+c)\left((\cos(dx+c))^2-1\right)d}\left(3\sqrt{-\sin(dx+c)+1}\sqrt{2+2\sin(dx+c)}(\sin(dx+c))^{7/2}\,\text{EllipticF}\left(\sqrt{-\sin(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -2/3/a^2/(e*sin(d*x+c))^(3/2)/cos(d*x+c)/(cos(d*x+c)^2-1)*e^3*(3*(-sin(d*x+
c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(7/2)*EllipticF((-sin(d*x+c)+
1)^(1/2),1/2*2^(1/2))-cos(d*x+c)^6+6*cos(d*x+c)^5+4*cos(d*x+c)^4-14*cos(d*x
+c)^3-3*cos(d*x+c)^2+8*cos(d*x+c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\sin(dx+c)}e\sin(dx+c)}{a^2\sec(dx+c)^2+2\,a^2\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*sin(d*x + c))*e*sin(d*x + c)/(a^2*sec(d*x + c)^2 + 2*a^2*se
c(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

3.130 $\int \frac{\sqrt{e\sin(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=188

$$\frac{4e^3}{5a^2d(e\sin(c+dx))^{5/2}}-\frac{2e^3\cos^3(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}}-\frac{2e^3\cos(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}}-\frac{4e}{a^2d\sqrt{e\sin(c+dx)}}+\frac{16e\cos(c+dx)}{5a^2d\sqrt{e\sin(c+dx)}}+\frac{28E}{}$$

```
[Out] (4*e^3)/(5*a^2*d*(e*Sin[c + d*x])^(5/2)) - (2*e^3*Cos[c + d*x])/(5*a^2*d*(e
*Sin[c + d*x])^(5/2)) - (2*e^3*Cos[c + d*x]^3)/(5*a^2*d*(e*Sin[c + d*x])^(5
/2)) - (4*e)/(a^2*d*Sqrt[e*Sin[c + d*x]]) + (16*e*Cos[c + d*x])/(5*a^2*d*Sq
rt[e*Sin[c + d*x]]) + (28*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d
*x]])/(5*a^2*d*Sqrt[Sin[c + d*x]])
```

Rubi [A] time = 0.589679, antiderivative size = 188, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3872, 2875, 2873, 2567, 2636, 2640, 2639, 2564, 14}

$$\frac{4e^3}{5a^2d(e\sin(c+dx))^{5/2}}-\frac{2e^3\cos^3(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}}-\frac{2e^3\cos(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}}-\frac{4e}{a^2d\sqrt{e\sin(c+dx)}}+\frac{16e\cos(c+dx)}{5a^2d\sqrt{e\sin(c+dx)}}+\frac{28E}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sin[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*e^3)/(5*a^2*d*(e*Sin[c + d*x])^(5/2)) - (2*e^3*Cos[c + d*x])/(5*a^2*d*(e
*Sin[c + d*x])^(5/2)) - (2*e^3*Cos[c + d*x]^3)/(5*a^2*d*(e*Sin[c + d*x])^(5
/2)) - (4*e)/(a^2*d*Sqrt[e*Sin[c + d*x]]) + (16*e*Cos[c + d*x])/(5*a^2*d*Sq
rt[e*Sin[c + d*x]]) + (28*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d
*x]])/(5*a^2*d*Sqrt[Sin[c + d*x]])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
```

```
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sqrt{e\sin(c+dx)}}{(-a-a\cos(c+dx))^2}\,dx$$
$$= \frac{e^4\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{(e\sin(c+dx))^{7/2}}\,dx}{a^4}$$
$$= \frac{e^4\int \left(\frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{7/2}} - \frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{7/2}} + \frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{7/2}}\right)dx}{a^4}$$
$$= \frac{e^4\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx}{a^2} + \frac{e^4\int \frac{\cos^4(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx}{a^2} - \frac{(2e^4)\int \frac{\cos^3(c+dx)}{(e\sin(c+dx))^{7/2}}\,dx}{a^2}$$
$$= -\frac{2e^3\cos(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos^3(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{(2e^2)\int \frac{1}{(e\sin(c+dx))^{3/2}}\,dx}{5a^2} - \frac{(6e^2)\int \frac{\cos^2(c+}{(e\sin(c+d}}{5a^2}$$
$$= -\frac{2e^3\cos(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos^3(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} + \frac{16e\cos(c+dx)}{5a^2d\sqrt{e\sin(c+dx)}} + \frac{2\int\sqrt{e\sin(c+dx)}}{5a^2}$$
$$= \frac{4e^3}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos^3(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{4e}{a^2d\sqrt{e\sin(c+dx)}}$$
$$= \frac{4e^3}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{2e^3\cos^3(c+dx)}{5a^2d(e\sin(c+dx))^{5/2}} - \frac{4e}{a^2d\sqrt{e\sin(c+dx)}}$$

Mathematica [C] time = 1.32651, size = 222, normalized size = 1.18

$$\frac{4\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{e\sin(c+dx)}\left(\frac{3}{4}\sec(c)\left(49\sin\left(\frac{1}{2}(c-dx)\right)+35\sin\left(\frac{1}{2}(3c+dx)\right)-23\sin\left(\frac{1}{2}(c+3dx)\right)+\right.\right.}{15a^2d(\sec(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Sin[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (4*Cos[(c + d*x)/2]^4*Sec[c + d*x]^2*Sqrt[e*Sin[c + d*x]]*(((56*I)*E^((2*I)
*c)*(3*Hypergeometric2F1[-1/4, 1/2, 3/4, E^((2*I)*(c + d*x))] + E^((2*I)*d*
x)*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))]))/((1 + E^((2*I)*c
))*Sqrt[1 - E^((2*I)*(c + d*x))]) + (3*Sec[c]*Sec[(c + d*x)/2]^3*(49*Sin[(c
 - d*x)/2] + 35*Sin[(3*c + d*x)/2] - 23*Sin[(c + 3*d*x)/2] + 5*Sin[(5*c + 3
*d*x)/2]))/4))/(15*a^2*d*(1 + Sec[c + d*x])^2)
```

Maple [A] time = 1.688, size = 205, normalized size = 1.1

$$\frac{1}{d}\left(-2\frac{e}{a^2}\left(2\frac{1}{\sqrt{e\sin(dx+c)}} - 2/5\frac{e^2}{(e\sin(dx+c))^{5/2}}\right) - \frac{2e}{5a^2(\sin(dx+c))^3\cos(dx+c)}\left(14\sqrt{-\sin(dx+c)+1}\sqrt{2+2\sin}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] (-2*e/a^2*(2/(e*sin(d*x+c))^(1/2)-2/5*e^2/(e*sin(d*x+c))^(5/2))-2/5*e*(14*(
-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(7/2)*EllipticE((-si
n(d*x+c)+1)^(1/2),1/2*2^(1/2))-7*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/
```

```
2)*sin(d*x+c)^(7/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+9*sin(d*x+
c)^5-11*sin(d*x+c)^3+2*sin(d*x+c))/a^2/sin(d*x+c)^3/cos(d*x+c)/(e*sin(d*x+c
))^(1/2))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\sin\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*sin(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{e\sin\left(dx+c\right)}}{a^{2}\sec\left(dx+c\right)^{2}+2\,a^{2}\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*sin(d*x + c))/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^
2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{e\sin\left(c+dx\right)}}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(1/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(e*sin(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a*
*2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\sin\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*sin(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

3.131 $\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$

Optimal. Leaf size=190

$$\frac{20\sqrt{\sin(c+dx)}\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{21a^2d\sqrt{e\sin(c+dx)}}+\frac{4e^3}{7a^2d(e\sin(c+dx))^{7/2}}-\frac{2e^3\cos^3(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}}-\frac{2e^3\cos(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}}-$$

```
[Out] (4*e^3)/(7*a^2*d*(e*Sin[c + d*x])^(7/2)) - (2*e^3*Cos[c + d*x])/(7*a^2*d*(e
*Sin[c + d*x])^(7/2)) - (2*e^3*Cos[c + d*x]^3)/(7*a^2*d*(e*Sin[c + d*x])^(7
/2)) - (4*e)/(3*a^2*d*(e*Sin[c + d*x])^(3/2)) + (16*e*Cos[c + d*x])/(21*a^2
*d*(e*Sin[c + d*x])^(3/2)) + (20*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[
c + d*x]])/(21*a^2*d*Sqrt[e*Sin[c + d*x]])
```

Rubi [A] time = 0.590883, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3872, 2875, 2873, 2567, 2636, 2642, 2641, 2564, 14}

$$\frac{4e^3}{7a^2d(e\sin(c+dx))^{7/2}}-\frac{2e^3\cos^3(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}}-\frac{2e^3\cos(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}}-\frac{4e}{3a^2d(e\sin(c+dx))^{3/2}}+\frac{16e\cos(c+dx)}{21a^2d(e\sin(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] (4*e^3)/(7*a^2*d*(e*Sin[c + d*x])^(7/2)) - (2*e^3*Cos[c + d*x])/(7*a^2*d*(e
*Sin[c + d*x])^(7/2)) - (2*e^3*Cos[c + d*x]^3)/(7*a^2*d*(e*Sin[c + d*x])^(7
/2)) - (4*e)/(3*a^2*d*(e*Sin[c + d*x])^(3/2)) + (16*e*Cos[c + d*x])/(21*a^2
*d*(e*Sin[c + d*x])^(3/2)) + (20*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[
c + d*x]])/(21*a^2*d*Sqrt[e*Sin[c + d*x]])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
```

```
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \int \frac{\cos^2(c+dx)}{(-a-a\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$$

$$= \frac{e^4\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{(e\sin(c+dx))^{9/2}}\,dx}{a^4}$$

$$= \frac{e^4\int \left(\frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{9/2}} - \frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{9/2}} + \frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{9/2}}\right)dx}{a^4}$$

$$= \frac{e^4\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{9/2}}\,dx}{a^2} + \frac{e^4\int \frac{\cos^4(c+dx)}{(e\sin(c+dx))^{9/2}}\,dx}{a^2} - \frac{(2e^4)\int \frac{\cos^3(c+dx)}{(e\sin(c+dx))^{9/2}}\,dx}{a^2}$$

$$= -\frac{2e^3\cos(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}} - \frac{2e^3\cos^3(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}} - \frac{(2e^2)\int \frac{1}{(e\sin(c+dx))^{5/2}}\,dx}{7a^2} - \ldots$$

$$= -\frac{2e^3\cos(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}} - \frac{2e^3\cos^3(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}} + \frac{16e\cos(c+dx)}{21a^2d(e\sin(c+dx))^{3/2}} - \ldots$$

$$= \frac{4e^3}{7a^2d(e\sin(c+dx))^{7/2}} - \frac{2e^3\cos(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}} - \frac{2e^3\cos^3(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}} - \ldots$$

$$= \frac{4e^3}{7a^2d(e\sin(c+dx))^{7/2}} - \frac{2e^3\cos(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}} - \frac{2e^3\cos^3(c+dx)}{7a^2d(e\sin(c+dx))^{7/2}} - \ldots$$

Mathematica [A] time = 1.37354, size = 82, normalized size = 0.43

$$-\frac{\csc^3(c+dx)\left(40\sin^{\frac{7}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{4}(-2c-2dx+\pi),2\right)+16\sin^4\left(\frac{1}{2}(c+dx)\right)(11\cos(c+dx)+8)\right)}{42a^2d\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] -(Csc[c + d*x]^3*(16*(8 + 11*Cos[c + d*x])*Sin[(c + d*x)/2]^4 + 40*Elliptic
F[(-2*c + Pi - 2*d*x)/4, 2]*Sin[c + d*x]^(7/2)))/(42*a^2*d*Sqrt[e*Sin[c + d
*x]])
```

Maple [A] time = 1.707, size = 148, normalized size = 0.8

$$\frac{1}{d}\left(\frac{4e^3\left(7(\cos(dx+c))^2-4\right)}{21a^2}(e\sin(dx+c))^{-\frac{7}{2}} - \frac{2}{21a^2(\sin(dx+c))^4\cos(dx+c)}\left(5\sqrt{-\sin(dx+c)+1}\sqrt{2+2\sin(dx\ldots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x)
```

```
[Out] (4/21/a^2*e^3/(e*sin(d*x+c))^(7/2)*(7*cos(d*x+c)^2-4)-2/21*(5*(-sin(d*x+c)+
1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(9/2)*EllipticF((-sin(d*x+c)+1)^
(1/2),1/2*2^(1/2))+11*sin(d*x+c)^5-17*sin(d*x+c)^3+6*sin(d*x+c))/a^2/sin(d*
x+c)^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\sin(dx+c)}}{\left(a^2e\sec(dx+c)^2+2\,a^2e\sec(dx+c)+a^2e\right)\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*sin(d*x + c))/((a^2*e*sec(d*x + c)^2 + 2*a^2*e*sec(d*x + c)
 + a^2*e)*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{(a\sec(dx+c)+a)^2\sqrt{e\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*sqrt(e*sin(d*x + c))), x)
```

3.132 $\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=224

$$\frac{4e^3}{9a^2d(e\sin(c+dx))^{9/2}}-\frac{2e^3\cos^3(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}}-\frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}}-\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{15a^2de^2\sqrt{\sin(c+dx)}}-\frac{}{5a^2d(e}$$

```
[Out] (4*e^3)/(9*a^2*d*(e*Sin[c + d*x])^(9/2)) - (2*e^3*Cos[c + d*x])/(9*a^2*d*(e
*Sin[c + d*x])^(9/2)) - (2*e^3*Cos[c + d*x]^3)/(9*a^2*d*(e*Sin[c + d*x])^(9
/2)) - (4*e)/(5*a^2*d*(e*Sin[c + d*x])^(5/2)) + (16*e*Cos[c + d*x])/(45*a^2
*d*(e*Sin[c + d*x])^(5/2)) - (4*Cos[c + d*x])/(15*a^2*d*e*Sqrt[e*Sin[c + d*
x]]) - (4*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(15*a^2*d*
e^2*Sqrt[Sin[c + d*x]])
```

Rubi [A] time = 0.6638, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3872, 2875, 2873, 2567, 2636, 2640, 2639, 2564, 14}

$$\frac{4e^3}{9a^2d(e\sin(c+dx))^{9/2}}-\frac{2e^3\cos^3(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}}-\frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}}-\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{15a^2de^2\sqrt{\sin(c+dx)}}-\frac{}{5a^2d(e}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (4*e^3)/(9*a^2*d*(e*Sin[c + d*x])^(9/2)) - (2*e^3*Cos[c + d*x])/(9*a^2*d*(e
*Sin[c + d*x])^(9/2)) - (2*e^3*Cos[c + d*x]^3)/(9*a^2*d*(e*Sin[c + d*x])^(9
/2)) - (4*e)/(5*a^2*d*(e*Sin[c + d*x])^(5/2)) + (16*e*Cos[c + d*x])/(45*a^2
*d*(e*Sin[c + d*x])^(5/2)) - (4*Cos[c + d*x])/(15*a^2*d*e*Sqrt[e*Sin[c + d*
x]]) - (4*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(15*a^2*d*
e^2*Sqrt[Sin[c + d*x]])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \int \frac{\cos^2(c+dx)}{(-a-a\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx$$

$$= \frac{e^4\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{(e\sin(c+dx))^{11/2}}\,dx}{a^4}$$

$$= \frac{e^4\int \left(\frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{11/2}} - \frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{11/2}} + \frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{11/2}}\right)dx}{a^4}$$

$$= \frac{e^4\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{11/2}}\,dx}{a^2} + \frac{e^4\int \frac{\cos^4(c+dx)}{(e\sin(c+dx))^{11/2}}\,dx}{a^2} - \frac{(2e^4)\int \frac{\cos^3(c+dx)}{(e\sin(c+dx))^{11/2}}\,dx}{a^2}$$

$$= -\frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos^3(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{(2e^2)\int \frac{1}{(e\sin(c+dx))^{7/2}}\,dx}{9a^2} -$$

$$= -\frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos^3(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} + \frac{16e\cos(c+dx)}{45a^2d(e\sin(c+dx))^{5/2}} -$$

$$= \frac{4e^3}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos^3(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{}{5}$$

$$= \frac{4e^3}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos^3(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{}{5}$$

$$= \frac{4e^3}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{2e^3\cos^3(c+dx)}{9a^2d(e\sin(c+dx))^{9/2}} - \frac{}{5}$$

Mathematica [C] time = 1.42102, size = 163, normalized size = 0.73

$$\frac{\sec^4\left(\frac{1}{2}(c+dx)\right)(\cos(c+dx)+i\sin(c+dx))\left(e^{-2i(c+dx)}\sqrt{1-e^{2i(c+dx)}}\left(1+e^{i(c+dx)}\right)^4 \text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},e^{2i(c+dx}\right.\right.}{180a^2de\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (Sec[(c + d*x)/2]^4*(Cos[c + d*x] + I*Sin[c + d*x])*(-31 - 40*Cos[c + d*x]
- 19*Cos[2*(c + d*x)] + ((1 + E^(I*(c + d*x)))^4*Sqrt[1 - E^((2*I)*(c + d*x
))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))])/E^((2*I)*(c + d*
x)) + (16*I)*Sin[c + d*x] + (13*I)*Sin[2*(c + d*x)]))/(180*a^2*d*e*Sqrt[e*S
in[c + d*x]])
```

Maple [A] time = 1.753, size = 213, normalized size = 1.

$$\frac{1}{d}\left(\frac{4\,e^3\left(9\,(\cos(dx+c))^2-4\right)}{45\,a^2}(e\sin(dx+c))^{-\frac{9}{2}}+\frac{2}{45\,ea^2\,(\sin(dx+c))^5\cos(dx+c)}\left(6\,\sqrt{-\sin(dx+c)+1}\sqrt{2+2\,\sin(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x)
```

```
[Out] (4/45*e^3/a^2/(e*sin(d*x+c))^(9/2)*(9*cos(d*x+c)^2-4)+2/45/e*(6*(-sin(d*x+c
)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(11/2)*EllipticE((-sin(d*x+c)+
```

```
1)^(1/2),1/2*2^(1/2))-3*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*
x+c)^(11/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+6*sin(d*x+c)^7-19*
sin(d*x+c)^5+23*sin(d*x+c)^3-10*sin(d*x+c))/a^2/sin(d*x+c)^5/cos(d*x+c)/(e*
sin(d*x+c))^(1/2))/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{e\sin\left(dx+c\right)}}{a^2e^2\cos\left(dx+c\right)^2-a^2e^2+\left(a^2e^2\cos\left(dx+c\right)^2-a^2e^2\right)\sec\left(dx+c\right)^2+2\left(a^2e^2\cos\left(dx+c\right)^2-a^2e^2\right)\sec\left(dx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*sin(d*x + c))/(a^2*e^2*cos(d*x + c)^2 - a^2*e^2 + (a^2*e^2
*cos(d*x + c)^2 - a^2*e^2)*sec(d*x + c)^2 + 2*(a^2*e^2*cos(d*x + c)^2 - a^2
*e^2)*sec(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)^2\left(e\sin\left(dx+c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2)), x)
```

3.133 $\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=224

$$\frac{4\sqrt{\sin(c+dx)}\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{231a^2de^2\sqrt{e\sin(c+dx)}}+\frac{4e^3}{11a^2d(e\sin(c+dx))^{11/2}}-\frac{2e^3\cos^3(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}}-\frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{1}}$$

```
[Out] (4*e^3)/(11*a^2*d*(e*Sin[c + d*x])^(11/2)) - (2*e^3*Cos[c + d*x])/(11*a^2*d
*(e*Sin[c + d*x])^(11/2)) - (2*e^3*Cos[c + d*x]^3)/(11*a^2*d*(e*Sin[c + d*x
])^(11/2)) - (4*e)/(7*a^2*d*(e*Sin[c + d*x])^(7/2)) + (16*e*Cos[c + d*x])/(
77*a^2*d*(e*Sin[c + d*x])^(7/2)) - (4*Cos[c + d*x])/(231*a^2*d*e*(e*Sin[c +
 d*x])^(3/2)) + (4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(23
1*a^2*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Rubi [A] time = 0.670105, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3872, 2875, 2873, 2567, 2636, 2642, 2641, 2564, 14}

$$\frac{4e^3}{11a^2d(e\sin(c+dx))^{11/2}}-\frac{2e^3\cos^3(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}}-\frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}}+\frac{4\sqrt{\sin(c+dx)}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{231a^2de^2\sqrt{e\sin(c+dx)}}-\frac{}{7a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```



```
[Out] (4*e^3)/(11*a^2*d*(e*Sin[c + d*x])^(11/2)) - (2*e^3*Cos[c + d*x])/(11*a^2*d
*(e*Sin[c + d*x])^(11/2)) - (2*e^3*Cos[c + d*x]^3)/(11*a^2*d*(e*Sin[c + d*x
])^(11/2)) - (4*e)/(7*a^2*d*(e*Sin[c + d*x])^(7/2)) + (16*e*Cos[c + d*x])/(
77*a^2*d*(e*Sin[c + d*x])^(7/2)) - (4*Cos[c + d*x])/(231*a^2*d*e*(e*Sin[c +
 d*x])^(3/2)) + (4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(23
1*a^2*d*e^2*Sqrt[e*Sin[c + d*x]])
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{1}{(a+a\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \int \frac{\cos^2(c+dx)}{(-a-a\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$$
$$= \frac{e^4\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{(e\sin(c+dx))^{13/2}}\,dx}{a^4}$$
$$= \frac{e^4\int \left(\frac{a^2\cos^2(c+dx)}{(e\sin(c+dx))^{13/2}} - \frac{2a^2\cos^3(c+dx)}{(e\sin(c+dx))^{13/2}} + \frac{a^2\cos^4(c+dx)}{(e\sin(c+dx))^{13/2}}\right)dx}{a^4}$$
$$= \frac{e^4\int \frac{\cos^2(c+dx)}{(e\sin(c+dx))^{13/2}}\,dx}{a^2} + \frac{e^4\int \frac{\cos^4(c+dx)}{(e\sin(c+dx))^{13/2}}\,dx}{a^2} - \frac{(2e^4)\int \frac{\cos^3(c+dx)}{(e\sin(c+dx))^{13/2}}\,dx}{a^2}$$
$$= -\frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos^3(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{(2e^2)\int \frac{1}{(e\sin(c+dx))^{9/2}}\,d}{11a^2}$$
$$= -\frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos^3(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} + \frac{16e\cos(c+dx)}{77a^2d(e\sin(c+dx))^{7}}$$
$$= \frac{4e^3}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos^3(c+dx)}{11a^2d(e\sin(c+dx))^{11/}}$$
$$= \frac{4e^3}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos^3(c+dx)}{11a^2d(e\sin(c+dx))^{11/}}$$
$$= \frac{4e^3}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos(c+dx)}{11a^2d(e\sin(c+dx))^{11/2}} - \frac{2e^3\cos^3(c+dx)}{11a^2d(e\sin(c+dx))^{11/}}$$

Mathematica [A] time = 0.960652, size = 113, normalized size = 0.5

$$-\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(\sin^{\frac{11}{2}}(c+dx)\csc^4\left(\frac{1}{2}(c+dx)\right)\text{EllipticF}\left(\frac{1}{4}(-2c-2dx+\pi),2\right)+97\cos(c+dx)+4\,c\right.}{1848a^2de^2\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] -(Csc[(c + d*x)/2]*Sec[(c + d*x)/2]^5*(52 + 97*Cos[c + d*x] + 4*Cos[2*(c +
d*x)] + Cos[3*(c + d*x)] + Csc[(c + d*x)/2]^4*EllipticF[(-2*c + Pi - 2*d*x)
/4, 2]*Sin[c + d*x]^(11/2)))/(1848*a^2*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Maple [A] time = 1.817, size = 160, normalized size = 0.7

$$\frac{1}{d}\left(\frac{4e^3\left(11(\cos(dx+c))^2-4\right)}{77a^2}(e\sin(dx+c))^{-\frac{11}{2}} - \frac{2}{231e^2a^2(\sin(dx+c))^6\cos(dx+c)}\left(\sqrt{-\sin(dx+c)+1}\sqrt{2+2\sin(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x)
```

```
[Out] (4/77*e^3/a^2/(e*sin(d*x+c))^(11/2)*(11*cos(d*x+c)^2-4)-2/231/e^2*((-sin(d*
x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(13/2)*EllipticF((-sin(d*x+
c)+1)^(1/2),1/2*2^(1/2))-2*sin(d*x+c)^7+47*sin(d*x+c)^5-87*sin(d*x+c)^3+42*
```

```
sin(d*x+c))/a^2/sin(d*x+c)^6/cos(d*x+c)/(e*sin(d*x+c))^(1/2))/d
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{\sqrt{e\sin\left(dx+c\right)}}{\left(a^2e^3\cos\left(dx+c\right)^2-a^2e^3+\left(a^2e^3\cos\left(dx+c\right)^2-a^2e^3\right)\sec\left(dx+c\right)^2+2\left(a^2e^3\cos\left(dx+c\right)^2-a^2e^3\right)\sec\left(dx\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(e*sin(d*x + c))/((a^2*e^3*cos(d*x + c)^2 - a^2*e^3 + (a^2*e^
3*cos(d*x + c)^2 - a^2*e^3)*sec(d*x + c)^2 + 2*(a^2*e^3*cos(d*x + c)^2 - a^
2*e^3)*sec(d*x + c))*sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**2/(e*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a\sec\left(dx+c\right)+a\right)^2\left(e\sin\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(5/2)), x)
```

3.134 $\int (a+a\sec(c+dx))^3(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=247

$$\frac{3a^3(e\sin(c+dx))^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}+\frac{a^3(e\sin(c+dx))^{m+1}\text{Hypergeometric2F1}\left(2,\right.}{de(m+1)}$$

```
[Out] (a^3*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (3*a^3*H
ypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^
(1 + m))/(d*e*(1 + m)) + (a^3*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, Si
n[c + d*x]^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)) + (3*a^3*Sqrt[Cos[c +
 d*x]^2]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c
 + d*x]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m))
```

Rubi [A] time = 0.352641, antiderivative size = 247, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3872, 2873, 2643, 2564, 364, 2577}

$$\frac{3a^3(e\sin(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}+\frac{a^3(e\sin(c+dx))^{m+1}\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}+\frac{a^3\cos(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*(e*Sin[c + d*x])^m,x]
```

```
[Out] (a^3*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (3*a^3*H
ypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^
(1 + m))/(d*e*(1 + m)) + (a^3*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, Si
n[c + d*x]^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)) + (3*a^3*Sqrt[Cos[c +
 d*x]^2]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c
 + d*x]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^3(e\sin(c+dx))^m\,dx &= -\int (-a-a\cos(c+dx))^3\sec^3(c+dx)(e\sin(c+dx))^m\,dx\\ &= -\int \left(-a^3(e\sin(c+dx))^m-3a^3\sec(c+dx)(e\sin(c+dx))^m-3a^3\sec^2(c+\right.\\ &= a^3\int (e\sin(c+dx))^m\,dx+a^3\int \sec^3(c+dx)(e\sin(c+dx))^m\,dx+\left(3a^3\right)\int\\ &= \frac{a^3\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}}+\frac{3a^3\sqrt{c}}{}\\ &= \frac{a^3\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}}+\frac{3a^3\,{}_2F_1}{}\end{aligned}$$

Mathematica [F] time = 2.15499, size = 0, normalized size = 0.

$$\int (a+a\sec(c+dx))^3(e\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*(e*Sin[c + d*x])^m,x]

[Out] Integrate[(a + a*Sec[c + d*x])^3*(e*Sin[c + d*x])^m, x]
```

Maple [F] time = 0.942, size = 0, normalized size = 0.

$$\int (a+a\sec(dx+c))^3(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))^3*(e*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^3 (e\sin(dx+c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*(e*sin(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a^3\sec(dx+c)^3+3\,a^3\sec(dx+c)^2+3\,a^3\sec(dx+c)+a^3\right)(e\sin(dx+c))^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*a^3*sec(d*x + c) +
a^3)*(e*sin(d*x + c))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(e*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^3 (e\sin(dx+c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^3*(e*sin(d*x + c))^m, x)
```

3.135 $\int (a+a\sec(c+dx))^2(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=195

$$\frac{2a^2(e\sin(c+dx))^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}+\frac{a^2\cos(c+dx)(e\sin(c+dx))^{m+1}\text{Hypergeo}}{de(m+1)\sqrt{\text{co}}}$$

```
[Out] (a^2*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (2*a^2*H
ypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^
(1 + m))/(d*e*(1 + m)) + (a^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (
1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Sin[c + d*x])^(1 + m))
/(d*e*(1 + m))
```

Rubi [A] time = 0.285148, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3872, 2873, 2643, 2564, 364, 2577}

$$\frac{2a^2(e\sin(c+dx))^{m+1}{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}+\frac{a^2\cos(c+dx)(e\sin(c+dx))^{m+1}{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right.}{de(m+1)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^m,x]
```

```
[Out] (a^2*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (2*a^2*H
ypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^
(1 + m))/(d*e*(1 + m)) + (a^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (
1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*Sec[c + d*x]*(e*Sin[c + d*x])^(1 + m))
/(d*e*(1 + m))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
```

```
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (a+a\sec(c+dx))^2(e\sin(c+dx))^m\,dx = \int (-a-a\cos(c+dx))^2\sec^2(c+dx)(e\sin(c+dx))^m\,dx$$
$$= \int \left(a^2(e\sin(c+dx))^m + 2a^2\sec(c+dx)(e\sin(c+dx))^m + a^2\sec^2(c+dx)(e\,\mathrm{si}\right.$$
$$= a^2\int (e\sin(c+dx))^m\,dx + a^2\int \sec^2(c+dx)(e\sin(c+dx))^m\,dx + \left(2a^2\right)\int \mathrm{se}$$
$$= \frac{a^2\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} + \frac{a^2\sqrt{\cos^2(c}}{}$$
$$= \frac{a^2\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} + \frac{2a^2\,{}_2F_1\left(1\right.}{}$$

Mathematica [F] time = 0.93236, size = 0, normalized size = 0.

$$\int (a+a\sec(c+dx))^2(e\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^m,x]

[Out] Integrate[(a + a*Sec[c + d*x])^2*(e*Sin[c + d*x])^m, x]
```

Maple [F] time = 0.79, size = 0, normalized size = 0.

$$\int \left(a+a\sec\left(dx+c\right)\right)^2\left(e\sin\left(dx+c\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^m,x)

[Out] int((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^2 (e\sin(dx+c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2\sec(dx+c)^2+2\,a^2\sec(dx+c)+a^2\right)(e\sin(dx+c))^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*(e*sin(d*x + c))^m
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^2 (e\sin(dx+c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2*(e*sin(d*x + c))^m, x)
```

3.136 $\int (a+a\sec(c+dx))(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=119

$$\frac{a(e\sin(c+dx))^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}+\frac{a\cos(c+dx)(e\sin(c+dx))^{m+1}\text{Hypergeometri}}{de(m+1)\sqrt{\cos^2(c+}}$$

```
[Out] (a*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2
]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (a*Hyperge
ometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1 + m
))/(d*e*(1 + m))
```

Rubi [A] time = 0.143024, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2838, 2564, 364, 2643}

$$\frac{a(e\sin(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}+\frac{a\cos(c+dx)(e\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^m,x]
```

```
[Out] (a*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2
]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (a*Hyperge
ometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1 + m
))/(d*e*(1 + m))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (a + a\sec(c+dx))(e\sin(c+dx))^m\,dx = -\int (-a - a\cos(c+dx))\sec(c+dx)(e\sin(c+dx))^m\,dx$$
$$= a\int (e\sin(c+dx))^m\,dx + a\int \sec(c+dx)(e\sin(c+dx))^m\,dx$$
$$= \frac{a\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} + \frac{a\,\mathrm{Subst}\Big(}{}$$
$$= \frac{a\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} + \frac{a\,{}_2F_1\Big(1,}{}$$

Mathematica [A] time = 0.145446, size = 97, normalized size = 0.82

$$\frac{a(e\sin(c+dx))^m\left(\sin(c+dx)\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)+\sqrt{\cos^2(c+dx)}\tan(c+dx)\text{Hyperg}\right.}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*(e*Sin[c + d*x])^m,x]
```

```
[Out] (a*(e*Sin[c + d*x])^m*(Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d
*x]^2]*Sin[c + d*x] + Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + m)/2
, (3 + m)/2, Sin[c + d*x]^2]*Tan[c + d*x]))/(d*(1 + m))
```

Maple [F] time = 0.651, size = 0, normalized size = 0.

$$\int (a + a\sec(dx+c))(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))*(e*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c) + a)(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sec\left(dx+c\right)+a)\left(e\sin\left(dx+c\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\left(e\sin\left(c+dx\right)\right)^{m}\,dx+\int\left(e\sin\left(c+dx\right)\right)^{m}\sec\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))**m,x)
```

```
[Out] a*(Integral((e*sin(c + d*x))**m, x) + Integral((e*sin(c + d*x))**m*sec(c +
d*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sec\left(dx+c\right)+a\right)\left(e\sin\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

3.137 $\int \frac{(e\sin(c+dx))^m}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=100

$$\frac{e\cos(c+dx)(e\sin(c+dx))^{m-1}\text{Hypergeometric2F1}\left(-\frac{1}{2},\frac{m-1}{2},\frac{m+1}{2},\sin^2(c+dx)\right)}{ad(1-m)\sqrt{\cos^2(c+dx)}}-\frac{e(e\sin(c+dx))^{m-1}}{ad(1-m)}$$

```
[Out] -((e*(e*Sin[c + d*x])^(-1 + m))/(a*d*(1 - m))) + (e*Cos[c + d*x]*Hypergeome
tric2F1[-1/2, (-1 + m)/2, (1 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(-1 +
 m))/(a*d*(1 - m)*Sqrt[Cos[c + d*x]^2])
```

Rubi [A] time = 0.198809, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3872, 2839, 2564, 30, 2577}

$$\frac{e\cos(c+dx)(e\sin(c+dx))^{m-1}\,{}_2F_1\left(-\frac{1}{2},\frac{m-1}{2};\frac{m+1}{2};\sin^2(c+dx)\right)}{ad(1-m)\sqrt{\cos^2(c+dx)}}-\frac{e(e\sin(c+dx))^{m-1}}{ad(1-m)}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((e*(e*Sin[c + d*x])^(-1 + m))/(a*d*(1 - m))) + (e*Cos[c + d*x]*Hypergeome
tric2F1[-1/2, (-1 + m)/2, (1 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(-1 +
 m))/(a*d*(1 - m)*Sqrt[Cos[c + d*x]^2])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^m}{a+a\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)(e\sin(c+dx))^m}{-a-a\cos(c+dx)}\,dx$$
$$= \frac{e^2\int \cos(c+dx)(e\sin(c+dx))^{-2+m}\,dx}{a} - \frac{e^2\int \cos^2(c+dx)(e\sin(c+dx))^{-2+m}\,dx}{a}$$
$$= \frac{e\cos(c+dx)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(-1+m);\frac{1+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{-1+m}}{ad(1-m)\sqrt{\cos^2(c+dx)}} + \frac{e\,\mathrm{Subst}\left(\int x^{-2+m}\,dx\right.}{ad}$$
$$= -\frac{e(e\sin(c+dx))^{-1+m}}{ad(1-m)} + \frac{e\cos(c+dx)\,{}_2F_1\left(-\frac{1}{2},\frac{1}{2}(-1+m);\frac{1+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{-1+}}{ad(1-m)\sqrt{\cos^2(c+dx)}}$$

Mathematica [F] time = 29.8222, size = 0, normalized size = 0.

$$\int \frac{(e\sin(c+dx))^m}{a+a\sec(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x]),x]

[Out] Integrate[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x]), x]
```

Maple [F] time = 0.619, size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^m}{a+a\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c)),x)

[Out] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^m}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="maxima")

[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e\sin(dx+c))^m}{a\sec(dx+c)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((e*sin(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{(e\sin(c+dx))^m}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral((e*sin(c + d*x))**m/(sec(c + d*x) + 1), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^m}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a), x)
```

3.138 $\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=207

$$-\frac{e^3\cos(c+dx)(e\sin(c+dx))^{m-3}\text{Hypergeometric2F1}\left(-\frac{3}{2},\frac{m-3}{2},\frac{m-1}{2},\sin^2(c+dx)\right)}{a^2d(3-m)\sqrt{\cos^2(c+dx)}}-\frac{e^3\cos(c+dx)(e\sin(c+dx))^{m-3}\text{H}}{a^2d(}$$

```
[Out] (2*e^3*(e*Sin[c + d*x])^(-3 + m))/(a^2*d*(3 - m)) - (e^3*Cos[c + d*x]*Hyper
geometric2F1[-3/2, (-3 + m)/2, (-1 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])
^(-3 + m))/(a^2*d*(3 - m)*Sqrt[Cos[c + d*x]^2]) - (e^3*Cos[c + d*x]*Hyperge
ometric2F1[-1/2, (-3 + m)/2, (-1 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(
-3 + m))/(a^2*d*(3 - m)*Sqrt[Cos[c + d*x]^2]) - (2*e*(e*Sin[c + d*x])^(-1 +
 m))/(a^2*d*(1 - m))
```

Rubi [A] time = 0.525747, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3872, 2875, 2873, 2577, 2564, 14}

$$-\frac{e^3\cos(c+dx)(e\sin(c+dx))^{m-3}{}_2F_1\left(-\frac{3}{2},\frac{m-3}{2};\frac{m-1}{2};\sin^2(c+dx)\right)}{a^2d(3-m)\sqrt{\cos^2(c+dx)}}-\frac{e^3\cos(c+dx)(e\sin(c+dx))^{m-3}{}_2F_1\left(-\frac{1}{2},\frac{m-3}{2};\frac{m-1}{2};\text{s}\right.}{a^2d(3-m)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*e^3*(e*Sin[c + d*x])^(-3 + m))/(a^2*d*(3 - m)) - (e^3*Cos[c + d*x]*Hyper
geometric2F1[-3/2, (-3 + m)/2, (-1 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])
^(-3 + m))/(a^2*d*(3 - m)*Sqrt[Cos[c + d*x]^2]) - (e^3*Cos[c + d*x]*Hyperge
ometric2F1[-1/2, (-3 + m)/2, (-1 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(
-3 + m))/(a^2*d*(3 - m)*Sqrt[Cos[c + d*x]^2]) - (2*e*(e*Sin[c + d*x])^(-1 +
 m))/(a^2*d*(1 - m))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)(e\sin(c+dx))^m}{(-a-a\cos(c+dx))^2}\,dx\\
&= \frac{e^4\int \cos^2(c+dx)(-a+a\cos(c+dx))^2(e\sin(c+dx))^{-4+m}\,dx}{a^4}\\
&= \frac{e^4\int \left(a^2\cos^2(c+dx)(e\sin(c+dx))^{-4+m}-2a^2\cos^3(c+dx)(e\sin(c+dx))^{-4+m}+a^2\cos^4(\right.}{a^4}\\
&= \frac{e^4\int \cos^2(c+dx)(e\sin(c+dx))^{-4+m}\,dx}{a^2}+\frac{e^4\int \cos^4(c+dx)(e\sin(c+dx))^{-4+m}\,dx}{a^2}-\frac{(2e^4}{}\\
&= -\frac{e^3\cos(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);\sin^2(c+dx)\right)(e\sin(c+dx))^{-3+m}}{a^2d(3-m)\sqrt{\cos^2(c+dx)}}-\frac{e^3\text{co}}{}\\
&= -\frac{e^3\cos(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);\sin^2(c+dx)\right)(e\sin(c+dx))^{-3+m}}{a^2d(3-m)\sqrt{\cos^2(c+dx)}}-\frac{e^3\text{co}}{}\\
&= \frac{2e^3(e\sin(c+dx))^{-3+m}}{a^2d(3-m)}-\frac{e^3\cos(c+dx)\,{}_2F_1\left(-\frac{3}{2},\frac{1}{2}(-3+m);\frac{1}{2}(-1+m);\sin^2(c+dx)\right)(e\,\text{s}}{a^2d(3-m)\sqrt{\cos^2(c+dx)}}
\end{aligned}$$

Mathematica [F] time = 0.673564, size = 0, normalized size = 0.

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] Integrate[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^2, x]
```

Maple [F] time = 0.364, size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^m}{(a+a\sec(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^2,x)
```

```
[Out] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^m}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{(e\sin(dx+c))^m}{a^2\sec(dx+c)^2+2\,a^2\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((e*sin(d*x + c))^m/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{(e\sin(c+dx))^m}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral((e*sin(c + d*x))**m/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a**
2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^m}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a)^2, x)
```

3.139 $\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=236

$$\frac{e^5\cos(c+dx)(e\sin(c+dx))^{m-5}\text{Hypergeometric2F1}\left(-\frac{5}{2},\frac{m-5}{2},\frac{m-3}{2},\sin^2(c+dx)\right)}{a^3d(5-m)\sqrt{\cos^2(c+dx)}}+\frac{3e^5\cos(c+dx)(e\sin(c+dx))^{m-}}{a^3}$$

```
[Out] (-4*e^5*(e*Sin[c + d*x])^(-5 + m))/(a^3*d*(5 - m)) + (e^5*Cos[c + d*x]*Hype
rgeometric2F1[-5/2, (-5 + m)/2, (-3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x]
)^(-5 + m))/(a^3*d*(5 - m)*Sqrt[Cos[c + d*x]^2]) + (3*e^5*Cos[c + d*x]*Hype
rgeometric2F1[-3/2, (-5 + m)/2, (-3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x]
)^(-5 + m))/(a^3*d*(5 - m)*Sqrt[Cos[c + d*x]^2]) + (7*e^3*(e*Sin[c + d*x])^
(-3 + m))/(a^3*d*(3 - m)) - (3*e*(e*Sin[c + d*x])^(-1 + m))/(a^3*d*(1 - m))
```

Rubi [A] time = 0.635327, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3872, 2875, 2873, 2564, 14, 2577, 270}

$$\frac{e^5\cos(c+dx)(e\sin(c+dx))^{m-5}\,{}_2F_1\left(-\frac{5}{2},\frac{m-5}{2};\frac{m-3}{2};\sin^2(c+dx)\right)}{a^3d(5-m)\sqrt{\cos^2(c+dx)}}+\frac{3e^5\cos(c+dx)(e\sin(c+dx))^{m-5}\,{}_2F_1\left(-\frac{3}{2},\frac{m-5}{2};\frac{m-}{2}\right.}{a^3d(5-m)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^3,x]
```


```
[Out] (-4*e^5*(e*Sin[c + d*x])^(-5 + m))/(a^3*d*(5 - m)) + (e^5*Cos[c + d*x]*Hype
rgeometric2F1[-5/2, (-5 + m)/2, (-3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x]
)^(-5 + m))/(a^3*d*(5 - m)*Sqrt[Cos[c + d*x]^2]) + (3*e^5*Cos[c + d*x]*Hype
rgeometric2F1[-3/2, (-5 + m)/2, (-3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x]
)^(-5 + m))/(a^3*d*(5 - m)*Sqrt[Cos[c + d*x]^2]) + (7*e^3*(e*Sin[c + d*x])^
(-3 + m))/(a^3*d*(3 - m)) - (3*e*(e*Sin[c + d*x])^(-1 + m))/(a^3*d*(1 - m))
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx &= -\int \frac{\cos^3(c+dx)(e\sin(c+dx))^m}{(-a-a\cos(c+dx))^3}\,dx\\
&= -\frac{e^6\int \cos^3(c+dx)(-a+a\cos(c+dx))^3(e\sin(c+dx))^{-6+m}\,dx}{a^6}\\
&= -\frac{e^6\int \left(-a^3\cos^3(c+dx)(e\sin(c+dx))^{-6+m}+3a^3\cos^4(c+dx)(e\sin(c+dx))^{-6+m}-3a^3\cos^5\right.}{a^6}\\
&= \frac{e^6\int \cos^3(c+dx)(e\sin(c+dx))^{-6+m}\,dx}{a^3}-\frac{e^6\int \cos^6(c+dx)(e\sin(c+dx))^{-6+m}\,dx}{a^3}-\frac{\left(3e^6\right)\int}{}\\
&= \frac{e^5\cos(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}(-5+m);\frac{1}{2}(-3+m);\sin^2(c+dx)\right)(e\sin(c+dx))^{-5+m}}{a^3d(5-m)\sqrt{\cos^2(c+dx)}}+\frac{3e^5\cos(c}{}\\
&= \frac{e^5\cos(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}(-5+m);\frac{1}{2}(-3+m);\sin^2(c+dx)\right)(e\sin(c+dx))^{-5+m}}{a^3d(5-m)\sqrt{\cos^2(c+dx)}}+\frac{3e^5\cos(c}{}\\
&= -\frac{4e^5(e\sin(c+dx))^{-5+m}}{a^3d(5-m)}+\frac{e^5\cos(c+dx)\,{}_2F_1\left(-\frac{5}{2},\frac{1}{2}(-5+m);\frac{1}{2}(-3+m);\sin^2(c+dx)\right)(e\sin}{a^3d(5-m)\sqrt{\cos^2(c+dx)}}
\end{aligned}$$

Mathematica [F] time = 1.21067, size = 0, normalized size = 0.

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^3}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] Integrate[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^3, x]
```

Maple [F] time = 0.409, size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^m}{(a+a\sec(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^3,x)
```

```
[Out] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^m}{(a\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e\sin(dx+c))^m}{a^3\sec(dx+c)^3+3\,a^3\sec(dx+c)^2+3\,a^3\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((e*sin(d*x + c))^m/(a^3*sec(d*x + c)^3 + 3*a^3*sec(d*x + c)^2 + 3*
a^3*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \sin(dx+c))^m}{(a \sec(dx+c)+a)^3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a)^3, x)
```

3.140 $\int (a + a\sec(c + dx))^{3/2}(e\sin(c + dx))^m\,dx$

Optimal. Leaf size=106

$$\frac{2ae\sqrt{a\sec(c+dx)+a}\,(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{-m/2}F_1\left(-\frac{1}{2};\frac{1-m}{2},\frac{1}{2}(-m-2);\frac{1}{2};\cos(c+dx),-\cos(c+dx)\right)}{d}$$

```
[Out] (2*a*e*AppellF1[-1/2, (1 - m)/2, (-2 - m)/2, 1/2, Cos[c + d*x], -Cos[c + d*
x]]*(1 - Cos[c + d*x])^((1 - m)/2)*Sqrt[a + a*Sec[c + d*x]]*(e*Sin[c + d*x]
)^(-1 + m))/(d*(1 + Cos[c + d*x])^(m/2))
```

Rubi [A] time = 0.37476, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3876, 2886, 135, 133}

$$\frac{2ae\sqrt{a\sec(c+dx)+a}\,(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{-m/2}F_1\left(-\frac{1}{2};\frac{1-m}{2},\frac{1}{2}(-m-2);\frac{1}{2};\cos(c+dx),-\cos(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m,x]
```

```
[Out] (2*a*e*AppellF1[-1/2, (1 - m)/2, (-2 - m)/2, 1/2, Cos[c + d*x], -Cos[c + d*
x]]*(1 - Cos[c + d*x])^((1 - m)/2)*Sqrt[a + a*Sec[c + d*x]]*(e*Sin[c + d*x]
)^(-1 + m))/(d*(1 + Cos[c + d*x])^(m/2))
```

Rule 3876

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(Sin[e + f*x]^FracPart[m]*(a + b*Csc[e + f*x]
)^FracPart[m])/(b + a*Sin[e + f*x])^FracPart[m], Int[((g*Cos[e + f*x])^p*(b
 + a*Sin[e + f*x])^m)/Sin[e + f*x]^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rule 2886

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos
[e + f*x])^(p - 1))/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x]
)^((p - 1)/2)), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
```

```
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx = \frac{\left(\sqrt{-\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\right)\int \frac{(-a-a\cos(c+dx))^{3/2}(e\sin(c+dx))^m}{(-\cos(c+dx))^{3/2}}\,dx}{\sqrt{-a-a\cos(c+dx)}}$$

$$= -\frac{\left(e\sqrt{-\cos(c+dx)}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}}(-a+a\cos(c+dx))^{\frac{1-m}{2}}\sqrt{a+\right.}{}$$

$$= \frac{\left(ae\sqrt{-\cos(c+dx)}(1+\cos(c+dx))^{-m/2}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}+\frac{m}{2}}(-a+\right.}{}$$

$$= \frac{\left(ae(1-\cos(c+dx))^{\frac{1}{2}-\frac{m}{2}}\sqrt{-\cos(c+dx)}(1+\cos(c+dx))^{-m/2}(-a-a\cos(c+\right.}{}$$

$$= \frac{2aeF_1\left(-\frac{1}{2};\frac{1-m}{2},\frac{1}{2}(-2-m);\frac{1}{2};\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{\frac{1}{}}}{d}$$

Mathematica [B] time = 9.70993, size = 1243, normalized size = 11.73

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m,x]
```

```
[Out] (4*(3 + m)*(AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + AppellF1[(1 + m)/2, 1/2, m, (3 + m)/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[(1 + m)/2, 3/2, m, (3 + m)/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[(c + d*x)/2]^3*(a*(1 + Sec[c + d
*x]))^(3/2)*Sin[(c + d*x)/2]*(e*Sin[c + d*x])^m)/(d*(1 + m)*(6*AppellF1[(1
+ m)/2, 3/2, m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*m*A
ppellF1[(1 + m)/2, 3/2, m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2] - 2*AppellF1[(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2] - 2*m*AppellF1[(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[(3 + m)/2, 1/2, 1 + m, (5 + m)
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*m*AppellF1[(3 + m)/2, 1/2,
 1 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[(3 +
 m)/2, 3/2, m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 4*m*Ap
pellF1[(3 + m)/2, 3/2, 1 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2] + 6*AppellF1[(3 + m)/2, 5/2, m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2] + 6*AppellF1[(1 + m)/2, 3/2, m, (3 + m)/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2]*Cos[c + d*x] + 2*m*AppellF1[(1 + m)/2, 3/2, m, (3
+ m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[c + d*x] + 2*AppellF1[
(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
*Cos[c + d*x] + 2*m*AppellF1[(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[c + d*x] + AppellF1[(3 + m)/2, 1/2, 1 + m
, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[c + d*x] + 2*m*Ap
pellF1[(3 + m)/2, 1/2, 1 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2]*Cos[c + d*x] - AppellF1[(3 + m)/2, 3/2, m, (5 + m)/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2]*Cos[c + d*x] + 4*m*AppellF1[(3 + m)/2, 3/2, 1 +
m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[c + d*x] - 6*App
ellF1[(3 + m)/2, 5/2, m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
```

```
]*Cos[c + d*x] + (3 + m)*AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x]) + (3 + m)*AppellF1[(1
 + m)/2, 1/2, m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + C
os[c + d*x])))
```

Maple [F] time = 0.191, size = 0, normalized size = 0.

$$\int (a + a\sec(dx + c))^{\frac{3}{2}} (e\sin(dx + c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx + c) + a)^{\frac{3}{2}} (e\sin(dx + c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sec(dx + c) + a)^{\frac{3}{2}} (e\sin(dx + c))^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(e*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{\frac{3}{2}}(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)
```

3.141 $\int \sqrt{a+a\sec(c+dx)}(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=107

$$-\frac{2e\cos(c+dx)\sqrt{a\sec(c+dx)+a}(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{-m/2}F_1\left(\frac{1}{2};\frac{1-m}{2},-\frac{m}{2};\frac{3}{2};\cos(c+dx),-\cos(c+d\right.}{d}$$

```
[Out] (-2*e*AppellF1[1/2, (1 - m)/2, -m/2, 3/2, Cos[c + d*x], -Cos[c + d*x]]*(1 -
 Cos[c + d*x])^((1 - m)/2)*Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(e*Sin[c +
 d*x])^(-1 + m))/(d*(1 + Cos[c + d*x])^(m/2))
```

Rubi [A] time = 0.313668, antiderivative size = 107, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3876, 2886, 135, 133}

$$-\frac{2e\cos(c+dx)\sqrt{a\sec(c+dx)+a}(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{-m/2}F_1\left(\frac{1}{2};\frac{1-m}{2},-\frac{m}{2};\frac{3}{2};\cos(c+dx),-\cos(c+d\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*(e*Sin[c + d*x])^m,x]
```

```
[Out] (-2*e*AppellF1[1/2, (1 - m)/2, -m/2, 3/2, Cos[c + d*x], -Cos[c + d*x]]*(1 -
 Cos[c + d*x])^((1 - m)/2)*Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(e*Sin[c +
 d*x])^(-1 + m))/(d*(1 + Cos[c + d*x])^(m/2))
```

Rule 3876

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(Sin[e + f*x]^FracPart[m]*(a + b*Csc[e + f*x]
)^FracPart[m])/(b + a*Sin[e + f*x])^FracPart[m], Int[((g*Cos[e + f*x])^p*(b
 + a*Sin[e + f*x])^m)/Sin[e + f*x]^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rule 2886

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos
[e + f*x])^(p - 1))/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x]
)^((p - 1)/2)), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
```

```
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}(e\sin(c+dx))^m\,dx = \frac{\left(\sqrt{-\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\right)\int \frac{\sqrt{-a-a\cos(c+dx)}(e\sin(c+dx))^m}{\sqrt{-\cos(c+dx)}}\,dx}{\sqrt{-a-a\cos(c+dx)}}$$

$$= -\frac{\left(e\sqrt{-\cos(c+dx)}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}}(-a+a\cos(c+dx))^{\frac{1-m}{2}}\sqrt{a+a\ldots}\right.}{\ldots}$$

$$= -\frac{\left(e\sqrt{-\cos(c+dx)}(1+\cos(c+dx))^{-m/2}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}+\frac{m}{2}}(-a+a\ldots\right.}{\ldots}$$

$$= -\frac{\left(e(1-\cos(c+dx))^{\frac{1}{2}-\frac{m}{2}}\sqrt{-\cos(c+dx)}(1+\cos(c+dx))^{-m/2}(-a-a\cos(c+\ldots\right.}{\ldots}$$

$$= -\frac{2eF_1\left(\frac{1}{2};\frac{1-m}{2},-\frac{m}{2};\frac{3}{2};\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{\frac{1-m}{2}}\cos(c+\ldots}{d}$$

Mathematica [B] time = 2.81868, size = 433, normalized size = 4.05

$$\frac{4(m+3)\sin\left(\frac{1}{2}(c\ldots\right.}{d(m+1)\left((\cos(c+dx)-1)\left(2(m+1)F_1\left(\frac{m+3}{2};-\frac{1}{2},m+2;\frac{m+5}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)+(2m+1)F_1\left(\frac{m+3}{2};\ldots\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*(e*Sin[c + d*x])^m,x]
```

```
[Out] (4*(3 + m)*(AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + AppellF1[(1 + m)/2, 1/2, m, (3 + m)/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[(c + d*x)/2]^3*Sqrt[a*(1 + Sec[c + d*x])]
*Sin[(c + d*x)/2]*(e*Sin[c + d*x])^m)/(d*(1 + m)*((2*(1 + m)*AppellF1[(3 +
m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (1
 + 2*m)*AppellF1[(3 + m)/2, 1/2, 1 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2] - AppellF1[(3 + m)/2, 3/2, m, (5 + m)/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + (3 + m)*AppellF1[(1 + m)/2,
-1/2, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c
 + d*x]) + (3 + m)*AppellF1[(1 + m)/2, 1/2, m, (3 + m)/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x])))
```

Maple [F] time = 0.211, size = 0, normalized size = 0.

$$\int \sqrt{a+a\sec(dx+c)}\,(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))^(1/2)*(e*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sec (dx + c) + a} \, (e \sin (dx + c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{a \sec (dx + c) + a} \, (e \sin (dx + c))^m \, , x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*(e*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \sec (dx + c) + a} \, (e \sin (dx + c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(e*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

3.142 $\int \frac{(e\sin(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=115

$$-\frac{2e\cos(c+dx)(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{1-\frac{m}{2}}F_1\left(\frac{3}{2};\frac{1-m}{2},\frac{2-m}{2};\frac{5}{2};\cos(c+dx),-\cos(c+dx)\right)(e\sin(c+dx))^{m-1}}{3d\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (-2*e*AppellF1[3/2, (1 - m)/2, (2 - m)/2, 5/2, Cos[c + d*x], -Cos[c + d*x]]
*(1 - Cos[c + d*x])^((1 - m)/2)*Cos[c + d*x]*(1 + Cos[c + d*x])^(1 - m/2)*(
e*Sin[c + d*x])^(-1 + m))/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.32496, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3876, 2886, 135, 133}

$$-\frac{2e\cos(c+dx)(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{1-\frac{m}{2}}F_1\left(\frac{3}{2};\frac{1-m}{2},\frac{2-m}{2};\frac{5}{2};\cos(c+dx),-\cos(c+dx)\right)(e\sin(c+dx))^{m-1}}{3d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^m/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (-2*e*AppellF1[3/2, (1 - m)/2, (2 - m)/2, 5/2, Cos[c + d*x], -Cos[c + d*x]]
*(1 - Cos[c + d*x])^((1 - m)/2)*Cos[c + d*x]*(1 + Cos[c + d*x])^(1 - m/2)*(
e*Sin[c + d*x])^(-1 + m))/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3876

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(Sin[e + f*x]^FracPart[m]*(a + b*Csc[e + f*x]
)^FracPart[m])/(b + a*Sin[e + f*x])^FracPart[m], Int[((g*Cos[e + f*x])^p*(b
 + a*Sin[e + f*x])^m)/Sin[e + f*x]^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rule 2886

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos
[e + f*x])^(p - 1))/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x]
)^((p - 1)/2)), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
```

```
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{\sqrt{-a-a\cos(c+dx)}\int \frac{\sqrt{-\cos(c+dx)}(e\sin(c+dx))^m}{\sqrt{-a-a\cos(c+dx)}}\,dx}{\sqrt{-\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{\left(e(-a-a\cos(c+dx))^{\frac{1}{2}+\frac{1-m}{2}}(-a+a\cos(c+dx))^{\frac{1-m}{2}}(e\sin(c+dx))^{-1+m}\right)\text{Subst}\left(\int \sqrt{-x}(\right.}{d\sqrt{-\cos(c+dx)}\sqrt{a+a\sec(c+dx}}$$

$$= -\frac{\left(e(1+\cos(c+dx))^{1-\frac{m}{2}}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}+\frac{m}{2}}(-a+a\cos(c+dx))^{\frac{1-m}{2}}(e\sin(c+d\right.}{d\sqrt{-\cos(c+dx)}\sqrt{a+a}}$$

$$= -\frac{\left(e(1-\cos(c+dx))^{\frac{1}{2}-\frac{m}{2}}(1+\cos(c+dx))^{1-\frac{m}{2}}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}+\frac{m}{2}}(-a+a\cos(c\right.}{d\sqrt{-\cos(c}}$$

$$= -\frac{2eF_1\left(\frac{3}{2};\frac{1-m}{2},\frac{2-m}{2};\frac{5}{2};\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{\frac{1-m}{2}}\cos(c+dx)(1+\text{co}}{3d\sqrt{a+a\sec(c+dx)}}$$

Mathematica [B] time = 2.0373, size = 277, normalized size = 2.41

$$\frac{4(m+3)\sin\left(\frac{1}{2}(c+dx)\right)\cos^3\left(\frac{1}{2}(c+dx\right.}{d(m+1)\sqrt{a(\sec(c+dx)+1)}\left((\cos(c+dx)-1)\left(2(m+1)F_1\left(\frac{m+3}{2};-\frac{1}{2},m+2;\frac{m+5}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx\right.\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^m/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (4*(3 + m)*AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^3*Sin[(c + d*x)/2]*(e*Sin[c + d*x])^m
)/(d*(1 + m)*((2*(1 + m)*AppellF1[(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[(3 + m)/2, 1/2, 1 + m, (5 + m
)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + (3 + m
)*AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2]*(1 + Cos[c + d*x]))*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [F] time = 0.208, size = 0, normalized size = 0.

$$\int (e\sin(dx+c))^m \frac{1}{\sqrt{a+a\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x)
```

```
[Out] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^m}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e\sin(dx+c))^m}{\sqrt{a\sec(dx+c)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*sin(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*sin(c + d*x))**m/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^m}{\sqrt{a\sec(dx+c)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^m/sqrt(a*sec(d*x + c) + a), x)
```

3.143 $\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=120

$$-\frac{2e\cos^2(c+dx)(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{1-\frac{m}{2}}F_1\left(\frac{5}{2};\frac{1-m}{2},\frac{4-m}{2};\frac{7}{2};\cos(c+dx),-\cos(c+dx)\right)(e\sin(c+dx)}{5ad\sqrt{a\sec(c+dx)+a}}$$

```
[Out] (-2*e*AppellF1[5/2, (1 - m)/2, (4 - m)/2, 7/2, Cos[c + d*x], -Cos[c + d*x]]
*(1 - Cos[c + d*x])^((1 - m)/2)*Cos[c + d*x]^2*(1 + Cos[c + d*x])^(1 - m/2)
*(e*Sin[c + d*x])^(-1 + m))/(5*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rubi [A] time = 0.373547, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3876, 2886, 135, 133}

$$-\frac{2e\cos^2(c+dx)(1-\cos(c+dx))^{\frac{1-m}{2}}(\cos(c+dx)+1)^{1-\frac{m}{2}}F_1\left(\frac{5}{2};\frac{1-m}{2},\frac{4-m}{2};\frac{7}{2};\cos(c+dx),-\cos(c+dx)\right)(e\sin(c+dx)}{5ad\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*e*AppellF1[5/2, (1 - m)/2, (4 - m)/2, 7/2, Cos[c + d*x], -Cos[c + d*x]]
*(1 - Cos[c + d*x])^((1 - m)/2)*Cos[c + d*x]^2*(1 + Cos[c + d*x])^(1 - m/2)
*(e*Sin[c + d*x])^(-1 + m))/(5*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3876

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(Sin[e + f*x]^FracPart[m]*(a + b*Csc[e + f*x]
)^FracPart[m])/(b + a*Sin[e + f*x])^FracPart[m], Int[((g*Cos[e + f*x])^p*(b
 + a*Sin[e + f*x])^m)/Sin[e + f*x]^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rule 2886

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos
[e + f*x])^(p - 1))/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x]
)^((p - 1)/2)), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
```

```
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^m}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{\sqrt{-a-a\cos(c+dx)}\int \frac{(-\cos(c+dx))^{3/2}(e\sin(c+dx))^m}{(-a-a\cos(c+dx))^{3/2}}\,dx}{\sqrt{-\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{\left(e(-a-a\cos(c+dx))^{\frac{1}{2}+\frac{1-m}{2}}(-a+a\cos(c+dx))^{\frac{1-m}{2}}(e\sin(c+dx))^{-1+m}\right)\operatorname{Subst}\left(\int(-x)^{3/2}\right.}{d\sqrt{-\cos(c+dx)}\sqrt{a+a\sec(c+dx}}$$

$$= \frac{\left(e(1+\cos(c+dx))^{1-\frac{m}{2}}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}+\frac{m}{2}}(-a+a\cos(c+dx))^{\frac{1-m}{2}}(e\sin(c+dx)\right.}{ad\sqrt{-\cos(c+dx)}\sqrt{a+a}}$$

$$= \frac{\left(e(1-\cos(c+dx))^{\frac{1}{2}-\frac{m}{2}}(1+\cos(c+dx))^{1-\frac{m}{2}}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}+\frac{m}{2}}(-a+a\cos(c+\right.}{ad\sqrt{-\cos(c}}$$

$$= -\frac{2eF_1\left(\frac{5}{2};\frac{1-m}{2},\frac{4-m}{2};\frac{7}{2};\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{\frac{1-m}{2}}\cos^2(c+dx)(1+\text{co}}{5ad\sqrt{a+a\sec(c+dx)}}$$

Mathematica [B] time = 2.86349, size = 484, normalized size = 4.03

$$\frac{}{d(m+1)(a(\sec(c+dx)+1))^{3/2}\left(-4(m+3)\cos^2\left(\frac{1}{2}(c+dx)\right)F_1\left(\frac{m+1}{2};-\frac{1}{2},m+1;\frac{m+3}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (4*(3 + m)*(AppellF1[(1 + m)/2, -1/2, m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2] - 2*AppellF1[(1 + m)/2, -1/2, 1 + m, (3 + m)/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[(c + d*x)/2]^3*Sin[(c + d*x)/2]*(e*Sin
[c + d*x])^m)/(d*(1 + m)*(-4*(3 + m)*AppellF1[(1 + m)/2, -1/2, 1 + m, (3 +
m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2 + (2*m*Ap
pellF1[(3 + m)/2, -1/2, 1 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2] - 4*(1 + m)*AppellF1[(3 + m)/2, -1/2, 2 + m, (5 + m)/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2] + AppellF1[(3 + m)/2, 1/2, m, (5 + m)/2, Tan[
(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*AppellF1[(3 + m)/2, 1/2, 1 + m, (5
 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + (3
 + m)*AppellF1[(1 + m)/2, -1/2, m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2]*(1 + Cos[c + d*x]))*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [F] time = 0.175, size = 0, normalized size = 0.

$$\int (e\sin(dx+c))^m(a+a\sec(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x)
```

```
[Out] int((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^m}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{a\sec(dx+c)+a}\,(e\sin(dx+c))^m}{a^2\sec(dx+c)^2+2\,a^2\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(a*sec(d*x + c) + a)*(e*sin(d*x + c))^m/(a^2*sec(d*x + c)^2 +
2*a^2*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^m}{(a\sec(dx+c)+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^m/(a*sec(d*x + c) + a)^(3/2), x)
```

3.144 $\int (a+a\sec(c+dx))^n(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=130

$$-\frac{e\cos(c+dx)(1-\cos(c+dx))^{\frac{1-m}{2}}(a\sec(c+dx)+a)^n(e\sin(c+dx))^{m-1}(\cos(c+dx)+1)^{\frac{1}{2}(-m-2n+1)}F_1\left(1-n;\frac{1-m}{2},\frac{1}{2}(-m\right.}{d(1-n)}$$

```
[Out] -((e*AppellF1[1 - n, (1 - m)/2, (1 - m - 2*n)/2, 2 - n, Cos[c + d*x], -Cos[
c + d*x]]*(1 - Cos[c + d*x])^((1 - m)/2)*Cos[c + d*x]*(1 + Cos[c + d*x])^((
1 - m - 2*n)/2)*(a + a*Sec[c + d*x])^n*(e*Sin[c + d*x])^(-1 + m))/(d*(1 - n
)))
```

Rubi [A] time = 0.276536, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3876, 2886, 135, 133}

$$-\frac{e\cos(c+dx)(1-\cos(c+dx))^{\frac{1-m}{2}}(a\sec(c+dx)+a)^n(e\sin(c+dx))^{m-1}(\cos(c+dx)+1)^{\frac{1}{2}(-m-2n+1)}F_1\left(1-n;\frac{1-m}{2},\frac{1}{2}(-m\right.}{d(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*(e*Sin[c + d*x])^m,x]
```


```
[Out] -((e*AppellF1[1 - n, (1 - m)/2, (1 - m - 2*n)/2, 2 - n, Cos[c + d*x], -Cos[
c + d*x]]*(1 - Cos[c + d*x])^((1 - m)/2)*Cos[c + d*x]*(1 + Cos[c + d*x])^((
1 - m - 2*n)/2)*(a + a*Sec[c + d*x])^n*(e*Sin[c + d*x])^(-1 + m))/(d*(1 - n
)))
```


Rule 3876

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(Sin[e + f*x]^FracPart[m]*(a + b*Csc[e + f*x]
)^FracPart[m])/(b + a*Sin[e + f*x])^FracPart[m], Int[((g*Cos[e + f*x])^p*(b
 + a*Sin[e + f*x])^m)/Sin[e + f*x]^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rule 2886

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos
[e + f*x])^(p - 1))/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x]
)^((p - 1)/2)), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
```

```
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^n(e\sin(c+dx))^m\,dx = ((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n)\int (-\cos(c+$$

$$= -\frac{\left(e(-\cos(c+dx))^n(-a-a\cos(c+dx))^{\frac{1-m}{2}-n}(-a+a\cos(c+dx))^{\frac{1-m}{2}}(a+\right.}{}$$

$$= -\frac{\left(e(-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{1}{2}-\frac{m}{2}-n}(-a-a\cos(c+dx))^{-\frac{1}{2}+\frac{1-m}{2}+\frac{m}{2}}(\right.}{}$$

$$= -\frac{\left(e(1-\cos(c+dx))^{\frac{1}{2}-\frac{m}{2}}(-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{1}{2}-\frac{m}{2}-n}(-a-a\,\mathrm{co}\right.}{}$$

$$= -\frac{eF_1\left(1-n;\frac{1-m}{2},\frac{1}{2}(1-m-2n);2-n;\cos(c+dx),-\cos(c+dx)\right)(1-\mathrm{co}}{}$$

Mathematica [B] time = 1.84603, size = 276, normalized size = 2.12

$$\frac{4(m+3)\sin\left(\frac{1}{2}(c+dx)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)(a(\sec(c+dx)+}{d(m+1)\left((m+3)(\cos(c+dx)+1)F_1\left(\frac{m+1}{2};n,m+1;\frac{m+3}{2};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-4\sin^2\left(\frac{1}{2}(c+dx)\right)(\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*(e*Sin[c + d*x])^m,x]
```

```
[Out] (4*(3 + m)*AppellF1[(1 + m)/2, n, 1 + m, (3 + m)/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^3*(a*(1 + Sec[c + d*x]))^n*Sin[(c + d*x)
/2]*(e*Sin[c + d*x])^m)/(d*(1 + m)*((3 + m)*AppellF1[(1 + m)/2, n, 1 + m, (
3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x]) - 4*(
(1 + m)*AppellF1[(3 + m)/2, n, 2 + m, (5 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2] - n*AppellF1[(3 + m)/2, 1 + n, 1 + m, (5 + m)/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2])*Sin[(c + d*x)/2]^2))
```

Maple [F] time = 0.728, size = 0, normalized size = 0.

$$\int (a+a\sec(dx+c))^n (e\sin(dx+c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+a*sec(d*x+c))^n*(e*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n (e\sin(dx+c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sec\left(dx+c\right)+a\right)^{n}\left(e\sin\left(dx+c\right)\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*(e*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a\sec\left(dx+c\right)+a\right)^{n}\left(e\sin\left(dx+c\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

3.145 $\int (a+a\sec(c+dx))^n \sin^7(c+dx)\,dx$

Optimal. Leaf size=180

$$-\frac{(3-n)(8-n)(16-n)(a\sec(c+dx)+a)^{n+4}\text{Hypergeometric2F1}(6,n+4,n+5,\sec(c+dx)+1)}{42a^4d(1-n)(n+4)}+\frac{\cos^7(c+dx)\,(\ldots}{}$$

```
[Out] -((3 - n)*(8 - n)*(16 - n)*Hypergeometric2F1[6, 4 + n, 5 + n, 1 + Sec[c + d
*x]]*(a + a*Sec[c + d*x])^(4 + n))/(42*a^4*d*(1 - n)*(4 + n)) - (Cos[c + d*
x]^7*(1 - Sec[c + d*x])^2*(a + a*Sec[c + d*x])^(4 + n))/(a^4*d*(1 - n)) + (
Cos[c + d*x]^7*(a + a*Sec[c + d*x])^(4 + n)*(6*(8 - n) - (108 - 25*n + n^2)
*Sec[c + d*x]))/(42*a^4*d*(1 - n))
```

Rubi [A] time = 0.1687, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3873, 100, 145, 65}

$$-\frac{(3-n)(8-n)(16-n)(a\sec(c+dx)+a)^{n+4}\,{}_2F_1(6,n+4;n+5;\sec(c+dx)+1)}{42a^4d(1-n)(n+4)}+\frac{\cos^7(c+dx)\left(6(8-n)-\left(n^2-25\ldots\right.\right.}{4\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^7,x]
```

```
[Out] -((3 - n)*(8 - n)*(16 - n)*Hypergeometric2F1[6, 4 + n, 5 + n, 1 + Sec[c + d
*x]]*(a + a*Sec[c + d*x])^(4 + n))/(42*a^4*d*(1 - n)*(4 + n)) - (Cos[c + d*
x]^7*(1 - Sec[c + d*x])^2*(a + a*Sec[c + d*x])^(4 + n))/(a^4*d*(1 - n)) + (
Cos[c + d*x]^7*(a + a*Sec[c + d*x])^(4 + n)*(6*(8 - n) - (108 - 25*n + n^2)
*Sec[c + d*x]))/(42*a^4*d*(1 - n))
```

Rule 3873

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> -Dist[(f*b^(p - 1))^(-1), Subst[Int[((-a + b*x)^((p - 1)/2
)*(a + b*x)^(m + (p - 1)/2))/x^(p + 1), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 145

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)*(
a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), x]
 + Dist[(f*h)/b^2 - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
```

```
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^n \sin^7(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(-a-ax)^3(a-ax)^{3+n}}{x^8}\,dx, x, -\sec(c+dx)\right)}{a^6 d}$$

$$= -\frac{\cos^7(c+dx)(1-\sec(c+dx))^2(a+a\sec(c+dx))^{4+n}}{a^4 d(1-n)} - \frac{\text{Subst}\left(\int \frac{(-a-ax)(a-ax)^{3+}}{\cdots}\right.}{\cdots}$$

$$= -\frac{\cos^7(c+dx)(1-\sec(c+dx))^2(a+a\sec(c+dx))^{4+n}}{a^4 d(1-n)} + \frac{\cos^7(c+dx)(a+a\sec(\cdots}{\cdots}$$

$$= -\frac{(3-n)(8-n)(16-n)\,{}_2F_1(6,4+n;5+n;1+\sec(c+dx))(a+a\sec(c+dx))^{4+\cdots}}{42a^4 d(1-n)(4+n)}$$

Mathematica [A] time = 1.5236, size = 113, normalized size = 0.63

$$\frac{(\sec(c+dx)+1)^4(a(\sec(c+dx)+1))^n\left((n+4)\cos^5(c+dx)\left(\left(n^2-25n+24\right)\cos(c+dx)+6(n-1)\cos^2(c+dx)+42\right)\cdots\right.}{42d(n-1)(n+4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^7,x]
```

```
[Out] (((4 + n)*Cos[c + d*x]^5*(42 + (24 - 25*n + n^2)*Cos[c + d*x] + 6*(-1 + n)*
Cos[c + d*x]^2) - (-384 + 200*n - 27*n^2 + n^3)*Hypergeometric2F1[6, 4 + n,
 5 + n, 1 + Sec[c + d*x]])*(1 + Sec[c + d*x])^4*(a*(1 + Sec[c + d*x]))^n)/(
42*d*(-1 + n)*(4 + n))
```

Maple [F] time = 0.701, size = 0, normalized size = 0.

$$\int (a+a\sec(dx+c))^n (\sin(dx+c))^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^7,x)
```

```
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^7,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n \sin(dx+c)^7\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^7, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos(dx+c)^6 - 3\cos(dx+c)^4 + 3\cos(dx+c)^2 - 1\right)(a\sec(dx+c)+a)^n \sin(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1)*(a*sec
(d*x + c) + a)^n*sin(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**7,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n \sin(dx+c)^7\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^7,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^7, x)
```

3.146 $\int (a+a\sec(c+dx))^n \sin^5(c+dx)\,dx$

Optimal. Leaf size=123

$$\frac{(n^2-13n+32)\,(a\sec(c+dx)+a)^{n+3}\text{Hypergeometric2F1}(4,n+3,n+4,\sec(c+dx)+1)}{20a^3d(n+3)}-\frac{\cos^5(c+dx)(a\sec(c+d}{5a^3d}$$

```
[Out] ((12 - n)*Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(3 + n))/(20*a^3*d) - (Cos[c
+ d*x]^5*(a + a*Sec[c + d*x])^(3 + n))/(5*a^3*d) + ((32 - 13*n + n^2)*Hyper
geometric2F1[4, 3 + n, 4 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3 + n
))/(20*a^3*d*(3 + n))
```

Rubi [A] time = 0.108252, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3873, 89, 78, 65}

$$\frac{(n^2-13n+32)\,(a\sec(c+dx)+a)^{n+3}\,{}_2F_1(4,n+3;n+4;\sec(c+dx)+1)}{20a^3d(n+3)}-\frac{\cos^5(c+dx)(a\sec(c+dx)+a)^{n+3}}{5a^3d}+\frac{(12-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^5,x]
```

```
[Out] ((12 - n)*Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(3 + n))/(20*a^3*d) - (Cos[c
+ d*x]^5*(a + a*Sec[c + d*x])^(3 + n))/(5*a^3*d) + ((32 - 13*n + n^2)*Hyper
geometric2F1[4, 3 + n, 4 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3 + n
))/(20*a^3*d*(3 + n))
```

Rule 3873

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> -Dist[(f*b^(p - 1))^(-1), Subst[Int[((-a + b*x)^((p - 1)/2
)*(a + b*x)^(m + (p - 1)/2))/x^(p + 1), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (a + a\sec(c+dx))^n \sin^5(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(-a-ax)^2(a-ax)^{2+n}}{x^6}\,dx, x, -\sec(c+dx)\right)}{a^4 d}$$

$$= -\frac{\cos^5(c+dx)(a+a\sec(c+dx))^{3+n}}{5a^3 d} - \frac{\text{Subst}\left(\int \frac{(a-ax)^{2+n}\left(a^3(12-n)+5a^3x\right)}{x^5}\,dx, x, \cdots\right.}{5a^5 d}$$

$$= \frac{(12-n)\cos^4(c+dx)(a+a\sec(c+dx))^{3+n}}{20a^3 d} - \frac{\cos^5(c+dx)(a+a\sec(c+dx))\cdots}{5a^3 d}$$

$$= \frac{(12-n)\cos^4(c+dx)(a+a\sec(c+dx))^{3+n}}{20a^3 d} - \frac{\cos^5(c+dx)(a+a\sec(c+dx))\cdots}{5a^3 d}$$

Mathematica [A] time = 0.497712, size = 84, normalized size = 0.68

$$-\frac{(\sec(c+dx)+1)^3(a(\sec(c+dx)+1))^n\left((n+3)\cos^4(c+dx)(4\cos(c+dx)+n-12)-\left(n^2-13n+32\right)\text{Hypergeom}\cdots\right.}{20d(n+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^5,x]
```

```
[Out] -(((3 + n)*Cos[c + d*x]^4*(-12 + n + 4*Cos[c + d*x]) - (32 - 13*n + n^2)*Hy
pergeometric2F1[4, 3 + n, 4 + n, 1 + Sec[c + d*x]])*(1 + Sec[c + d*x])^3*(a
*(1 + Sec[c + d*x]))^n)/(20*d*(3 + n))
```

Maple [F] time = 0.658, size = 0, normalized size = 0.

$$\int (a + a\sec(dx+c))^n (\sin(dx+c))^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^5,x)
```

```
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c) + a)^n \sin(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1\right)\left(a\sec\left(dx+c\right)+a\right)^{n}\sin\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(a*sec(d*x + c) + a)^n*sin
(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(dx+c\right)+a\right)^{n}\sin\left(dx+c\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^5, x)
```

3.147 $\int (a + a\sec(c + dx))^n \sin^3(c + dx)\,dx$

Optimal. Leaf size=83

$$\frac{\cos^3(c+dx)(a\sec(c+dx)+a)^{n+2}}{3a^2d} - \frac{(4-n)(a\sec(c+dx)+a)^{n+2}\text{Hypergeometric2F1}(3,n+2,n+3,\sec(c+dx)+}{3a^2d(n+2)}$$

```
[Out] (Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(2 + n))/(3*a^2*d) - ((4 - n)*Hypergeo
metric2F1[3, 2 + n, 3 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(2 + n))/
(3*a^2*d*(2 + n))
```

Rubi [A] time = 0.0726852, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3873, 78, 65}

$$\frac{\cos^3(c+dx)(a\sec(c+dx)+a)^{n+2}}{3a^2d} - \frac{(4-n)(a\sec(c+dx)+a)^{n+2}\,{}_2F_1(3,n+2;n+3;\sec(c+dx)+1)}{3a^2d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^3,x]
```


```
[Out] (Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(2 + n))/(3*a^2*d) - ((4 - n)*Hypergeo
metric2F1[3, 2 + n, 3 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(2 + n))/
(3*a^2*d*(2 + n))
```


Rule 3873

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> -Dist[(f*b^(p - 1))^(-1), Subst[Int[((-a + b*x)^((p - 1)/2
)*(a + b*x)^(m + (p - 1)/2))/x^(p + 1), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (a+a\sec(c+dx))^n\sin^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(-a-ax)(a-ax)^{1+n}}{x^4}\,dx, x, -\sec(c+dx)\right)}{a^2d}$$
$$= \frac{\cos^3(c+dx)(a+a\sec(c+dx))^{2+n}}{3a^2d} + \frac{(4-n)\,\text{Subst}\left(\int \frac{(a-ax)^{1+n}}{x^3}\,dx, x, -\sec(c+c\right.}{3ad}$$
$$= \frac{\cos^3(c+dx)(a+a\sec(c+dx))^{2+n}}{3a^2d} - \frac{(4-n)\,{}_2F_1(3,2+n;3+n;1+\sec(c+dx))}{3a^2d(2+n)}$$

Mathematica [A] time = 0.138474, size = 67, normalized size = 0.81

$$\frac{(\sec(c+dx)+1)^2(a(\sec(c+dx)+1))^n\left((n-4)\text{Hypergeometric2F1}(3,n+2,n+3,\sec(c+dx)+1)+(n+2)\cos^3(c+\right.}{3d(n+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^3,x]
```

```
[Out] (((2 + n)*Cos[c + d*x]^3 + (-4 + n)*Hypergeometric2F1[3, 2 + n, 3 + n, 1 +
Sec[c + d*x]])*(1 + Sec[c + d*x])^2*(a*(1 + Sec[c + d*x]))^n)/(3*d*(2 + n))
```

Maple [F] time = 0.581, size = 0, normalized size = 0.

$$\int (a+a\sec(dx+c))^n (\sin(dx+c))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^3,x)
```

```
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n\sin(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos(dx+c)^2-1\right)(a\sec(dx+c)+a)^n\sin(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(a*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n \sin(dx+c)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

3.148 $\int (a+a\sec(c+dx))^n \sin(c+dx)\,dx$

Optimal. Leaf size=42

$$\frac{(a\sec(c+dx)+a)^{n+1}\text{Hypergeometric2F1}(2,n+1,n+2,\sec(c+dx)+1)}{ad(n+1)}$$

```
[Out] (Hypergeometric2F1[2, 1 + n, 2 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^
(1 + n))/(a*d*(1 + n))
```

Rubi [A] time = 0.0373294, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3873, 65}

$$\frac{(a\sec(c+dx)+a)^{n+1}\,{}_2F_1(2,n+1;n+2;\sec(c+dx)+1)}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x],x]
```

```
[Out] (Hypergeometric2F1[2, 1 + n, 2 + n, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^
(1 + n))/(a*d*(1 + n))
```

Rule 3873

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> -Dist[(f*b^(p - 1))^(-1), Subst[Int[((-a + b*x)^((p - 1)/2
)*(a + b*x)^(m + (p - 1)/2))/x^(p + 1), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^n\sin(c+dx)\,dx &= -\frac{\text{Subst}\left(\int \frac{(a-ax)^n}{x^2}\,dx,x,-\sec(c+dx)\right)}{d}\\ &= \frac{{}_2F_1(2,1+n;2+n;1+\sec(c+dx))(a+a\sec(c+dx))^{1+n}}{ad(1+n)}\end{aligned}$$

Mathematica [A] time = 0.0370683, size = 42, normalized size = 1.

$$\frac{(a(\sec(c+dx)+1))^{n+1}\text{Hypergeometric2F1}(2,n+1,n+2,\sec(c+dx)+1)}{ad(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x],x]
```

```
[Out] (Hypergeometric2F1[2, 1 + n, 2 + n, 1 + Sec[c + d*x]]*(a*(1 + Sec[c + d*x])
)^(1 + n))/(a*d*(1 + n))
```

Maple [F] time = 0.337, size = 0, normalized size = 0.

$$\int \left(a + a \sec \left(dx + c\right)\right)^{n} \sin \left(dx + c\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c),x)
```

```
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sec \left(dx + c\right) + a\right)^{n} \sin \left(dx + c\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(a \sec \left(dx + c\right) + a\right)^{n} \sin \left(dx + c\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \left(\sec \left(c + dx\right) + 1\right)\right)^{n} \sin \left(c + dx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**n*sin(c + d*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a \sec \left(dx + c\right) + a\right)^{n} \sin \left(dx + c\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

3.149 $\int \csc(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal. Leaf size=40

$$-\frac{(a\sec(c+dx)+a)^n\text{Hypergeometric2F1}\left(1,n,n+1,\frac{1}{2}(\sec(c+dx)+1)\right)}{2dn}$$

```
[Out] -(Hypergeometric2F1[1, n, 1 + n, (1 + Sec[c + d*x])/2]*(a + a*Sec[c + d*x])
^n)/(2*d*n)
```

Rubi [A] time = 0.0459735, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3873, 68}

$$-\frac{(a\sec(c+dx)+a)^n\,{}_2F_1\left(1,n;n+1;\frac{1}{2}(\sec(c+dx)+1)\right)}{2dn}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -(Hypergeometric2F1[1, n, 1 + n, (1 + Sec[c + d*x])/2]*(a + a*Sec[c + d*x])
^n)/(2*d*n)
```

Rule 3873

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> -Dist[(f*b^(p - 1))^(-1), Subst[Int[((-a + b*x)^((p - 1)/2
)*(a + b*x)^(m + (p - 1)/2))/x^(p + 1), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \csc(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{a^2\,\text{Subst}\left(\int \frac{(a-ax)^{-1+n}}{-a-ax}\,dx,x,-\sec(c+dx)\right)}{d}$$
$$= -\frac{{}_2F_1\left(1,n;1+n;\frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))^n}{2dn}$$

Mathematica [B] time = 0.768554, size = 92, normalized size = 2.3

$$\frac{2^{n-1}(\sec(c+dx)+1)^{-n}(a(\sec(c+dx)+1))^n\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{n-1}\text{Hypergeometric2F1}\left(1,1-n,2-n,\right.}{d(n-1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] (2^(-1 + n)*Hypergeometric2F1[1, 1 - n, 2 - n, Cos[c + d*x]*Sec[(c + d*x)/2
]^2]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n)*(a*(1 + Sec[c + d*x]))^n)/(
d*(-1 + n)*(1 + Sec[c + d*x])^n)
```

Maple [F] time = 0.254, size = 0, normalized size = 0.

$$\int \csc(dx+c)\,(a+a\sec(dx+c))^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+a*sec(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)*(a+a*sec(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^{n}\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((a\sec(dx+c)+a)^{n}\csc(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a \sec (dx + c) + a)^n \csc (dx + c) \ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+a*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

3.150 $\int \csc^3(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal. Leaf size=112

$$-\frac{(n+2)(a\sec(c+dx)+a)^n\text{Hypergeometric2F1}\left(1,n,n+1,\frac{1}{2}(\sec(c+dx)+1)\right)}{8dn}-\frac{a(2-n)(a\sec(c+dx)+a)^{n-1}}{4d(1-n)}+\frac{a(}{}$$

```
[Out] -(a*(2 - n)*(a + a*Sec[c + d*x])^(-1 + n))/(4*d*(1 - n)) + (a*(a + a*Sec[c
+ d*x])^(-1 + n))/(2*d*(1 - Sec[c + d*x])) - ((2 + n)*Hypergeometric2F1[1,
n, 1 + n, (1 + Sec[c + d*x])/2]*(a + a*Sec[c + d*x])^n)/(8*d*n)
```

Rubi [A] time = 0.0968238, antiderivative size = 112, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3873, 89, 79, 68}

$$-\frac{(n+2)(a\sec(c+dx)+a)^n\,{}_2F_1\left(1,n;n+1;\frac{1}{2}(\sec(c+dx)+1)\right)}{8dn}-\frac{a(2-n)(a\sec(c+dx)+a)^{n-1}}{4d(1-n)}+\frac{a(a\sec(c+dx)+a)^n}{2d(1-\sec(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -(a*(2 - n)*(a + a*Sec[c + d*x])^(-1 + n))/(4*d*(1 - n)) + (a*(a + a*Sec[c
+ d*x])^(-1 + n))/(2*d*(1 - Sec[c + d*x])) - ((2 + n)*Hypergeometric2F1[1,
n, 1 + n, (1 + Sec[c + d*x])/2]*(a + a*Sec[c + d*x])^n)/(8*d*n)
```

Rule 3873

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> -Dist[(f*b^(p - 1))^(-1), Subst[Int[((-a + b*x)^((p - 1)/2
)*(a + b*x)^(m + (p - 1)/2))/x^(p + 1), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^Simplify[p +
1], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] &&  !RationalQ[p] && SumSi
mplerQ[p, 1]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \csc^3(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{a^4\,\mathrm{Subst}\left(\int \frac{x^2(a-ax)^{-2+n}}{(-a-ax)^2}\,dx,x,-\sec(c+dx)\right)}{d}$$

$$= \frac{a(a+a\sec(c+dx))^{-1+n}}{2d(1-\sec(c+dx))} + \frac{\mathrm{Subst}\left(\int \frac{(a-ax)^{-2+n}\left(-a^3n+2a^3x\right)}{-a-ax}\,dx,x,-\sec(c+dx)\right)}{2d}$$

$$= -\frac{a(2-n)(a+a\sec(c+dx))^{-1+n}}{4d(1-n)} + \frac{a(a+a\sec(c+dx))^{-1+n}}{2d(1-\sec(c+dx))} - \frac{\left(a^2(2+n)\right)\mathrm{S}}{}$$

$$= -\frac{a(2-n)(a+a\sec(c+dx))^{-1+n}}{4d(1-n)} + \frac{a(a+a\sec(c+dx))^{-1+n}}{2d(1-\sec(c+dx))} - \frac{(2+n)\,{}_2F_1\left(1\right.}{}$$

Mathematica [A] time = 1.68421, size = 123, normalized size = 1.1

$$\frac{2^{n-4}(\sec(c+dx)+1)^{-n}(a(\sec(c+dx)+1))^n\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{n-1}\left(2(n+2)\mathrm{Hypergeometric2F1}\left(1,1-\right.\right.}{d(n-1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^3*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] (2^(-4 + n)*(-((-2 + n + n*Cos[c + d*x])*Csc[(c + d*x)/2]^2) + 2*(2 + n)*Hy
pergeometric2F1[1, 1 - n, 2 - n, Cos[c + d*x]*Sec[(c + d*x)/2]^2])*(Cos[(c
+ d*x)/2]^2*Sec[c + d*x])^(-1 + n)*(a*(1 + Sec[c + d*x]))^n)/(d*(-1 + n)*(1
 + Sec[c + d*x])^n)
```

Maple [F] time = 0.275, size = 0, normalized size = 0.

$$\int (\csc(dx+c))^3 (a+a\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+a*sec(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^3*(a+a*sec(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n \csc(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a\sec\left(dx+c\right)+a\right)^{n}\csc\left(dx+c\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+a*sec(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(dx+c\right)+a\right)^{n}\csc\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+a*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

3.151 $\int \csc^5(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal. Leaf size=240

$$\frac{a^2\left(n^2+9n+12\right)(a\sec(c+dx)+a)^{n-2}\text{Hypergeometric2F1}\left(1,n-2,n-1,\frac{1}{2}(\sec(c+dx)+1)\right)}{16d(2-n)}-\frac{a^2\left(-2(1-n)(n\right.}{}$$

```
[Out] (a^2*(12 + 9*n + n^2)*Hypergeometric2F1[1, -2 + n, -1 + n, (1 + Sec[c + d*x
])/2]*(a + a*Sec[c + d*x])^(-2 + n))/(16*d*(2 - n)) + (a^2*(3 + n)*Sec[c +
d*x]^2*(a + a*Sec[c + d*x])^(-2 + n))/(4*d*(1 - n)*(1 - Sec[c + d*x])^2) -
(a^2*Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(-2 + n))/(d*(1 - n)*(1 - Sec[c +
d*x])^2) - (a^2*(a + a*Sec[c + d*x])^(-2 + n)*(12 + 4*n - 7*n^2 - n^3 - 2*(
1 - n)*(6 + n)*Sec[c + d*x]))/(8*d*(2 - 3*n + n^2)*(1 - Sec[c + d*x]))
```

Rubi [A] time = 0.223967, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3873, 100, 149, 146, 68}

$$\frac{a^2\left(n^2+9n+12\right)(a\sec(c+dx)+a)^{n-2}\,{}_2F_1\left(1,n-2;n-1;\frac{1}{2}(\sec(c+dx)+1)\right)}{16d(2-n)}-\frac{a^2\left(-2(1-n)(n+6)\sec(c+dx)-n\right.}{8d\left(n^2-3n+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] (a^2*(12 + 9*n + n^2)*Hypergeometric2F1[1, -2 + n, -1 + n, (1 + Sec[c + d*x
])/2]*(a + a*Sec[c + d*x])^(-2 + n))/(16*d*(2 - n)) + (a^2*(3 + n)*Sec[c +
d*x]^2*(a + a*Sec[c + d*x])^(-2 + n))/(4*d*(1 - n)*(1 - Sec[c + d*x])^2) -
(a^2*Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(-2 + n))/(d*(1 - n)*(1 - Sec[c +
d*x])^2) - (a^2*(a + a*Sec[c + d*x])^(-2 + n)*(12 + 4*n - 7*n^2 - n^3 - 2*(
1 - n)*(6 + n)*Sec[c + d*x]))/(8*d*(2 - 3*n + n^2)*(1 - Sec[c + d*x]))
```

Rule 3873

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> -Dist[(f*b^(p - 1))^(-1), Subst[Int[((-a + b*x)^((p - 1)/2
)*(a + b*x)^(m + (p - 1)/2))/x^(p + 1), x], x, Csc[e + f*x]], x] /; FreeQ[{
a, b, e, f, m}, x] && IntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0]
```

Rule 100

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m - 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/(d*f*(m + n + p + 1)), x] + Dist[1/(d*f*(m + n + p + 1)), Int[(a
 + b*x)^(m - 2)*(c + d*x)^n*(e + f*x)^p*Simp[a^2*d*f*(m + n + p + 1) - b*(b
*c*e*(m - 1) + a*(d*e*(n + 1) + c*f*(p + 1))) + b*(a*d*f*(2*m + n + p) - b*
(d*e*(m + n) + c*f*(m + p)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p
}, x] && GtQ[m, 1] && NeQ[m + n + p + 1, 0] && IntegerQ[m]
```

Rule 149

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^n*(e + f*x)^(p + 1))/(b*(b*e - a*f)*(m + 1)), x] - Dist[1/(b*(
b*e - a*f)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1)*(e + f*x)^p*Si
```

```
mp[b*c*(f*g - e*h)*(m + 1) + (b*g - a*h)*(d*e*n + c*f*(p + 1)) + d*(b*(f*g
- e*h)*(m + 1) + f*(b*g - a*h)*(n + p + 1))*x, x], x], x] /; FreeQ[{a, b, c
, d, e, f, g, h, p}, x] && LtQ[m, -1] && GtQ[n, 0] && IntegerQ[m]
```

Rule 146

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((a^2*d*f*h*(n + 2) + b^2*d*e*g*(
m + n + 3) + a*b*(c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b*f*h*(b*c -
 a*d)*(m + 1)*x)*(a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*d*(b*c - a*d)*(m
 + 1)*(m + n + 3)), x] - Dist[(a^2*d^2*f*h*(n + 1)*(n + 2) + a*b*d*(n + 1)*
(2*c*f*h*(m + 1) - d*(f*g + e*h)*(m + n + 3)) + b^2*(c^2*f*h*(m + 1)*(m + 2
) - c*d*(f*g + e*h)*(m + 1)*(m + n + 3) + d^2*e*g*(m + n + 2)*(m + n + 3)))
/(b^2*d*(b*c - a*d)*(m + 1)*(m + n + 3)), Int[(a + b*x)^(m + 1)*(c + d*x)^n
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && ((GeQ[m, -2] && LtQ
[m, -1]) || SumSimplerQ[m, 1]) && NeQ[m, -1] && NeQ[m + n + 3, 0]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \csc^5(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{a^6\,\text{Subst}\left(\int \frac{x^4(a-ax)^{-3+n}}{(-a-ax)^3}\,dx, x, -\sec(c+dx)\right)}{d}$$

$$= -\frac{a^2\sec^3(c+dx)(a+a\sec(c+dx))^{-2+n}}{d(1-n)(1-\sec(c+dx))^2} + \frac{a^4\,\text{Subst}\left(\int \frac{x^2(a-ax)^{-3+n}\left(3a^2-a^2nx\right)}{(-a-ax)^3}\,dx, x\right.}{d(1-n)}$$

$$= \frac{a^2(3+n)\sec^2(c+dx)(a+a\sec(c+dx))^{-2+n}}{4d(1-n)(1-\sec(c+dx))^2} - \frac{a^2\sec^3(c+dx)(a+a\sec(c+dx}{d(1-n)(1-\sec(c+dx))^2}$$

$$= \frac{a^2(3+n)\sec^2(c+dx)(a+a\sec(c+dx))^{-2+n}}{4d(1-n)(1-\sec(c+dx))^2} - \frac{a^2\sec^3(c+dx)(a+a\sec(c+dx}{d(1-n)(1-\sec(c+dx))^2}$$

$$= \frac{a^2\left(12+9n+n^2\right)\,{}_2F_1\left(1,-2+n;-1+n;\frac{1}{2}(1+\sec(c+dx))\right)(a+a\sec(c+dx))}{16d(2-n)}$$

Mathematica [A] time = 5.72126, size = 316, normalized size = 1.32

$$-\frac{2^{n-6}\tan^4\left(\frac{1}{2}(c+dx)\right)\left(\cot^2\left(\frac{1}{2}(c+dx)\right)-1\right)(\sec(c+dx)+1)^{-n}(a(\sec(c+dx)+1))^n\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^n\left(-\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^5*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -((2^(-6 + n)*(-1 + Cot[(c + d*x)/2]^2)*(-((-46 + 41*n - 6*n^2 - n^3 + (9 +
 2*n - 5*n^2)*Cos[c + d*x] + (30 - 21*n + 2*n^2 + n^3)*Cos[2*(c + d*x)] - 9
*Cos[3*(c + d*x)] + 2*n*Cos[3*(c + d*x)] + n^2*Cos[3*(c + d*x)])*Csc[(c + d
*x)/2]^6)/8 - 2*(6 - 15*n + 4*n^2 + n^3)*Cot[(c + d*x)/2]^2*Hypergeometric2
```

```
F1[1, 1 - n, 2 - n, Cos[c + d*x]*Sec[(c + d*x)/2]^2] + (18 - 21*n + 2*n^2 +
 n^3)*Cos[c + d*x]*Csc[(c + d*x)/2]^2*Hypergeometric2F1[1, 2 - n, 3 - n, Co
s[c + d*x]*Sec[(c + d*x)/2]^2])*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(a*(1 +
 Sec[c + d*x]))^n*Tan[(c + d*x)/2]^4)/(d*(-2 + n)*(-1 + n)*(1 + Sec[c + d*x
])^n))
```

Maple [F] time = 0.296, size = 0, normalized size = 0.

$$\int (\csc (dx+c))^5 (a+a\sec (dx+c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+a*sec(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^5*(a+a*sec(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec (dx+c)+a)^n \csc (dx+c)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sec (dx+c)+a)^n \csc (dx+c)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*csc(d*x + c)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+a*sec(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n \csc(dx+c)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+a*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^5, x)
```

3.152 $\int (a + a \sec(c + dx))^n \sin^4(c + dx) \, dx$

Optimal. Leaf size=230

$$\frac{2^{n+\frac{1}{2}} \sin(c+dx) \cos^n(c+dx) (\cos(c+dx)+1)^{-n-\frac{1}{2}} (a \sec(c+dx)+a)^n F_1\left(\frac{1}{2}; n-4, \frac{1}{2}-n; \frac{3}{2}; 1-\cos(c+dx), \frac{1}{2}(1-\cos(c+dx))\right)}{d}$$

```
[Out] -((AppellF1[1 - n, -1/2, 1/2 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*(1 +
Cos[c + d*x])^(1/2 - n)*(n - n*Cos[c + d*x])*Cot[c + d*x]*(a + a*Sec[c + d*
x])^n)/(d*(1 - n)*Sqrt[1 - Cos[c + d*x]])) - (Cos[c + d*x]*(a + a*Sec[c + d
*x])^n*Sin[c + d*x])/d + (2^(1/2 + n)*AppellF1[1/2, -4 + n, 1/2 - n, 3/2, 1
 - Cos[c + d*x], (1 - Cos[c + d*x])/2]*Cos[c + d*x]^n*(1 + Cos[c + d*x])^(-
1/2 - n)*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/d
```

Rubi [A] time = 0.667858, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3876, 2881, 2787, 2786, 2785, 133, 3046, 3008, 135}

$$\frac{2^{n+\frac{1}{2}} \sin(c+dx) \cos^n(c+dx) (\cos(c+dx)+1)^{-n-\frac{1}{2}} (a \sec(c+dx)+a)^n F_1\left(\frac{1}{2}; n-4, \frac{1}{2}-n; \frac{3}{2}; 1-\cos(c+dx), \frac{1}{2}(1-\cos(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^4,x]
```

```
[Out] -((AppellF1[1 - n, -1/2, 1/2 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*(1 +
Cos[c + d*x])^(1/2 - n)*(n - n*Cos[c + d*x])*Cot[c + d*x]*(a + a*Sec[c + d*
x])^n)/(d*(1 - n)*Sqrt[1 - Cos[c + d*x]])) - (Cos[c + d*x]*(a + a*Sec[c + d
*x])^n*Sin[c + d*x])/d + (2^(1/2 + n)*AppellF1[1/2, -4 + n, 1/2 - n, 3/2, 1
 - Cos[c + d*x], (1 - Cos[c + d*x])/2]*Cos[c + d*x]^n*(1 + Cos[c + d*x])^(-
1/2 - n)*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/d
```

Rule 3876

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(Sin[e + f*x]^FracPart[m]*(a + b*Csc[e + f*x]
)^FracPart[m])/(b + a*Sin[e + f*x])^FracPart[m], Int[((g*Cos[e + f*x])^p*(b
 + a*Sin[e + f*x])^m)/Sin[e + f*x]^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rule 2881

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/d^4, Int[(d*Sin[e
+ f*x])^(n + 4)*(a + b*Sin[e + f*x])^m, x], x] + Int[(d*Sin[e + f*x])^n*(a
+ b*Sin[e + f*x])^m*(1 - 2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m,
n}, x] && EqQ[a^2 - b^2, 0] &&  !IGtQ[m, 0]
```

Rule 2787

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Sin[e + f*x])^FracPart[m
])/(1 + (b*Sin[e + f*x])/a)^FracPart[m], Int[(1 + (b*Sin[e + f*x])/a)^m*(d*
Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 2786

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)])^(m_), x_Symbol] :> Dist[((d/b)^IntPart[n]*(d*Sin[e + f*x])^FracPart[n
])/(b*Sin[e + f*x])^FracPart[n], Int[(a + b*Sin[e + f*x])^m*(b*Sin[e + f*x]
)^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !In
tegerQ[m] && GtQ[a, 0] &&  !GtQ[d/b, 0]
```

Rule 2785

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Dist[(b*(d/b)^n*Cos[e + f*x])/(f*Sqrt[a + b*Sin[e
+ f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[((a - x)^n*(2*a - x)^(m - 1/2)
)/Sqrt[x], x], x, a - b*Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] && GtQ[d/b, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3046

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
 -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))
/(d*f*(m + n + 2)), x] + Dist[1/(b*d*(m + n + 2)), Int[(a + b*Sin[e + f*x])
^m*(c + d*Sin[e + f*x])^n*Simp[A*b*d*(m + n + 2) + C*(a*c*m + b*d*(n + 1))
+ C*(a*d*m - b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e,
 f, A, C, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 -
d^2, 0] &&  !LtQ[m, -2^(-1)] && NeQ[m + n + 2, 0]
```

Rule 3008

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(f*Cos[e + f*x]
), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, Si
n[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c +
 a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rubi steps

$$\int (a+a\sec(c+dx))^n \sin^4(c+dx)\,dx = ((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n)\int (-\cos(c+dx)$$
$$= ((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n)\int (-\cos(c+dx)$$
$$= -\frac{\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d} + ((-\cos(c+dx))^n(1+\cos(c+$$
$$= -\frac{\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d} + (\cos^n(c+dx)(1+\cos(c+dx)$$
$$= -\frac{\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d} - \frac{\left((-\cos(c+dx))^n(1+\cos(c+\right.}{}$$
$$= -\frac{\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d} + \frac{2^{\frac{1}{2}+n}F_1\left(\frac{1}{2};-4+n,\frac{1}{2}-n;\frac{3}{2};1\right.}{}$$
$$= -\frac{F_1\left(1-n;-\frac{1}{2},\frac{1}{2}-n;2-n;\cos(c+dx),-\cos(c+dx)\right)(1+\cos(c+dx))^{\frac{1}{2}-n}(}{d(1-n)\sqrt{1-\cos(c+dx)}}$$

Mathematica [C] time = 23.103, size = 7069, normalized size = 30.73

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^4,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.66, size = 0, normalized size = 0.

$$\int (a+a\sec(dx+c))^n (\sin(dx+c))^4\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^4,x)
```

```
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n \sin(dx+c)^4\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1\right)\left(a\sec\left(dx+c\right)+a\right)^{n},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(a*sec(d*x + c) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a\sec\left(dx+c\right)+a\right)^{n}\sin\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

3.153 $\int (a + a\sec(c + dx))^n \sin^2(c + dx)\, dx$

Optimal. Leaf size=95

$$-\frac{\sqrt{1-\cos(c+dx)}\cot(c+dx)(\cos(c+dx)+1)^{\frac{1}{2}-n}(a\sec(c+dx)+a)^n F_1\left(1-n;-\frac{1}{2},-n-\frac{1}{2};2-n;\cos(c+dx),-\cos(\right.}{d(1-n)}$$

```
[Out] -((AppellF1[1 - n, -1/2, -1/2 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*Sqrt
[1 - Cos[c + d*x]]*(1 + Cos[c + d*x])^(1/2 - n)*Cot[c + d*x]*(a + a*Sec[c +
 d*x])^n)/(d*(1 - n)))
```

Rubi [A] time = 0.353077, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3876, 2874, 3008, 135, 133}

$$-\frac{\sqrt{1-\cos(c+dx)}\cot(c+dx)(\cos(c+dx)+1)^{\frac{1}{2}-n}(a\sec(c+dx)+a)^n F_1\left(1-n;-\frac{1}{2},-n-\frac{1}{2};2-n;\cos(c+dx),-\cos(\right.}{d(1-n)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^2,x]
```

```
[Out] -((AppellF1[1 - n, -1/2, -1/2 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*Sqrt
[1 - Cos[c + d*x]]*(1 + Cos[c + d*x])^(1/2 - n)*Cot[c + d*x]*(a + a*Sec[c +
 d*x])^n)/(d*(1 - n)))
```

Rule 3876

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(Sin[e + f*x]^FracPart[m]*(a + b*Csc[e + f*x]
)^FracPart[m])/(b + a*Sin[e + f*x])^FracPart[m], Int[((g*Cos[e + f*x])^p*(b
 + a*Sin[e + f*x])^m)/Sin[e + f*x]^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rule 2874

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[1/b^2, Int[(d*Sin[e
+ f*x])^n*(a + b*Sin[e + f*x])^(m + 1)*(a - b*Sin[e + f*x]), x], x] /; Free
Q[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && (ILtQ[m, 0] ||  !IGtQ[n
, 0])
```

Rule 3008

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])^(p_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol]
:> Dist[(Sqrt[a + b*Sin[e + f*x]]*Sqrt[c + d*Sin[e + f*x]])/(f*Cos[e + f*x]
), Subst[Int[(a + b*x)^(m - 1/2)*(c + d*x)^(n - 1/2)*(A + B*x)^p, x], x, Si
n[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, A, B, m, n, p}, x] && EqQ[b*c +
 a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
```

```
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (a + a\sec(c+dx))^n \sin^2(c+dx)\,dx = ((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n)\int(-\cos(c+dx))^{-n}($$

$$= \frac{((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n)\int(-\cos(c+dx))^{-n}(}{a^2}$$

$$= -\frac{\left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{\frac{1}{2}-n}\sqrt{-a+a\cos(c+dx)}\csc(c+dx)(a+a\right.}{a^2d}$$

$$= -\frac{\left((-\cos(c+dx))^n(1+\cos(c+dx))^{-\frac{1}{2}-n}(-a-a\cos(c+dx))\sqrt{-a+a\cos(c+dx}\right.}{}$$

$$= -\frac{\left((-\cos(c+dx))^n(1+\cos(c+dx))^{-\frac{1}{2}-n}(-a-a\cos(c+dx))(-a+a\cos(c+dx)\right.}{a^2d\sqrt{1}}$$

$$= -\frac{F_1\left(1-n;-\frac{1}{2},-\frac{1}{2}-n;2-n;\cos(c+dx),-\cos(c+dx)\right)\sqrt{1-\cos(c+dx)}(1+c}{d(1-n)}$$

Mathematica [C] time = 17.3173, size = 4297, normalized size = 45.23

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^2,x]
```

```
[Out] (2^(3 + n)*Cos[(c + d*x)/2]^5*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(a*(1 + S
ec[c + d*x]))^n*Sin[(c + d*x)/2]*(Cos[2*(c + d*x)]*(-(1 + Sec[c + d*x])^n/4
 - ((1 + Sec[c + d*x])^n*Sin[c + d*x]^2)/2 - ((1 + Sec[c + d*x])^n*Sin[c +
d*x]^4)/4) + (I/4)*(1 + Sec[c + d*x])^n*Sin[2*(c + d*x)] + (I/2)*(1 + Sec[c
 + d*x])^n*Sin[c + d*x]^2*Sin[2*(c + d*x)] + (I/4)*(1 + Sec[c + d*x])^n*Sin
[c + d*x]^4*Sin[2*(c + d*x)] + Cos[c + d*x]^4*(-(Cos[2*(c + d*x)]*(1 + Sec[
c + d*x])^n)/4 + (I/4)*(1 + Sec[c + d*x])^n*Sin[2*(c + d*x)]) + Cos[c + d*x
]^3*((-I)*Cos[2*(c + d*x)]*(1 + Sec[c + d*x])^n*Sin[c + d*x] - (1 + Sec[c +
 d*x])^n*Sin[c + d*x]*Sin[2*(c + d*x)]) + Cos[c + d*x]^2*(Cos[2*(c + d*x)]*
((1 + Sec[c + d*x])^n/2 + (3*(1 + Sec[c + d*x])^n*Sin[c + d*x]^2)/2) - (I/2
)*(1 + Sec[c + d*x])^n*Sin[2*(c + d*x)] - ((3*I)/2)*(1 + Sec[c + d*x])^n*Si
n[c + d*x]^2*Sin[2*(c + d*x)]) + Cos[c + d*x]*(Cos[2*(c + d*x)]*(I*(1 + Sec
[c + d*x])^n*Sin[c + d*x] + I*(1 + Sec[c + d*x])^n*Sin[c + d*x]^3) + (1 + S
ec[c + d*x])^n*Sin[c + d*x]*Sin[2*(c + d*x)] + (1 + Sec[c + d*x])^n*Sin[c +
 d*x]^3*Sin[2*(c + d*x)]))*((3*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2)/(3*AppellF1[1/2, n, 2, 3/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-2*AppellF1[3/2, n, 3, 5/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*AppellF1[3/2, 1 + n, 2, 5/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) - AppellF1[1/2, n, 3
, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]/(AppellF1[1/2, n, 3, 3/2, T
```

```
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-3*AppellF1[3/2, n, 4, 5/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*AppellF1[3/2, 1 + n, 3, 5/2, Ta
n[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/3)))/(d*(1 + Se
c[c + d*x])^n*(2^(2 + n)*Cos[(c + d*x)/2]^6*(Cos[(c + d*x)/2]^2*Sec[c + d*x
])^n*((3*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*
Sec[(c + d*x)/2]^2)/(3*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2] + 2*(-2*AppellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c
+ d*x)/2]^2] + n*AppellF1[3/2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2])*Tan[(c + d*x)/2]^2) - AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)/2
]^2, -Tan[(c + d*x)/2]^2]/(AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2] + (2*(-3*AppellF1[3/2, n, 4, 5/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2] + n*AppellF1[3/2, 1 + n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/3)) - 5*2^(2 + n)*Cos[(c + d*x)/2]^4*
(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*Sin[(c + d*x)/2]^2*((3*AppellF1[1/2, n,
 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2)/(3*Ap
pellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-2*App
ellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*AppellF1
[3/2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x
)/2]^2) - AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
/(AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-
3*AppellF1[3/2, n, 4, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*App
ellF1[3/2, 1 + n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c
+ d*x)/2]^2)/3)) + 2^(3 + n)*Cos[(c + d*x)/2]^5*(Cos[(c + d*x)/2]^2*Sec[c +
 d*x])^n*Sin[(c + d*x)/2]*((3*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3*AppellF1[1/2,
n, 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-2*AppellF1[3/2, n
, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*AppellF1[3/2, 1 + n,
 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) + (3
*Sec[(c + d*x)/2]^2*((-2*AppellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (n*AppellF1[3/2, 1
 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2])/3))/(3*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2] + 2*(-2*AppellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2] + n*AppellF1[3/2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c
+ d*x)/2]^2])*Tan[(c + d*x)/2]^2) - (-(AppellF1[3/2, n, 4, 5/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (n*App
ellF1[3/2, 1 + n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c +
 d*x)/2]^2*Tan[(c + d*x)/2])/3)/(AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2] + (2*(-3*AppellF1[3/2, n, 4, 5/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2] + n*AppellF1[3/2, 1 + n, 3, 5/2, Tan[(c + d*x)/2]^2
, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)/3) - (3*AppellF1[1/2, n, 2, 3/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*(2*(-2*Appell
F1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*AppellF1[3/
2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2
]^2*Tan[(c + d*x)/2] + 3*((-2*AppellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (n*AppellF1[3
/2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2
]^2*Tan[(c + d*x)/2])/3) + 2*Tan[(c + d*x)/2]^2*(-2*((-9*AppellF1[5/2, n, 4
, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c +
 d*x)/2])/5 + (3*n*AppellF1[5/2, 1 + n, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5) + n*((-6*AppellF1[5/2
, 1 + n, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^
2*Tan[(c + d*x)/2])/5 + (3*(1 + n)*AppellF1[5/2, 2 + n, 2, 7/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5))))/(3
*AppellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-2*
AppellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*Appel
lF1[3/2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c +
d*x)/2]^2)^2 + (AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2]*(-(AppellF1[3/2, n, 4, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
```

```
*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (n*AppellF1[3/2, 1 + n, 3, 5/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/
3 + (2*(-3*AppellF1[3/2, n, 4, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
] + n*AppellF1[3/2, 1 + n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
)*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3 + (2*Tan[(c + d*x)/2]^2*(-3*((-12*
AppellF1[5/2, n, 5, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2])/5 + (3*n*AppellF1[5/2, 1 + n, 4, 7/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5) + n
*((-9*AppellF1[5/2, 1 + n, 4, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (3*(1 + n)*AppellF1[5/2, 2 + n, 3
, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c +
 d*x)/2])/5)))/3))/(AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2] + (2*(-3*AppellF1[3/2, n, 4, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2] + n*AppellF1[3/2, 1 + n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d
*x)/2]^2])*Tan[(c + d*x)/2]^2)/3)^2) + 2^(3 + n)*n*Cos[(c + d*x)/2]^5*(Cos[
(c + d*x)/2]^2*Sec[c + d*x])^(-1 + n)*Sin[(c + d*x)/2]*((3*AppellF1[1/2, n,
 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2)/(3*Ap
pellF1[1/2, n, 2, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-2*App
ellF1[3/2, n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*AppellF1
[3/2, 1 + n, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x
)/2]^2) - AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
/(AppellF1[1/2, n, 3, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + (2*(-
3*AppellF1[3/2, n, 4, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + n*App
ellF1[3/2, 1 + n, 3, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c
+ d*x)/2]^2)/3))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(
c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x])))
```

Maple [F] time = 0.569, size = 0, normalized size = 0.

$$\int (a + a\sec(dx + c))^n (\sin(dx + c))^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^2,x)
```

```
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx + c) + a)^n \sin(dx + c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\left(\cos(dx + c)^2 - 1\right)(a\sec(dx + c) + a)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(a*sec(d*x + c) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n \sin(dx+c)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

3.154 $\int \csc^2(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal. Leaf size=98

$$\frac{2^{n-\frac{1}{2}}n\tan(c+dx)(\sec(c+dx)+1)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^n\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{2}-n,\frac{3}{2},\frac{1}{2}(1-\sec(c+dx))\right)}{d}-\text{cot}$$

```
[Out] -((Cot[c + d*x]*(a + a*Sec[c + d*x])^n)/d) + (2^(-1/2 + n)*n*Hypergeometric
2F1[1/2, 3/2 - n, 3/2, (1 - Sec[c + d*x])/2]*(1 + Sec[c + d*x])^(-1/2 - n)*
(a + a*Sec[c + d*x])^n*Tan[c + d*x])/d
```

Rubi [A] time = 0.132263, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3875, 3828, 3827, 69}

$$\frac{2^{n-\frac{1}{2}}n\tan(c+dx)(\sec(c+dx)+1)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)}{d}-\frac{\cot(c+dx)(a\sec(c+}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -((Cot[c + d*x]*(a + a*Sec[c + d*x])^n)/d) + (2^(-1/2 + n)*n*Hypergeometric
2F1[1/2, 3/2 - n, 3/2, (1 - Sec[c + d*x])/2]*(1 + Sec[c + d*x])^(-1/2 - n)*
(a + a*Sec[c + d*x])^n*Tan[c + d*x])/d
```

Rule 3875

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(Tan[e + f*x]*(a + b*Csc[e + f*x])^m)/f, x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3828

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^IntPart[m]*(a + b*Csc[e + f*x])^FracPart[m
])/(1 + (b*Csc[e + f*x])/a)^FracPart[m], Int[(1 + (b*Csc[e + f*x])/a)^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 3827

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(a^2*d*Cot[e + f*x])/(f*Sqrt[a + b*Csc[e + f*
x]]*Sqrt[a - b*Csc[e + f*x]]), Subst[Int[((d*x)^(n - 1)*(a + b*x)^(m - 1/2)
)/Sqrt[a - b*x], x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 69

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a + b*x))/(b*c -
 a*d))])/(b*(m + 1)*(b/(b*c - a*d))^n), x] /; FreeQ[{a, b, c, d, m, n}, x]
&& NeQ[b*c - a*d, 0] &&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[b/(b*c - a*d)
```

```
, 0] && (RationalQ[m] ||  !(RationalQ[n] && GtQ[-(d/(b*c - a*d)), 0]))
```

Rubi steps

$$\int \csc^2(c+dx)(a+a\sec(c+dx))^n\,dx = -\frac{\cot(c+dx)(a+a\sec(c+dx))^n}{d} + (an)\int \sec(c+dx)(a+a\sec(c+dx))^{-1+}$$

$$= -\frac{\cot(c+dx)(a+a\sec(c+dx))^n}{d} + \left(n(1+\sec(c+dx))^{-n}(a+a\sec(c+dx))^n\right.$$

$$= -\frac{\cot(c+dx)(a+a\sec(c+dx))^n}{d} - \frac{\left(n(1+\sec(c+dx))^{-\frac{1}{2}-n}(a+a\sec(c+dx\right.}{d\sqrt{}}$$

$$= -\frac{\cot(c+dx)(a+a\sec(c+dx))^n}{d} + \frac{2^{-\frac{1}{2}+n}n\,{}_2F_1\left(\frac{1}{2},\frac{3}{2}-n;\frac{3}{2};\frac{1}{2}(1-\sec(c+dx\right.}{}$$

Mathematica [A] time = 1.05219, size = 87, normalized size = 0.89

$$-\frac{\tan\left(\frac{1}{2}(c+dx)\right)(a(\sec(c+dx)+1))^n\left(-2n\left(\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\right)^n \text{Hypergeometric2F1}\left(\frac{1}{2},n,\frac{3}{2},\tan^2\left(\frac{1}{2}(c\right.\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -((-1 + Cot[(c + d*x)/2]^2 - 2*n*Hypergeometric2F1[1/2, n, 3/2, Tan[(c + d*
x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n)*(a*(1 + Sec[c + d*x]))^n*Tan[
(c + d*x)/2])/(2*d)
```

Maple [F] time = 0.276, size = 0, normalized size = 0.

$$\int (\csc(dx+c))^2 (a+a\sec(dx+c))^n \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+a*sec(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^2*(a+a*sec(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n \csc(dx+c)^2 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sec(dx+c)+a)^n\csc(dx+c)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+a*sec(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n\csc(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+a*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

3.155 $\int \csc^4(c+dx)(a+a\sec(c+dx))^n\,dx$

Optimal. Leaf size=349

$$\frac{n\left(-n^2-3n+7\right)\sin(c+dx)\cos(c+dx)\left(\frac{\cos(c+dx)+1}{1-\cos(c+dx)}\right)^{-n-\frac{1}{2}}(a\sec(c+dx)+a)^n\text{Hypergeometric2F1}\left(-n-\frac{1}{2},1-n,2\right.}{d(1-2n)(3-2n)(1-n)(2n+1)(1-\cos(c+dx))^2}$$

```
[Out] ((2 - n + n^2)*Cos[c + d*x]*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*
n)*(1 - 4*n^2)*(1 - Cos[c + d*x])^2) - (a^4*Cos[c + d*x]*(a + a*Sec[c + d*x
])^n*Sin[c + d*x])/(d*(3 - 2*n)*(a - a*Cos[c + d*x])^2*(a + a*Cos[c + d*x])
^2) - (a^3*(4 - n)*Cos[c + d*x]*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3
- 8*n + 4*n^2)*(a - a*Cos[c + d*x])^2*(a + a*Cos[c + d*x])) + (n*(7 - 3*n -
 n^2)*Cos[c + d*x]*((1 + Cos[c + d*x])/(1 - Cos[c + d*x]))^(-1/2 - n)*Hyper
geometric2F1[-1/2 - n, 1 - n, 2 - n, (-2*Cos[c + d*x])/(1 - Cos[c + d*x])]*
(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*(3 - 2*n)*(1 - n)*(1 + 2*
n)*(1 - Cos[c + d*x])^2)
```

Rubi [A] time = 0.541432, antiderivative size = 349, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3876, 2883, 129, 155, 12, 132}

$$-\frac{a^3(4-n)\sin(c+dx)\cos(c+dx)(a\sec(c+dx)+a)^n}{d\left(4n^2-8n+3\right)(a-a\cos(c+dx))^2(a\cos(c+dx)+a)}-\frac{a^4\sin(c+dx)\cos(c+dx)(a\sec(c+dx)+a)^n}{d(3-2n)(a-a\cos(c+dx))^2(a\cos(c+dx)+a)^2}+\frac{n\left(-n\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^n,x]
```


```
[Out] ((2 - n + n^2)*Cos[c + d*x]*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*
n)*(1 - 4*n^2)*(1 - Cos[c + d*x])^2) - (a^4*Cos[c + d*x]*(a + a*Sec[c + d*x
])^n*Sin[c + d*x])/(d*(3 - 2*n)*(a - a*Cos[c + d*x])^2*(a + a*Cos[c + d*x])
^2) - (a^3*(4 - n)*Cos[c + d*x]*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3
- 8*n + 4*n^2)*(a - a*Cos[c + d*x])^2*(a + a*Cos[c + d*x])) + (n*(7 - 3*n -
 n^2)*Cos[c + d*x]*((1 + Cos[c + d*x])/(1 - Cos[c + d*x]))^(-1/2 - n)*Hyper
geometric2F1[-1/2 - n, 1 - n, 2 - n, (-2*Cos[c + d*x])/(1 - Cos[c + d*x])]*
(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*(3 - 2*n)*(1 - n)*(1 + 2*
n)*(1 - Cos[c + d*x])^2)
```


Rule 3876

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(Sin[e + f*x]^FracPart[m]*(a + b*Csc[e + f*x]
)^FracPart[m])/(b + a*Sin[e + f*x])^FracPart[m], Int[((g*Cos[e + f*x])^p*(b
 + a*Sin[e + f*x])^m)/Sin[e + f*x]^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rule 2883

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[Cos[e + f*x]/(a^(
p - 2)*f*Sqrt[a + b*Sin[e + f*x]]*Sqrt[a - b*Sin[e + f*x]]), Subst[Int[(d*x
)^n*(a + b*x)^(m + p/2 - 1/2)*(a - b*x)^(p/2 - 1/2), x], x, Sin[e + f*x]],
x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[p/2]
```

```
&& !IntegerQ[m]
```

Rule 129

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_
))^(p_.), x_Symbol] :> Simp[(b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*(e + f*x
)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)), x] + Dist[1/((m + 1)*(b*c - a
*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*
(m + 1) - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && ILtQ[m + n + p + 2,
0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
1]) && !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 155

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b*g - a*h)*(a + b*x)^(m +
1)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/((m + 1)*(b*c - a*d)*(b*e - a*f)),
x] + Dist[1/((m + 1)*(b*c - a*d)*(b*e - a*f)), Int[(a + b*x)^(m + 1)*(c + d
*x)^n*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g
 - a*h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m + n + p + 2,
 0] && NeQ[m, -1] && (SumSimplerQ[m, 1] || ( !(NeQ[n, -1] && SumSimplerQ[n,
 1]) && !(NeQ[p, -1] && SumSimplerQ[p, 1])))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 132

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^(p + 1)*
Hypergeometric2F1[m + 1, -n, m + 2, -(((d*e - c*f)*(a + b*x))/((b*c - a*d)*
(e + f*x)))])/(((b*e - a*f)*(m + 1))*(((b*e - a*f)*(c + d*x))/((b*c - a*d)*
(e + f*x)))^n), x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] && !IntegerQ[n]
```

Rubi steps

$$\int \csc^4(c+dx)(a+a\sec(c+dx))^n\,dx = ((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n)\int(-\cos(c+dx)$$

$$= -\frac{\left(a^6(-\cos(c+dx))^n(-a-a\cos(c+dx))^{-\frac{1}{2}-n}(a+a\sec(c+dx))^n\sin(c+dx)\right.}{d\sqrt{-a+a\cos(c+dx)}}$$

$$= -\frac{a^4\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(3-2n)(a-a\cos(c+dx))^2(a+a\cos(c+dx))^2} - \frac{\left(a^3(-\cos(c+dx))^n(-a\right.}{}$$

$$= -\frac{a^4\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(3-2n)(a-a\cos(c+dx))^2(a+a\cos(c+dx))^2} - \frac{a^3(4-n)\cos(c+dx)(}{d(1-2n)(3-2n)(a-a}$$

$$= \frac{(2-n+n^2)\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1-2n)(3-2n)(1+2n)(1-\cos(c+dx))^2} - \frac{a^4\cos(c+dx)(a}{d(3-2n)(a-a\,\mathrm{co}}$$

$$= \frac{(2-n+n^2)\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1-2n)(3-2n)(1+2n)(1-\cos(c+dx))^2} - \frac{a^4\cos(c+dx)(a}{d(3-2n)(a-a\,\mathrm{co}}$$

$$= \frac{(2-n+n^2)\cos(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1-2n)(3-2n)(1+2n)(1-\cos(c+dx))^2} - \frac{a^4\cos(c+dx)(a}{d(3-2n)(a-a\,\mathrm{co}}$$

Mathematica [A] time = 6.76317, size = 214, normalized size = 0.61

$$-\frac{2^{n-3}\tan^3\left(\frac{1}{2}(c+dx)\right)(\sec(c+dx)+1)^{-n}(a(\sec(c+dx)+1))^n\left(\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\right)^n\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^4*(a + a*Sec[c + d*x])^n,x]
```

```
[Out] -(2^(-3 + n)*(Cot[(c + d*x)/2]^6*Hypergeometric2F1[-3/2, n, -1/2, Tan[(c +
d*x)/2]^2] + 9*Cot[(c + d*x)/2]^4*Hypergeometric2F1[-1/2, n, 1/2, Tan[(c +
d*x)/2]^2] - 9*Cot[(c + d*x)/2]^2*Hypergeometric2F1[1/2, n, 3/2, Tan[(c + d
*x)/2]^2] - Hypergeometric2F1[3/2, n, 5/2, Tan[(c + d*x)/2]^2])*(Cos[c + d*
x]*Sec[(c + d*x)/2]^2)^n*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(a*(1 + Sec[c
+ d*x]))^n*Tan[(c + d*x)/2]^3)/(3*d*(1 + Sec[c + d*x])^n)
```

Maple [F] time = 0.321, size = 0, normalized size = 0.

$$\int (\csc(dx+c))^4 (a+a\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+a*sec(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^4*(a+a*sec(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n \csc(dx+c)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sec(dx+c)+a)^n \csc(dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+a*sec(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n \csc(dx+c)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+a*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

3.156 $\int (a+a\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=105

$$-\frac{\sqrt{\sin(c+dx)}\cos(c+dx)(\cos(c+dx)+1)^{-n-\frac{1}{4}}(a\sec(c+dx)+a)^n F_1\left(1-n;-\frac{1}{4},-n-\frac{1}{4};2-n;\cos(c+dx),-\cos(c+dx)\right)}{d(1-n)\sqrt[4]{1-\cos(c+dx)}}$$

```
[Out] -((AppellF1[1 - n, -1/4, -1/4 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*Cos[
c + d*x]*(1 + Cos[c + d*x])^(-1/4 - n)*(a + a*Sec[c + d*x])^n*Sqrt[Sin[c +
d*x]])/(d*(1 - n)*(1 - Cos[c + d*x])^(1/4)))
```

Rubi [A] time = 0.261743, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3876, 2886, 135, 133}

$$-\frac{\sqrt{\sin(c+dx)}\cos(c+dx)(\cos(c+dx)+1)^{-n-\frac{1}{4}}(a\sec(c+dx)+a)^n F_1\left(1-n;-\frac{1}{4},-n-\frac{1}{4};2-n;\cos(c+dx),-\cos(c+dx)\right)}{d(1-n)\sqrt[4]{1-\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^(3/2),x]
```


```
[Out] -((AppellF1[1 - n, -1/4, -1/4 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*Cos[
c + d*x]*(1 + Cos[c + d*x])^(-1/4 - n)*(a + a*Sec[c + d*x])^n*Sqrt[Sin[c +
d*x]])/(d*(1 - n)*(1 - Cos[c + d*x])^(1/4)))
```


Rule 3876

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(Sin[e + f*x]^FracPart[m]*(a + b*Csc[e + f*x]
)^FracPart[m])/(b + a*Sin[e + f*x])^FracPart[m], Int[((g*Cos[e + f*x])^p*(b
 + a*Sin[e + f*x])^m)/Sin[e + f*x]^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rule 2886

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos
[e + f*x])^(p - 1))/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x]
)^((p - 1)/2)), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
```

```
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (a + a\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx = ((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n)\int(-\cos(c+dx))^{-n}$$

$$= -\frac{\left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-\frac{1}{4}-n}(a+a\sec(c+dx))^n\sqrt{\sin(c+dx)}\right)\text{Su}}{d\sqrt[4]{-a+a\cos(c+dx)}}$$

$$= -\frac{\left((-\cos(c+dx))^n(1+\cos(c+dx))^{-\frac{1}{4}-n}(a+a\sec(c+dx))^n\sqrt{\sin(c+dx)}\right)\text{Subs}}{d\sqrt[4]{-a+a\cos(c+dx)}}$$

$$= -\frac{\left((-\cos(c+dx))^n(1+\cos(c+dx))^{-\frac{1}{4}-n}(a+a\sec(c+dx))^n\sqrt{\sin(c+dx)}\right)\text{Subs}}{d\sqrt[4]{1-\cos(c+dx)}}$$

$$= -\frac{F_1\left(1-n;-\frac{1}{4},-\frac{1}{4}-n;2-n;\cos(c+dx),-\cos(c+dx)\right)\cos(c+dx)(1+\cos(c}{d(1-n)\sqrt[4]{1-\cos(c+dx)}}$$

Mathematica [B] time = 3.18768, size = 382, normalized size = 3.64

$$\frac{1\text{(}}{d\left(2(\cos(c+dx)-1)\left(3F_1\left(\frac{5}{4};n,\frac{5}{2};\frac{9}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-5F_1\left(\frac{5}{4};n,\frac{7}{2};\frac{9}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}\right.\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sin[c + d*x]^(3/2),x]
```

```
[Out] (10*(AppellF1[1/4, n, 3/2, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] -
AppellF1[1/4, n, 5/2, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(1 + C
os[c + d*x])*(a*(1 + Sec[c + d*x]))^n*Sin[c + d*x]^(5/2))/(d*(2*(3*AppellF1
[5/4, n, 5/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 5*AppellF1[5/
4, n, 7/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*n*AppellF1[5/4
, 1 + n, 3/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*n*AppellF1[
5/4, 1 + n, 5/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c
 + d*x]) + 5*AppellF1[1/4, n, 3/2, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2]*(1 + Cos[c + d*x]) - 5*AppellF1[1/4, n, 5/2, 5/4, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2]*(1 + Cos[c + d*x])))
```

Maple [F] time = 0.179, size = 0, normalized size = 0.

$$\int (a + a\sec(dx+c))^n (\sin(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^(3/2),x)
```

```
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n \sin(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((a\sec(dx+c)+a)^n \sin(dx+c)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n \sin(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

3.157 $\int (a+a\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx$

Optimal. Leaf size=105

$$-\frac{\sqrt[4]{1-\cos(c+dx)}\cos(c+dx)(\cos(c+dx)+1)^{\frac{1}{4}-n}(a\sec(c+dx)+a)^n F_1\left(1-n;\frac{1}{4},\frac{1}{4}-n;2-n;\cos(c+dx),-\cos(c+dx\right.}{d(1-n)\sqrt{\sin(c+dx)}}$$

```
[Out] -((AppellF1[1 - n, 1/4, 1/4 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*(1 - C
os[c + d*x])^(1/4)*Cos[c + d*x]*(1 + Cos[c + d*x])^(1/4 - n)*(a + a*Sec[c +
 d*x])^n)/(d*(1 - n)*Sqrt[Sin[c + d*x]]))
```

Rubi [A] time = 0.272398, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3876, 2886, 135, 133}

$$-\frac{\sqrt[4]{1-\cos(c+dx)}\cos(c+dx)(\cos(c+dx)+1)^{\frac{1}{4}-n}(a\sec(c+dx)+a)^n F_1\left(1-n;\frac{1}{4},\frac{1}{4}-n;2-n;\cos(c+dx),-\cos(c+dx\right.}{d(1-n)\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]],x]
```

```
[Out] -((AppellF1[1 - n, 1/4, 1/4 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*(1 - C
os[c + d*x])^(1/4)*Cos[c + d*x]*(1 + Cos[c + d*x])^(1/4 - n)*(a + a*Sec[c +
 d*x])^n)/(d*(1 - n)*Sqrt[Sin[c + d*x]]))
```

Rule 3876

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(Sin[e + f*x]^FracPart[m]*(a + b*Csc[e + f*x]
)^FracPart[m])/(b + a*Sin[e + f*x])^FracPart[m], Int[((g*Cos[e + f*x])^p*(b
 + a*Sin[e + f*x])^m)/Sin[e + f*x]^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rule 2886

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos
[e + f*x])^(p - 1))/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x]
)^((p - 1)/2)), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
```

```
& !IntegerQ[m] && !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int (a + a\sec(c + dx))^n \sqrt{\sin(c + dx)}\,dx = ((-\cos(c + dx))^n(-a - a\cos(c + dx))^{-n}(a + a\sec(c + dx))^n) \int (-\cos(c + dx$$

$$= -\frac{\left((-\cos(c + dx))^n(-a - a\cos(c + dx))^{\frac{1}{4}-n}\sqrt[4]{-a + a\cos(c + dx)}(a + a\sec(c +\right.}{d\sqrt{\sin(c + dx)}}$$

$$= -\frac{\left((-\cos(c + dx))^n(1 + \cos(c + dx))^{\frac{1}{4}-n}\sqrt[4]{-a + a\cos(c + dx)}(a + a\sec(c + dx\right.}{d\sqrt{\sin(c + dx)}}$$

$$= -\frac{\left(\sqrt[4]{1 - \cos(c + dx)}(-\cos(c + dx))^n(1 + \cos(c + dx))^{\frac{1}{4}-n}(a + a\sec(c + dx))^n\right.}{d\sqrt{\sin(c + dx)}}$$

$$= -\frac{F_1\left(1 - n; \frac{1}{4}, \frac{1}{4} - n; 2 - n; \cos(c + dx), -\cos(c + dx)\right)\sqrt[4]{1 - \cos(c + dx)}\cos(}{d(1 - n)\sqrt{\sin(c + dx)}}$$

Mathematica [B] time = 1.34085, size = 214, normalized size = 2.04

$$\frac{14\sin^{\frac{3}{2}}(c + dx)(\cos(c + dx) + 1)(a(\sec(c + dx) + 1))^n F_1\left(\frac{3}{4}; n, \frac{3}{2}\right.}{d\left(6(\cos(c + dx) - 1)\left(3F_1\left(\frac{7}{4}; n, \frac{5}{2}; \frac{11}{4}; \tan^2\left(\frac{1}{2}(c + dx)\right), -\tan^2\left(\frac{1}{2}(c + dx)\right)\right) - 2nF_1\left(\frac{7}{4}; n + 1, \frac{3}{2}; \frac{11}{4}; \tan^2\left(\frac{1}{2}(c + dx)\right),\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]],x]
```

```
[Out] (14*AppellF1[3/4, n, 3/2, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1
+ Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^n*Sin[c + d*x]^(3/2))/(d*(6*(3*Appel
lF1[7/4, n, 5/2, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*n*Appel
lF1[7/4, 1 + n, 3/2, 11/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 +
Cos[c + d*x]) + 21*AppellF1[3/4, n, 3/2, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2]*(1 + Cos[c + d*x])))
```

Maple [F] time = 0.161, size = 0, normalized size = 0.

$$\int (a + a\sec(dx + c))^n \sqrt{\sin(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2),x)
```

```
[Out] int((a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx + c) + a)^n \sqrt{\sin(dx + c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((a\sec(dx+c)+a)^n\sqrt{\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n*sin(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (a\sec(dx+c)+a)^n\sqrt{\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

3.158 $\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx$

Optimal. Leaf size=105

$$-\frac{(1-\cos(c+dx))^{3/4}\cos(c+dx)(\cos(c+dx)+1)^{\frac{3}{4}-n}(a\sec(c+dx)+a)^n F_1\left(1-n;\frac{3}{4},\frac{3}{4}-n;2-n;\cos(c+dx),-\cos(\right.}{d(1-n)\sin^{\frac{3}{2}}(c+dx)}$$

```
[Out] -((AppellF1[1 - n, 3/4, 3/4 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*(1 - C
os[c + d*x])^(3/4)*Cos[c + d*x]*(1 + Cos[c + d*x])^(3/4 - n)*(a + a*Sec[c +
 d*x])^n)/(d*(1 - n)*Sin[c + d*x]^(3/2)))
```

Rubi [A] time = 0.24954, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3876, 2886, 135, 133}

$$-\frac{(1-\cos(c+dx))^{3/4}\cos(c+dx)(\cos(c+dx)+1)^{\frac{3}{4}-n}(a\sec(c+dx)+a)^n F_1\left(1-n;\frac{3}{4},\frac{3}{4}-n;2-n;\cos(c+dx),-\cos(\right.}{d(1-n)\sin^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]],x]
```

```
[Out] -((AppellF1[1 - n, 3/4, 3/4 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*(1 - C
os[c + d*x])^(3/4)*Cos[c + d*x]*(1 + Cos[c + d*x])^(3/4 - n)*(a + a*Sec[c +
 d*x])^n)/(d*(1 - n)*Sin[c + d*x]^(3/2)))
```

Rule 3876

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(Sin[e + f*x]^FracPart[m]*(a + b*Csc[e + f*x]
)^FracPart[m])/(b + a*Sin[e + f*x])^FracPart[m], Int[((g*Cos[e + f*x])^p*(b
 + a*Sin[e + f*x])^m)/Sin[e + f*x]^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rule 2886

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos
[e + f*x])^(p - 1))/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x]
)^((p - 1)/2)), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[m]
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !GtQ[c, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
```

```
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx = ((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n)\int \frac{(-\cos(c+dx))^{-n}(-a-a\cos}{\sqrt{\sin(c+dx)}}$$

$$= -\frac{\left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{\frac{3}{4}-n}(-a+a\cos(c+dx))^{3/4}(a+a\sec(c+dx))^n\right)\text{Subs}}{d\sin^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{\left((-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{3}{4}-n}(-a+a\cos(c+dx))^{3/4}(a+a\sec(c+dx))^n\right)\text{Subst}\Big(}{d\sin^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{\left((1-\cos(c+dx))^{3/4}(-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{3}{4}-n}(a+a\sec(c+dx))^n\right)\text{Subst}\Big(\int \frac{(-}{}}{d\sin^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{F_1\left(1-n;\frac{3}{4},\frac{3}{4}-n;2-n;\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{3/4}\cos(c+dx)(1+\text{c}}{d(1-n)\sin^{\frac{3}{2}}(c+dx)}$$

Mathematica [B] time = 1.01211, size = 212, normalized size = 2.02

$$\frac{10\sqrt{\sin(c+dx)}(\cos(c+dx)+1)(a(\sec(c+dx)+1))^nF_1\left(\frac{1}{4};n,\frac{1}{2};\frac{5}{4};\text{t}\right.}{d\left(2(\cos(c+dx)-1)\left(F_1\left(\frac{5}{4};n,\frac{3}{2};\frac{9}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-2nF_1\left(\frac{5}{4};n+1,\frac{1}{2};\frac{9}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]],x]
```

```
[Out] (10*AppellF1[1/4, n, 1/2, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1
+ Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^n*Sqrt[Sin[c + d*x]])/(d*(2*(AppellF
1[5/4, n, 3/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - 2*n*AppellF1
[5/4, 1 + n, 1/2, 9/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[
c + d*x]) + 5*AppellF1[1/4, n, 1/2, 5/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2]*(1 + Cos[c + d*x])))
```

Maple [F] time = 0.166, size = 0, normalized size = 0.

$$\int (a+a\sec(dx+c))^n\frac{1}{\sqrt{\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n/sin(d*x+c)^(1/2),x)
```

```
[Out] int((a+a*sec(d*x+c))^n/sin(d*x+c)^(1/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^n}{\sqrt{\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(a\sec(dx+c)+a)^n}{\sqrt{\sin(dx+c)}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a(\sec(c+dx)+1))^n}{\sqrt{\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n/sin(d*x+c)**(1/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**n/sqrt(sin(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^n}{\sqrt{\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

3.159 $\int \frac{(a+a\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=105

$$-\frac{(1-\cos(c+dx))^{5/4}\cos(c+dx)(\cos(c+dx)+1)^{\frac{5}{4}-n}(a\sec(c+dx)+a)^n F_1\left(1-n;\frac{5}{4},\frac{5}{4}-n;2-n;\cos(c+dx),-\cos(c+\right.}{d(1-n)\sin^{\frac{5}{2}}(c+dx)}$$

```
[Out] -((AppellF1[1 - n, 5/4, 5/4 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*(1 - C
os[c + d*x])^(5/4)*Cos[c + d*x]*(1 + Cos[c + d*x])^(5/4 - n)*(a + a*Sec[c +
 d*x])^n)/(d*(1 - n)*Sin[c + d*x]^(5/2)))
```

Rubi [A] time = 0.266506, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3876, 2886, 135, 133}

$$-\frac{(1-\cos(c+dx))^{5/4}\cos(c+dx)(\cos(c+dx)+1)^{\frac{5}{4}-n}(a\sec(c+dx)+a)^n F_1\left(1-n;\frac{5}{4},\frac{5}{4}-n;2-n;\cos(c+dx),-\cos(c+\right.}{d(1-n)\sin^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^n/Sin[c + d*x]^(3/2),x]
```

```
[Out] -((AppellF1[1 - n, 5/4, 5/4 - n, 2 - n, Cos[c + d*x], -Cos[c + d*x]]*(1 - C
os[c + d*x])^(5/4)*Cos[c + d*x]*(1 + Cos[c + d*x])^(5/4 - n)*(a + a*Sec[c +
 d*x])^n)/(d*(1 - n)*Sin[c + d*x]^(5/2)))
```

Rule 3876

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[(Sin[e + f*x]^FracPart[m]*(a + b*Csc[e + f*x]
)^FracPart[m])/(b + a*Sin[e + f*x])^FracPart[m], Int[((g*Cos[e + f*x])^p*(b
 + a*Sin[e + f*x])^m)/Sin[e + f*x]^m, x], x] /; FreeQ[{a, b, e, f, g, m, p}
, x] && (EqQ[a^2 - b^2, 0] || IntegersQ[2*m, p])
```

Rule 2886

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(g*(g*Cos
[e + f*x])^(p - 1))/(f*(a + b*Sin[e + f*x])^((p - 1)/2)*(a - b*Sin[e + f*x]
)^((p - 1)/2)), Subst[Int[(d*x)^n*(a + b*x)^(m + (p - 1)/2)*(a - b*x)^((p -
 1)/2), x], x, Sin[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n, p}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[m]
```

Rule 135

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Dist[(c^IntPart[n]*(c + d*x)^FracPart[n])/(1 + (d*x)/c)^FracPart
[n], Int[(b*x)^m*(1 + (d*x)/c)^n*(e + f*x)^p, x], x] /; FreeQ[{b, c, d, e,
f, m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !GtQ[c, 0]
```

Rule 133

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[(c^n*e^p*(b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2, -((d*
```

```
x)/c), -((f*x)/e)])/(b*(m + 1)), x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx = \left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{-n}(a+a\sec(c+dx))^n\right)\int \frac{(-\cos(c+dx))^{-n}(-a-a}{\sin^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{\left((-\cos(c+dx))^n(-a-a\cos(c+dx))^{\frac{5}{4}-n}(-a+a\cos(c+dx))^{5/4}(a+a\sec(c+dx))^n\right)\mathrm{Su}}{d\sin^{\frac{5}{2}}(c+dx)}$$

$$= -\left(-\frac{\left((-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{1}{4}-n}(-a-a\cos(c+dx))(-a+a\cos(c+dx))^{5/4}(a+\right.}{ad\sin^{\frac{5}{2}}(c+dx)}\right.$$

$$= -\frac{\left(\sqrt[4]{1-\cos(c+dx)}(-\cos(c+dx))^n(1+\cos(c+dx))^{\frac{1}{4}-n}(-a-a\cos(c+dx))(-a+a\cos(\right.}{a^2d\sin^{\frac{5}{2}}(c+dx}$$

$$= -\frac{F_1\left(1-n;\frac{5}{4},\frac{5}{4}-n;2-n;\cos(c+dx),-\cos(c+dx)\right)(1-\cos(c+dx))^{5/4}\cos(c+dx)(1-}{d(1-n)\sin^{\frac{5}{2}}(c+dx)}$$

Mathematica [B] time = 1.17057, size = 212, normalized size = 2.02

$$-\frac{6(\cos(c+dx)+1)(a(\sec(c+dx)+1))^nF_1\left(-\frac{1}{4};n,\right.}{d\sqrt{\sin(c+dx)}\left(3(\cos(c+dx)+1)F_1\left(-\frac{1}{4};n,-\frac{1}{2};\frac{3}{4};\tan^2\left(\frac{1}{2}(c+dx)\right),-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-2(\cos(c+dx)-1)\left(F_1\left(\frac{3}{4}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^n/Sin[c + d*x]^(3/2),x]
```

```
[Out] (-6*AppellF1[-1/4, n, -1/2, 3/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(
1 + Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^n)/(d*(-2*(AppellF1[3/4, n, 1/2, 7
/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*n*AppellF1[3/4, 1 + n, -1/
2, 7/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(-1 + Cos[c + d*x]) + 3*A
ppellF1[-1/4, n, -1/2, 3/4, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(1 + C
os[c + d*x]))*Sqrt[Sin[c + d*x]])
```

Maple [F] time = 0.16, size = 0, normalized size = 0.

$$\int (a+a\sec(dx+c))^n(\sin(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^n/sin(d*x+c)^(3/2),x)
```

```
[Out] int((a+a*sec(d*x+c))^n/sin(d*x+c)^(3/2),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^n}{\sin(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n/sin(d*x + c)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(a\sec(dx+c)+a)^n\sqrt{\sin(dx+c)}}{\cos(dx+c)^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(a*sec(d*x + c) + a)^n*sqrt(sin(d*x + c))/(cos(d*x + c)^2 - 1), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**n/sin(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^n}{\sin(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^n/sin(d*x + c)^(3/2), x)
```

3.160 $\int (a+b\sec(c+dx))\sin^7(c+dx)\,dx$

Optimal. Leaf size=119

$$\frac{a\cos^7(c+dx)}{7d}-\frac{3a\cos^5(c+dx)}{5d}+\frac{a\cos^3(c+dx)}{d}-\frac{a\cos(c+dx)}{d}+\frac{b\cos^6(c+dx)}{6d}-\frac{3b\cos^4(c+dx)}{4d}+\frac{3b\cos^2(c+dx)}{2d}$$

```
[Out] -((a*Cos[c + d*x])/d) + (3*b*Cos[c + d*x]^2)/(2*d) + (a*Cos[c + d*x]^3)/d -
 (3*b*Cos[c + d*x]^4)/(4*d) - (3*a*Cos[c + d*x]^5)/(5*d) + (b*Cos[c + d*x]^
6)/(6*d) + (a*Cos[c + d*x]^7)/(7*d) - (b*Log[Cos[c + d*x]])/d
```

Rubi [A] time = 0.109538, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3872, 2837, 12, 766}

$$\frac{a\cos^7(c+dx)}{7d}-\frac{3a\cos^5(c+dx)}{5d}+\frac{a\cos^3(c+dx)}{d}-\frac{a\cos(c+dx)}{d}+\frac{b\cos^6(c+dx)}{6d}-\frac{3b\cos^4(c+dx)}{4d}+\frac{3b\cos^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^7,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (3*b*Cos[c + d*x]^2)/(2*d) + (a*Cos[c + d*x]^3)/d -
 (3*b*Cos[c + d*x]^4)/(4*d) - (3*a*Cos[c + d*x]^5)/(5*d) + (b*Cos[c + d*x]^
6)/(6*d) + (a*Cos[c + d*x]^7)/(7*d) - (b*Log[Cos[c + d*x]])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))\sin^7(c+dx)\,dx = -\int (-b-a\cos(c+dx))\sin^6(c+dx)\tan(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a(-b+x)(a^2-x^2)^3}{x}\,dx, x, -a\cos(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-b+x)(a^2-x^2)^3}{x}\,dx, x, -a\cos(c+dx)\right)}{a^6 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^6 - \frac{a^6 b}{x} + 3a^4bx - 3a^4x^2 - 3a^2bx^3 + 3a^2x^4 + bx^5 - x^6\right) dx, x, -a\cos(c\right.}{a^6 d}$$
$$= -\frac{a\cos(c+dx)}{d} + \frac{3b\cos^2(c+dx)}{2d} + \frac{a\cos^3(c+dx)}{d} - \frac{3b\cos^4(c+dx)}{4d} - \frac{3a\cos^5(c}{5d}$$

Mathematica [A] time = 0.13935, size = 115, normalized size = 0.97

$$-\frac{35a\cos(c+dx)}{64d} + \frac{7a\cos(3(c+dx))}{64d} - \frac{7a\cos(5(c+dx))}{320d} + \frac{a\cos(7(c+dx))}{448d} - \frac{b\left(-\frac{1}{3}\cos^6(c+dx) + \frac{3}{2}\cos^4(c+dx) - 3\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^7,x]
```

```
[Out] (-35*a*Cos[c + d*x])/(64*d) + (7*a*Cos[3*(c + d*x)])/(64*d) - (7*a*Cos[5*(c
 + d*x)])/(320*d) + (a*Cos[7*(c + d*x)])/(448*d) - (b*(-3*Cos[c + d*x]^2 +
(3*Cos[c + d*x]^4)/2 - Cos[c + d*x]^6/3 + 2*Log[Cos[c + d*x]]))/(2*d)
```

Maple [A] time = 0.041, size = 129, normalized size = 1.1

$$-\frac{16\,a\cos(dx+c)}{35\,d} - \frac{a\cos(dx+c)(\sin(dx+c))^6}{7\,d} - \frac{6\,a\cos(dx+c)(\sin(dx+c))^4}{35\,d} - \frac{8\,a\cos(dx+c)(\sin(dx+c))^2}{35\,d} - \frac{b(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*sin(d*x+c)^7,x)
```

```
[Out] -16/35*a*cos(d*x+c)/d-1/7/d*a*cos(d*x+c)*sin(d*x+c)^6-6/35/d*a*cos(d*x+c)*s
in(d*x+c)^4-8/35/d*a*cos(d*x+c)*sin(d*x+c)^2-1/6/d*b*sin(d*x+c)^6-1/4/d*b*s
in(d*x+c)^4-1/2/d*b*sin(d*x+c)^2-b*ln(cos(d*x+c))/d
```

Maxima [A] time = 0.96798, size = 123, normalized size = 1.03

$$\frac{60\,a\cos(dx+c)^7 + 70\,b\cos(dx+c)^6 - 252\,a\cos(dx+c)^5 - 315\,b\cos(dx+c)^4 + 420\,a\cos(dx+c)^3 + 630\,b\cos(dx+c}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="maxima")
```

```
[Out] 1/420*(60*a*cos(d*x + c)^7 + 70*b*cos(d*x + c)^6 - 252*a*cos(d*x + c)^5 - 3
15*b*cos(d*x + c)^4 + 420*a*cos(d*x + c)^3 + 630*b*cos(d*x + c)^2 - 420*a*c
```

```
os(d*x + c) - 420*b*log(cos(d*x + c)))/d
```

Fricas [A] time = 1.83308, size = 261, normalized size = 2.19

$$\frac{60\,a\cos\left(dx+c\right)^{7}+70\,b\cos\left(dx+c\right)^{6}-252\,a\cos\left(dx+c\right)^{5}-315\,b\cos\left(dx+c\right)^{4}+420\,a\cos\left(dx+c\right)^{3}+630\,b\cos\left(dx\right.}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="fricas")
```

```
[Out] 1/420*(60*a*cos(d*x + c)^7 + 70*b*cos(d*x + c)^6 - 252*a*cos(d*x + c)^5 - 3
15*b*cos(d*x + c)^4 + 420*a*cos(d*x + c)^3 + 630*b*cos(d*x + c)^2 - 420*a*c
os(d*x + c) - 420*b*log(-cos(d*x + c)))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)**7,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35996, size = 428, normalized size = 3.6

$$\frac{420\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-420\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{384\,a+1089\,b-\frac{2688\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{8463\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{8064\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{}}{420\,}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^7,x, algorithm="giac")
```

```
[Out] 1/420*(420*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 420*b*l
og(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (384*a + 1089*b - 268
8*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 8463*b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + 8064*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 28749*b*
(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 13440*a*(cos(d*x + c) - 1)^3/(c
os(d*x + c) + 1)^3 - 56035*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56
035*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 28749*b*(cos(d*x + c) - 1
)^5/(cos(d*x + c) + 1)^5 + 8463*b*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6
 - 1089*b*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/((cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - 1)^7)/d
```

3.161 $\int (a+b\sec(c+dx))\sin^5(c+dx)\,dx$

Optimal. Leaf size=87

$$-\frac{a\cos^5(c+dx)}{5d}+\frac{2a\cos^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{d}-\frac{b\cos^4(c+dx)}{4d}+\frac{b\cos^2(c+dx)}{d}-\frac{b\log(\cos(c+dx))}{d}$$

```
[Out] -((a*Cos[c + d*x])/d) + (b*Cos[c + d*x]^2)/d + (2*a*Cos[c + d*x]^3)/(3*d) -
 (b*Cos[c + d*x]^4)/(4*d) - (a*Cos[c + d*x]^5)/(5*d) - (b*Log[Cos[c + d*x]]
)/d
```

Rubi [A] time = 0.0981964, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3872, 2837, 12, 766}

$$-\frac{a\cos^5(c+dx)}{5d}+\frac{2a\cos^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{d}-\frac{b\cos^4(c+dx)}{4d}+\frac{b\cos^2(c+dx)}{d}-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^5,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (b*Cos[c + d*x]^2)/d + (2*a*Cos[c + d*x]^3)/(3*d) -
 (b*Cos[c + d*x]^4)/(4*d) - (a*Cos[c + d*x]^5)/(5*d) - (b*Log[Cos[c + d*x]]
)/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))\sin^5(c+dx)\,dx = -\int (-b-a\cos(c+dx))\sin^4(c+dx)\tan(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a(-b+x)\left(a^2-x^2\right)^2}{x}\,dx, x, -a\cos(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-b+x)\left(a^2-x^2\right)^2}{x}\,dx, x, -a\cos(c+dx)\right)}{a^4 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^4-\frac{a^4 b}{x}+2a^2bx-2a^2x^2-bx^3+x^4\right)dx, x, -a\cos(c+dx)\right)}{a^4 d}$$
$$= -\frac{a\cos(c+dx)}{d}+\frac{b\cos^2(c+dx)}{d}+\frac{2a\cos^3(c+dx)}{3d}-\frac{b\cos^4(c+dx)}{4d}-\frac{a\cos^5(}{5}$$

Mathematica [A] time = 0.0818673, size = 83, normalized size = 0.95

$$-\frac{5a\cos(c+dx)}{8d}+\frac{5a\cos(3(c+dx))}{48d}-\frac{a\cos(5(c+dx))}{80d}-\frac{b\left(\frac{1}{4}\cos^4(c+dx)-\cos^2(c+dx)+\log(\cos(c+dx))\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^5,x]
```

```
[Out] (-5*a*Cos[c + d*x])/(8*d) + (5*a*Cos[3*(c + d*x)])/(48*d) - (a*Cos[5*(c + d
*x)])/(80*d) - (b*(-Cos[c + d*x]^2 + Cos[c + d*x]^4/4 + Log[Cos[c + d*x]]))
/d
```

Maple [A] time = 0.035, size = 95, normalized size = 1.1

$$-\frac{8\,a\cos(dx+c)}{15\,d}-\frac{a\cos(dx+c)(\sin(dx+c))^4}{5\,d}-\frac{4\,a\cos(dx+c)(\sin(dx+c))^2}{15\,d}-\frac{b(\sin(dx+c))^4}{4\,d}-\frac{b(\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*sin(d*x+c)^5,x)
```

```
[Out] -8/15*a*cos(d*x+c)/d-1/5/d*a*cos(d*x+c)*sin(d*x+c)^4-4/15/d*a*cos(d*x+c)*si
n(d*x+c)^2-1/4/d*b*sin(d*x+c)^4-1/2/d*b*sin(d*x+c)^2-b*ln(cos(d*x+c))/d
```

Maxima [A] time = 0.967445, size = 93, normalized size = 1.07

$$-\frac{12\,a\cos(dx+c)^5+15\,b\cos(dx+c)^4-40\,a\cos(dx+c)^3-60\,b\cos(dx+c)^2+60\,a\cos(dx+c)+60\,b\log(\cos(dx}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(12*a*cos(d*x + c)^5 + 15*b*cos(d*x + c)^4 - 40*a*cos(d*x + c)^3 - 60
*b*cos(d*x + c)^2 + 60*a*cos(d*x + c) + 60*b*log(cos(d*x + c)))/d
```

Fricas [A] time = 1.80906, size = 193, normalized size = 2.22

$$-\frac{12\,a\cos\left(dx+c\right)^{5}+15\,b\cos\left(dx+c\right)^{4}-40\,a\cos\left(dx+c\right)^{3}-60\,b\cos\left(dx+c\right)^{2}+60\,a\cos\left(dx+c\right)+60\,b\log\left(-\cos\left(dx+\right.\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/60*(12*a*cos(d*x + c)^5 + 15*b*cos(d*x + c)^4 - 40*a*cos(d*x + c)^3 - 60
*b*cos(d*x + c)^2 + 60*a*cos(d*x + c) + 60*b*log(-cos(d*x + c)))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.37916, size = 335, normalized size = 3.85

$$\frac{60\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right)-60\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{64\,a+137\,b-\frac{320\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{805\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{640\,a(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{1970\,b(\cos(\ldots}{(\cos(dx+\ldots}}{\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-\ldots}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*b*log(
abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (64*a + 137*b - 320*a*(c
os(d*x + c) - 1)/(cos(d*x + c) + 1) - 805*b*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 640*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1970*b*(cos(d*x
+ c) - 1)^2/(cos(d*x + c) + 1)^2 - 1970*b*(cos(d*x + c) - 1)^3/(cos(d*x + c
) + 1)^3 + 805*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 137*b*(cos(d*x
 + c) - 1)^5/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) -
 1)^5)/d
```

3.162 $\int (a+b\sec(c+dx))\sin^3(c+dx)\,dx$

Optimal. Leaf size=58

$$\frac{a\cos^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{d}+\frac{b\cos^2(c+dx)}{2d}-\frac{b\log(\cos(c+dx))}{d}$$

```
[Out] -((a*Cos[c + d*x])/d) + (b*Cos[c + d*x]^2)/(2*d) + (a*Cos[c + d*x]^3)/(3*d)
 - (b*Log[Cos[c + d*x]])/d
```

Rubi [A] time = 0.0857308, antiderivative size = 58, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3872, 2837, 12, 766}

$$\frac{a\cos^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{d}+\frac{b\cos^2(c+dx)}{2d}-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^3,x]
```

```
[Out] -((a*Cos[c + d*x])/d) + (b*Cos[c + d*x]^2)/(2*d) + (a*Cos[c + d*x]^3)/(3*d)
 - (b*Log[Cos[c + d*x]])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 766

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))\sin^3(c+dx)\,dx = -\int (-b-a\cos(c+dx))\sin^2(c+dx)\tan(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a(-b+x)(a^2-x^2)}{x}\,dx, x, -a\cos(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-b+x)(a^2-x^2)}{x}\,dx, x, -a\cos(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2-\frac{a^2 b}{x}+bx-x^2\right)dx, x, -a\cos(c+dx)\right)}{a^2 d}$$
$$= -\frac{a\cos(c+dx)}{d}+\frac{b\cos^2(c+dx)}{2d}+\frac{a\cos^3(c+dx)}{3d}-\frac{b\log(\cos(c+dx))}{d}$$

Mathematica [A] time = 0.0453399, size = 57, normalized size = 0.98

$$-\frac{3a\cos(c+dx)}{4d}+\frac{a\cos(3(c+dx))}{12d}-\frac{b\left(\log(\cos(c+dx))-\frac{1}{2}\cos^2(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^3,x]
```

```
[Out] (-3*a*Cos[c + d*x])/(4*d) + (a*Cos[3*(c + d*x)])/(12*d) - (b*(-Cos[c + d*x]
^2/2 + Log[Cos[c + d*x]]))/d
```

Maple [A] time = 0.033, size = 61, normalized size = 1.1

$$-\frac{a\cos(dx+c)(\sin(dx+c))^2}{3d}-\frac{2a\cos(dx+c)}{3d}-\frac{b(\sin(dx+c))^2}{2d}-\frac{b\ln(\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*sin(d*x+c)^3,x)
```

```
[Out] -1/3/d*a*cos(d*x+c)*sin(d*x+c)^2-2/3*a*cos(d*x+c)/d-1/2/d*b*sin(d*x+c)^2-b*
ln(cos(d*x+c))/d
```

Maxima [A] time = 0.952518, size = 63, normalized size = 1.09

$$\frac{2a\cos(dx+c)^3+3b\cos(dx+c)^2-6a\cos(dx+c)-6b\log(\cos(dx+c))}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a*cos(d*x + c)^3 + 3*b*cos(d*x + c)^2 - 6*a*cos(d*x + c) - 6*b*log(c
os(d*x + c)))/d
```

Fricas [A] time = 1.78341, size = 126, normalized size = 2.17

$$\frac{2\,a\cos\left(dx+c\right)^{3}+3\,b\cos\left(dx+c\right)^{2}-6\,a\cos\left(dx+c\right)-6\,b\log\left(-\cos\left(dx+c\right)\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a*cos(d*x + c)^3 + 3*b*cos(d*x + c)^2 - 6*a*cos(d*x + c) - 6*b*log(-
cos(d*x + c)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a+b\sec\left(c+dx\right)\right)\sin^{3}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sin(c + d*x)**3, x)
```

Giac [A] time = 1.31767, size = 89, normalized size = 1.53

$$-\frac{b\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{2\,ad^{2}\cos\left(dx+c\right)^{3}+3\,bd^{2}\cos\left(dx+c\right)^{2}-6\,ad^{2}\cos\left(dx+c\right)}{6\,d^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -b*log(abs(cos(d*x + c))/abs(d))/d + 1/6*(2*a*d^2*cos(d*x + c)^3 + 3*b*d^2*
cos(d*x + c)^2 - 6*a*d^2*cos(d*x + c))/d^3
```

3.163 $\int (a+b\sec(c+dx))\sin(c+dx)\,dx$

Optimal. Leaf size=26

$$-\frac{a\cos(c+dx)}{d}-\frac{b\log(\cos(c+dx))}{d}$$

```
[Out] -((a*Cos[c + d*x])/d) - (b*Log[Cos[c + d*x]])/d
```

Rubi [A] time = 0.0329812, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {3872, 2721, 43}

$$-\frac{a\cos(c+dx)}{d}-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x],x]
```

```
[Out] -((a*Cos[c + d*x])/d) - (b*Log[Cos[c + d*x]])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\begin{aligned}
\int (a+b\sec(c+dx))\sin(c+dx)\,dx &= -\int (-b-a\cos(c+dx))\tan(c+dx)\,dx\\
&= \frac{\text{Subst}\left(\int \frac{-b+x}{x}\,dx,x,-a\cos(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(1-\frac{b}{x}\right)\,dx,x,-a\cos(c+dx)\right)}{d}\\
&= -\frac{a\cos(c+dx)}{d}-\frac{b\log(\cos(c+dx))}{d}
\end{aligned}$$

Mathematica [A] time = 0.0258105, size = 37, normalized size = 1.42

$$\frac{a\sin(c)\sin(dx)}{d}-\frac{a\cos(c)\cos(dx)}{d}-\frac{b\log(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x],x]
```

```
[Out] -((a*Cos[c]*Cos[d*x])/d) - (b*Log[Cos[c + d*x]])/d + (a*Sin[c]*Sin[d*x])/d
```

Maple [A] time = 0.018, size = 28, normalized size = 1.1

$$\frac{b\ln(\sec(dx+c))}{d}-\frac{a}{d\sec(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*sin(d*x+c),x)
```

```
[Out] 1/d*b*ln(sec(d*x+c))-1/d*a/sec(d*x+c)
```

Maxima [A] time = 0.983567, size = 31, normalized size = 1.19

$$-\frac{a\cos(dx+c)+b\log(\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -(a*cos(d*x + c) + b*log(cos(d*x + c)))/d
```

Fricas [A] time = 1.72309, size = 59, normalized size = 2.27

$$-\frac{a\cos(dx+c)+b\log(-\cos(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -(a*cos(d*x + c) + b*log(-cos(d*x + c)))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int(a+b\sec(c+dx))\sin(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sin(c + d*x), x)
```

Giac [A] time = 1.29375, size = 43, normalized size = 1.65

$$-\frac{a\cos(dx+c)}{d}-\frac{b\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -a*cos(d*x + c)/d - b*log(abs(cos(d*x + c))/abs(d))/d
```

3.164 $\int \csc(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=26

$$\frac{b\log(\tan(c+dx))}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) + (b*Log[Tan[c + d*x]])/d
```

Rubi [A] time = 0.0725366, antiderivative size = 26, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3872, 2834, 2620, 29, 3770}

$$\frac{b\log(\tan(c+dx))}{d}-\frac{a\tanh^{-1}(\cos(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Sec[c + d*x]),x]

[Out] -((a*ArcTanh[Cos[c + d*x]])/d) + (b*Log[Tan[c + d*x]])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)(a+b\sec(c+dx))\,dx &= -\int(-b-a\cos(c+dx))\csc(c+dx)\sec(c+dx)\,dx\\ &= a\int \csc(c+dx)\,dx + b\int \csc(c+dx)\sec(c+dx)\,dx\\ &= -\frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{b\,\text{Subst}\left(\int \frac{1}{x}\,dx, x, \tan(c+dx)\right)}{d}\\ &= -\frac{a\tanh^{-1}(\cos(c+dx))}{d} + \frac{b\log(\tan(c+dx))}{d}\end{aligned}$$

Mathematica [B] time = 0.0347635, size = 63, normalized size = 2.42

$$\frac{a\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{a\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{b(\log(\cos(c+dx)) - \log(\sin(c+dx)))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + b*Sec[c + d*x]),x]
```

```
[Out] -((a*Log[Cos[c/2 + (d*x)/2]])/d) + (a*Log[Sin[c/2 + (d*x)/2]])/d - (b*(Log[
Cos[c + d*x]] - Log[Sin[c + d*x]]))/d
```

Maple [A] time = 0.033, size = 35, normalized size = 1.4

$$\frac{a\ln(\csc(dx+c) - \cot(dx+c))}{d} + \frac{b\ln(\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*a*ln(csc(d*x+c)-cot(d*x+c))+b*ln(tan(d*x+c))/d
```

Maxima [A] time = 0.953032, size = 61, normalized size = 2.35

$$-\frac{(a-b)\log(\cos(dx+c)+1) - (a+b)\log(\cos(dx+c)-1) + 2\,b\log(\cos(dx+c))}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*((a - b)*log(cos(d*x + c) + 1) - (a + b)*log(cos(d*x + c) - 1) + 2*b*l
og(cos(d*x + c)))/d
```

Fricas [A] time = 1.72636, size = 149, normalized size = 5.73

$$-\frac{2\,b\log(-\cos(dx+c)) + (a-b)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) - (a+b)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*log(-cos(d*x + c)) + (a - b)*log(1/2*cos(d*x + c) + 1/2) - (a + b
)*log(-1/2*cos(d*x + c) + 1/2))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sec (c + dx)) \csc (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*csc(c + d*x), x)
```

Giac [B] time = 1.34591, size = 82, normalized size = 3.15

$$\frac{(a+b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 2\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((a + b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 2*b*log(ab
s(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)))/d
```

3.165 $\int \csc^3(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=64

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}-\frac{b\cot^2(c+dx)}{2d}+\frac{b\log(\tan(c+dx))}{d}$$

```
[Out] -(a*ArcTanh[Cos[c + d*x]])/(2*d) - (b*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*
x]*Csc[c + d*x])/(2*d) + (b*Log[Tan[c + d*x]])/d
```

Rubi [A] time = 0.103521, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3872, 2834, 2620, 14, 3768, 3770}

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}-\frac{b\cot^2(c+dx)}{2d}+\frac{b\log(\tan(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

```
[Out] -(a*ArcTanh[Cos[c + d*x]])/(2*d) - (b*Cot[c + d*x]^2)/(2*d) - (a*Cot[c + d*
x]*Csc[c + d*x])/(2*d) + (b*Log[Tan[c + d*x]])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
```

```
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^3(c+dx)(a+b\sec(c+dx))\,dx &= -\int (-b-a\cos(c+dx))\csc^3(c+dx)\sec(c+dx)\,dx\\ &= a\int \csc^3(c+dx)\,dx + b\int \csc^3(c+dx)\sec(c+dx)\,dx\\ &= -\frac{a\cot(c+dx)\csc(c+dx)}{2d}+\frac{1}{2}a\int \csc(c+dx)\,dx+\frac{b\,\text{Subst}\left(\int \frac{1+x^2}{x^3}\,dx,x,\text{ta}\right.}{d}\\ &= -\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}+\frac{b\,\text{Subst}\left(\int\left(\frac{1}{x^3}+\frac{1}{x}\right)dx\right.}{d}\\ &= -\frac{a\tanh^{-1}(\cos(c+dx))}{2d}-\frac{b\cot^2(c+dx)}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}+\frac{b\log(}{}\end{aligned}$$

Mathematica [A] time = 0.52254, size = 114, normalized size = 1.78

$$-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{b\left(\csc^2(c+dx)-2\log\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Sec[c + d*x]),x]
```

```
[Out] -(a*Csc[(c + d*x)/2]^2)/(8*d) - (a*Log[Cos[(c + d*x)/2]])/(2*d) + (a*Log[Si
n[(c + d*x)/2]])/(2*d) - (b*(Csc[c + d*x]^2 + 2*Log[Cos[c + d*x]] - 2*Log[S
in[c + d*x]]))/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

Maple [A] time = 0.094, size = 68, normalized size = 1.1

$$-\frac{a\cot(dx+c)\csc(dx+c)}{2d}+\frac{a\ln(\csc(dx+c)-\cot(dx+c))}{2d}-\frac{b}{2d(\sin(dx+c))^2}+\frac{b\ln(\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*sec(d*x+c)),x)
```

```
[Out] -1/2*a*cot(d*x+c)*csc(d*x+c)/d+1/2/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/2/d*b/si
n(d*x+c)^2+b*ln(tan(d*x+c))/d
```

Maxima [A] time = 0.972545, size = 96, normalized size = 1.5

$$-\frac{(a-2b)\log(\cos(dx+c)+1)-(a+2b)\log(\cos(dx+c)-1)+4b\log(\cos(dx+c))-\frac{2(a\cos(dx+c)+b)}{\cos(dx+c)^2-1}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/4*((a - 2*b)*log(cos(d*x + c) + 1) - (a + 2*b)*log(cos(d*x + c) - 1) + 4
*b*log(cos(d*x + c)) - 2*(a*cos(d*x + c) + b)/(cos(d*x + c)^2 - 1))/d
```

Fricas [B] time = 1.82602, size = 316, normalized size = 4.94

$$\frac{2\,a\cos\left(dx+c\right)-4\left(b\cos\left(dx+c\right)^{2}-b\right)\log\left(-\cos\left(dx+c\right)\right)-\left(\left(a-2\,b\right)\cos\left(dx+c\right)^{2}-a+2\,b\right)\log\left(\frac{1}{2}\cos\left(dx+c\right)+\frac{1}{2}\right)\cdots}{4\left(d\cos\left(dx+c\right)^{2}-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*cos(d*x + c) - 4*(b*cos(d*x + c)^2 - b)*log(-cos(d*x + c)) - ((a -
 2*b)*cos(d*x + c)^2 - a + 2*b)*log(1/2*cos(d*x + c) + 1/2) + ((a + 2*b)*co
s(d*x + c)^2 - a - 2*b)*log(-1/2*cos(d*x + c) + 1/2) + 2*b)/(d*cos(d*x + c)
^2 - d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec\left(c+dx\right)\right)\csc^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*csc(c + d*x)**3, x)
```

Giac [B] time = 1.357, size = 228, normalized size = 3.56

$$\frac{2\left(a+2\,b\right)\log\left(\frac{\left|-\cos\left(dx+c\right)+1\right|}{\left|\cos\left(dx+c\right)+1\right|}\right)-8\,b\log\left(\left|-\frac{\cos\left(dx+c\right)-1}{\cos\left(dx+c\right)+1}-1\right|\right)+\frac{\left(a+b-\frac{2\,a\left(\cos\left(dx+c\right)-1\right)}{\cos\left(dx+c\right)+1}-\frac{4\,b\left(\cos\left(dx+c\right)-1\right)}{\cos\left(dx+c\right)+1}\right)\left(\cos\left(dx+c\right)+1\right)}{\cos\left(dx+c\right)-1}-\frac{a\left(\cos\left(dx+c\right)-1\right)}{\cos\left(dx+c\right)+1}+\cdots}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(2*(a + 2*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 8*b*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a + b - 2*a*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) - 4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*(
cos(d*x + c) + 1)/(cos(d*x + c) - 1) - a*(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/d
```

3.166 $\int \csc^5(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=100

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\cot^4(c+dx)}{4d}-\frac{b\cot^2(c+dx)}{d}$$

```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(8*d) - (b*Cot[c + d*x]^2)/d - (b*Cot[c + d*x]
^4)/(4*d) - (3*a*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]*Csc[c +
 d*x]^3)/(4*d) + (b*Log[Tan[c + d*x]])/d
```

Rubi [A] time = 0.124194, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3872, 2834, 2620, 266, 43, 3768, 3770}

$$-\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}-\frac{3a\cot(c+dx)\csc(c+dx)}{8d}-\frac{b\cot^4(c+dx)}{4d}-\frac{b\cot^2(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```



```
[Out] (-3*a*ArcTanh[Cos[c + d*x]])/(8*d) - (b*Cot[c + d*x]^2)/d - (b*Cot[c + d*x]
^4)/(4*d) - (3*a*Cot[c + d*x]*Csc[c + d*x])/(8*d) - (a*Cot[c + d*x]*Csc[c +
 d*x]^3)/(4*d) + (b*Log[Tan[c + d*x]])/d
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
```

```
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^5(c+dx)(a+b\sec(c+dx))\,dx &= -\int (-b-a\cos(c+dx))\csc^5(c+dx)\sec(c+dx)\,dx\\ &= a\int \csc^5(c+dx)\,dx+b\int \csc^5(c+dx)\sec(c+dx)\,dx\\ &= -\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{1}{4}(3a)\int \csc^3(c+dx)\,dx+\frac{b\,\mathrm{Subst}\left(\int \frac{(1+x^2)^2}{x^5}\,dx,x\right.}{d}\\ &= -\frac{3a\cot(c+dx)\csc(c+dx)}{8d}-\frac{a\cot(c+dx)\csc^3(c+dx)}{4d}+\frac{1}{8}(3a)\int \csc(c+dx)\,d\\ &= -\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{3a\cot(c+dx)\csc(c+dx)}{8d}-\frac{a\cot(c+dx)\csc^3(c+d}{4d}\\ &= -\frac{3a\tanh^{-1}(\cos(c+dx))}{8d}-\frac{b\cot^2(c+dx)}{d}-\frac{b\cot^4(c+dx)}{4d}-\frac{3a\cot(c+dx)\csc(}{8d}\end{aligned}$$

Mathematica [A] time = 0.592156, size = 164, normalized size = 1.64

$$-\frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{3a\csc^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d}+\frac{3a\sec^2\left(\frac{1}{2}(c+dx)\right)}{32d}+\frac{3a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}-\frac{3a\,\mathrm{lo}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-3*a*Csc[(c + d*x)/2]^2)/(32*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (3*a*Log
[Cos[(c + d*x)/2]])/(8*d) + (3*a*Log[Sin[(c + d*x)/2]])/(8*d) - (b*(2*Csc[c
 + d*x]^2 + Csc[c + d*x]^4 + 4*Log[Cos[c + d*x]] - 4*Log[Sin[c + d*x]]))/(4
*d) + (3*a*Sec[(c + d*x)/2]^2)/(32*d) + (a*Sec[(c + d*x)/2]^4)/(64*d)
```

Maple [A] time = 0.091, size = 102, normalized size = 1.

$$-\frac{a\cot(dx+c)(\csc(dx+c))^3}{4d}-\frac{3a\cot(dx+c)\csc(dx+c)}{8d}+\frac{3a\ln(\csc(dx+c)-\cot(dx+c))}{8d}-\frac{b}{4d(\sin(dx+c))^4}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5*(a+b*sec(d*x+c)),x)
```

```
[Out] -1/4*a*cot(d*x+c)*csc(d*x+c)^3/d-3/8*a*cot(d*x+c)*csc(d*x+c)/d+3/8/d*a*ln(c
sc(d*x+c)-cot(d*x+c))-1/4/d*b/sin(d*x+c)^4-1/2/d*b/sin(d*x+c)^2+b*ln(tan(d*
x+c))/d
```

Maxima [A] time = 0.959158, size = 149, normalized size = 1.49

$$-\frac{(3a-8b)\log(\cos(dx+c)+1)-(3a+8b)\log(\cos(dx+c)-1)+16b\log(\cos(dx+c))-\frac{2\left(3a\cos(dx+c)^3+4b\cos(dx+\cdots\right.}{\cos(dx+c)^4-2\cos\cdots}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/16*((3*a - 8*b)*log(cos(d*x + c) + 1) - (3*a + 8*b)*log(cos(d*x + c) - 1
) + 16*b*log(cos(d*x + c)) - 2*(3*a*cos(d*x + c)^3 + 4*b*cos(d*x + c)^2 - 5
*a*cos(d*x + c) - 6*b)/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1))/d
```

Fricas [B] time = 1.841, size = 529, normalized size = 5.29

$$\frac{6a\cos(dx+c)^3+8b\cos(dx+c)^2-10a\cos(dx+c)-16\left(b\cos(dx+c)^4-2b\cos(dx+c)^2+b\right)\log(-\cos(dx+c)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(6*a*cos(d*x + c)^3 + 8*b*cos(d*x + c)^2 - 10*a*cos(d*x + c) - 16*(b*c
os(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)*log(-cos(d*x + c)) - ((3*a - 8*b)*c
os(d*x + c)^4 - 2*(3*a - 8*b)*cos(d*x + c)^2 + 3*a - 8*b)*log(1/2*cos(d*x +
 c) + 1/2) + ((3*a + 8*b)*cos(d*x + c)^4 - 2*(3*a + 8*b)*cos(d*x + c)^2 + 3
*a + 8*b)*log(-1/2*cos(d*x + c) + 1/2) - 12*b)/(d*cos(d*x + c)^4 - 2*d*cos(
d*x + c)^2 + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5*(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.39645, size = 359, normalized size = 3.59

$$\frac{4(3a+8b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-64b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\frac{\left(a+b-\frac{8a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{12b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{18a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{48b(\cos(dx+\cdots}{(\cos(dx+c)+\cdots}\right.}{(\cos(dx+c)-1)^2}}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/64*(4*(3*a + 8*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 64*
b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a + b - 8*a*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) - 12*b*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 18*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 48*b*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^2/(cos(d*x + c) - 1)^2 - 8*a*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12*b*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2)/d
```

3.167 $\int \csc^7(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=140

$$-\frac{5a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{5a\cot(c+dx)\csc^3(c+dx)}{24d}-\frac{5a\cot(c+dx)\csc(c+dx)}{16d}-\frac{b}{}$$

```
[Out] (-5*a*ArcTanh[Cos[c + d*x]])/(16*d) - (3*b*Cot[c + d*x]^2)/(2*d) - (3*b*Cot
[c + d*x]^4)/(4*d) - (b*Cot[c + d*x]^6)/(6*d) - (5*a*Cot[c + d*x]*Csc[c + d
*x])/(16*d) - (5*a*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a*Cot[c + d*x]*Cs
c[c + d*x]^5)/(6*d) + (b*Log[Tan[c + d*x]])/d
```

Rubi [A] time = 0.144351, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3872, 2834, 2620, 266, 43, 3768, 3770}

$$-\frac{5a\tanh^{-1}(\cos(c+dx))}{16d}-\frac{a\cot(c+dx)\csc^5(c+dx)}{6d}-\frac{5a\cot(c+dx)\csc^3(c+dx)}{24d}-\frac{5a\cot(c+dx)\csc(c+dx)}{16d}-\frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^7*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-5*a*ArcTanh[Cos[c + d*x]])/(16*d) - (3*b*Cot[c + d*x]^2)/(2*d) - (3*b*Cot
[c + d*x]^4)/(4*d) - (b*Cot[c + d*x]^6)/(6*d) - (5*a*Cot[c + d*x]*Csc[c + d
*x])/(16*d) - (5*a*Cot[c + d*x]*Csc[c + d*x]^3)/(24*d) - (a*Cot[c + d*x]*Cs
c[c + d*x]^5)/(6*d) + (b*Log[Tan[c + d*x]])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2834

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)*((a_
) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[Cos[e + f*x]^p
*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[Cos[e + f*x]^p*(d*Sin[e + f*x])
^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n, p}, x] && IntegerQ[(p - 1)/2]
&& IntegerQ[n] && ((LtQ[p, 0] && NeQ[a^2 - b^2, 0]) || LtQ[0, n, p - 1] ||
LtQ[p + 1, -n, 2*p + 1])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \csc^7(c+dx)(a+b\sec(c+dx))\,dx &= -\int(-b-a\cos(c+dx))\csc^7(c+dx)\sec(c+dx)\,dx \\
&= a\int \csc^7(c+dx)\,dx + b\int \csc^7(c+dx)\sec(c+dx)\,dx \\
&= -\frac{a\cot(c+dx)\csc^5(c+dx)}{6d} + \frac{1}{6}(5a)\int \csc^5(c+dx)\,dx + \frac{b\,\mathrm{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^7}\,dx, x\right.}{d} \\
&= -\frac{5a\cot(c+dx)\csc^3(c+dx)}{24d} - \frac{a\cot(c+dx)\csc^5(c+dx)}{6d} + \frac{1}{8}(5a)\int \csc^3(c+dx \\
&= -\frac{5a\cot(c+dx)\csc(c+dx)}{16d} - \frac{5a\cot(c+dx)\csc^3(c+dx)}{24d} - \frac{a\cot(c+dx)\csc^5(c}{6d} \\
&= -\frac{5a\tanh^{-1}(\cos(c+dx))}{16d} - \frac{3b\cot^2(c+dx)}{2d} - \frac{3b\cot^4(c+dx)}{4d} - \frac{b\cot^6(c+dx)}{6d} -
\end{aligned}$$

Mathematica [A] time = 0.601131, size = 216, normalized size = 1.54

$$-\frac{a\csc^6\left(\frac{1}{2}(c+dx)\right)}{384d} - \frac{a\csc^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{5a\csc^2\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{a\sec^6\left(\frac{1}{2}(c+dx)\right)}{384d} + \frac{a\sec^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{5a\sec^2\left(\frac{1}{2}(c\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^7*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-5*a*Csc[(c + d*x)/2]^2)/(64*d) - (a*Csc[(c + d*x)/2]^4)/(64*d) - (a*Csc[(
c + d*x)/2]^6)/(384*d) - (5*a*Log[Cos[(c + d*x)/2]])/(16*d) + (5*a*Log[Sin[
(c + d*x)/2]])/(16*d) - (b*(6*Csc[c + d*x]^2 + 3*Csc[c + d*x]^4 + 2*Csc[c +
 d*x]^6 + 12*Log[Cos[c + d*x]] - 12*Log[Sin[c + d*x]]))/(12*d) + (5*a*Sec[(
c + d*x)/2]^2)/(64*d) + (a*Sec[(c + d*x)/2]^4)/(64*d) + (a*Sec[(c + d*x)/2]
^6)/(384*d)
```

Maple [A] time = 0.097, size = 136, normalized size = 1.

$$-\frac{a\cot(dx+c)(\csc(dx+c))^5}{6\,d} - \frac{5\,a\cot(dx+c)(\csc(dx+c))^3}{24\,d} - \frac{5\,a\cot(dx+c)\csc(dx+c)}{16\,d} + \frac{5\,a\ln(\csc(dx+c)-\cot}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^7*(a+b*sec(d*x+c)),x)
```

```
[Out] -1/6*a*cot(d*x+c)*csc(d*x+c)^5/d-5/24*a*cot(d*x+c)*csc(d*x+c)^3/d-5/16*a*co
t(d*x+c)*csc(d*x+c)/d+5/16/d*a*ln(csc(d*x+c)-cot(d*x+c))-1/6/d*b/sin(d*x+c)
^6-1/4/d*b/sin(d*x+c)^4-1/2/d*b/sin(d*x+c)^2+b*ln(tan(d*x+c))/d
```

Maxima [A] time = 0.964925, size = 193, normalized size = 1.38

$$-\frac{3\,(5\,a-16\,b)\log\left(\cos\left(dx+c\right)+1\right)-3\,(5\,a+16\,b)\log\left(\cos\left(dx+c\right)-1\right)+96\,b\log\left(\cos\left(dx+c\right)\right)-\frac{2\left(15\,a\cos(dx+c)^{5}+24\right.}{}}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/96*(3*(5*a - 16*b)*log(cos(d*x + c) + 1) - 3*(5*a + 16*b)*log(cos(d*x +
c) - 1) + 96*b*log(cos(d*x + c)) - 2*(15*a*cos(d*x + c)^5 + 24*b*cos(d*x +
c)^4 - 40*a*cos(d*x + c)^3 - 60*b*cos(d*x + c)^2 + 33*a*cos(d*x + c) + 44*b
)/(cos(d*x + c)^6 - 3*cos(d*x + c)^4 + 3*cos(d*x + c)^2 - 1))/d
```

Fricas [B] time = 2.18366, size = 749, normalized size = 5.35

$$\frac{30\,a\cos\left(dx+c\right)^{5}+48\,b\cos\left(dx+c\right)^{4}-80\,a\cos\left(dx+c\right)^{3}-120\,b\cos\left(dx+c\right)^{2}+66\,a\cos\left(dx+c\right)-96\left(b\cos\left(dx+c\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/96*(30*a*cos(d*x + c)^5 + 48*b*cos(d*x + c)^4 - 80*a*cos(d*x + c)^3 - 120
*b*cos(d*x + c)^2 + 66*a*cos(d*x + c) - 96*(b*cos(d*x + c)^6 - 3*b*cos(d*x
+ c)^4 + 3*b*cos(d*x + c)^2 - b)*log(-cos(d*x + c)) - 3*((5*a - 16*b)*cos(d
*x + c)^6 - 3*(5*a - 16*b)*cos(d*x + c)^4 + 3*(5*a - 16*b)*cos(d*x + c)^2 -
 5*a + 16*b)*log(1/2*cos(d*x + c) + 1/2) + 3*((5*a + 16*b)*cos(d*x + c)^6 -
 3*(5*a + 16*b)*cos(d*x + c)^4 + 3*(5*a + 16*b)*cos(d*x + c)^2 - 5*a - 16*b
)*log(-1/2*cos(d*x + c) + 1/2) + 88*b)/(d*cos(d*x + c)^6 - 3*d*cos(d*x + c)
^4 + 3*d*cos(d*x + c)^2 - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**7*(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.42245, size = 482, normalized size = 3.44

$$\frac{12\,(5\,a+16\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-384\,b\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{\left(a+b-\frac{9\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{12\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{45\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{87\,b(\cos(dx+}{(\cos(dx+c)}}{(\cos(dx+c)-}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^7*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/384*(12*(5*a + 16*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) -
384*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + (a + b - 9*a*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*b*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 45*a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 87*b*(cos(d*x + c
) - 1)^2/(cos(d*x + c) + 1)^2 - 110*a*(cos(d*x + c) - 1)^3/(cos(d*x + c) +
1)^3 - 352*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)*(cos(d*x + c) + 1)^
3/(cos(d*x + c) - 1)^3 - 45*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 87*b*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2 - 12*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - a*(cos(d*x +
c) - 1)^3/(cos(d*x + c) + 1)^3 + b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^
3)/d
```

3.168 $\int (a+b\sec(c+dx))\sin^6(c+dx)\,dx$

Optimal. Leaf size=127

$$-\frac{a\sin^5(c+dx)\cos(c+dx)}{6d}-\frac{5a\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{5a\sin(c+dx)\cos(c+dx)}{16d}+\frac{5ax}{16}-\frac{b\sin^5(c+dx)}{5d}-\frac{b\sin}{}$$

```
[Out] (5*a*x)/16 + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d - (5*a*Cos[c
+ d*x]*Sin[c + d*x])/(16*d) - (b*Sin[c + d*x]^3)/(3*d) - (5*a*Cos[c + d*x]*
Sin[c + d*x]^3)/(24*d) - (b*Sin[c + d*x]^5)/(5*d) - (a*Cos[c + d*x]*Sin[c +
 d*x]^5)/(6*d)
```

Rubi [A] time = 0.128401, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3872, 2838, 2592, 302, 206, 2635, 8}

$$-\frac{a\sin^5(c+dx)\cos(c+dx)}{6d}-\frac{5a\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{5a\sin(c+dx)\cos(c+dx)}{16d}+\frac{5ax}{16}-\frac{b\sin^5(c+dx)}{5d}-\frac{b\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^6,x]
```


```
[Out] (5*a*x)/16 + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d - (5*a*Cos[c
+ d*x]*Sin[c + d*x])/(16*d) - (b*Sin[c + d*x]^3)/(3*d) - (5*a*Cos[c + d*x]*
Sin[c + d*x]^3)/(24*d) - (b*Sin[c + d*x]^5)/(5*d) - (a*Cos[c + d*x]*Sin[c +
 d*x]^5)/(6*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))\sin^6(c+dx)\,dx &= -\int (-b-a\cos(c+dx))\sin^5(c+dx)\tan(c+dx)\,dx\\ &= a\int \sin^6(c+dx)\,dx + b\int \sin^5(c+dx)\tan(c+dx)\,dx\\ &= -\frac{a\cos(c+dx)\sin^5(c+dx)}{6d} + \frac{1}{6}(5a)\int \sin^4(c+dx)\,dx + \frac{b\,\mathrm{Subst}\left(\int \frac{x^6}{1-x^2}\,dx, x, \mathrm{si}\right.}{d}\\ &= -\frac{5a\cos(c+dx)\sin^3(c+dx)}{24d} - \frac{a\cos(c+dx)\sin^5(c+dx)}{6d} + \frac{1}{8}(5a)\int \sin^2(c+dx\\ &= -\frac{b\sin(c+dx)}{d} - \frac{5a\cos(c+dx)\sin(c+dx)}{16d} - \frac{b\sin^3(c+dx)}{3d} - \frac{5a\cos(c+dx)\sin}{24d}\\ &= \frac{5ax}{16} + \frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{b\sin(c+dx)}{d} - \frac{5a\cos(c+dx)\sin(c+dx)}{16d} - \frac{b\sin}{}\end{aligned}$$

Mathematica [A] time = 0.208805, size = 118, normalized size = 0.93

$$\frac{5a(c+dx)}{16d} - \frac{15a\sin(2(c+dx))}{64d} + \frac{3a\sin(4(c+dx))}{64d} - \frac{a\sin(6(c+dx))}{192d} - \frac{b\sin^5(c+dx)}{5d} - \frac{b\sin^3(c+dx)}{3d} - \frac{b\sin(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^6,x]
```

```
[Out] (5*a*(c + d*x))/(16*d) + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d -
 (b*Sin[c + d*x]^3)/(3*d) - (b*Sin[c + d*x]^5)/(5*d) - (15*a*Sin[2*(c + d*x
)])/(64*d) + (3*a*Sin[4*(c + d*x)])/(64*d) - (a*Sin[6*(c + d*x)])/(192*d)
```

Maple [A] time = 0.039, size = 130, normalized size = 1.

$$-\frac{a\cos(dx+c)(\sin(dx+c))^5}{6d} - \frac{5a\cos(dx+c)(\sin(dx+c))^3}{24d} - \frac{5a\cos(dx+c)\sin(dx+c)}{16d} + \frac{5ax}{16} + \frac{5ac}{16d} - \frac{b(\sin(dx}{5d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*sin(d*x+c)^6,x)
```

```
[Out] -1/6*a*cos(d*x+c)*sin(d*x+c)^5/d-5/24*a*cos(d*x+c)*sin(d*x+c)^3/d-5/16*a*co
s(d*x+c)*sin(d*x+c)/d+5/16*a*x+5/16/d*a*c-1/5*b*sin(d*x+c)^5/d-1/3*b*sin(d*
x+c)^3/d-b*sin(d*x+c)/d+1/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.986187, size = 143, normalized size = 1.13

$$\frac{5\left(4\sin\left(2\,dx+2\,c\right)^3+60\,dx+60\,c+9\,\sin\left(4\,dx+4\,c\right)-48\,\sin\left(2\,dx+2\,c\right)\right)a-32\left(6\,\sin\left(dx+c\right)^5+10\,\sin\left(dx+c\right)^3\right.}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/960*(5*(4*sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*si
n(2*d*x + 2*c))*a - 32*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin(d
*x + c) + 1) + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*b)/d
```

Fricas [A] time = 1.98949, size = 290, normalized size = 2.28

$$\frac{75\,adx+120\,b\log\left(\sin\left(dx+c\right)+1\right)-120\,b\log\left(-\sin\left(dx+c\right)+1\right)-\left(40\,a\cos\left(dx+c\right)^5+48\,b\cos\left(dx+c\right)^4-130\,a\,c\right.}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/240*(75*a*d*x + 120*b*log(sin(d*x + c) + 1) - 120*b*log(-sin(d*x + c) + 1
) - (40*a*cos(d*x + c)^5 + 48*b*cos(d*x + c)^4 - 130*a*cos(d*x + c)^3 - 176
*b*cos(d*x + c)^2 + 165*a*cos(d*x + c) + 368*b)*sin(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35849, size = 308, normalized size = 2.43

$$75\,(dx+c)a+240\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-240\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(75\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-240\,b\tan\left(\frac{1}{2}\,dx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/240*(75*(d*x + c)*a + 240*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 240*b*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(75*a*tan(1/2*d*x + 1/2*c)^11 - 240*b*
tan(1/2*d*x + 1/2*c)^11 + 425*a*tan(1/2*d*x + 1/2*c)^9 - 1520*b*tan(1/2*d*x
 + 1/2*c)^9 + 990*a*tan(1/2*d*x + 1/2*c)^7 - 4128*b*tan(1/2*d*x + 1/2*c)^7
- 990*a*tan(1/2*d*x + 1/2*c)^5 - 4128*b*tan(1/2*d*x + 1/2*c)^5 - 425*a*tan(
1/2*d*x + 1/2*c)^3 - 1520*b*tan(1/2*d*x + 1/2*c)^3 - 75*a*tan(1/2*d*x + 1/2
*c) - 240*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.169 $\int (a+b\sec(c+dx))\sin^4(c+dx)\,dx$

Optimal. Leaf size=89

$$-\frac{a\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (3*a*x)/8 + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d - (3*a*Cos[c +
 d*x]*Sin[c + d*x])/(8*d) - (b*Sin[c + d*x]^3)/(3*d) - (a*Cos[c + d*x]*Sin[
c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.110673, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3872, 2838, 2592, 302, 206, 2635, 8}

$$-\frac{a\sin^3(c+dx)\cos(c+dx)}{4d}-\frac{3a\sin(c+dx)\cos(c+dx)}{8d}+\frac{3ax}{8}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^4,x]
```

```
[Out] (3*a*x)/8 + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d - (3*a*Cos[c +
 d*x]*Sin[c + d*x])/(8*d) - (b*Sin[c + d*x]^3)/(3*d) - (a*Cos[c + d*x]*Sin[
c + d*x]^3)/(4*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))\sin^4(c+dx)\,dx &= -\int (-b-a\cos(c+dx))\sin^3(c+dx)\tan(c+dx)\,dx\\ &= a\int \sin^4(c+dx)\,dx + b\int \sin^3(c+dx)\tan(c+dx)\,dx\\ &= -\frac{a\cos(c+dx)\sin^3(c+dx)}{4d}+\frac{1}{4}(3a)\int \sin^2(c+dx)\,dx+\frac{b\,\mathrm{Subst}\left(\int \frac{x^4}{1-x^2}\,dx,x,\mathrm{si}\right.}{d}\\ &= -\frac{3a\cos(c+dx)\sin(c+dx)}{8d}-\frac{a\cos(c+dx)\sin^3(c+dx)}{4d}+\frac{1}{8}(3a)\int 1\,dx+\frac{b\,\mathrm{Sub}}{}\\ &= \frac{3ax}{8}-\frac{b\sin(c+dx)}{d}-\frac{3a\cos(c+dx)\sin(c+dx)}{8d}-\frac{b\sin^3(c+dx)}{3d}-\frac{a\cos(c+dx)}{4}\\ &= \frac{3ax}{8}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{b\sin(c+dx)}{d}-\frac{3a\cos(c+dx)\sin(c+dx)}{8d}-\frac{b\sin}{}\end{aligned}$$

Mathematica [A] time = 0.149137, size = 86, normalized size = 0.97

$$\frac{3a(c+dx)}{8d}-\frac{a\sin(2(c+dx))}{4d}+\frac{a\sin(4(c+dx))}{32d}-\frac{b\sin^3(c+dx)}{3d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^4,x]
```

```
[Out] (3*a*(c + d*x))/(8*d) + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d -
(b*Sin[c + d*x]^3)/(3*d) - (a*Sin[2*(c + d*x)])/(4*d) + (a*Sin[4*(c + d*x)]
)/(32*d)
```

Maple [A] time = 0.035, size = 96, normalized size = 1.1

$$-\frac{a\cos(dx+c)(\sin(dx+c))^3}{4d}-\frac{3a\cos(dx+c)\sin(dx+c)}{8d}+\frac{3ax}{8}+\frac{3ac}{8d}-\frac{b(\sin(dx+c))^3}{3d}-\frac{b\sin(dx+c)}{d}+\frac{b\ln(\mathrm{se}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*sin(d*x+c)^4,x)
```

```
[Out] -1/4*a*cos(d*x+c)*sin(d*x+c)^3/d-3/8*a*cos(d*x+c)*sin(d*x+c)/d+3/8*a*x+3/8/
d*a*c-1/3*b*sin(d*x+c)^3/d-b*sin(d*x+c)/d+1/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.957505, size = 109, normalized size = 1.22

$$\frac{3\,(12\,dx+12\,c+\sin(4\,dx+4\,c)-8\,\sin(2\,dx+2\,c))a-16\left(2\,\sin(dx+c)^3-3\,\log(\sin(dx+c)+1)+3\,\log(\sin(dx\right.}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*a - 16*(2*s
in(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d
*x + c))*b)/d
```

Fricas [A] time = 1.81185, size = 217, normalized size = 2.44

$$\frac{9\,adx+12\,b\log(\sin(dx+c)+1)-12\,b\log(-\sin(dx+c)+1)+\left(6\,a\cos(dx+c)^3+8\,b\cos(dx+c)^2-15\,a\cos(dx\right.}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(9*a*d*x + 12*b*log(sin(d*x + c) + 1) - 12*b*log(-sin(d*x + c) + 1) +
(6*a*cos(d*x + c)^3 + 8*b*cos(d*x + c)^2 - 15*a*cos(d*x + c) - 32*b)*sin(d*
x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31595, size = 232, normalized size = 2.61

$$\frac{9\,(dx+c)a+24\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-24\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(9\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-24\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+\right.}{}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/24*(9*(d*x + c)*a + 24*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*b*log(ab
s(tan(1/2*d*x + 1/2*c) - 1)) + 2*(9*a*tan(1/2*d*x + 1/2*c)^7 - 24*b*tan(1/2
*d*x + 1/2*c)^7 + 33*a*tan(1/2*d*x + 1/2*c)^5 - 104*b*tan(1/2*d*x + 1/2*c)^
```

```
5 - 33*a*tan(1/2*d*x + 1/2*c)^3 - 104*b*tan(1/2*d*x + 1/2*c)^3 - 9*a*tan(1/
2*d*x + 1/2*c) - 24*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)
/d
```

3.170 $\int (a + b\sec(c + dx))\sin^2(c + dx)\,dx$

Optimal. Leaf size=51

$$-\frac{a\sin(c+dx)\cos(c+dx)}{2d} + \frac{ax}{2} - \frac{b\sin(c+dx)}{d} + \frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (a*x)/2 + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d - (a*Cos[c + d*x
]*Sin[c + d*x])/(2*d)
```

Rubi [A] time = 0.0825193, antiderivative size = 51, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3872, 2838, 2592, 321, 206, 2635, 8}

$$-\frac{a\sin(c+dx)\cos(c+dx)}{2d} + \frac{ax}{2} - \frac{b\sin(c+dx)}{d} + \frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*Sin[c + d*x]^2,x]
```

```
[Out] (a*x)/2 + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d - (a*Cos[c + d*x
]*Sin[c + d*x])/(2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))\sin^2(c+dx)\,dx &= -\int (-b-a\cos(c+dx))\sin(c+dx)\tan(c+dx)\,dx\\ &= a\int \sin^2(c+dx)\,dx + b\int \sin(c+dx)\tan(c+dx)\,dx\\ &= -\frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}a\int 1\,dx+\frac{b\,\mathrm{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d}\\ &= \frac{ax}{2}-\frac{b\sin(c+dx)}{d}-\frac{a\cos(c+dx)\sin(c+dx)}{2d}+\frac{b\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sin(c+d\right.}{d}\\ &= \frac{ax}{2}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}-\frac{b\sin(c+dx)}{d}-\frac{a\cos(c+dx)\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0591425, size = 54, normalized size = 1.06

$$\frac{a(c+dx)}{2d}-\frac{a\sin(2(c+dx))}{4d}-\frac{b\sin(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*Sin[c + d*x]^2,x]
```

```
[Out] (a*(c + d*x))/(2*d) + (b*ArcTanh[Sin[c + d*x]])/d - (b*Sin[c + d*x])/d - (a
*Sin[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.033, size = 62, normalized size = 1.2

$$-\frac{a\cos(dx+c)\sin(dx+c)}{2d}+\frac{ax}{2}+\frac{ac}{2d}+\frac{b\ln(\sec(dx+c)+\tan(dx+c))}{d}-\frac{b\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*sin(d*x+c)^2,x)
```

```
[Out] -1/2*a*cos(d*x+c)*sin(d*x+c)/d+1/2*a*x+1/2/d*a*c+1/d*b*ln(sec(d*x+c)+tan(d*
x+c))-b*sin(d*x+c)/d
```

Maxima [A] time = 0.947176, size = 80, normalized size = 1.57

$$\frac{(2\,dx + 2\,c - \sin(2\,dx + 2\,c))a + 2\,b(\log(\sin(dx + c) + 1) - \log(\sin(dx + c) - 1) - 2\,\sin(dx + c))}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a + 2*b*(log(sin(d*x + c) + 1) - log(
sin(d*x + c) - 1) - 2*sin(d*x + c)))/d
```

Fricas [A] time = 1.80418, size = 143, normalized size = 2.8

$$\frac{adx + b\log(\sin(dx + c) + 1) - b\log(-\sin(dx + c) + 1) - (a\cos(dx + c) + 2\,b)\sin(dx + c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(a*d*x + b*log(sin(d*x + c) + 1) - b*log(-sin(d*x + c) + 1) - (a*cos(d*
x + c) + 2*b)*sin(d*x + c))/d
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sec(c + dx))\sin^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))*sin(c + d*x)**2, x)
```

Giac [B] time = 1.35116, size = 154, normalized size = 3.02

$$\frac{(dx + c)a + 2\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 2\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 2\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - a\tan\left(\frac{1}{2}\right.}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*((d*x + c)*a + 2*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*b*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) + 2*(a*tan(1/2*d*x + 1/2*c)^3 - 2*b*tan(1/2*d*x + 1
/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - 2*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
 1/2*c)^2 + 1)^2)/d
```

3.171 $\int \csc^2(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=37

$$-\frac{a\cot(c+dx)}{d}-\frac{b\csc(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (b*Csc[c + d*x])/d
```

Rubi [A] time = 0.0959633, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.368, Rules used = {3872, 2838, 2621, 321, 207, 3767, 8}

$$-\frac{a\cot(c+dx)}{d}-\frac{b\csc(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (b*Csc[c + d*x])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \csc^2(c+dx)(a+b\sec(c+dx))\,dx &= -\int (-b-a\cos(c+dx))\csc^2(c+dx)\sec(c+dx)\,dx \\
&= a\int \csc^2(c+dx)\,dx + b\int \csc^2(c+dx)\sec(c+dx)\,dx \\
&= -\frac{a\,\text{Subst}\left(\int 1\,dx, x, \cot(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} \\
&= -\frac{a\cot(c+dx)}{d} - \frac{b\csc(c+dx)}{d} - \frac{b\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} \\
&= \frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{b\csc(c+dx)}{d}
\end{aligned}$$

Mathematica [C] time = 0.0269862, size = 41, normalized size = 1.11

$$-\frac{b\csc(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\sin^2(c+dx)\right)}{d} - \frac{a\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Sec[c + d*x]),x]
```

```
[Out] -((a*Cot[c + d*x])/d) - (b*Csc[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin
[c + d*x]^2])/d
```

Maple [A] time = 0.034, size = 47, normalized size = 1.3

$$-\frac{a\cot(dx+c)}{d} - \frac{b}{d\sin(dx+c)} + \frac{b\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+b*sec(d*x+c)),x)
```

```
[Out] -a*cot(d*x+c)/d-1/d*b/sin(d*x+c)+1/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.952336, size = 68, normalized size = 1.84

$$-\frac{b\left(\frac{2}{\sin(dx+c)} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + \frac{2a}{\tan(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/2*(b*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) +
2*a/tan(d*x + c))/d
```

Fricas [A] time = 1.73875, size = 170, normalized size = 4.59

$$\frac{b\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-b\log\left(-\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)-2\,a\cos\left(dx+c\right)-2\,b}{2\,d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(b*log(sin(d*x + c) + 1)*sin(d*x + c) - b*log(-sin(d*x + c) + 1)*sin(d*
x + c) - 2*a*cos(d*x + c) - 2*b)/(d*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec\left(c+dx\right)\right)\csc^2\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((a + b*sec(c + d*x))*csc(c + d*x)**2, x)
```

Giac [B] time = 1.28673, size = 104, normalized size = 2.81

$$\frac{2\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\frac{a+b}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*b*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) + a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c) - (a + b)/tan(1
/2*d*x + 1/2*c))/d
```

3.172 $\int \csc^4(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=69

$$-\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}-\frac{b\csc(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (a*Cot[c + d*x]^3)/(3*d)
 - (b*Csc[c + d*x])/d - (b*Csc[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.104665, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3872, 2838, 2621, 302, 207, 3767}

$$-\frac{a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{b\csc^3(c+dx)}{3d}-\frac{b\csc(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (a*Cot[c + d*x]^3)/(3*d)
 - (b*Csc[c + d*x])/d - (b*Csc[c + d*x]^3)/(3*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \csc^4(c+dx)(a+b\sec(c+dx))\,dx &= -\int (-b-a\cos(c+dx))\csc^4(c+dx)\sec(c+dx)\,dx \\
&= a\int \csc^4(c+dx)\,dx + b\int \csc^4(c+dx)\sec(c+dx)\,dx \\
&= -\frac{a\,\mathrm{Subst}\left(\int \left(1+x^2\right)dx, x, \cot(c+dx)\right)}{d} - \frac{b\,\mathrm{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \csc(c+dx)\right)}{d} \\
&= -\frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{b\,\mathrm{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \csc(c+dx)\right)}{d} \\
&= -\frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{b\csc(c+dx)}{d} - \frac{b\csc^3(c+dx)}{3d} - \frac{b\,\mathrm{Subst}\left(\int \frac{}{}\right.}{} \\
&= \frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{a\cot^3(c+dx)}{3d} - \frac{b\csc(c+dx)}{d} - \frac{b\csc^3(}{3}
\end{aligned}$$

Mathematica [C] time = 0.0244176, size = 69, normalized size = 1.

$$-\frac{b\csc^3(c+dx)\mathrm{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\sin^2(c+dx)\right)}{3d} - \frac{2a\cot(c+dx)}{3d} - \frac{a\cot(c+dx)\csc^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*a*Cot[c + d*x])/(3*d) - (a*Cot[c + d*x]*Csc[c + d*x]^2)/(3*d) - (b*Csc[
c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Sin[c + d*x]^2])/(3*d)
```

Maple [A] time = 0.041, size = 81, normalized size = 1.2

$$-\frac{2\,a\cot(dx+c)}{3\,d} - \frac{a\cot(dx+c)\,(\csc(dx+c))^2}{3\,d} - \frac{b}{3\,d\,(\sin(dx+c))^3} - \frac{b}{d\sin(dx+c)} + \frac{b\ln(\sec(dx+c)+\tan(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*sec(d*x+c)),x)
```

```
[Out] -2/3*a*cot(d*x+c)/d-1/3/d*a*cot(d*x+c)*csc(d*x+c)^2-1/3/d*b/sin(d*x+c)^3-1/
d*b/sin(d*x+c)+1/d*b*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.977636, size = 103, normalized size = 1.49

$$-\frac{b\left(\frac{2\left(3\,\sin(dx+c)^2+1\right)}{\sin(dx+c)^3} - 3\,\log(\sin(dx+c)+1) + 3\,\log(\sin(dx+c)-1)\right) + \frac{2\left(3\,\tan(dx+c)^2+1\right)a}{\tan(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(b*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1)
+ 3*log(sin(d*x + c) - 1)) + 2*(3*tan(d*x + c)^2 + 1)*a/tan(d*x + c)^3)/d
```

Fricas [A] time = 1.73392, size = 319, normalized size = 4.62

$$-\frac{4\,a\cos\left(dx+c\right)^{3}+6\,b\cos\left(dx+c\right)^{2}-3\left(b\cos\left(dx+c\right)^{2}-b\right)\log\left(\sin\left(dx+c\right)+1\right)\sin\left(dx+c\right)+3\left(b\cos\left(dx+c\right)^{2}-b\right)}{6\left(d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/6*(4*a*cos(d*x + c)^3 + 6*b*cos(d*x + c)^2 - 3*(b*cos(d*x + c)^2 - b)*lo
g(sin(d*x + c) + 1)*sin(d*x + c) + 3*(b*cos(d*x + c)^2 - b)*log(-sin(d*x +
c) + 1)*sin(d*x + c) - 6*a*cos(d*x + c) - 8*b)/((d*cos(d*x + c)^2 - d)*sin(
d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35754, size = 180, normalized size = 2.61

$$\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+24\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-24\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+9\,a\,\mathrm{ta}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1/2*d*x + 1/2*c)^3 + 24*b*log(abs(ta
n(1/2*d*x + 1/2*c) + 1)) - 24*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 9*a*ta
n(1/2*d*x + 1/2*c) - 15*b*tan(1/2*d*x + 1/2*c) - (9*a*tan(1/2*d*x + 1/2*c)^
2 + 15*b*tan(1/2*d*x + 1/2*c)^2 + a + b)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.173 $\int \csc^6(c+dx)(a+b\sec(c+dx))\,dx$

Optimal. Leaf size=101

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{2a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{b\csc^5(c+dx)}{5d}-\frac{b\csc^3(c+dx)}{3d}-\frac{b\csc(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+}{d}$$

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (2*a*Cot[c + d*x]^3)/(3*
d) - (a*Cot[c + d*x]^5)/(5*d) - (b*Csc[c + d*x])/d - (b*Csc[c + d*x]^3)/(3*
d) - (b*Csc[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.110891, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3872, 2838, 2621, 302, 207, 3767}

$$-\frac{a\cot^5(c+dx)}{5d}-\frac{2a\cot^3(c+dx)}{3d}-\frac{a\cot(c+dx)}{d}-\frac{b\csc^5(c+dx)}{5d}-\frac{b\csc^3(c+dx)}{3d}-\frac{b\csc(c+dx)}{d}+\frac{b\tanh^{-1}(\sin(c+}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*ArcTanh[Sin[c + d*x]])/d - (a*Cot[c + d*x])/d - (2*a*Cot[c + d*x]^3)/(3*
d) - (a*Cot[c + d*x]^5)/(5*d) - (b*Csc[c + d*x])/d - (b*Csc[c + d*x]^3)/(3*
d) - (b*Csc[c + d*x]^5)/(5*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \csc^6(c+dx)(a+b\sec(c+dx))\,dx = -\int (-b-a\cos(c+dx))\csc^6(c+dx)\sec(c+dx)\,dx$$
$$= a\int \csc^6(c+dx)\,dx + b\int \csc^6(c+dx)\sec(c+dx)\,dx$$
$$= -\frac{a\,\text{Subst}\left(\int \left(1+2x^2+x^4\right)dx, x, \cot(c+dx)\right)}{d} - \frac{b\,\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx, x, \csc(c$$
$$= -\frac{a\cot(c+dx)}{d} - \frac{2a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} - \frac{b\,\text{Subst}\left(\int \left(1+x^2+x^4\right.\right.}{}$$
$$= -\frac{a\cot(c+dx)}{d} - \frac{2a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} - \frac{b\csc(c+dx)}{d} - \frac{b\csc^3(c}{3d}$$
$$= \frac{b\tanh^{-1}(\sin(c+dx))}{d} - \frac{a\cot(c+dx)}{d} - \frac{2a\cot^3(c+dx)}{3d} - \frac{a\cot^5(c+dx)}{5d} - \frac{b}{}$$

Mathematica [C] time = 0.0257432, size = 91, normalized size = 0.9

$$-\frac{b\csc^5(c+dx)\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\sin^2(c+dx)\right)}{5d} - \frac{8a\cot(c+dx)}{15d} - \frac{a\cot(c+dx)\csc^4(c+dx)}{5d} - \frac{4a\,\text{cc}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Sec[c + d*x]),x]
```

```
[Out] (-8*a*Cot[c + d*x])/(15*d) - (4*a*Cot[c + d*x]*Csc[c + d*x]^2)/(15*d) - (a*
Cot[c + d*x]*Csc[c + d*x]^4)/(5*d) - (b*Csc[c + d*x]^5*Hypergeometric2F1[-5
/2, 1, -3/2, Sin[c + d*x]^2])/(5*d)
```

Maple [A] time = 0.042, size = 115, normalized size = 1.1

$$-\frac{8a\cot(dx+c)}{15d} - \frac{a\cot(dx+c)(\csc(dx+c))^4}{5d} - \frac{4a\cot(dx+c)(\csc(dx+c))^2}{15d} - \frac{b}{5d(\sin(dx+c))^5} - \frac{b}{3d(\sin(dx-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+b*sec(d*x+c)),x)
```

```
[Out] -8/15*a*cot(d*x+c)/d-1/5/d*a*cot(d*x+c)*csc(d*x+c)^4-4/15/d*a*cot(d*x+c)*cs
c(d*x+c)^2-1/5/d*b/sin(d*x+c)^5-1/3/d*b/sin(d*x+c)^3-1/d*b/sin(d*x+c)+1/d*b
*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.969416, size = 130, normalized size = 1.29

$$-\frac{b\left(\frac{2\left(15\,\sin\left(dx+c\right)^{4}+5\,\sin\left(dx+c\right)^{2}+3\right)}{\sin\left(dx+c\right)^{5}}-15\,\log\left(\sin\left(dx+c\right)+1\right)+15\,\log\left(\sin\left(dx+c\right)-1\right)\right)+\frac{2\left(15\,\tan\left(dx+c\right)^{4}+10\,\tan\left(dx+c\right)^{2}+3\right)a}{\tan\left(dx+c\right)^{5}}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/30*(b*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)/sin(d*x + c)^5 - 15*
log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 2*(15*tan(d*x + c)^4 +
10*tan(d*x + c)^2 + 3)*a/tan(d*x + c)^5)/d
```

Fricas [A] time = 1.7965, size = 473, normalized size = 4.68

$$-\frac{16\,a\cos\left(dx+c\right)^{5}+30\,b\cos\left(dx+c\right)^{4}-40\,a\cos\left(dx+c\right)^{3}-70\,b\cos\left(dx+c\right)^{2}-15\,\left(b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+\right.}{30\,\left(d\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/30*(16*a*cos(d*x + c)^5 + 30*b*cos(d*x + c)^4 - 40*a*cos(d*x + c)^3 - 70
*b*cos(d*x + c)^2 - 15*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)*log(sin(
d*x + c) + 1)*sin(d*x + c) + 15*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + b)
*log(-sin(d*x + c) + 1)*sin(d*x + c) + 30*a*cos(d*x + c) + 46*b)/((d*cos(d*
x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.3389, size = 262, normalized size = 2.59

$$\frac{3\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+25\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-35\,b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+480\,b\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/480*(3*a*tan(1/2*d*x + 1/2*c)^5 - 3*b*tan(1/2*d*x + 1/2*c)^5 + 25*a*tan(1
/2*d*x + 1/2*c)^3 - 35*b*tan(1/2*d*x + 1/2*c)^3 + 480*b*log(abs(tan(1/2*d*x
```

```
 + 1/2*c) + 1)) - 480*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 150*a*tan(1/2*
d*x + 1/2*c) - 330*b*tan(1/2*d*x + 1/2*c) - (150*a*tan(1/2*d*x + 1/2*c)^4 +
 330*b*tan(1/2*d*x + 1/2*c)^4 + 25*a*tan(1/2*d*x + 1/2*c)^2 + 35*b*tan(1/2*
d*x + 1/2*c)^2 + 3*a + 3*b)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.174 $\int (a+b\sec(c+dx))^2\sin^5(c+dx)\,dx$

Optimal. Leaf size=124

$$\frac{(2a^2-b^2)\cos^3(c+dx)}{3d}-\frac{(a^2-2b^2)\cos(c+dx)}{d}-\frac{a^2\cos^5(c+dx)}{5d}-\frac{ab\cos^4(c+dx)}{2d}+\frac{2ab\cos^2(c+dx)}{d}-\frac{2ab\log(\cos(}{d}$$

```
[Out] -(((a^2 - 2*b^2)*Cos[c + d*x])/d) + (2*a*b*Cos[c + d*x]^2)/d + ((2*a^2 - b^
2)*Cos[c + d*x]^3)/(3*d) - (a*b*Cos[c + d*x]^4)/(2*d) - (a^2*Cos[c + d*x]^5
)/(5*d) - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Sec[c + d*x])/d
```

Rubi [A] time = 0.195642, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2837, 12, 948}

$$\frac{(2a^2-b^2)\cos^3(c+dx)}{3d}-\frac{(a^2-2b^2)\cos(c+dx)}{d}-\frac{a^2\cos^5(c+dx)}{5d}-\frac{ab\cos^4(c+dx)}{2d}+\frac{2ab\cos^2(c+dx)}{d}-\frac{2ab\log(\cos(}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^5,x]
```



```
[Out] -(((a^2 - 2*b^2)*Cos[c + d*x])/d) + (2*a*b*Cos[c + d*x]^2)/d + ((2*a^2 - b^
2)*Cos[c + d*x]^3)/(3*d) - (a*b*Cos[c + d*x]^4)/(2*d) - (a^2*Cos[c + d*x]^5
)/(5*d) - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Sec[c + d*x])/d
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\sin^5(c+dx)\,dx = \int (-b-a\cos(c+dx))^2\sin^3(c+dx)\tan^2(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^2(-b+x)^2(a^2-x^2)^2}{x^2}dx, x, -a\cos(c+dx)\right)}{a^5 d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-b+x)^2(a^2-x^2)^2}{x^2}dx, x, -a\cos(c+dx)\right)}{a^3 d}$$
$$= \frac{\text{Subst}\left(\int \left(a^4\left(1-\frac{2b^2}{a^2}\right)+\frac{a^4b^2}{x^2}-\frac{2a^4b}{x}+4a^2bx-\left(2a^2-b^2\right)x^2-2bx^3+x^4\right)dx\right.}{a^3 d}$$
$$= -\frac{\left(a^2-2b^2\right)\cos(c+dx)}{d}+\frac{2ab\cos^2(c+dx)}{d}+\frac{\left(2a^2-b^2\right)\cos^3(c+dx)}{3d}-\frac{ab\,c}{}$$

Mathematica [A] time = 0.36251, size = 112, normalized size = 0.9

$$-\frac{30\left(5a^2-14b^2\right)\cos(c+dx)-25a^2\cos(3(c+dx))+3a^2\cos(5(c+dx))-180ab\cos(2(c+dx))+15ab\cos(4(c+dx))}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^5,x]
```

```
[Out] -(30*(5*a^2 - 14*b^2)*Cos[c + d*x] - 180*a*b*Cos[2*(c + d*x)] - 25*a^2*Cos[
3*(c + d*x)] + 20*b^2*Cos[3*(c + d*x)] + 15*a*b*Cos[4*(c + d*x)] + 3*a^2*Co
s[5*(c + d*x)] + 480*a*b*Log[Cos[c + d*x]] - 240*b^2*Sec[c + d*x])/(240*d)
```

Maple [A] time = 0.041, size = 184, normalized size = 1.5

$$-\frac{8a^2\cos(dx+c)}{15d}-\frac{a^2\cos(dx+c)(\sin(dx+c))^4}{5d}-\frac{4a^2\cos(dx+c)(\sin(dx+c))^2}{15d}-\frac{ab(\sin(dx+c))^4}{2d}-\frac{ab(\sin(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*sin(d*x+c)^5,x)
```

```
[Out] -8/15*a^2*cos(d*x+c)/d-1/5/d*a^2*cos(d*x+c)*sin(d*x+c)^4-4/15/d*a^2*cos(d*x
+c)*sin(d*x+c)^2-1/2/d*a*b*sin(d*x+c)^4-1/d*a*b*sin(d*x+c)^2-2*a*b*ln(cos(d
*x+c))/d+1/d*b^2*sin(d*x+c)^6/cos(d*x+c)+8/3/d*b^2*cos(d*x+c)+1/d*b^2*sin(d
*x+c)^4*cos(d*x+c)+4/3/d*b^2*cos(d*x+c)*sin(d*x+c)^2
```

Maxima [A] time = 0.968799, size = 142, normalized size = 1.15

$$-\frac{6a^2\cos(dx+c)^5+15ab\cos(dx+c)^4-60ab\cos(dx+c)^2-10\left(2a^2-b^2\right)\cos(dx+c)^3+60ab\log(\cos(dx+c))-}{30d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/30*(6*a^2*cos(d*x + c)^5 + 15*a*b*cos(d*x + c)^4 - 60*a*b*cos(d*x + c)^2
 - 10*(2*a^2 - b^2)*cos(d*x + c)^3 + 60*a*b*log(cos(d*x + c)) + 30*(a^2 - 2
*b^2)*cos(d*x + c) - 30*b^2/cos(d*x + c))/d
```

Fricas [A] time = 1.80296, size = 328, normalized size = 2.65

$$-\frac{48\,a^2\cos(dx+c)^6 + 120\,ab\cos(dx+c)^5 - 480\,ab\cos(dx+c)^3 - 80\left(2\,a^2-b^2\right)\cos(dx+c)^4 + 480\,ab\cos(dx+c)\log(}{240\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/240*(48*a^2*cos(d*x + c)^6 + 120*a*b*cos(d*x + c)^5 - 480*a*b*cos(d*x +
c)^3 - 80*(2*a^2 - b^2)*cos(d*x + c)^4 + 480*a*b*cos(d*x + c)*log(-cos(d*x
+ c)) + 195*a*b*cos(d*x + c) + 240*(a^2 - 2*b^2)*cos(d*x + c)^2 - 240*b^2)/
(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*sin(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.39652, size = 564, normalized size = 4.55

$$\frac{60\,ab\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right|\right) - 60\,ab\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right) + \frac{60\left(ab+b^2+\frac{ab(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1} + \frac{32\,a^2+137\,ab-100\,b^2-\frac{160\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-805}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/30*(60*a*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) - 60*a*b*
log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + 60*(a*b + b^2 + a*b*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 1) + (32*a^2 + 137*a*b - 100*b^2 - 160*a^2*(cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) - 805*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 440*b^2*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 320*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c
) + 1)^2 + 1970*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 640*b^2*(co
s(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1970*a*b*(cos(d*x + c) - 1)^3/(cos
(d*x + c) + 1)^3 + 360*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 805*
a*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 60*b^2*(cos(d*x + c) - 1)^4
/(cos(d*x + c) + 1)^4 - 137*a*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5)/
((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5)/d
```

3.175 $\int (a+b\sec(c+dx))^2\sin^3(c+dx)\,dx$

Optimal. Leaf size=80

$$-\frac{(a^2-b^2)\cos(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{ab\cos^2(c+dx)}{d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

```
[Out] -(((a^2 - b^2)*Cos[c + d*x])/d) + (a*b*Cos[c + d*x]^2)/d + (a^2*Cos[c + d*x
]^3)/(3*d) - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Sec[c + d*x])/d
```

Rubi [A] time = 0.144795, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2837, 12, 894}

$$-\frac{(a^2-b^2)\cos(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}+\frac{ab\cos^2(c+dx)}{d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^3,x]
```

```
[Out] -(((a^2 - b^2)*Cos[c + d*x])/d) + (a*b*Cos[c + d*x]^2)/d + (a^2*Cos[c + d*x
]^3)/(3*d) - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Sec[c + d*x])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\sin^3(c+dx)\,dx = \int (-b-a\cos(c+dx))^2\sin(c+dx)\tan^2(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^2(-b+x)^2(a^2-x^2)}{x^2}\,dx, x, -a\cos(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{(-b+x)^2(a^2-x^2)}{x^2}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{b^2}{a^2}\right)+\frac{a^2b^2}{x^2}-\frac{2a^2b}{x}+2bx-x^2\right)dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= -\frac{\left(a^2-b^2\right)\cos(c+dx)}{d}+\frac{ab\cos^2(c+dx)}{d}+\frac{a^2\cos^3(c+dx)}{3d}-\frac{2ab\log(\cos(c+d$$

Mathematica [A] time = 0.176215, size = 72, normalized size = 0.9

$$\frac{\left(12b^2-9a^2\right)\cos(c+dx)+a^2\cos(3(c+dx))+6ab\cos(2(c+dx))-24ab\log(\cos(c+dx))+12b^2\sec(c+dx)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^3,x]
```

```
[Out] ((-9*a^2 + 12*b^2)*Cos[c + d*x] + 6*a*b*Cos[2*(c + d*x)] + a^2*Cos[3*(c + d
*x)] - 24*a*b*Log[Cos[c + d*x]] + 12*b^2*Sec[c + d*x])/(12*d)
```

Maple [A] time = 0.039, size = 125, normalized size = 1.6

$$-\frac{a^2\cos(dx+c)(\sin(dx+c))^2}{3d}-\frac{2a^2\cos(dx+c)}{3d}-\frac{ab(\sin(dx+c))^2}{d}-2\frac{ab\ln(\cos(dx+c))}{d}+\frac{b^2(\sin(dx+c))^4}{d\cos(dx+c)}+\frac{b^2 c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*sin(d*x+c)^3,x)
```

```
[Out] -1/3/d*a^2*cos(d*x+c)*sin(d*x+c)^2-2/3*a^2*cos(d*x+c)/d-1/d*a*b*sin(d*x+c)^
2-2*a*b*ln(cos(d*x+c))/d+1/d*b^2*sin(d*x+c)^4/cos(d*x+c)+1/d*b^2*cos(d*x+c)
*sin(d*x+c)^2+2/d*b^2*cos(d*x+c)
```

Maxima [A] time = 0.962144, size = 96, normalized size = 1.2

$$\frac{a^2\cos(dx+c)^3+3ab\cos(dx+c)^2-6ab\log(\cos(dx+c))-3\left(a^2-b^2\right)\cos(dx+c)+\frac{3b^2}{\cos(dx+c)}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/3*(a^2*cos(d*x + c)^3 + 3*a*b*cos(d*x + c)^2 - 6*a*b*log(cos(d*x + c)) -
3*(a^2 - b^2)*cos(d*x + c) + 3*b^2/cos(d*x + c))/d
```

Fricas [A] time = 1.81172, size = 228, normalized size = 2.85

$$\frac{2\,a^2\cos\left(dx+c\right)^4+6\,ab\cos\left(dx+c\right)^3-12\,ab\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)-3\,ab\cos\left(dx+c\right)-6\left(a^2-b^2\right)\cos\left(dx\right.}{6\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^2*cos(d*x + c)^4 + 6*a*b*cos(d*x + c)^3 - 12*a*b*cos(d*x + c)*log(
-cos(d*x + c)) - 3*a*b*cos(d*x + c) - 6*(a^2 - b^2)*cos(d*x + c)^2 + 6*b^2)
/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*sin(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.3138, size = 135, normalized size = 1.69

$$-\frac{2\,ab\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{b^2}{d\cos\left(dx+c\right)}+\frac{a^2d^5\cos\left(dx+c\right)^3+3\,abd^5\cos\left(dx+c\right)^2-3\,a^2d^5\cos\left(dx+c\right)+3\,b^2d^5\cos\left(d\right.}{3\,d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -2*a*b*log(abs(cos(d*x + c))/abs(d))/d + b^2/(d*cos(d*x + c)) + 1/3*(a^2*d^
5*cos(d*x + c)^3 + 3*a*b*d^5*cos(d*x + c)^2 - 3*a^2*d^5*cos(d*x + c) + 3*b^
2*d^5*cos(d*x + c))/d^6
```

3.176 $\int (a+b\sec(c+dx))^2\sin(c+dx)\,dx$

Optimal. Leaf size=42

$$-\frac{a^2\cos(c+dx)}{d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

```
[Out] -((a^2*Cos[c + d*x])/d) - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Sec[c + d*x])/
d
```

Rubi [A] time = 0.0775606, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3872, 2833, 12, 43}

$$-\frac{a^2\cos(c+dx)}{d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x],x]
```

```
[Out] -((a^2*Cos[c + d*x])/d) - (2*a*b*Log[Cos[c + d*x]])/d + (b^2*Sec[c + d*x])/
d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\sin(c+dx)\,dx = \int (-b-a\cos(c+dx))^2\sec(c+dx)\tan(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^2(-b+x)^2}{x^2}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{(-b+x)^2}{x^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a\,\text{Subst}\left(\int \left(1+\frac{b^2}{x^2}-\frac{2b}{x}\right)dx, x, -a\cos(c+dx)\right)}{d}$$
$$= -\frac{a^2\cos(c+dx)}{d} - \frac{2ab\log(\cos(c+dx))}{d} + \frac{b^2\sec(c+dx)}{d}$$

Mathematica [A] time = 0.0565242, size = 37, normalized size = 0.88

$$\frac{b(b\sec(c+dx)-2a\log(\cos(c+dx)))-a^2\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x],x]

[Out] (-(a^2*Cos[c + d*x]) + b*(-2*a*Log[Cos[c + d*x]] + b*Sec[c + d*x]))/d
```

Maple [A] time = 0.022, size = 45, normalized size = 1.1

$$\frac{b^2\sec(dx+c)}{d}+2\frac{ab\ln(\sec(dx+c))}{d}-\frac{a^2}{d\sec(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*sin(d*x+c),x)

[Out] b^2*sec(d*x+c)/d+2/d*a*b*ln(sec(d*x+c))-1/d*a^2/sec(d*x+c)
```

Maxima [A] time = 0.940664, size = 54, normalized size = 1.29

$$-\frac{a^2\cos(dx+c)+2\,ab\log(\cos(dx+c))-\frac{b^2}{\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c),x, algorithm="maxima")

[Out] -(a^2*cos(d*x + c) + 2*a*b*log(cos(d*x + c)) - b^2/cos(d*x + c))/d
```

Fricas [A] time = 1.75211, size = 116, normalized size = 2.76

$$-\frac{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)\log(-\cos(dx+c))-b^2}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c)*log(-cos(d*x + c)) - b^2)/(d*cos(
d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b \sec(c + dx))^2 \sin(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*sin(d*x+c),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sin(c + d*x), x)
```

Giac [A] time = 1.28743, size = 68, normalized size = 1.62

$$-\frac{a^2 \cos(dx + c)}{d} - \frac{2\, ab \log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d} + \frac{b^2}{d \cos(dx + c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -a^2*cos(d*x + c)/d - 2*a*b*log(abs(cos(d*x + c))/abs(d))/d + b^2/(d*cos(d*
x + c))
```

3.177 $\int \csc(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=74

$$-\frac{(a-b)^2\log(\cos(c+dx)+1)}{2d}+\frac{(a+b)^2\log(1-\cos(c+dx))}{2d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

```
[Out] ((a + b)^2*Log[1 - Cos[c + d*x]])/(2*d) - (2*a*b*Log[Cos[c + d*x]])/d - ((a
 - b)^2*Log[1 + Cos[c + d*x]])/(2*d) + (b^2*Sec[c + d*x])/d
```

Rubi [A] time = 0.180278, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3872, 2837, 12, 1802}

$$-\frac{(a-b)^2\log(\cos(c+dx)+1)}{2d}+\frac{(a+b)^2\log(1-\cos(c+dx))}{2d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{b^2\sec(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((a + b)^2*Log[1 - Cos[c + d*x]])/(2*d) - (2*a*b*Log[Cos[c + d*x]])/d - ((a
 - b)^2*Log[1 + Cos[c + d*x]])/(2*d) + (b^2*Sec[c + d*x])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \csc(c+dx)(a+b\sec(c+dx))^2\,dx = \int (-b-a\cos(c+dx))^2\csc(c+dx)\sec^2(c+dx)\,dx$$
$$= \frac{a\,\text{Subst}\left(\int \frac{a^2(-b+x)^2}{x^2(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{(-b+x)^2}{x^2(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{(a-b)^2}{2a^3(a-x)}+\frac{b^2}{a^2x^2}-\frac{2b}{a^2x}+\frac{(a+b)^2}{2a^3(a+x)}\right)dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{(a+b)^2\log(1-\cos(c+dx))}{2d}-\frac{2ab\log(\cos(c+dx))}{d}-\frac{(a-b)^2\log(1+\cos(c+d\cdots}{2d}$$

Mathematica [A] time = 0.151346, size = 91, normalized size = 1.23

$$\frac{a^2\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+2ab\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-2ab\log(\cos(c+dx))-(a-b)^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+b^2\sec(c+\cdots}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-((a - b)^2*Log[Cos[(c + d*x)/2]]) - 2*a*b*Log[Cos[c + d*x]] + a^2*Log[Sin
[(c + d*x)/2]] + 2*a*b*Log[Sin[(c + d*x)/2]] + b^2*Log[Sin[(c + d*x)/2]] +
b^2*Sec[c + d*x])/d
```

Maple [A] time = 0.035, size = 77, normalized size = 1.

$$\frac{a^2\ln(\csc(dx+c)-\cot(dx+c))}{d}+2\,\frac{ab\ln(\tan(dx+c))}{d}+\frac{b^2}{d\cos(dx+c)}+\frac{b^2\ln(\csc(dx+c)-\cot(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*a^2*ln(csc(d*x+c)-cot(d*x+c))+2/d*a*b*ln(tan(d*x+c))+1/d*b^2/cos(d*x+c)
+1/d*b^2*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 0.944199, size = 99, normalized size = 1.34

$$-\frac{4\,ab\log(\cos(dx+c))+\left(a^2-2\,ab+b^2\right)\log(\cos(dx+c)+1)-\left(a^2+2\,ab+b^2\right)\log(\cos(dx+c)-1)-\frac{2\,b^2}{\cos(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/2*(4*a*b*log(cos(d*x + c)) + (a^2 - 2*a*b + b^2)*log(cos(d*x + c) + 1) -
 (a^2 + 2*a*b + b^2)*log(cos(d*x + c) - 1) - 2*b^2/cos(d*x + c))/d
```

Fricas [A] time = 1.79424, size = 267, normalized size = 3.61

$$-\frac{4\,ab\cos\left(dx+c\right)\log\left(-\cos\left(dx+c\right)\right)+\left(a^2-2\,ab+b^2\right)\cos\left(dx+c\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-\left(a^2+2\,ab+b^2\right)\cos\left(dx+c\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-2\,b^2}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/2*(4*a*b*cos(d*x + c)*log(-cos(d*x + c)) + (a^2 - 2*a*b + b^2)*cos(d*x +
 c)*log(1/2*cos(d*x + c) + 1/2) - (a^2 + 2*a*b + b^2)*cos(d*x + c)*log(-1/2
*cos(d*x + c) + 1/2) - 2*b^2)/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec\left(c+dx\right)\right)^2\csc\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*csc(c + d*x), x)
```

Giac [A] time = 1.36733, size = 167, normalized size = 2.26

$$-\frac{4\,ab\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)-\left(a^2+2\,ab+b^2\right)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-\frac{4\left(ab+b^2+\frac{ab(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*a*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a^2 + 2
*a*b + b^2)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - 4*(a*b + b^
2 + a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((cos(d*x + c) - 1)/(cos(d*x
 + c) + 1) + 1))/d
```

3.178 $\int \csc^3(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=114

$$-\frac{\csc^2(c+dx)\left(\left(a^2+b^2\right)\cos(c+dx)+2ab\right)}{2d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{(a+b)(a+3b)\log(1-\cos(c+dx))}{4d}-\frac{(a-3b)(a-$$

```
[Out] -((2*a*b + (a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x]^2)/(2*d) + ((a + b)*(a +
3*b)*Log[1 - Cos[c + d*x]])/(4*d) - (2*a*b*Log[Cos[c + d*x]])/d - ((a - 3*b
)*(a - b)*Log[1 + Cos[c + d*x]])/(4*d) + (b^2*Sec[c + d*x])/d
```

Rubi [A] time = 0.294116, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2837, 12, 1805, 1802}

$$-\frac{\csc^2(c+dx)\left(\left(a^2+b^2\right)\cos(c+dx)+2ab\right)}{2d}-\frac{2ab\log(\cos(c+dx))}{d}+\frac{(a+b)(a+3b)\log(1-\cos(c+dx))}{4d}-\frac{(a-3b)(a-$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((2*a*b + (a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x]^2)/(2*d) + ((a + b)*(a +
3*b)*Log[1 - Cos[c + d*x]])/(4*d) - (2*a*b*Log[Cos[c + d*x]])/d - ((a - 3*b
)*(a - b)*Log[1 + Cos[c + d*x]])/(4*d) + (b^2*Sec[c + d*x])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^2\,dx = \int(-b-a\cos(c+dx))^2\csc^3(c+dx)\sec^2(c+dx)\,dx$$

$$= \frac{a^3\,\text{Subst}\left(\int \frac{a^2(-b+x)^2}{x^2\left(a^2-x^2\right)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a^5\,\text{Subst}\left(\int \frac{(-b+x)^2}{x^2\left(a^2-x^2\right)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= -\frac{a\left(2b+\frac{\left(a^2+b^2\right)\cos(c+dx)}{a}\right)\csc^2(c+dx)}{2d} - \frac{a^3\,\text{Subst}\left(\int \frac{-2b^2+4bx-\frac{\left(a^2+b^2\right)x^2}{a^2}}{x^2\left(a^2-x^2\right)}\,dx, x, \ldots\right)}{2d}$$

$$= -\frac{a\left(2b+\frac{\left(a^2+b^2\right)\cos(c+dx)}{a}\right)\csc^2(c+dx)}{2d} - \frac{a^3\,\text{Subst}\left(\int\left(\frac{(a-3b)(-a+b)}{2a^3(a-x)} - \frac{2b^2}{a^2x^2} + \frac{4b}{a^2}\ldots\right)\ldots\right)}{2d}$$

$$= -\frac{a\left(2b+\frac{\left(a^2+b^2\right)\cos(c+dx)}{a}\right)\csc^2(c+dx)}{2d} + \frac{(a+b)(a+3b)\log(1-\cos(c+dx))}{4d}\ldots$$

Mathematica [B] time = 0.615619, size = 329, normalized size = 2.89

$$-\frac{\csc^4(c+dx)\left(2\left(a^2+3b^2\right)\cos(2(c+dx))+\cos(c+dx)\left(\left(a^2-4ab+3b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)+a^2\left(-\log\left(\sin\left(\frac{1}{2}(c\ldots\right.\right.\right.\right.\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(Csc[c + d*x]^4*(2*a^2 - 2*b^2 + 2*(a^2 + 3*b^2)*Cos[2*(c + d*x)] - a^2*Co
s[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] + 4*a*b*Cos[3*(c + d*x)]*Log[Cos[(c +
d*x)/2]] - 3*b^2*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 4*a*b*Cos[3*(c +
d*x)]*Log[Cos[c + d*x]] + a^2*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 4*a*
b*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] + 3*b^2*Cos[3*(c + d*x)]*Log[Sin[(
c + d*x)/2]] + Cos[c + d*x]*(8*a*b + (a^2 - 4*a*b + 3*b^2)*Log[Cos[(c + d*x
)/2]] + 4*a*b*Log[Cos[c + d*x]] - a^2*Log[Sin[(c + d*x)/2]] - 4*a*b*Log[Sin
[(c + d*x)/2]] - 3*b^2*Log[Sin[(c + d*x)/2]])))/(2*d*(Csc[(c + d*x)/2]^2 -
Sec[(c + d*x)/2]^2))
```

Maple [A] time = 0.042, size = 139, normalized size = 1.2

$$-\frac{a^2\csc(dx+c)\cot(dx+c)}{2d}+\frac{a^2\ln(\csc(dx+c)-\cot(dx+c))}{2d}-\frac{ab}{d(\sin(dx+c))^2}+2\frac{ab\ln(\tan(dx+c))}{d}-\frac{\ldots}{2d(\sin\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/2/d*a^2*csc(d*x+c)*cot(d*x+c)+1/2/d*a^2*ln(csc(d*x+c)-cot(d*x+c))-1/d*a*
b/sin(d*x+c)^2+2/d*a*b*ln(tan(d*x+c))-1/2/d*b^2/sin(d*x+c)^2/cos(d*x+c)+3/2
/d*b^2/cos(d*x+c)+3/2/d*b^2*ln(csc(d*x+c)-cot(d*x+c))
```

Maxima [A] time = 0.958066, size = 161, normalized size = 1.41

$$-\frac{8\,ab\log\left(\cos\left(dx+c\right)\right)+\left(a^2-4\,ab+3\,b^2\right)\log\left(\cos\left(dx+c\right)+1\right)-\left(a^2+4\,ab+3\,b^2\right)\log\left(\cos\left(dx+c\right)-1\right)-\frac{2\left(2\,ab\cos(dx+\cdots}{\cos\cdots}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/4*(8*a*b*log(cos(d*x + c)) + (a^2 - 4*a*b + 3*b^2)*log(cos(d*x + c) + 1)
 - (a^2 + 4*a*b + 3*b^2)*log(cos(d*x + c) - 1) - 2*(2*a*b*cos(d*x + c) + (a
^2 + 3*b^2)*cos(d*x + c)^2 - 2*b^2)/(cos(d*x + c)^3 - cos(d*x + c)))/d
```

Fricas [A] time = 1.86678, size = 512, normalized size = 4.49

$$\frac{4\,ab\cos\left(dx+c\right)+2\left(a^2+3\,b^2\right)\cos\left(dx+c\right)^2-4\,b^2-8\left(ab\cos\left(dx+c\right)^3-ab\cos\left(dx+c\right)\right)\log\left(-\cos\left(dx+c\right)\right)-\left(\left(a^2-4\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*a*b*cos(d*x + c) + 2*(a^2 + 3*b^2)*cos(d*x + c)^2 - 4*b^2 - 8*(a*b*c
os(d*x + c)^3 - a*b*cos(d*x + c))*log(-cos(d*x + c)) - ((a^2 - 4*a*b + 3*b^
2)*cos(d*x + c)^3 - (a^2 - 4*a*b + 3*b^2)*cos(d*x + c))*log(1/2*cos(d*x + c
) + 1/2) + ((a^2 + 4*a*b + 3*b^2)*cos(d*x + c)^3 - (a^2 + 4*a*b + 3*b^2)*co
s(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/(d*cos(d*x + c)^3 - d*cos(d*x + c
))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.38369, size = 424, normalized size = 3.72

$$-\frac{16\,ab\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)+\frac{a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-2\left(a^2+4\,ab+3\,b^2\right)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)-\cdots}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/8*(16*a*b*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) + a^2*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a*b*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) + b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*(a^2 + 4*a*b + 3*b^2)
*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) - (a^2 + 2*a*b + b^2 + 6
*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 14*b^2*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) - a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 4*a*b*(cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x
 + c) + 1)^2)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + (cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2))/d
```

3.179 $\int (a+b\sec(c+dx))^2\sin^6(c+dx)\,dx$

Optimal. Leaf size=175

$$\frac{(13a^2-6b^2)\sin(c+dx)\cos^3(c+dx)}{24d}-\frac{(11a^2-18b^2)\sin(c+dx)\cos(c+dx)}{16d}+\frac{5}{16}x\left(a^2-6b^2\right)-\frac{a^2\sin(c+dx)\cos^5(c+}{6d}$$

```
[Out] (5*(a^2 - 6*b^2)*x)/16 + (2*a*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Sin[c + d
*x])/d - ((11*a^2 - 18*b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + ((13*a^2 -
6*b^2)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (a^2*Cos[c + d*x]^5*Sin[c + d*
x])/(6*d) - (2*a*b*Sin[c + d*x]^3)/(3*d) - (2*a*b*Sin[c + d*x]^5)/(5*d) + (
b^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.461275, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3872, 2911, 2592, 302, 206, 455, 1814, 1157, 388, 203}

$$\frac{(13a^2-6b^2)\sin(c+dx)\cos^3(c+dx)}{24d}-\frac{(11a^2-18b^2)\sin(c+dx)\cos(c+dx)}{16d}+\frac{5}{16}x\left(a^2-6b^2\right)-\frac{a^2\sin(c+dx)\cos^5(c+}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^6,x]
```

```
[Out] (5*(a^2 - 6*b^2)*x)/16 + (2*a*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Sin[c + d
*x])/d - ((11*a^2 - 18*b^2)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + ((13*a^2 -
6*b^2)*Cos[c + d*x]^3*Sin[c + d*x])/(24*d) - (a^2*Cos[c + d*x]^5*Sin[c + d*
x])/(6*d) - (2*a*b*Sin[c + d*x]^3)/(3*d) - (2*a*b*Sin[c + d*x]^5)/(5*d) + (
b^2*Tan[c + d*x])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\sin^6(c+dx)\,dx = \int (-b-a\cos(c+dx))^2\sin^4(c+dx)\tan^2(c+dx)\,dx$$
$$= (2ab)\int \sin^5(c+dx)\tan(c+dx)\,dx + \int \left(b^2+a^2\cos^2(c+dx)\right)\sin^4(c+dx)\tan$$
$$= \frac{\text{Subst}\left(\int \frac{x^6\left(a^2+b^2+b^2x^2\right)}{\left(1+x^2\right)^4}\,dx, x, \tan(c+dx)\right)}{d} + \frac{(2ab)\,\text{Subst}\left(\int \frac{x^6}{1-x^2}\,dx, x, \sin(c+\right.}{d}$$
$$= -\frac{a^2\cos^5(c+dx)\sin(c+dx)}{6d} - \frac{\text{Subst}\left(\int \frac{-a^2+6a^2x^2-6a^2x^4-6b^2x^6}{\left(1+x^2\right)^3}\,dx, x, \tan(c+dx)\right)}{6d}$$
$$= -\frac{2ab\sin(c+dx)}{d} + \frac{\left(13a^2-6b^2\right)\cos^3(c+dx)\sin(c+dx)}{24d} - \frac{a^2\cos^5(c+dx)\sin(}{6d}$$
$$= \frac{2ab\tanh^{-1}(\sin(c+dx))}{d} - \frac{2ab\sin(c+dx)}{d} - \frac{\left(11a^2-18b^2\right)\cos(c+dx)\sin(c+}{16d}$$
$$= \frac{2ab\tanh^{-1}(\sin(c+dx))}{d} - \frac{2ab\sin(c+dx)}{d} - \frac{\left(11a^2-18b^2\right)\cos(c+dx)\sin(c+}{16d}$$
$$= \frac{5}{16}\left(a^2-6b^2\right)x + \frac{2ab\tanh^{-1}(\sin(c+dx))}{d} - \frac{2ab\sin(c+dx)}{d} - \frac{\left(11a^2-18b^2\right)\text{co}}{}$$

Mathematica [A] time = 1.6659, size = 193, normalized size = 1.1

$$\frac{\tan(c+dx)\left(-5\left(29a^2-84b^2\right)\cos(2(c+dx))+35a^2\cos(4(c+dx))-5a^2\cos(6(c+dx))-185a^2+232ab\cos(3(c+dx))\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^6,x]
```

```
[Out] (60*(5*(a^2 - 6*b^2)*(c + d*x) - 32*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] + 32*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 2128*a*b*Sin[c +
 d*x] + (-185*a^2 + 1410*b^2 - 5*(29*a^2 - 84*b^2)*Cos[2*(c + d*x)] + 232*a
*b*Cos[3*(c + d*x)] + 35*a^2*Cos[4*(c + d*x)] - 30*b^2*Cos[4*(c + d*x)] - 2
4*a*b*Cos[5*(c + d*x)] - 5*a^2*Cos[6*(c + d*x)])*Tan[c + d*x])/(960*d)
```

Maple [A] time = 0.044, size = 246, normalized size = 1.4

$$-\frac{a^2\cos(dx+c)(\sin(dx+c))^5}{6d} - \frac{5a^2\cos(dx+c)(\sin(dx+c))^3}{24d} - \frac{5a^2\cos(dx+c)\sin(dx+c)}{16d} + \frac{5a^2x}{16} + \frac{5a^2c}{16d} - \frac{2ab(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*sin(d*x+c)^6,x)
```

```
[Out] -1/6/d*a^2*cos(d*x+c)*sin(d*x+c)^5-5/24/d*a^2*cos(d*x+c)*sin(d*x+c)^3-5/16*
a^2*cos(d*x+c)*sin(d*x+c)/d+5/16*a^2*x+5/16/d*a^2*c-2/5*a*b*sin(d*x+c)^5/d-
2/3*a*b*sin(d*x+c)^3/d-2*a*b*sin(d*x+c)/d+2/d*a*b*ln(sec(d*x+c)+tan(d*x+c))
+1/d*b^2*sin(d*x+c)^7/cos(d*x+c)+1/d*b^2*sin(d*x+c)^5*cos(d*x+c)+5/4/d*b^2*
cos(d*x+c)*sin(d*x+c)^3+15/8/d*b^2*cos(d*x+c)*sin(d*x+c)-15/8*b^2*x-15/8/d*
b^2*c
```

Maxima [A] time = 1.46326, size = 234, normalized size = 1.34

$$5\left(4\sin\left(2dx+2c\right)^{3}+60dx+60c+9\sin\left(4dx+4c\right)-48\sin\left(2dx+2c\right)\right)a^{2}-64\left(6\sin\left(dx+c\right)^{5}+10\sin\left(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/960*(5*(4*sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*si
n(2*d*x + 2*c))*a^2 - 64*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin
(d*x + c) + 1) + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*a*b - 120*(15*
d*x + 15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(d*
x + c)^2 + 1) - 8*tan(d*x + c))*b^2)/d
```

Fricas [A] time = 1.95119, size = 468, normalized size = 2.67

$$75\left(a^{2}-6b^{2}\right)dx\cos\left(dx+c\right)+240ab\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-240ab\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/240*(75*(a^2 - 6*b^2)*d*x*cos(d*x + c) + 240*a*b*cos(d*x + c)*log(sin(d*x
 + c) + 1) - 240*a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) - (40*a^2*cos(d*x
+ c)^6 + 96*a*b*cos(d*x + c)^5 - 352*a*b*cos(d*x + c)^3 - 10*(13*a^2 - 6*b^
2)*cos(d*x + c)^4 + 736*a*b*cos(d*x + c) + 15*(11*a^2 - 18*b^2)*cos(d*x + c
)^2 - 240*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*sin(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.36594, size = 512, normalized size = 2.93

$$480ab\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-480ab\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+75\left(a^{2}-6b^{2}\right)\left(dx+c\right)-\frac{480b^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/240*(480*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 480*a*b*log(abs(tan(1/2
*d*x + 1/2*c) - 1)) + 75*(a^2 - 6*b^2)*(d*x + c) - 480*b^2*tan(1/2*d*x + 1/
2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(75*a^2*tan(1/2*d*x + 1/2*c)^11 - 480
*a*b*tan(1/2*d*x + 1/2*c)^11 - 210*b^2*tan(1/2*d*x + 1/2*c)^11 + 425*a^2*ta
n(1/2*d*x + 1/2*c)^9 - 3040*a*b*tan(1/2*d*x + 1/2*c)^9 - 870*b^2*tan(1/2*d*
x + 1/2*c)^9 + 990*a^2*tan(1/2*d*x + 1/2*c)^7 - 8256*a*b*tan(1/2*d*x + 1/2*
c)^7 - 660*b^2*tan(1/2*d*x + 1/2*c)^7 - 990*a^2*tan(1/2*d*x + 1/2*c)^5 - 82
56*a*b*tan(1/2*d*x + 1/2*c)^5 + 660*b^2*tan(1/2*d*x + 1/2*c)^5 - 425*a^2*ta
n(1/2*d*x + 1/2*c)^3 - 3040*a*b*tan(1/2*d*x + 1/2*c)^3 + 870*b^2*tan(1/2*d*
x + 1/2*c)^3 - 75*a^2*tan(1/2*d*x + 1/2*c) - 480*a*b*tan(1/2*d*x + 1/2*c) +
 210*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.180 $\int (a+b\sec(c+dx))^2\sin^4(c+dx)\,dx$

Optimal. Leaf size=178

$$-\frac{b\left(28a^2+b^2\right)\sin(c+dx)}{6ad}-\frac{\left(12a^2+b^2\right)\sin(c+dx)(a\cos(c+dx)+b)^2}{12abd}-\frac{\left(39a^2+2b^2\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{3}{8}x$$

```
[Out] (3*(a^2 - 4*b^2)*x)/8 + (2*a*b*ArcTanh[Sin[c + d*x]])/d - (b*(28*a^2 + b^2)
*Sin[c + d*x])/(6*a*d) - ((39*a^2 + 2*b^2)*Cos[c + d*x]*Sin[c + d*x])/(24*d
) - ((12*a^2 + b^2)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/(12*a*b*d) + ((b +
 a*Cos[c + d*x])^3*Sin[c + d*x])/(4*a*d) + ((b + a*Cos[c + d*x])^3*Tan[c +
d*x])/(b*d)
```

Rubi [A] time = 0.556468, antiderivative size = 178, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2894, 3049, 3033, 3023, 2735, 3770}

$$-\frac{b\left(28a^2+b^2\right)\sin(c+dx)}{6ad}-\frac{\left(12a^2+b^2\right)\sin(c+dx)(a\cos(c+dx)+b)^2}{12abd}-\frac{\left(39a^2+2b^2\right)\sin(c+dx)\cos(c+dx)}{24d}+\frac{3}{8}x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^4,x]
```


```
[Out] (3*(a^2 - 4*b^2)*x)/8 + (2*a*b*ArcTanh[Sin[c + d*x]])/d - (b*(28*a^2 + b^2)
*Sin[c + d*x])/(6*a*d) - ((39*a^2 + 2*b^2)*Cos[c + d*x]*Sin[c + d*x])/(24*d
) - ((12*a^2 + b^2)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/(12*a*b*d) + ((b +
 a*Cos[c + d*x])^3*Sin[c + d*x])/(4*a*d) + ((b + a*Cos[c + d*x])^3*Tan[c +
d*x])/(b*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2894

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a*b*d*(n + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x]
)^(n + 1)*Simp[a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4) + a*b*(m +
 3)*Sin[e + f*x] - (a^2*(n + 1)*(n + 3) - b^2*(m + n + 3)*(m + n + 4))*Sin[
e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d
*Sin[e + f*x])^(n + 2))/(b*d^2*f*(m + n + 4)), x]) /; FreeQ[{a, b, d, e, f,
 m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n]) &&  !m
< -1 && LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
```

```
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2\sin^4(c+dx)\,dx = \int (-b-a\cos(c+dx))^2\sin^2(c+dx)\tan^2(c+dx)\,dx$$

$$= \frac{(b+a\cos(c+dx))^3\sin(c+dx)}{4ad} + \frac{(b+a\cos(c+dx))^3\tan(c+dx)}{bd} - \int(-b-a\,\mathrm{c}$$

$$= -\frac{\left(12a^2+b^2\right)(b+a\cos(c+dx))^2\sin(c+dx)}{12abd} + \frac{(b+a\cos(c+dx))^3\sin(c+dx)}{4ad}$$

$$= -\frac{\left(39a^2+2b^2\right)\cos(c+dx)\sin(c+dx)}{24d} - \frac{\left(12a^2+b^2\right)(b+a\cos(c+dx))^2\sin(c+}{12abd}$$

$$= -\frac{b\left(28a^2+b^2\right)\sin(c+dx)}{6ad} - \frac{\left(39a^2+2b^2\right)\cos(c+dx)\sin(c+dx)}{24d} - \frac{\left(12a^2+b^2\right)}{}$$

$$= \frac{3}{8}\left(a^2-4b^2\right)x - \frac{b\left(28a^2+b^2\right)\sin(c+dx)}{6ad} - \frac{\left(39a^2+2b^2\right)\cos(c+dx)\sin(c+dx}{24d}$$

$$= \frac{3}{8}\left(a^2-4b^2\right)x + \frac{2ab\tanh^{-1}(\sin(c+dx))}{d} - \frac{b\left(28a^2+b^2\right)\sin(c+dx)}{6ad} - \frac{\left(39a^2+\right.}{}$$

Mathematica [A] time = 1.01218, size = 157, normalized size = 0.88

$$\frac{\tan(c+dx)\left(-6\left(3a^2-4b^2\right)\cos(2(c+dx))+3\left(a^2\cos(4(c+dx))-7a^2+40b^2\right)+16ab\cos(3(c+dx))\right)+12\left(3\left(a^2-4\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^4,x]
```

```
[Out] (12*(3*(a^2 - 4*b^2)*(c + d*x) - 16*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]] + 16*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - 208*a*b*Sin[c +
d*x] + (-6*(3*a^2 - 4*b^2)*Cos[2*(c + d*x)] + 16*a*b*Cos[3*(c + d*x)] + 3*(
-7*a^2 + 40*b^2 + a^2*Cos[4*(c + d*x)]))*Tan[c + d*x])/(96*d)
```

Maple [A] time = 0.041, size = 187, normalized size = 1.1

$$-\frac{a^2\cos(dx+c)(\sin(dx+c))^3}{4d}-\frac{3a^2\cos(dx+c)\sin(dx+c)}{8d}+\frac{3a^2x}{8}+\frac{3a^2c}{8d}-\frac{2ab(\sin(dx+c))^3}{3d}-2\frac{ab\sin(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*sin(d*x+c)^4,x)
```

```
[Out] -1/4/d*a^2*cos(d*x+c)*sin(d*x+c)^3-3/8*a^2*cos(d*x+c)*sin(d*x+c)/d+3/8*a^2*
x+3/8/d*a^2*c-2/3*a*b*sin(d*x+c)^3/d-2*a*b*sin(d*x+c)/d+2/d*a*b*ln(sec(d*x+
c)+tan(d*x+c))+1/d*b^2*sin(d*x+c)^5/cos(d*x+c)+1/d*b^2*cos(d*x+c)*sin(d*x+c
)^3+3/2/d*b^2*cos(d*x+c)*sin(d*x+c)-3/2*b^2*x-3/2/d*b^2*c
```

Maxima [A] time = 1.49883, size = 169, normalized size = 0.95

$$\frac{3(12dx+12c+\sin(4dx+4c)-8\sin(2dx+2c))a^2-32\left(2\sin(dx+c)^3-3\log(\sin(dx+c)+1)+3\log(\sin(dx\right.}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*a^2 - 32*(2
*sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin
(d*x + c))*a*b - 48*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*ta
n(d*x + c))*b^2)/d
```

Fricas [A] time = 1.88299, size = 371, normalized size = 2.08

$$\frac{9\left(a^2-4b^2\right)dx\cos(dx+c)+24ab\cos(dx+c)\log(\sin(dx+c)+1)-24ab\cos(dx+c)\log(-\sin(dx+c)+1)+\left(6\right.}{24d\cos(dx-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/24*(9*(a^2 - 4*b^2)*d*x*cos(d*x + c) + 24*a*b*cos(d*x + c)*log(sin(d*x +
c) + 1) - 24*a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) + (6*a^2*cos(d*x + c)^
4 + 16*a*b*cos(d*x + c)^3 - 64*a*b*cos(d*x + c) - 3*(5*a^2 - 4*b^2)*cos(d*x
 + c)^2 + 24*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*sin(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31842, size = 385, normalized size = 2.16

$$48\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 48\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + 9\left(a^2 - 4\,b^2\right)(dx + c) - \frac{48\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 1} + \frac{2\left(9\,a^2\,\mathrm{ta}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/24*(48*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 48*a*b*log(abs(tan(1/2*d*
x + 1/2*c) - 1)) + 9*(a^2 - 4*b^2)*(d*x + c) - 48*b^2*tan(1/2*d*x + 1/2*c)/
(tan(1/2*d*x + 1/2*c)^2 - 1) + 2*(9*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*a*b*tan
(1/2*d*x + 1/2*c)^7 - 12*b^2*tan(1/2*d*x + 1/2*c)^7 + 33*a^2*tan(1/2*d*x +
1/2*c)^5 - 208*a*b*tan(1/2*d*x + 1/2*c)^5 - 12*b^2*tan(1/2*d*x + 1/2*c)^5 -
 33*a^2*tan(1/2*d*x + 1/2*c)^3 - 208*a*b*tan(1/2*d*x + 1/2*c)^3 + 12*b^2*ta
n(1/2*d*x + 1/2*c)^3 - 9*a^2*tan(1/2*d*x + 1/2*c) - 48*a*b*tan(1/2*d*x + 1/
2*c) + 12*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.181 $\int (a+b\sec(c+dx))^2\sin^2(c+dx)\,dx$

Optimal. Leaf size=77

$$-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{a^2x}{2}-\frac{2ab\sin(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}-b^2x$$

```
[Out] (a^2*x)/2 - b^2*x + (2*a*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Sin[c + d*x])/
d - (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (b^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.131229, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2722, 2635, 8, 2592, 321, 206, 3473}

$$-\frac{a^2\sin(c+dx)\cos(c+dx)}{2d}+\frac{a^2x}{2}-\frac{2ab\sin(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}-b^2x$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^2,x]
```

```
[Out] (a^2*x)/2 - b^2*x + (2*a*b*ArcTanh[Sin[c + d*x]])/d - (2*a*b*Sin[c + d*x])/
d - (a^2*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (b^2*Tan[c + d*x])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((g_.)*tan[(e_.) + (f_.)*(
x_)])^(p_.), x_Symbol] :> Int[ExpandIntegrand[(g*Tan[e + f*x])^p, (a + b*Si
n[e + f*x])^m, x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2, 0]
&& IGtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int (a+b\sec(c+dx))^2\sin^2(c+dx)\,dx &= \int (-b-a\cos(c+dx))^2\tan^2(c+dx)\,dx\\
&= \int \left(a^2\sin^2(c+dx)+2ab\sin(c+dx)\tan(c+dx)+b^2\tan^2(c+dx)\right)dx\\
&= a^2\int \sin^2(c+dx)\,dx+(2ab)\int \sin(c+dx)\tan(c+dx)\,dx+b^2\int \tan^2(c+dx)\,d\\
&= -\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}+\frac{b^2\tan(c+dx)}{d}+\frac{1}{2}a^2\int 1\,dx-b^2\int 1\,dx+\frac{(2ab)}{}\\
&= \frac{a^2x}{2}-b^2x-\frac{2ab\sin(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c+dx)}{2d}+\frac{b^2\tan(c+dx)}{d}+\frac{(2}{}\\
&= \frac{a^2x}{2}-b^2x+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}-\frac{2ab\sin(c+dx)}{d}-\frac{a^2\cos(c+dx)\sin(c+}{2d}
\end{aligned}$$

Mathematica [A] time = 0.575742, size = 121, normalized size = 1.57

$$-\frac{a^2\sin(2(c+dx))-2a^2c-2a^2dx+8ab\sin(c+dx)+8ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-8ab\log\left(\sin\left(\frac{1}{2}(c+\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*Sin[c + d*x]^2,x]
```

```
[Out] -(-2*a^2*c + 4*b^2*c - 2*a^2*d*x + 4*b^2*d*x + 8*a*b*Log[Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2]] - 8*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 8*a*b
*Sin[c + d*x] + a^2*Sin[2*(c + d*x)] - 4*b^2*Tan[c + d*x])/(4*d)
```

Maple [A] time = 0.037, size = 99, normalized size = 1.3

$$-\frac{a^2\cos(dx+c)\sin(dx+c)}{2d}+\frac{a^2x}{2}+\frac{a^2c}{2d}+2\frac{ab\ln(\sec(dx+c)+\tan(dx+c))}{d}-2\frac{ab\sin(dx+c)}{d}-b^2x+\frac{b^2\tan(dx+}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*sin(d*x+c)^2,x)
```

```
[Out] -1/2*a^2*cos(d*x+c)*sin(d*x+c)/d+1/2*a^2*x+1/2/d*a^2*c+2/d*a*b*ln(sec(d*x+c
)+tan(d*x+c))-2*a*b*sin(d*x+c)/d-b^2*x+b^2*tan(d*x+c)/d-1/d*b^2*c
```

Maxima [A] time = 1.54745, size = 108, normalized size = 1.4

$$\frac{(2\,dx+2\,c-\sin\left(2\,dx+2\,c\right))a^2-4\left(dx+c-\tan\left(dx+c\right)\right)b^2+4\,ab(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)-2\,\mathrm{s}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a^2 - 4*(d*x + c - tan(d*x + c))*b^2
+ 4*a*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1) - 2*sin(d*x + c)))/d
```

Fricas [A] time = 1.83019, size = 279, normalized size = 3.62

$$\frac{\left(a^2-2\,b^2\right)dx\cos\left(dx+c\right)+2\,ab\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-2\,ab\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)-\left(a^2\cos\right.}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((a^2 - 2*b^2)*d*x*cos(d*x + c) + 2*a*b*cos(d*x + c)*log(sin(d*x + c) +
 1) - 2*a*b*cos(d*x + c)*log(-sin(d*x + c) + 1) - (a^2*cos(d*x + c)^2 + 4*a
*b*cos(d*x + c) - 2*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec\left(c+dx\right)\right)^2\sin^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*sin(d*x+c)**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*sin(c + d*x)**2, x)
```

Giac [B] time = 1.37719, size = 215, normalized size = 2.79

$$\frac{4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\left(a^2-2\,b^2\right)(dx+c)-\frac{4\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}+\frac{2\left(a^2\tan\left(\frac{1}{2}\right.\right.}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 4*a*b*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + (a^2 - 2*b^2)*(d*x + c) - 4*b^2*tan(1/2*d*x + 1/2*c)/(tan(1
/2*d*x + 1/2*c)^2 - 1) + 2*(a^2*tan(1/2*d*x + 1/2*c)^3 - 4*a*b*tan(1/2*d*x
+ 1/2*c)^3 - a^2*tan(1/2*d*x + 1/2*c) - 4*a*b*tan(1/2*d*x + 1/2*c))/(tan(1/
2*d*x + 1/2*c)^2 + 1)^2)/d
```

3.182 $\int \csc^2(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=59

$$-\frac{(a^2+b^2)\cot(c+dx)}{d}-\frac{2ab\csc(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

```
[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - ((a^2 + b^2)*Cot[c + d*x])/d - (2*a*b*Csc
[c + d*x])/d + (b^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.414007, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3872, 2911, 2621, 321, 207, 14}

$$-\frac{(a^2+b^2)\cot(c+dx)}{d}-\frac{2ab\csc(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{b^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```



```
[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - ((a^2 + b^2)*Cot[c + d*x])/d - (2*a*b*Csc
[c + d*x])/d + (b^2*Tan[c + d*x])/d
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^2\,dx = \int (-b-a\cos(c+dx))^2\csc^2(c+dx)\sec^2(c+dx)\,dx$$
$$= (2ab)\int \csc^2(c+dx)\sec(c+dx)\,dx + \int \left(b^2+a^2\cos^2(c+dx)\right)\csc^2(c+dx)\sec^2$$
$$= \frac{\text{Subst}\left(\int \frac{a^2+b^2+b^2x^2}{x^2}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \csc(c+dx\right.}{d}$$
$$= -\frac{2ab\csc(c+dx)}{d} + \frac{\text{Subst}\left(\int \left(b^2+\frac{a^2+b^2}{x^2}\right)dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{2ab\tanh^{-1}(\sin(c+dx))}{d} - \frac{\left(a^2+b^2\right)\cot(c+dx)}{d} - \frac{2ab\csc(c+dx)}{d} + \frac{b^2\tan(c+}{d}$$

Mathematica [B] time = 0.46498, size = 138, normalized size = 2.34

$$-\frac{\csc^3\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\left(a^2+2b^2\right)\cos(2(c+dx))+4ab\cos(c+dx)+a\left(a+2b\sin(2(c+dx))\left(\log\left(\cos\left(\frac{1}{2}(c+\right.\right.\right.\right.\right.}{4d\left(\cot^2\left(\frac{1}{2}(c+dx)\right)-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(Csc[(c + d*x)/2]^3*Sec[(c + d*x)/2]*(4*a*b*Cos[c + d*x] + (a^2 + 2*b^2)*C
os[2*(c + d*x)] + a*(a + 2*b*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Lo
g[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])*Sin[2*(c + d*x)])))/(4*d*(-1 + Cot[
(c + d*x)/2]^2))
```

Maple [A] time = 0.036, size = 89, normalized size = 1.5

$$-\frac{a^2\cot(dx+c)}{d} - 2\frac{ab}{d\sin(dx+c)} + 2\frac{ab\ln(\sec(dx+c)+\tan(dx+c))}{d} + \frac{b^2}{d\sin(dx+c)\cos(dx+c)} - 2\frac{b^2\cot(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+b*sec(d*x+c))^2,x)
```

```
[Out] -a^2*cot(d*x+c)/d-2/d*a*b/sin(d*x+c)+2/d*a*b*ln(sec(d*x+c)+tan(d*x+c))+1/d*
b^2/sin(d*x+c)/cos(d*x+c)-2/d*b^2*cot(d*x+c)
```

Maxima [A] time = 1.02287, size = 99, normalized size = 1.68

$$-\frac{ab\left(\frac{2}{\sin(dx+c)} - \log(\sin(dx+c)+1) + \log(\sin(dx+c)-1)\right) + b^2\left(\frac{1}{\tan(dx+c)} - \tan(dx+c)\right) + \frac{a^2}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -(a*b*(2/sin(d*x + c) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + b^
2*(1/tan(d*x + c) - tan(d*x + c)) + a^2/tan(d*x + c))/d
```

Fricas [A] time = 1.7732, size = 267, normalized size = 4.53

$$\frac{ab\cos(dx+c)\log(\sin(dx+c)+1)\sin(dx+c) - ab\cos(dx+c)\log(-\sin(dx+c)+1)\sin(dx+c) - 2\,ab\cos(dx+}{d\cos(dx+c)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] (a*b*cos(d*x + c)*log(sin(d*x + c) + 1)*sin(d*x + c) - a*b*cos(d*x + c)*log
(-sin(d*x + c) + 1)*sin(d*x + c) - 2*a*b*cos(d*x + c) - (a^2 + 2*b^2)*cos(d
*x + c)^2 + b^2)/(d*cos(d*x + c)*sin(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sec(c+dx))^2 \csc^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**2*csc(c + d*x)**2, x)
```

Giac [B] time = 1.32974, size = 225, normalized size = 3.81

$$\frac{4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 4\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + b^2\,\mathrm{t}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 4*a*b*log(abs(tan(1/2*d*x +
 1/2*c) - 1)) + a^2*tan(1/2*d*x + 1/2*c) - 2*a*b*tan(1/2*d*x + 1/2*c) + b^2
*tan(1/2*d*x + 1/2*c) - (a^2*tan(1/2*d*x + 1/2*c)^2 + 2*a*b*tan(1/2*d*x + 1
/2*c)^2 + 5*b^2*tan(1/2*d*x + 1/2*c)^2 - a^2 - 2*a*b - b^2)/(tan(1/2*d*x +
1/2*c)^3 - tan(1/2*d*x + 1/2*c)))/d
```

3.183 $\int \csc^4(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=100

$$-\frac{(a^2+b^2)\cot^3(c+dx)}{3d}-\frac{(a^2+2b^2)\cot(c+dx)}{d}-\frac{2ab\csc^3(c+dx)}{3d}-\frac{2ab\csc(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+b^2$$

```
[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - ((a^2 + 2*b^2)*Cot[c + d*x])/d - ((a^2 +
b^2)*Cot[c + d*x]^3)/(3*d) - (2*a*b*Csc[c + d*x])/d - (2*a*b*Csc[c + d*x]^3
)/(3*d) + (b^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.321917, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3872, 2911, 2621, 302, 207, 448}

$$-\frac{(a^2+b^2)\cot^3(c+dx)}{3d}-\frac{(a^2+2b^2)\cot(c+dx)}{d}-\frac{2ab\csc^3(c+dx)}{3d}-\frac{2ab\csc(c+dx)}{d}+\frac{2ab\tanh^{-1}(\sin(c+dx))}{d}+b^2$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - ((a^2 + 2*b^2)*Cot[c + d*x])/d - ((a^2 +
b^2)*Cot[c + d*x]^3)/(3*d) - (2*a*b*Csc[c + d*x])/d - (2*a*b*Csc[c + d*x]^3
)/(3*d) + (b^2*Tan[c + d*x])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^2\,dx = \int (-b-a\cos(c+dx))^2\csc^4(c+dx)\sec^2(c+dx)\,dx$$
$$= (2ab)\int \csc^4(c+dx)\sec(c+dx)\,dx + \int \left(b^2+a^2\cos^2(c+dx)\right)\csc^4(c+dx)\,\mathrm{s}$$
$$= \frac{\mathrm{Subst}\left(\int \frac{(1+x^2)(a^2+b^2+b^2x^2)}{x^4}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\,\mathrm{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \mathrm{c}\right.}{d}$$
$$= \frac{\mathrm{Subst}\left(\int \left(b^2+\frac{a^2+b^2}{x^4}+\frac{a^2+2b^2}{x^2}\right)dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\,\mathrm{Subst}\left(\int \left(1+x^2\right.\right.}{}$$
$$= -\frac{(a^2+2b^2)\cot(c+dx)}{d} - \frac{(a^2+b^2)\cot^3(c+dx)}{3d} - \frac{2ab\csc(c+dx)}{d} - \frac{2ab\,\mathrm{cs}}{}$$
$$= \frac{2ab\tanh^{-1}(\sin(c+dx))}{d} - \frac{(a^2+2b^2)\cot(c+dx)}{d} - \frac{(a^2+b^2)\cot^3(c+dx)}{3d} -$$

Mathematica [B] time = 0.603211, size = 259, normalized size = 2.59

$$\frac{\csc^5\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(-2\left(a^2+4b^2\right)\cos(2(c+dx))+a^2\cos(4(c+dx))-3a^2-14ab\cos(c+dx)+6ab\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (Csc[(c + d*x)/2]^5*Sec[(c + d*x)/2]^3*(-3*a^2 - 14*a*b*Cos[c + d*x] - 2*(a
^2 + 4*b^2)*Cos[2*(c + d*x)] + 6*a*b*Cos[3*(c + d*x)] + a^2*Cos[4*(c + d*x)
] + 4*b^2*Cos[4*(c + d*x)] - 6*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
*Sin[2*(c + d*x)] + 6*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[2*(c
 + d*x)] + 3*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[4*(c + d*x)]
- 3*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[4*(c + d*x)]))/(96*d*(
-1 + Cot[(c + d*x)/2]^2))
```

Maple [A] time = 0.045, size = 151, normalized size = 1.5

$$-\frac{2a^2\cot(dx+c)}{3d} - \frac{a^2\cot(dx+c)(\csc(dx+c))^2}{3d} - \frac{2ab}{3d(\sin(dx+c))^3} - 2\frac{ab}{d\sin(dx+c)} + 2\frac{ab\ln(\sec(dx+c)+\mathrm{tan}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*sec(d*x+c))^2,x)
```

```
[Out] -2/3*a^2*cot(d*x+c)/d-1/3/d*a^2*cot(d*x+c)*csc(d*x+c)^2-2/3/d*a*b/sin(d*x+c
)^3-2/d*a*b/sin(d*x+c)+2/d*a*b*ln(sec(d*x+c)+tan(d*x+c))-1/3/d*b^2/sin(d*x+
c)^3/cos(d*x+c)+4/3/d*b^2/sin(d*x+c)/cos(d*x+c)-8/3/d*b^2*cot(d*x+c)
```

Maxima [A] time = 0.967585, size = 151, normalized size = 1.51

$$-\frac{ab\left(\frac{2\left(3\sin(dx+c)^2+1\right)}{\sin(dx+c)^3} - 3\log\left(\sin\left(dx+c\right)+1\right) + 3\log\left(\sin\left(dx+c\right)-1\right)\right) + b^2\left(\frac{6\tan(dx+c)^2+1}{\tan(dx+c)^3} - 3\tan\left(dx+c\right)\right) + \frac{(3\tan(dx+\cdots}{\tan(dx\cdots}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/3*(a*b*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1
) + 3*log(sin(d*x + c) - 1)) + b^2*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3 -
 3*tan(d*x + c)) + (3*tan(d*x + c)^2 + 1)*a^2/tan(d*x + c)^3)/d
```

Fricas [A] time = 1.78355, size = 451, normalized size = 4.51

$$-\frac{6\,ab\cos\left(dx+c\right)^3 + 2\left(a^2+4\,b^2\right)\cos\left(dx+c\right)^4 - 8\,ab\cos\left(dx+c\right) - 3\left(a^2+4\,b^2\right)\cos\left(dx+c\right)^2 - 3\left(ab\cos\left(dx+c\right)^3 - ab\cdots\right.}{3\left(d\cos\left(dx+c\right)^3 - \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/3*(6*a*b*cos(d*x + c)^3 + 2*(a^2 + 4*b^2)*cos(d*x + c)^4 - 8*a*b*cos(d*x
 + c) - 3*(a^2 + 4*b^2)*cos(d*x + c)^2 - 3*(a*b*cos(d*x + c)^3 - a*b*cos(d*
x + c))*log(sin(d*x + c) + 1)*sin(d*x + c) + 3*(a*b*cos(d*x + c)^3 - a*b*co
s(d*x + c))*log(-sin(d*x + c) + 1)*sin(d*x + c) + 3*b^2)/((d*cos(d*x + c)^3
 - d*cos(d*x + c))*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.41682, size = 305, normalized size = 3.05

$$\frac{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 2\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 48\,ab\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - 48\,ab\log\left(\left|\tan\cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(a^2*tan(1/2*d*x + 1/2*c)^3 - 2*a*b*tan(1/2*d*x + 1/2*c)^3 + b^2*tan(1
/2*d*x + 1/2*c)^3 + 48*a*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 48*a*b*log(
abs(tan(1/2*d*x + 1/2*c) - 1)) + 9*a^2*tan(1/2*d*x + 1/2*c) - 30*a*b*tan(1/
2*d*x + 1/2*c) + 21*b^2*tan(1/2*d*x + 1/2*c) - 48*b^2*tan(1/2*d*x + 1/2*c)/
(tan(1/2*d*x + 1/2*c)^2 - 1) - (9*a^2*tan(1/2*d*x + 1/2*c)^2 + 30*a*b*tan(1
/2*d*x + 1/2*c)^2 + 21*b^2*tan(1/2*d*x + 1/2*c)^2 + a^2 + 2*a*b + b^2)/tan(
1/2*d*x + 1/2*c)^3)/d
```

3.184 $\int \csc^6(c+dx)(a+b\sec(c+dx))^2\,dx$

Optimal. Leaf size=143

$$-\frac{(a^2+b^2)\cot^5(c+dx)}{5d}-\frac{(2a^2+3b^2)\cot^3(c+dx)}{3d}-\frac{(a^2+3b^2)\cot(c+dx)}{d}-\frac{2ab\csc^5(c+dx)}{5d}-\frac{2ab\csc^3(c+dx)}{3d}-2a$$

```
[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - ((a^2 + 3*b^2)*Cot[c + d*x])/d - ((2*a^2
+ 3*b^2)*Cot[c + d*x]^3)/(3*d) - ((a^2 + b^2)*Cot[c + d*x]^5)/(5*d) - (2*a*
b*Csc[c + d*x])/d - (2*a*b*Csc[c + d*x]^3)/(3*d) - (2*a*b*Csc[c + d*x]^5)/(
5*d) + (b^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.407985, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3872, 2911, 2621, 302, 207, 448}

$$-\frac{(a^2+b^2)\cot^5(c+dx)}{5d}-\frac{(2a^2+3b^2)\cot^3(c+dx)}{3d}-\frac{(a^2+3b^2)\cot(c+dx)}{d}-\frac{2ab\csc^5(c+dx)}{5d}-\frac{2ab\csc^3(c+dx)}{3d}-2a$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*a*b*ArcTanh[Sin[c + d*x]])/d - ((a^2 + 3*b^2)*Cot[c + d*x])/d - ((2*a^2
+ 3*b^2)*Cot[c + d*x]^3)/(3*d) - ((a^2 + b^2)*Cot[c + d*x]^5)/(5*d) - (2*a*
b*Csc[c + d*x])/d - (2*a*b*Csc[c + d*x]^3)/(3*d) - (2*a*b*Csc[c + d*x]^5)/(
5*d) + (b^2*Tan[c + d*x])/d
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 448

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^2\,dx = \int (-b-a\cos(c+dx))^2\csc^6(c+dx)\sec^2(c+dx)\,dx$$
$$= (2ab)\int \csc^6(c+dx)\sec(c+dx)\,dx + \int \left(b^2+a^2\cos^2(c+dx)\right)\csc^6(c+dx)\,\mathrm{s}$$
$$= \frac{\mathrm{Subst}\left(\int \frac{\left(1+x^2\right)^2\left(a^2+b^2+b^2x^2\right)}{x^6}\,dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\,\mathrm{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx, x,\right.}{d}$$
$$= \frac{\mathrm{Subst}\left(\int \left(b^2+\frac{a^2+b^2}{x^6}+\frac{2a^2+3b^2}{x^4}+\frac{a^2+3b^2}{x^2}\right)dx, x, \tan(c+dx)\right)}{d} - \frac{(2ab)\,\mathrm{Subst}\left(\right.}{}$$
$$= -\frac{\left(a^2+3b^2\right)\cot(c+dx)}{d} - \frac{\left(2a^2+3b^2\right)\cot^3(c+dx)}{3d} - \frac{\left(a^2+b^2\right)\cot^5(c+dx)}{5d}$$
$$= \frac{2ab\tanh^{-1}(\sin(c+dx))}{d} - \frac{\left(a^2+3b^2\right)\cot(c+dx)}{d} - \frac{\left(2a^2+3b^2\right)\cot^3(c+dx}{3d}$$

Mathematica [B] time = 0.721495, size = 368, normalized size = 2.57

$$-\frac{\csc^7\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(20\left(a^2+6b^2\right)\cos(2(c+dx))-16a^2\cos(4(c+dx))+4a^2\cos(6(c+dx))+40a^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(Csc[(c + d*x)/2]^7*Sec[(c + d*x)/2]^5*(40*a^2 + 196*a*b*Cos[c + d*x] + 20
*(a^2 + 6*b^2)*Cos[2*(c + d*x)] - 130*a*b*Cos[3*(c + d*x)] - 16*a^2*Cos[4*(
c + d*x)] - 96*b^2*Cos[4*(c + d*x)] + 30*a*b*Cos[5*(c + d*x)] + 4*a^2*Cos[6
*(c + d*x)] + 24*b^2*Cos[6*(c + d*x)] + 75*a*b*Log[Cos[(c + d*x)/2] - Sin[(
c + d*x)/2]]*Sin[2*(c + d*x)] - 75*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)
/2]]*Sin[2*(c + d*x)] - 60*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin
[4*(c + d*x)] + 60*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[4*(c +
d*x)] + 15*a*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[6*(c + d*x)] -
15*a*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[6*(c + d*x)]))/(7680*d*
(-1 + Cot[(c + d*x)/2]^2))
```

Maple [A] time = 0.048, size = 212, normalized size = 1.5

$$-\frac{8a^2\cot(dx+c)}{15d} - \frac{a^2\cot(dx+c)(\csc(dx+c))^4}{5d} - \frac{4a^2\cot(dx+c)(\csc(dx+c))^2}{15d} - \frac{2ab}{5d(\sin(dx+c))^5} - \frac{2a}{3d(\sin(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+b*sec(d*x+c))^2,x)
```

```
[Out] -8/15*a^2*cot(d*x+c)/d-1/5/d*a^2*cot(d*x+c)*csc(d*x+c)^4-4/15/d*a^2*cot(d*x
+c)*csc(d*x+c)^2-2/5/d*a*b/sin(d*x+c)^5-2/3/d*a*b/sin(d*x+c)^3-2/d*a*b/sin(
d*x+c)+2/d*a*b*ln(sec(d*x+c)+tan(d*x+c))-1/5/d*b^2/sin(d*x+c)^5/cos(d*x+c)-
2/5/d*b^2/sin(d*x+c)^3/cos(d*x+c)+8/5/d*b^2/sin(d*x+c)/cos(d*x+c)-16/5/d*b^
2*cot(d*x+c)
```

Maxima [A] time = 1.04026, size = 193, normalized size = 1.35

$$-\frac{ab\left(\frac{2\left(15\sin(dx+c)^4+5\sin(dx+c)^2+3\right)}{\sin(dx+c)^5}-15\log\left(\sin\left(dx+c\right)+1\right)+15\log\left(\sin\left(dx+c\right)-1\right)\right)+3b^2\left(\frac{15\tan(dx+c)^4+5\tan(dx+c)^2+1}{\tan(dx+c)^5}\right.}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/15*(a*b*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)/sin(d*x + c)^5 - 1
5*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 3*b^2*((15*tan(d*x +
c)^4 + 5*tan(d*x + c)^2 + 1)/tan(d*x + c)^5 - 5*tan(d*x + c)) + (15*tan(d*x
 + c)^4 + 10*tan(d*x + c)^2 + 3)*a^2/tan(d*x + c)^5)/d
```

Fricas [A] time = 1.86416, size = 629, normalized size = 4.4

$$-\frac{30\,ab\cos\left(dx+c\right)^5+8\left(a^2+6\,b^2\right)\cos\left(dx+c\right)^6-70\,ab\cos\left(dx+c\right)^3-20\left(a^2+6\,b^2\right)\cos\left(dx+c\right)^4+46\,ab\cos\left(dx+c\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/15*(30*a*b*cos(d*x + c)^5 + 8*(a^2 + 6*b^2)*cos(d*x + c)^6 - 70*a*b*cos(
d*x + c)^3 - 20*(a^2 + 6*b^2)*cos(d*x + c)^4 + 46*a*b*cos(d*x + c) + 15*(a^
2 + 6*b^2)*cos(d*x + c)^2 - 15*(a*b*cos(d*x + c)^5 - 2*a*b*cos(d*x + c)^3 +
 a*b*cos(d*x + c))*log(sin(d*x + c) + 1)*sin(d*x + c) + 15*(a*b*cos(d*x + c
)^5 - 2*a*b*cos(d*x + c)^3 + a*b*cos(d*x + c))*log(-sin(d*x + c) + 1)*sin(d
*x + c) - 15*b^2)/((d*cos(d*x + c)^5 - 2*d*cos(d*x + c)^3 + d*cos(d*x + c))
*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.36358, size = 440, normalized size = 3.08

$$\frac{3\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+25\,a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-70\,ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*a*b*tan(1/2*d*x + 1/2*c)^5 + 3*b^2*
tan(1/2*d*x + 1/2*c)^5 + 25*a^2*tan(1/2*d*x + 1/2*c)^3 - 70*a*b*tan(1/2*d*x
 + 1/2*c)^3 + 45*b^2*tan(1/2*d*x + 1/2*c)^3 + 960*a*b*log(abs(tan(1/2*d*x +
 1/2*c) + 1)) - 960*a*b*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 150*a^2*tan(1/
2*d*x + 1/2*c) - 660*a*b*tan(1/2*d*x + 1/2*c) + 570*b^2*tan(1/2*d*x + 1/2*c
) - 960*b^2*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (150*a^2*ta
n(1/2*d*x + 1/2*c)^4 + 660*a*b*tan(1/2*d*x + 1/2*c)^4 + 570*b^2*tan(1/2*d*x
 + 1/2*c)^4 + 25*a^2*tan(1/2*d*x + 1/2*c)^2 + 70*a*b*tan(1/2*d*x + 1/2*c)^2
 + 45*b^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2 + 6*a*b + 3*b^2)/tan(1/2*d*x + 1/2
*c)^5)/d
```

3.185 $\int (a+b\sec(c+dx))^3\sin^5(c+dx)\,dx$

Optimal. Leaf size=170

$$\frac{a(2a^2-3b^2)\cos^3(c+dx)}{3d}+\frac{b(6a^2-b^2)\cos^2(c+dx)}{2d}-\frac{a(a^2-6b^2)\cos(c+dx)}{d}-\frac{b(3a^2-2b^2)\log(\cos(c+dx))}{d}-\frac{3a^2}{}$$

```
[Out] -((a*(a^2 - 6*b^2)*Cos[c + d*x])/d) + (b*(6*a^2 - b^2)*Cos[c + d*x]^2)/(2*d
) + (a*(2*a^2 - 3*b^2)*Cos[c + d*x]^3)/(3*d) - (3*a^2*b*Cos[c + d*x]^4)/(4*
d) - (a^3*Cos[c + d*x]^5)/(5*d) - (b*(3*a^2 - 2*b^2)*Log[Cos[c + d*x]])/d +
 (3*a*b^2*Sec[c + d*x])/d + (b^3*Sec[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.255129, antiderivative size = 170, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2837, 12, 948}

$$\frac{a(2a^2-3b^2)\cos^3(c+dx)}{3d}+\frac{b(6a^2-b^2)\cos^2(c+dx)}{2d}-\frac{a(a^2-6b^2)\cos(c+dx)}{d}-\frac{b(3a^2-2b^2)\log(\cos(c+dx))}{d}-\frac{3a^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^5,x]
```

```
[Out] -((a*(a^2 - 6*b^2)*Cos[c + d*x])/d) + (b*(6*a^2 - b^2)*Cos[c + d*x]^2)/(2*d
) + (a*(2*a^2 - 3*b^2)*Cos[c + d*x]^3)/(3*d) - (3*a^2*b*Cos[c + d*x]^4)/(4*
d) - (a^3*Cos[c + d*x]^5)/(5*d) - (b*(3*a^2 - 2*b^2)*Log[Cos[c + d*x]])/d +
 (3*a*b^2*Sec[c + d*x])/d + (b^3*Sec[c + d*x]^2)/(2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int (a+b\sec(c+dx))^3\sin^5(c+dx)\,dx = -\int (-b-a\cos(c+dx))^3\sin^2(c+dx)\tan^3(c+dx)\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{a^3(-b+x)^3(a^2-x^2)^2}{x^3}dx, x, -a\cos(c+dx)\right)}{a^5d}$$

$$= \frac{\text{Subst}\left(\int \frac{(-b+x)^3(a^2-x^2)^2}{x^3}dx, x, -a\cos(c+dx)\right)}{a^2d}$$

$$= \frac{\text{Subst}\left(\int \left(a^4\left(1-\frac{6b^2}{a^2}\right)-\frac{a^4b^3}{x^3}+\frac{3a^4b^2}{x^2}+\frac{-3a^4b+2a^2b^3}{x}-b\left(-6a^2+b^2\right)x-\left(2a^2-\right.\right.\right.}{a^2d}$$

$$= -\frac{a\left(a^2-6b^2\right)\cos(c+dx)}{d}+\frac{b\left(6a^2-b^2\right)\cos^2(c+dx)}{2d}+\frac{a\left(2a^2-3b^2\right)\cos^3(c}{3d}$$

Mathematica [A] time = 0.63709, size = 154, normalized size = 0.91

$$-60a\left(5a^2-42b^2\right)\cos(c+dx)+60\left(9a^2b-2b^3\right)\cos(2(c+dx))-45a^2b\cos(4(c+dx))-1440a^2b\log(\cos(c+dx))+5$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^5,x]
```

```
[Out] (-60*a*(5*a^2 - 42*b^2)*Cos[c + d*x] + 60*(9*a^2*b - 2*b^3)*Cos[2*(c + d*x)
] + 50*a^3*Cos[3*(c + d*x)] - 120*a*b^2*Cos[3*(c + d*x)] - 45*a^2*b*Cos[4*(
c + d*x)] - 6*a^3*Cos[5*(c + d*x)] - 1440*a^2*b*Log[Cos[c + d*x]] + 960*b^3
*Log[Cos[c + d*x]] + 1440*a*b^2*Sec[c + d*x] + 240*b^3*Sec[c + d*x]^2)/(480
*d)
```

Maple [A] time = 0.049, size = 266, normalized size = 1.6

$$-\frac{8\,a^3\cos(dx+c)}{15\,d}-\frac{a^3\cos(dx+c)(\sin(dx+c))^4}{5\,d}-\frac{4\,a^3\cos(dx+c)(\sin(dx+c))^2}{15\,d}-\frac{3\,a^2b(\sin(dx+c))^4}{4\,d}-\frac{3\,a^2b\,(\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*sin(d*x+c)^5,x)
```

```
[Out] -8/15*a^3*cos(d*x+c)/d-1/5/d*a^3*cos(d*x+c)*sin(d*x+c)^4-4/15/d*a^3*cos(d*x
+c)*sin(d*x+c)^2-3/4/d*a^2*b*sin(d*x+c)^4-3/2/d*a^2*b*sin(d*x+c)^2-3*a^2*b*
ln(cos(d*x+c))/d+3/d*a*b^2*sin(d*x+c)^6/cos(d*x+c)+8/d*cos(d*x+c)*a*b^2+3/d
*a*b^2*sin(d*x+c)^4*cos(d*x+c)+4/d*a*b^2*cos(d*x+c)*sin(d*x+c)^2+1/2/d*b^3*
sin(d*x+c)^6/cos(d*x+c)^2+1/2/d*b^3*sin(d*x+c)^4+1/d*b^3*sin(d*x+c)^2+2/d*b
^3*ln(cos(d*x+c))
```

Maxima [A] time = 0.999803, size = 192, normalized size = 1.13

$$-\frac{12\,a^3\cos(dx+c)^5+45\,a^2b\cos(dx+c)^4-20\left(2\,a^3-3\,ab^2\right)\cos(dx+c)^3-30\left(6\,a^2b-b^3\right)\cos(dx+c)^2+60\left(a^3-6\right.}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] -1/60*(12*a^3*cos(d*x + c)^5 + 45*a^2*b*cos(d*x + c)^4 - 20*(2*a^3 - 3*a*b^
2)*cos(d*x + c)^3 - 30*(6*a^2*b - b^3)*cos(d*x + c)^2 + 60*(a^3 - 6*a*b^2)*
cos(d*x + c) + 60*(3*a^2*b - 2*b^3)*log(cos(d*x + c)) - 30*(6*a*b^2*cos(d*x
 + c) + b^3)/cos(d*x + c)^2)/d
```

Fricas [A] time = 1.92754, size = 437, normalized size = 2.57

$$-\frac{96\,a^3\cos\left(dx+c\right)^7+360\,a^2b\cos\left(dx+c\right)^6-160\left(2\,a^3-3\,ab^2\right)\cos\left(dx+c\right)^5-240\left(6\,a^2b-b^3\right)\cos\left(dx+c\right)^4-1440\,ab^2\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] -1/480*(96*a^3*cos(d*x + c)^7 + 360*a^2*b*cos(d*x + c)^6 - 160*(2*a^3 - 3*a
*b^2)*cos(d*x + c)^5 - 240*(6*a^2*b - b^3)*cos(d*x + c)^4 - 1440*a*b^2*cos(
d*x + c) + 480*(a^3 - 6*a*b^2)*cos(d*x + c)^3 + 480*(3*a^2*b - 2*b^3)*cos(d
*x + c)^2*log(-cos(d*x + c)) - 240*b^3 + 15*(39*a^2*b - 8*b^3)*cos(d*x + c)
^2)/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*sin(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34467, size = 938, normalized size = 5.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^5,x, algorithm="giac")
```

```
[Out] 1/60*(60*(3*a^2*b - 2*b^3)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 1)) - 60*(3*a^2*b - 2*b^3)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
- 1)) + 30*(9*a^2*b + 12*a*b^2 - 6*b^3 + 18*a^2*b*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 12*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 16*b^3*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^2*b*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2 - 6*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 1)^2 + (64*a^3 + 411*a^2*b - 600*a*b^2 - 274*b^
3 - 320*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2415*a^2*b*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 2640*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
) + 1490*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 640*a^3*(cos(d*x + c)
- 1)^2/(cos(d*x + c) + 1)^2 + 5910*a^2*b*(cos(d*x + c) - 1)^2/(cos(d*x + c)
```

```
 + 1)^2 - 3840*a*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3100*b^3*(
cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 5910*a^2*b*(cos(d*x + c) - 1)^3/
(cos(d*x + c) + 1)^3 + 2160*a*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3
 + 3100*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 2415*a^2*b*(cos(d*x
 + c) - 1)^4/(cos(d*x + c) + 1)^4 - 360*a*b^2*(cos(d*x + c) - 1)^4/(cos(d*x
 + c) + 1)^4 - 1490*b^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 411*a^2
*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 274*b^3*(cos(d*x + c) - 1)^5
/(cos(d*x + c) + 1)^5)/((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5)/d
```

3.186 $\int (a+b\sec(c+dx))^3 \sin^3(c+dx)\,dx$

Optimal. Leaf size=116

$$-\frac{a\left(a^2-3b^2\right)\cos(c+dx)}{d}-\frac{b\left(3a^2-b^2\right)\log(\cos(c+dx))}{d}+\frac{3a^2b\cos^2(c+dx)}{2d}+\frac{a^3\cos^3(c+dx)}{3d}+\frac{3ab^2\sec(c+dx)}{d}+b$$

```
[Out] -((a*(a^2 - 3*b^2)*Cos[c + d*x])/d) + (3*a^2*b*Cos[c + d*x]^2)/(2*d) + (a^3
*Cos[c + d*x]^3)/(3*d) - (b*(3*a^2 - b^2)*Log[Cos[c + d*x]])/d + (3*a*b^2*S
ec[c + d*x])/d + (b^3*Sec[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.128158, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3872, 2721, 894}

$$-\frac{a\left(a^2-3b^2\right)\cos(c+dx)}{d}-\frac{b\left(3a^2-b^2\right)\log(\cos(c+dx))}{d}+\frac{3a^2b\cos^2(c+dx)}{2d}+\frac{a^3\cos^3(c+dx)}{3d}+\frac{3ab^2\sec(c+dx)}{d}+b$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^3,x]
```

```
[Out] -((a*(a^2 - 3*b^2)*Cos[c + d*x])/d) + (3*a^2*b*Cos[c + d*x]^2)/(2*d) + (a^3
*Cos[c + d*x]^3)/(3*d) - (b*(3*a^2 - b^2)*Log[Cos[c + d*x]])/d + (3*a*b^2*S
ec[c + d*x])/d + (b^3*Sec[c + d*x]^2)/(2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int (a+b\sec(c+dx))^3\sin^3(c+dx)\,dx = -\int (-b-a\cos(c+dx))^3\tan^3(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(-b+x)^3(a^2-x^2)}{x^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{3b^2}{a^2}\right)-\frac{a^2b^3}{x^3}+\frac{3a^2b^2}{x^2}+\frac{-3a^2b+b^3}{x}+3bx-x^2\right)dx, x, -a\cos(c+\right.}{d}$$
$$= -\frac{a\left(a^2-3b^2\right)\cos(c+dx)}{d}+\frac{3a^2b\cos^2(c+dx)}{2d}+\frac{a^3\cos^3(c+dx)}{3d}-\frac{b\left(3a^2-\right.}{}$$

Mathematica [A] time = 0.33687, size = 102, normalized size = 0.88

$$\frac{-9a\left(a^2-4b^2\right)\cos(c+dx)+9a^2b\cos(2(c+dx))-36a^2b\log(\cos(c+dx))+a^3\cos(3(c+dx))+36ab^2\sec(c+dx)+6}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^3,x]
```

```
[Out] (-9*a*(a^2 - 4*b^2)*Cos[c + d*x] + 9*a^2*b*Cos[2*(c + d*x)] + a^3*Cos[3*(c
+ d*x)] - 36*a^2*b*Log[Cos[c + d*x]] + 12*b^3*Log[Cos[c + d*x]] + 36*a*b^2*
Sec[c + d*x] + 6*b^3*Sec[c + d*x]^2)/(12*d)
```

Maple [A] time = 0.044, size = 164, normalized size = 1.4

$$-\frac{a^3\cos(dx+c)(\sin(dx+c))^2}{3d}-\frac{2a^3\cos(dx+c)}{3d}-\frac{3a^2b(\sin(dx+c))^2}{2d}-3\frac{a^2b\ln(\cos(dx+c))}{d}+3\frac{ab^2(\sin(dx+c}{d\cos(dx+c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*sin(d*x+c)^3,x)
```

```
[Out] -1/3/d*a^3*cos(d*x+c)*sin(d*x+c)^2-2/3*a^3*cos(d*x+c)/d-3/2/d*a^2*b*sin(d*x
+c)^2-3*a^2*b*ln(cos(d*x+c))/d+3/d*a*b^2*sin(d*x+c)^4/cos(d*x+c)+3/d*a*b^2*
cos(d*x+c)*sin(d*x+c)^2+6/d*cos(d*x+c)*a*b^2+1/2/d*b^3*tan(d*x+c)^2+1/d*b^3
*ln(cos(d*x+c))
```

Maxima [A] time = 1.00247, size = 132, normalized size = 1.14

$$\frac{2a^3\cos(dx+c)^3+9a^2b\cos(dx+c)^2-6\left(a^3-3ab^2\right)\cos(dx+c)-6\left(3a^2b-b^3\right)\log(\cos(dx+c))+\frac{3\left(6ab^2\cos(dx+c)\right.}{\cos(dx+c)^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] 1/6*(2*a^3*cos(d*x + c)^3 + 9*a^2*b*cos(d*x + c)^2 - 6*(a^3 - 3*a*b^2)*cos(
d*x + c) - 6*(3*a^2*b - b^3)*log(cos(d*x + c)) + 3*(6*a*b^2*cos(d*x + c) +
b^3)/cos(d*x + c)^2)/d
```

Fricas [A] time = 1.76629, size = 300, normalized size = 2.59

$$\frac{4\,a^3\cos(dx+c)^5 + 18\,a^2b\cos(dx+c)^4 - 9\,a^2b\cos(dx+c)^2 + 36\,ab^2\cos(dx+c) - 12\left(a^3 - 3\,ab^2\right)\cos(dx+c)^3 - 12\left(3\,a\right.}{12\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*a^3*cos(d*x + c)^5 + 18*a^2*b*cos(d*x + c)^4 - 9*a^2*b*cos(d*x + c)
^2 + 36*a*b^2*cos(d*x + c) - 12*(a^3 - 3*a*b^2)*cos(d*x + c)^3 - 12*(3*a^2*
b - b^3)*cos(d*x + c)^2*log(-cos(d*x + c)) + 6*b^3)/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*sin(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36824, size = 173, normalized size = 1.49

$$-\frac{\left(3\,a^2b - b^3\right)\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d} + \frac{6\,ab^2\cos(dx+c) + b^3}{2\,d\cos(dx+c)^2} + \frac{2\,a^3d^8\cos(dx+c)^3 + 9\,a^2bd^8\cos(dx+c)^2 - 6\,a^3d^8\cos(dx+c}{6\,d^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] -(3*a^2*b - b^3)*log(abs(cos(d*x + c))/abs(d))/d + 1/2*(6*a*b^2*cos(d*x + c
) + b^3)/(d*cos(d*x + c)^2) + 1/6*(2*a^3*d^8*cos(d*x + c)^3 + 9*a^2*b*d^8*c
os(d*x + c)^2 - 6*a^3*d^8*cos(d*x + c) + 18*a*b^2*d^8*cos(d*x + c))/d^9
```

3.187 $\int (a + b\sec(c + dx))^3 \sin(c + dx)\, dx$

Optimal. Leaf size=64

$$-\frac{3a^2b\log(\cos(c+dx))}{d} - \frac{a^3\cos(c+dx)}{d} + \frac{3ab^2\sec(c+dx)}{d} + \frac{b^3\sec^2(c+dx)}{2d}$$

```
[Out] -((a^3*Cos[c + d*x])/d) - (3*a^2*b*Log[Cos[c + d*x]])/d + (3*a*b^2*Sec[c +
d*x])/d + (b^3*Sec[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.101173, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3872, 2833, 12, 43}

$$-\frac{3a^2b\log(\cos(c+dx))}{d} - \frac{a^3\cos(c+dx)}{d} + \frac{3ab^2\sec(c+dx)}{d} + \frac{b^3\sec^2(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x],x]
```

```
[Out] -((a^3*Cos[c + d*x])/d) - (3*a^2*b*Log[Cos[c + d*x]])/d + (3*a*b^2*Sec[c +
d*x])/d + (b^3*Sec[c + d*x]^2)/(2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int (a+b\sec(c+dx))^3\sin(c+dx)\,dx = -\int (-b-a\cos(c+dx))^3\sec^2(c+dx)\tan(c+dx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{a^3(-b+x)^3}{x^3}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{(-b+x)^3}{x^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^2\,\text{Subst}\left(\int \left(1-\frac{b^3}{x^3}+\frac{3b^2}{x^2}-\frac{3b}{x}\right)dx, x, -a\cos(c+dx)\right)}{d}$$
$$= -\frac{a^3\cos(c+dx)}{d}-\frac{3a^2b\log(\cos(c+dx))}{d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{b^3\sec^2(c+dx)}{2d}$$

Mathematica [A] time = 0.107135, size = 56, normalized size = 0.88

$$\frac{b\left(-6a^2\log(\cos(c+dx))+6ab\sec(c+dx)+b^2\sec^2(c+dx)\right)-2a^3\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x],x]
```

```
[Out] (-2*a^3*Cos[c + d*x] + b*(-6*a^2*Log[Cos[c + d*x]] + 6*a*b*Sec[c + d*x] + b
^2*Sec[c + d*x]^2))/(2*d)
```

Maple [A] time = 0.022, size = 65, normalized size = 1.

$$\frac{b^3(\sec(dx+c))^2}{2d}+3\,\frac{ab^2\sec(dx+c)}{d}+3\,\frac{a^2b\ln(\sec(dx+c))}{d}-\frac{a^3}{d\sec(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*sin(d*x+c),x)
```

```
[Out] 1/2*b^3*sec(d*x+c)^2/d+3*a*b^2*sec(d*x+c)/d+3/d*a^2*b*ln(sec(d*x+c))-1/d*a^
3/sec(d*x+c)
```

Maxima [A] time = 0.961768, size = 77, normalized size = 1.2

$$-\frac{2\,a^3\cos(dx+c)+6\,a^2b\log(\cos(dx+c))-\frac{6\,ab^2}{\cos(dx+c)}-\frac{b^3}{\cos(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a^3*cos(d*x + c) + 6*a^2*b*log(cos(d*x + c)) - 6*a*b^2/cos(d*x + c)
 - b^3/cos(d*x + c)^2)/d
```

Fricas [A] time = 1.82262, size = 163, normalized size = 2.55

$$-\frac{2\,a^3\cos\left(dx+c\right)^3+6\,a^2b\cos\left(dx+c\right)^2\log\left(-\cos\left(dx+c\right)\right)-6\,ab^2\cos\left(dx+c\right)-b^3}{2\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*cos(d*x + c)^3 + 6*a^2*b*cos(d*x + c)^2*log(-cos(d*x + c)) - 6*
a*b^2*cos(d*x + c) - b^3)/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sec\left(c+dx\right)\right)^3\sin\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*sin(d*x+c),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*sin(c + d*x), x)
```

Giac [A] time = 1.37943, size = 89, normalized size = 1.39

$$-\frac{a^3\cos\left(dx+c\right)}{d}-\frac{3\,a^2b\log\left(\frac{|\cos(dx+c)|}{|d|}\right)}{d}+\frac{6\,ab^2\cos\left(dx+c\right)+b^3}{2\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c),x, algorithm="giac")
```

```
[Out] -a^3*cos(d*x + c)/d - 3*a^2*b*log(abs(cos(d*x + c))/abs(d))/d + 1/2*(6*a*b^
2*cos(d*x + c) + b^3)/(d*cos(d*x + c)^2)
```

3.188 $\int \csc(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=102

$$-\frac{b\left(3a^2+b^2\right)\log(\cos(c+dx))}{d}+\frac{3ab^2\sec(c+dx)}{d}-\frac{(a-b)^3\log(\cos(c+dx)+1)}{2d}+\frac{(a+b)^3\log(1-\cos(c+dx))}{2d}+\frac{b^3\,\mathrm{se}}{}$$

```
[Out] ((a + b)^3*Log[1 - Cos[c + d*x]])/(2*d) - (b*(3*a^2 + b^2)*Log[Cos[c + d*x]
])/d - ((a - b)^3*Log[1 + Cos[c + d*x]])/(2*d) + (3*a*b^2*Sec[c + d*x])/d +
 (b^3*Sec[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.219213, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3872, 2837, 12, 1802}

$$-\frac{b\left(3a^2+b^2\right)\log(\cos(c+dx))}{d}+\frac{3ab^2\sec(c+dx)}{d}-\frac{(a-b)^3\log(\cos(c+dx)+1)}{2d}+\frac{(a+b)^3\log(1-\cos(c+dx))}{2d}+\frac{b^3\,\mathrm{se}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((a + b)^3*Log[1 - Cos[c + d*x]])/(2*d) - (b*(3*a^2 + b^2)*Log[Cos[c + d*x]
])/d - ((a - b)^3*Log[1 + Cos[c + d*x]])/(2*d) + (3*a*b^2*Sec[c + d*x])/d +
 (b^3*Sec[c + d*x]^2)/(2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \csc(c+dx)(a+b\sec(c+dx))^3\,dx = -\int(-b-a\cos(c+dx))^3\csc(c+dx)\sec^3(c+dx)\,dx$$
$$= \frac{a\,\mathrm{Subst}\left(\int \frac{a^3(-b+x)^3}{x^3(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^4\,\mathrm{Subst}\left(\int \frac{(-b+x)^3}{x^3(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{a^4\,\mathrm{Subst}\left(\int\left(\frac{(a-b)^3}{2a^4(a-x)} - \frac{b^3}{a^2x^3} + \frac{3b^2}{a^2x^2} + \frac{b(-3a^2-b^2)}{a^4x} + \frac{(a+b)^3}{2a^4(a+x)}\right)dx, x, -a\cos(c+dx\right.}{d}$$
$$= \frac{(a+b)^3\log(1-\cos(c+dx))}{2d} - \frac{b\left(3a^2+b^2\right)\log(\cos(c+dx))}{d} - \frac{(a-b)^3\log(1}{2}$$

Mathematica [A] time = 0.292273, size = 89, normalized size = 0.87

$$\frac{-2b\left(3a^2+b^2\right)\log(\cos(c+dx)) + 6ab^2\sec(c+dx) + 2(a+b)^3\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - 2(a-b)^3\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-2*(a - b)^3*Log[Cos[(c + d*x)/2]] - 2*b*(3*a^2 + b^2)*Log[Cos[c + d*x]] +
 2*(a + b)^3*Log[Sin[(c + d*x)/2]] + 6*a*b^2*Sec[c + d*x] + b^3*Sec[c + d*x
]^2)/(2*d)
```

Maple [A] time = 0.041, size = 113, normalized size = 1.1

$$\frac{a^3\ln(\csc(dx+c)-\cot(dx+c))}{d} + 3\,\frac{a^2b\ln(\tan(dx+c))}{d} + 3\,\frac{ab^2}{d\cos(dx+c)} + 3\,\frac{ab^2\ln(\csc(dx+c)-\cot(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*a^3*ln(csc(d*x+c)-cot(d*x+c))+3/d*a^2*b*ln(tan(d*x+c))+3/d*a*b^2/cos(d*
x+c)+3/d*a*b^2*ln(csc(d*x+c)-cot(d*x+c))+1/2/d*b^3/cos(d*x+c)^2+1/d*b^3*ln(
tan(d*x+c))
```

Maxima [A] time = 0.967473, size = 151, normalized size = 1.48

$$-\frac{\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\log(\cos(dx+c)+1) - \left(a^3+3\,a^2b+3\,ab^2+b^3\right)\log(\cos(dx+c)-1) + 2\left(3\,a^2b+b^3\right)\log(}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/2*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*log(cos(d*x + c) + 1) - (a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*log(cos(d*x + c) - 1) + 2*(3*a^2*b + b^3)*log(cos(d*x +
c)) - (6*a*b^2*cos(d*x + c) + b^3)/cos(d*x + c)^2)/d
```

Fricas [A] time = 1.93014, size = 354, normalized size = 3.47

$$\frac{6\,ab^2\cos(dx+c) - 2\left(3\,a^2b + b^3\right)\cos(dx+c)^2\log(-\cos(dx+c)) - \left(a^3 - 3\,a^2b + 3\,ab^2 - b^3\right)\cos(dx+c)^2\log\left(\frac{1}{2}\cos(d\right.}{2\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(6*a*b^2*cos(d*x + c) - 2*(3*a^2*b + b^3)*cos(d*x + c)^2*log(-cos(d*x +
 c)) - (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cos(d*x + c)^2*log(1/2*cos(d*x + c)
+ 1/2) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x + c)^2*log(-1/2*cos(d*x +
c) + 1/2) + b^3)/(d*cos(d*x + c)^2)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a + b\sec(c+dx))^3 \csc(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((a + b*sec(c + d*x))**3*csc(c + d*x), x)
```

Giac [B] time = 1.49241, size = 338, normalized size = 3.31

$$\frac{\left(a^3 + 3\,a^2b + 3\,ab^2 + b^3\right)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) - 2\left(3\,a^2b + b^3\right)\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1} - 1\right|\right) + \frac{9\,a^2b+12\,ab^2+3\,b^3+\frac{18\,a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{12\,ab^2(}{\cos}}{}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x
 + c) + 1)) - 2*(3*a^2*b + b^3)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) - 1)) + (9*a^2*b + 12*a*b^2 + 3*b^3 + 18*a^2*b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + 12*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b^3*(cos
(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^2*b*(cos(d*x + c) - 1)^2/(cos(d*x +
 c) + 1)^2 + 3*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

3.189 $\int \csc^3(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=162

$$-\frac{b\left(3a^2+2b^2\right)\log(\cos(c+dx))}{d}-\frac{a^2\csc^2(c+dx)\left(a\left(\frac{3b^2}{a^2}+1\right)\cos(c+dx)+b\left(\frac{b^2}{a^2}+3\right)\right)}{2d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{(a+b)}{}$$

```
[Out] -(a^2*(b*(3 + b^2/a^2) + a*(1 + (3*b^2)/a^2)*Cos[c + d*x])*Csc[c + d*x]^2)/
(2*d) + ((a + b)^2*(a + 4*b)*Log[1 - Cos[c + d*x]])/(4*d) - (b*(3*a^2 + 2*b
^2)*Log[Cos[c + d*x]])/d - ((a - 4*b)*(a - b)^2*Log[1 + Cos[c + d*x]])/(4*d
) + (3*a*b^2*Sec[c + d*x])/d + (b^3*Sec[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.348948, antiderivative size = 162, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2837, 12, 1805, 1802}

$$-\frac{b\left(3a^2+2b^2\right)\log(\cos(c+dx))}{d}-\frac{a^2\csc^2(c+dx)\left(a\left(\frac{3b^2}{a^2}+1\right)\cos(c+dx)+b\left(\frac{b^2}{a^2}+3\right)\right)}{2d}+\frac{3ab^2\sec(c+dx)}{d}+\frac{(a+b)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
```


```
[Out] -(a^2*(b*(3 + b^2/a^2) + a*(1 + (3*b^2)/a^2)*Cos[c + d*x])*Csc[c + d*x]^2)/
(2*d) + ((a + b)^2*(a + 4*b)*Log[1 - Cos[c + d*x]])/(4*d) - (b*(3*a^2 + 2*b
^2)*Log[Cos[c + d*x]])/d - ((a - 4*b)*(a - b)^2*Log[1 + Cos[c + d*x]])/(4*d
) + (3*a*b^2*Sec[c + d*x])/d + (b^3*Sec[c + d*x]^2)/(2*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
```

```
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^3\,dx = -\int (-b-a\cos(c+dx))^3\csc^3(c+dx)\sec^3(c+dx)\,dx$$

$$= \frac{a^3\,\text{Subst}\left(\int \frac{a^3(-b+x)^3}{x^3\left(a^2-x^2\right)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a^6\,\text{Subst}\left(\int \frac{(-b+x)^3}{x^3\left(a^2-x^2\right)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= -\frac{a^2\left(b\left(3+\frac{b^2}{a^2}\right)+a\left(1+\frac{3b^2}{a^2}\right)\cos(c+dx)\right)\csc^2(c+dx)}{2d} - \frac{a^4\,\text{Subst}\left(\int \frac{2b^3-6b^2x+2\ldots}{\ldots}\right.}{\ldots}$$

$$= -\frac{a^2\left(b\left(3+\frac{b^2}{a^2}\right)+a\left(1+\frac{3b^2}{a^2}\right)\cos(c+dx)\right)\csc^2(c+dx)}{2d} - \frac{a^4\,\text{Subst}\left(\int\left(-\frac{(a-4b)(a-\ldots}{2a^4(a-\ldots}\right.\right.}{\ldots}$$

$$= -\frac{a^2\left(b\left(3+\frac{b^2}{a^2}\right)+a\left(1+\frac{3b^2}{a^2}\right)\cos(c+dx)\right)\csc^2(c+dx)}{2d} + \frac{(a+b)^2(a+4b)\log(1\ldots}{4d}$$

Mathematica [B] time = 6.19619, size = 669, normalized size = 4.13

$$\frac{\left(-3a^2b+a^3+3ab^2-b^3\right)\cos^3(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)(a+b\sec(c+dx))^3}{8d(a\cos(c+dx)+b)^3} + \frac{\left(6a^2b-a^3-9ab^2+4b^3\right)\cos^3(c+dx)\log\left(\ldots\right.}{2d(a\cos(c+\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (3*a*b^2*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(d*(b + a*Cos[c + d*x])^3)
+ ((-a^3 - 3*a^2*b - 3*a*b^2 - b^3)*Cos[c + d*x]^3*Csc[(c + d*x)/2]^2*(a +
b*Sec[c + d*x])^3)/(8*d*(b + a*Cos[c + d*x])^3) + ((-a^3 + 6*a^2*b - 9*a*b^
2 + 4*b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2]]*(a + b*Sec[c + d*x])^3)/(2*
d*(b + a*Cos[c + d*x])^3) + ((-3*a^2*b - 2*b^3)*Cos[c + d*x]^3*Log[Cos[c +
d*x]]*(a + b*Sec[c + d*x])^3)/(d*(b + a*Cos[c + d*x])^3) + ((a^3 + 6*a^2*b
+ 9*a*b^2 + 4*b^3)*Cos[c + d*x]^3*Log[Sin[(c + d*x)/2]]*(a + b*Sec[c + d*x]
)^3)/(2*d*(b + a*Cos[c + d*x])^3) + ((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*Cos[c
+ d*x]^3*Sec[(c + d*x)/2]^2*(a + b*Sec[c + d*x])^3)/(8*d*(b + a*Cos[c + d*x
])^3) + (b^3*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(4*d*(b + a*Cos[c + d*x
])^3*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (3*a*b^2*Cos[c + d*x]^3*(a
+ b*Sec[c + d*x])^3*Sin[(c + d*x)/2])/(d*(b + a*Cos[c + d*x])^3*(Cos[(c + d
*x)/2] - Sin[(c + d*x)/2])) + (b^3*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(
4*d*(b + a*Cos[c + d*x])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) - (3*a*
b^2*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[(c + d*x)/2])/(d*(b + a*Cos[c
 + d*x])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))
```

Maple [A] time = 0.053, size = 201, normalized size = 1.2

$$-\frac{a^3\csc(dx+c)\cot(dx+c)}{2d}+\frac{a^3\ln(\csc(dx+c)-\cot(dx+c))}{2d}-\frac{3a^2b}{2d(\sin(dx+c))^2}+3\frac{a^2b\ln(\tan(dx+c))}{d}-\frac{}{2d(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*sec(d*x+c))^3,x)
```

```
[Out] -1/2/d*a^3*csc(d*x+c)*cot(d*x+c)+1/2/d*a^3*ln(csc(d*x+c)-cot(d*x+c))-3/2/d*
a^2*b/sin(d*x+c)^2+3/d*a^2*b*ln(tan(d*x+c))-3/2/d*a*b^2/sin(d*x+c)^2/cos(d*
x+c)+9/2/d*a*b^2/cos(d*x+c)+9/2/d*a*b^2*ln(csc(d*x+c)-cot(d*x+c))+1/2/d*b^3
/sin(d*x+c)^2/cos(d*x+c)^2-1/d*b^3/sin(d*x+c)^2+2/d*b^3*ln(tan(d*x+c))
```

Maxima [A] time = 1.02559, size = 231, normalized size = 1.43

$$-\frac{\left(a^3-6a^2b+9ab^2-4b^3\right)\log(\cos(dx+c)+1)-\left(a^3+6a^2b+9ab^2+4b^3\right)\log(\cos(dx+c)-1)+4\left(3a^2b+2b^3\right)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*((a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*log(cos(d*x + c) + 1) - (a^3 + 6*a^
2*b + 9*a*b^2 + 4*b^3)*log(cos(d*x + c) - 1) + 4*(3*a^2*b + 2*b^3)*log(cos(
d*x + c)) + 2*(6*a*b^2*cos(d*x + c) - (a^3 + 9*a*b^2)*cos(d*x + c)^3 + b^3
- (3*a^2*b + 2*b^3)*cos(d*x + c)^2)/(cos(d*x + c)^4 - cos(d*x + c)^2))/d
```

Fricas [A] time = 1.88695, size = 683, normalized size = 4.22

$$-\frac{12ab^2\cos(dx+c)-2\left(a^3+9ab^2\right)\cos(dx+c)^3+2b^3-2\left(3a^2b+2b^3\right)\cos(dx+c)^2+4\left(\left(3a^2b+2b^3\right)\cos(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(12*a*b^2*cos(d*x + c) - 2*(a^3 + 9*a*b^2)*cos(d*x + c)^3 + 2*b^3 - 2*
(3*a^2*b + 2*b^3)*cos(d*x + c)^2 + 4*((3*a^2*b + 2*b^3)*cos(d*x + c)^4 - (3
*a^2*b + 2*b^3)*cos(d*x + c)^2)*log(-cos(d*x + c)) + ((a^3 - 6*a^2*b + 9*a*
b^2 - 4*b^3)*cos(d*x + c)^4 - (a^3 - 6*a^2*b + 9*a*b^2 - 4*b^3)*cos(d*x + c
)^2)*log(1/2*cos(d*x + c) + 1/2) - ((a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cos(d
*x + c)^4 - (a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*cos(d*x + c)^2)*log(-1/2*cos(
d*x + c) + 1/2))/(d*cos(d*x + c)^4 - d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.5433, size = 651, normalized size = 4.02

$$-\frac{\frac{a^3(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{3\,a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{3\,ab^2(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{b^3(\cos(dx+c)-1)}{\cos(dx+c)+1} - 2\left(a^3 + 6\,a^2b + 9\,ab^2 + 4\,b^3\right)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) + 8\left(\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/8*(a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*a^2*b*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 3*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b^3*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)
*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) + 8*(3*a^2*b + 2*b^3)*lo
g(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (a^3 + 3*a^2*b + 3*a*b
^2 + b^3 - 2*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*a^2*b*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) - 18*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) - 8*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/(cos(d
*x + c) - 1) - 4*(9*a^2*b + 12*a*b^2 + 6*b^3 + 18*a^2*b*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1) + 12*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*b^3
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^2*b*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 + 6*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + 1)^2)/d
```

3.190 $\int (a+b\sec(c+dx))^3\sin^6(c+dx)\,dx$

Optimal. Leaf size=299

$$-\frac{3a^2b\sin^5(c+dx)}{5d}-\frac{a^2b\sin^3(c+dx)}{d}-\frac{3a^2b\sin(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\sin^5(c+dx)\cos(c+dx)}{6d}-$$

```
[Out] (5*a^3*x)/16 - (45*a*b^2*x)/8 + (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (5*b^3*
ArcTanh[Sin[c + d*x]])/(2*d) - (3*a^2*b*Sin[c + d*x])/d + (5*b^3*Sin[c + d*
x])/(2*d) - (5*a^3*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (a^2*b*Sin[c + d*x]^
3)/d + (5*b^3*Sin[c + d*x]^3)/(6*d) - (5*a^3*Cos[c + d*x]*Sin[c + d*x]^3)/(
24*d) - (3*a^2*b*Sin[c + d*x]^5)/(5*d) - (a^3*Cos[c + d*x]*Sin[c + d*x]^5)/
(6*d) + (45*a*b^2*Tan[c + d*x])/(8*d) - (15*a*b^2*Sin[c + d*x]^2*Tan[c + d*
x])/(8*d) - (3*a*b^2*Sin[c + d*x]^4*Tan[c + d*x])/(4*d) + (b^3*Sin[c + d*x]
^3*Tan[c + d*x]^2)/(2*d)
```

Rubi [A] time = 0.335932, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 21, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {3872, 2912, 2635, 8, 2592, 302, 206, 2591, 288, 321, 203}

$$-\frac{3a^2b\sin^5(c+dx)}{5d}-\frac{a^2b\sin^3(c+dx)}{d}-\frac{3a^2b\sin(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\sin^5(c+dx)\cos(c+dx)}{6d}-$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^6,x]
```

```
[Out] (5*a^3*x)/16 - (45*a*b^2*x)/8 + (3*a^2*b*ArcTanh[Sin[c + d*x]])/d - (5*b^3*
ArcTanh[Sin[c + d*x]])/(2*d) - (3*a^2*b*Sin[c + d*x])/d + (5*b^3*Sin[c + d*
x])/(2*d) - (5*a^3*Cos[c + d*x]*Sin[c + d*x])/(16*d) - (a^2*b*Sin[c + d*x]^
3)/d + (5*b^3*Sin[c + d*x]^3)/(6*d) - (5*a^3*Cos[c + d*x]*Sin[c + d*x]^3)/(
24*d) - (3*a^2*b*Sin[c + d*x]^5)/(5*d) - (a^3*Cos[c + d*x]*Sin[c + d*x]^5)/
(6*d) + (45*a*b^2*Tan[c + d*x])/(8*d) - (15*a*b^2*Sin[c + d*x]^2*Tan[c + d*
x])/(8*d) - (3*a*b^2*Sin[c + d*x]^4*Tan[c + d*x])/(4*d) + (b^3*Sin[c + d*x]
^3*Tan[c + d*x]^2)/(2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int (a+b\sec(c+dx))^3\sin^6(c+dx)\,dx = -\int (-b-a\cos(c+dx))^3\sin^3(c+dx)\tan^3(c+dx)\,dx$$
$$= -\int \left(-a^3\sin^6(c+dx) - 3a^2b\sin^5(c+dx)\tan(c+dx) - 3ab^2\sin^4(c+dx)\tan\right.$$
$$= a^3\int \sin^6(c+dx)\,dx + \left(3a^2b\right)\int \sin^5(c+dx)\tan(c+dx)\,dx + \left(3ab^2\right)\int \sin^4$$
$$= -\frac{a^3\cos(c+dx)\sin^5(c+dx)}{6d} + \frac{1}{6}\left(5a^3\right)\int \sin^4(c+dx)\,dx + \frac{\left(3a^2b\right)\text{Subst}\left(\int\right.}{}$$
$$= -\frac{5a^3\cos(c+dx)\sin^3(c+dx)}{24d} - \frac{a^3\cos(c+dx)\sin^5(c+dx)}{6d} - \frac{3ab^2\sin^4(c+}{4}$$
$$= -\frac{3a^2b\sin(c+dx)}{d} - \frac{5a^3\cos(c+dx)\sin(c+dx)}{16d} - \frac{a^2b\sin^3(c+dx)}{d} - \frac{5a^3\cos}{}$$
$$= \frac{5a^3x}{16} + \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d} - \frac{3a^2b\sin(c+dx)}{d} + \frac{5b^3\sin(c+dx)}{2d} - \frac{5a^3}{}$$
$$= \frac{5a^3x}{16} - \frac{45}{8}ab^2x + \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d} - \frac{5b^3\tanh^{-1}(\sin(c+dx))}{2d} - \frac{3a^2}{}$$

Mathematica [B] time = 6.24289, size = 818, normalized size = 2.74

$$\frac{\left(5b^3-6a^2b\right)\cos^3(c+dx)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)(a+b\sec(c+dx))^3}{2d(b+a\cos(c+dx))^3} + \frac{\left(6a^2b-5b^3\right)\cos^3(c+dx)\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^6,x]
```

```
[Out] (5*a*(a^2 - 18*b^2)*(c + d*x)*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(16*d*
(b + a*Cos[c + d*x])^3) + ((-6*a^2*b + 5*b^3)*Cos[c + d*x]^3*Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]]*(a + b*Sec[c + d*x])^3)/(2*d*(b + a*Cos[c + d*x]
)^3) + ((6*a^2*b - 5*b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]]*(a + b*Sec[c + d*x])^3)/(2*d*(b + a*Cos[c + d*x])^3) + (b^3*Cos[c +
d*x]^3*(a + b*Sec[c + d*x])^3)/(4*d*(b + a*Cos[c + d*x])^3*(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])^2) + (3*a*b^2*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*S
in[(c + d*x)/2])/(d*(b + a*Cos[c + d*x])^3*(Cos[(c + d*x)/2] - Sin[(c + d*x
)/2])) - (b^3*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(4*d*(b + a*Cos[c + d*
x])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (3*a*b^2*Cos[c + d*x]^3*(a
 + b*Sec[c + d*x])^3*Sin[(c + d*x)/2])/(d*(b + a*Cos[c + d*x])^3*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])) + (3*b*(-11*a^2 + 6*b^2)*Cos[c + d*x]^3*(a + b
*Sec[c + d*x])^3*Sin[c + d*x])/(8*d*(b + a*Cos[c + d*x])^3) - (3*a*(5*a^2 -
 32*b^2)*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[2*(c + d*x)])/(64*d*(b +
 a*Cos[c + d*x])^3) - (b*(-21*a^2 + 4*b^2)*Cos[c + d*x]^3*(a + b*Sec[c + d*
x])^3*Sin[3*(c + d*x)])/(48*d*(b + a*Cos[c + d*x])^3) + (3*a*(a^2 - 2*b^2)*
Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[4*(c + d*x)])/(64*d*(b + a*Cos[c
+ d*x])^3) - (3*a^2*b*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[5*(c + d*x)
])/(80*d*(b + a*Cos[c + d*x])^3) - (a^3*Cos[c + d*x]^3*(a + b*Sec[c + d*x])
^3*Sin[6*(c + d*x)])/(192*d*(b + a*Cos[c + d*x])^3)
```

Maple [A] time = 0.051, size = 354, normalized size = 1.2

$$-\frac{a^3\cos(dx+c)(\sin(dx+c))^5}{6d} - \frac{5a^3\cos(dx+c)(\sin(dx+c))^3}{24d} - \frac{5a^3\cos(dx+c)\sin(dx+c)}{16d} + \frac{5a^3x}{16} + \frac{5a^3c}{16d} - 3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*sin(d*x+c)^6,x)
```

```
[Out] -1/6*a^3*cos(d*x+c)*sin(d*x+c)^5/d-5/24*a^3*cos(d*x+c)*sin(d*x+c)^3/d-5/16*
a^3*cos(d*x+c)*sin(d*x+c)/d+5/16*a^3*x+5/16/d*a^3*c-3/5*a^2*b*sin(d*x+c)^5/
d-a^2*b*sin(d*x+c)^3/d-3*a^2*b*sin(d*x+c)/d+3/d*a^2*b*ln(sec(d*x+c)+tan(d*x
+c))+3/d*a*b^2*sin(d*x+c)^7/cos(d*x+c)+3/d*a*b^2*sin(d*x+c)^5*cos(d*x+c)+15
/4/d*a*b^2*cos(d*x+c)*sin(d*x+c)^3+45/8/d*cos(d*x+c)*sin(d*x+c)*a*b^2-45/8*
a*b^2*x-45/8/d*a*b^2*c+1/2/d*b^3*sin(d*x+c)^7/cos(d*x+c)^2+1/2/d*b^3*sin(d*
x+c)^5+5/6*b^3*sin(d*x+c)^3/d+5/2*b^3*sin(d*x+c)/d-5/2/d*b^3*ln(sec(d*x+c)+
tan(d*x+c))
```

Maxima [A] time = 1.53197, size = 327, normalized size = 1.09

$$5\left(4\,\sin\left(2\,dx+2\,c\right)^{3}+60\,dx+60\,c+9\,\sin\left(4\,dx+4\,c\right)-48\,\sin\left(2\,dx+2\,c\right)\right)a^{3}-96\left(6\,\sin\left(dx+c\right)^{5}+10\,\sin\left(dx+c\right)^{3}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="maxima")
```

```
[Out] 1/960*(5*(4*sin(2*d*x + 2*c)^3 + 60*d*x + 60*c + 9*sin(4*d*x + 4*c) - 48*si
n(2*d*x + 2*c))*a^3 - 96*(6*sin(d*x + c)^5 + 10*sin(d*x + c)^3 - 15*log(sin
(d*x + c) + 1) + 15*log(sin(d*x + c) - 1) + 30*sin(d*x + c))*a^2*b - 360*(1
5*d*x + 15*c - (9*tan(d*x + c)^3 + 7*tan(d*x + c))/(tan(d*x + c)^4 + 2*tan(
d*x + c)^2 + 1) - 8*tan(d*x + c))*a*b^2 + 80*(4*sin(d*x + c)^3 - 6*sin(d*x
+ c)/(sin(d*x + c)^2 - 1) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c)
- 1) + 24*sin(d*x + c))*b^3)/d
```

Fricas [A] time = 2.01143, size = 605, normalized size = 2.02

$$75\left(a^{3}-18\,ab^{2}\right)dx\cos\left(dx+c\right)^{2}+60\left(6\,a^{2}b-5\,b^{3}\right)\cos\left(dx+c\right)^{2}\log\left(\sin\left(dx+c\right)+1\right)-60\left(6\,a^{2}b-5\,b^{3}\right)\cos\left(dx+c\right)^{2}\log$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="fricas")
```

```
[Out] 1/240*(75*(a^3 - 18*a*b^2)*d*x*cos(d*x + c)^2 + 60*(6*a^2*b - 5*b^3)*cos(d*
x + c)^2*log(sin(d*x + c) + 1) - 60*(6*a^2*b - 5*b^3)*cos(d*x + c)^2*log(-s
in(d*x + c) + 1) - (40*a^3*cos(d*x + c)^7 + 144*a^2*b*cos(d*x + c)^6 - 10*(
13*a^3 - 18*a*b^2)*cos(d*x + c)^5 - 16*(33*a^2*b - 5*b^3)*cos(d*x + c)^4 -
720*a*b^2*cos(d*x + c) + 15*(11*a^3 - 54*a*b^2)*cos(d*x + c)^3 - 120*b^3 +
16*(69*a^2*b - 35*b^3)*cos(d*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*sin(d*x+c)**6,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.54781, size = 760, normalized size = 2.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^6,x, algorithm="giac")
```

```
[Out] 1/240*(75*(a^3 - 18*a*b^2)*(d*x + c) + 120*(6*a^2*b - 5*b^3)*log(abs(tan(1/
2*d*x + 1/2*c) + 1)) - 120*(6*a^2*b - 5*b^3)*log(abs(tan(1/2*d*x + 1/2*c) -
 1)) - 240*(6*a*b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6
*a*b^2*tan(1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*
c)^2 - 1)^2 + 2*(75*a^3*tan(1/2*d*x + 1/2*c)^11 - 720*a^2*b*tan(1/2*d*x + 1
/2*c)^11 - 630*a*b^2*tan(1/2*d*x + 1/2*c)^11 + 480*b^3*tan(1/2*d*x + 1/2*c)
^11 + 425*a^3*tan(1/2*d*x + 1/2*c)^9 - 4560*a^2*b*tan(1/2*d*x + 1/2*c)^9 -
2610*a*b^2*tan(1/2*d*x + 1/2*c)^9 + 2720*b^3*tan(1/2*d*x + 1/2*c)^9 + 990*a
^3*tan(1/2*d*x + 1/2*c)^7 - 12384*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 1980*a*b^2
*tan(1/2*d*x + 1/2*c)^7 + 5760*b^3*tan(1/2*d*x + 1/2*c)^7 - 990*a^3*tan(1/2
*d*x + 1/2*c)^5 - 12384*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 1980*a*b^2*tan(1/2*d
*x + 1/2*c)^5 + 5760*b^3*tan(1/2*d*x + 1/2*c)^5 - 425*a^3*tan(1/2*d*x + 1/2
*c)^3 - 4560*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 2610*a*b^2*tan(1/2*d*x + 1/2*c)
^3 + 2720*b^3*tan(1/2*d*x + 1/2*c)^3 - 75*a^3*tan(1/2*d*x + 1/2*c) - 720*a^
2*b*tan(1/2*d*x + 1/2*c) + 630*a*b^2*tan(1/2*d*x + 1/2*c) + 480*b^3*tan(1/2
*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

3.191 $\int (a + b\sec(c + dx))^3 \sin^4(c + dx)\, dx$

Optimal. Leaf size=236

$$-\frac{b(17a^2 - b^2)\sin(c + dx)}{2d} + \frac{3b(2a^2 - b^2)\tanh^{-1}(\sin(c + dx))}{2d} - \frac{(4a^2 - b^2)\sin(c + dx)(a\cos(c + dx) + b)^3}{4b^2d} - \frac{(6a^2 - b^2)}{}$$

```
[Out] (3*a*(a^2 - 12*b^2)*x)/8 + (3*b*(2*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(2*d)
- (b*(17*a^2 - b^2)*Sin[c + d*x])/(2*d) - (a*(21*a^2 - 2*b^2)*Cos[c + d*x]*
Sin[c + d*x])/(8*d) - ((6*a^2 - b^2)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/(
4*b*d) - ((4*a^2 - b^2)*(b + a*Cos[c + d*x])^3*Sin[c + d*x])/(4*b^2*d) + (a
*(b + a*Cos[c + d*x])^4*Tan[c + d*x])/(b^2*d) + ((b + a*Cos[c + d*x])^4*Sec
[c + d*x]*Tan[c + d*x])/(2*b*d)
```

Rubi [A] time = 0.74808, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2893, 3049, 3033, 3023, 2735, 3770}

$$-\frac{b(17a^2 - b^2)\sin(c + dx)}{2d} + \frac{3b(2a^2 - b^2)\tanh^{-1}(\sin(c + dx))}{2d} - \frac{(4a^2 - b^2)\sin(c + dx)(a\cos(c + dx) + b)^3}{4b^2d} - \frac{(6a^2 - b^2)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^4,x]
```


```
[Out] (3*a*(a^2 - 12*b^2)*x)/8 + (3*b*(2*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(2*d)
- (b*(17*a^2 - b^2)*Sin[c + d*x])/(2*d) - (a*(21*a^2 - 2*b^2)*Cos[c + d*x]*
Sin[c + d*x])/(8*d) - ((6*a^2 - b^2)*(b + a*Cos[c + d*x])^2*Sin[c + d*x])/(
4*b*d) - ((4*a^2 - b^2)*(b + a*Cos[c + d*x])^3*Sin[c + d*x])/(4*b^2*d) + (a
*(b + a*Cos[c + d*x])^4*Tan[c + d*x])/(b^2*d) + ((b + a*Cos[c + d*x])^4*Sec
[c + d*x]*Tan[c + d*x])/(2*b*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2893

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(a + b
*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*d*f*(n + 1)), x] + (-Di
st[1/(a^2*d^2*(n + 1)*(n + 2)), Int[(a + b*Sin[e + f*x])^m*(d*Sin[e + f*x])
^(n + 2)*Simp[a^2*n*(n + 2) - b^2*(m + n + 2)*(m + n + 3) + a*b*m*Sin[e + f
*x] - (a^2*(n + 1)*(n + 2) - b^2*(m + n + 2)*(m + n + 4))*Sin[e + f*x]^2, x
], x], x] - Simp[(b*(m + n + 2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*(
d*Sin[e + f*x])^(n + 2))/(a^2*d^2*f*(n + 1)*(n + 2)), x]) /; FreeQ[{a, b, d
, e, f, m}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
&&  !m < -1 && LtQ[n, -1] && (LtQ[n, -2] || EqQ[m + n + 4, 0])
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
```

```
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^3\sin^4(c+dx)\,dx = -\int (-b-a\cos(c+dx))^3\sin(c+dx)\tan^3(c+dx)\,dx$$
$$= \frac{a(b+a\cos(c+dx))^4\tan(c+dx)}{b^2d} + \frac{(b+a\cos(c+dx))^4\sec(c+dx)\tan(c+dx)}{2bd}$$
$$= -\frac{(4a^2-b^2)(b+a\cos(c+dx))^3\sin(c+dx)}{4b^2d} + \frac{a(b+a\cos(c+dx))^4\tan(c+dx)}{b^2d}$$
$$= -\frac{(6a^2-b^2)(b+a\cos(c+dx))^2\sin(c+dx)}{4bd} - \frac{(4a^2-b^2)(b+a\cos(c+dx))^3\sin}{4b^2d}$$
$$= -\frac{a(21a^2-2b^2)\cos(c+dx)\sin(c+dx)}{8d} - \frac{(6a^2-b^2)(b+a\cos(c+dx))^2\sin(c+}{4bd}$$
$$= -\frac{b(17a^2-b^2)\sin(c+dx)}{2d} - \frac{a(21a^2-2b^2)\cos(c+dx)\sin(c+dx)}{8d} - \frac{(6a^2-b^2)}{}$$
$$= \frac{3}{8}a(a^2-12b^2)x - \frac{b(17a^2-b^2)\sin(c+dx)}{2d} - \frac{a(21a^2-2b^2)\cos(c+dx)\sin(c+}{8d}$$
$$= \frac{3}{8}a(a^2-12b^2)x + \frac{3b(2a^2-b^2)\tanh^{-1}(\sin(c+dx))}{2d} - \frac{b(17a^2-b^2)\sin(c+dx)}{2d}$$

Mathematica [B] time = 6.16191, size = 696, normalized size = 2.95

$$\frac{3a(a^2-12b^2)(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^3}{8d(a\cos(c+dx)+b)^3} + \frac{b(4b^2-15a^2)\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^3}{4d(a\cos(c+dx)+b)^3} - a$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^4,x]
```

```
[Out] (3*a*(a^2 - 12*b^2)*(c + d*x)*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(8*d*(
b + a*Cos[c + d*x])^3) + (3*(-2*a^2*b + b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*
x)/2] - Sin[(c + d*x)/2]]*(a + b*Sec[c + d*x])^3)/(2*d*(b + a*Cos[c + d*x])
^3) - (3*(-2*a^2*b + b^3)*Cos[c + d*x]^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]]*(a + b*Sec[c + d*x])^3)/(2*d*(b + a*Cos[c + d*x])^3) + (b^3*Cos[c +
d*x]^3*(a + b*Sec[c + d*x])^3)/(4*d*(b + a*Cos[c + d*x])^3*(Cos[(c + d*x)/2
] - Sin[(c + d*x)/2])^2) + (3*a*b^2*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*S
in[(c + d*x)/2])/(d*(b + a*Cos[c + d*x])^3*(Cos[(c + d*x)/2] - Sin[(c + d*x
)/2])) - (b^3*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3)/(4*d*(b + a*Cos[c + d*
x])^3*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (3*a*b^2*Cos[c + d*x]^3*(a
 + b*Sec[c + d*x])^3*Sin[(c + d*x)/2])/(d*(b + a*Cos[c + d*x])^3*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2])) + (b*(-15*a^2 + 4*b^2)*Cos[c + d*x]^3*(a + b*S
ec[c + d*x])^3*Sin[c + d*x])/(4*d*(b + a*Cos[c + d*x])^3) - (a*(a^2 - 3*b^2
)*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[2*(c + d*x)])/(4*d*(b + a*Cos[c
 + d*x])^3) + (a^2*b*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[3*(c + d*x)]
)/(4*d*(b + a*Cos[c + d*x])^3) + (a^3*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3
*Sin[4*(c + d*x)])/(32*d*(b + a*Cos[c + d*x])^3)
```

Maple [A] time = 0.047, size = 276, normalized size = 1.2

$$-\frac{a^3\cos(dx+c)(\sin(dx+c))^3}{4d} - \frac{3a^3\cos(dx+c)\sin(dx+c)}{8d} + \frac{3a^3x}{8} + \frac{3a^3c}{8d} - \frac{a^2b(\sin(dx+c))^3}{d} + 3\frac{a^2b\ln(\sec(dx+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*sin(d*x+c)^4,x)
```

```
[Out] -1/4*a^3*cos(d*x+c)*sin(d*x+c)^3/d-3/8*a^3*cos(d*x+c)*sin(d*x+c)/d+3/8*a^3*
x+3/8/d*a^3*c-a^2*b*sin(d*x+c)^3/d+3/d*a^2*b*ln(sec(d*x+c)+tan(d*x+c))-3*a^
2*b*sin(d*x+c)/d+3/d*a*b^2*sin(d*x+c)^5/cos(d*x+c)+3/d*a*b^2*cos(d*x+c)*sin
(d*x+c)^3+9/2/d*cos(d*x+c)*sin(d*x+c)*a*b^2-9/2*a*b^2*x-9/2/d*a*b^2*c+1/2/d
*b^3*sin(d*x+c)^5/cos(d*x+c)^2+1/2*b^3*sin(d*x+c)^3/d+3/2*b^3*sin(d*x+c)/d-
3/2/d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.48542, size = 247, normalized size = 1.05

$$\frac{(12\,dx + 12\,c + \sin(4\,dx + 4\,c) - 8\,\sin(2\,dx + 2\,c))a^3 - 16\left(2\,\sin(dx + c)^3 - 3\,\log(\sin(dx + c) + 1) + 3\,\log(\sin(dx + \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/32*((12*d*x + 12*c + sin(4*d*x + 4*c) - 8*sin(2*d*x + 2*c))*a^3 - 16*(2*s
in(d*x + c)^3 - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1) + 6*sin(d
*x + c))*a^2*b - 48*(3*d*x + 3*c - tan(d*x + c)/(tan(d*x + c)^2 + 1) - 2*ta
n(d*x + c))*a*b^2 - 8*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + 3*log(sin(
d*x + c) + 1) - 3*log(sin(d*x + c) - 1) - 4*sin(d*x + c)))/d
```

Fricas [A] time = 1.9365, size = 466, normalized size = 1.97

$$\frac{3\left(a^3 - 12\,ab^2\right)dx\cos(dx + c)^2 + 6\left(2\,a^2b - b^3\right)\cos(dx + c)^2\log(\sin(dx + c) + 1) - 6\left(2\,a^2b - b^3\right)\cos(dx + c)^2\log(- \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(3*(a^3 - 12*a*b^2)*d*x*cos(d*x + c)^2 + 6*(2*a^2*b - b^3)*cos(d*x + c)
^2*log(sin(d*x + c) + 1) - 6*(2*a^2*b - b^3)*cos(d*x + c)^2*log(-sin(d*x +
c) + 1) + (2*a^3*cos(d*x + c)^5 + 8*a^2*b*cos(d*x + c)^4 + 24*a*b^2*cos(d*x
 + c) - (5*a^3 - 12*a*b^2)*cos(d*x + c)^3 + 4*b^3 - 8*(4*a^2*b - b^3)*cos(d
*x + c)^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*sin(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.53695, size = 582, normalized size = 2.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^4,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^3 - 12*a*b^2)*(d*x + c) + 12*(2*a^2*b - b^3)*log(abs(tan(1/2*d*x
+ 1/2*c) + 1)) - 12*(2*a^2*b - b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 8*
(6*a*b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(
1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2
 + 2*(3*a^3*tan(1/2*d*x + 1/2*c)^7 - 24*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 12*a
*b^2*tan(1/2*d*x + 1/2*c)^7 + 8*b^3*tan(1/2*d*x + 1/2*c)^7 + 11*a^3*tan(1/2
*d*x + 1/2*c)^5 - 104*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 12*a*b^2*tan(1/2*d*x +
 1/2*c)^5 + 24*b^3*tan(1/2*d*x + 1/2*c)^5 - 11*a^3*tan(1/2*d*x + 1/2*c)^3 -
 104*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 24*b^
3*tan(1/2*d*x + 1/2*c)^3 - 3*a^3*tan(1/2*d*x + 1/2*c) - 24*a^2*b*tan(1/2*d*
x + 1/2*c) + 12*a*b^2*tan(1/2*d*x + 1/2*c) + 8*b^3*tan(1/2*d*x + 1/2*c))/(t
an(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

3.192 $\int (a+b\sec(c+dx))^3 \sin^2(c+dx)\,dx$

Optimal. Leaf size=138

$$\frac{b\left(6a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{1}{2}ax\left(a^2-6b^2\right)-\frac{15a^2b\sin(c+dx)}{2d}-\frac{5a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a\tan(c+dx)}{}$$

```
[Out] (a*(a^2 - 6*b^2)*x)/2 + (b*(6*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(2*d) - (15
*a^2*b*Sin[c + d*x])/(2*d) - (5*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (3*a
*(b + a*Cos[c + d*x])^2*Tan[c + d*x])/(2*d) + ((b + a*Cos[c + d*x])^3*Sec[c
 + d*x]*Tan[c + d*x])/(2*d)
```

Rubi [A] time = 0.503821, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2889, 3048, 3047, 3033, 3023, 2735, 3770}

$$\frac{b\left(6a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{1}{2}ax\left(a^2-6b^2\right)-\frac{15a^2b\sin(c+dx)}{2d}-\frac{5a^3\sin(c+dx)\cos(c+dx)}{2d}+\frac{3a\tan(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^2,x]
```


```
[Out] (a*(a^2 - 6*b^2)*x)/2 + (b*(6*a^2 - b^2)*ArcTanh[Sin[c + d*x]])/(2*d) - (15
*a^2*b*Sin[c + d*x])/(2*d) - (5*a^3*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (3*a
*(b + a*Cos[c + d*x])^2*Tan[c + d*x])/(2*d) + ((b + a*Cos[c + d*x])^3*Sec[c
 + d*x]*Tan[c + d*x])/(2*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3033

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f
_.)*(x_)]^2), x_Symbol] :> -Simp[(C*d*Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[
e + f*x])^(m + 1))/(b*f*(m + 3)), x] + Dist[1/(b*(m + 3)), Int[(a + b*Sin[e
 + f*x])^m*Simp[a*C*d + A*b*c*(m + 3) + b*(B*c*(m + 3) + d*(C*(m + 2) + A*(
m + 3)))*Sin[e + f*x] - (2*a*C*d - b*(c*C + B*d)*(m + 3))*Sin[e + f*x]^2, x
], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, m}, x] && NeQ[b*c - a*d, 0]
&& NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^3\sin^2(c+dx)\,dx = -\int (-b-a\cos(c+dx))^3\sec(c+dx)\tan^2(c+dx)\,dx$$
$$= -\int (-b-a\cos(c+dx))^3\left(1-\cos^2(c+dx)\right)\sec^3(c+dx)\,dx$$
$$= \frac{(b+a\cos(c+dx))^3\sec(c+dx)\tan(c+dx)}{2d} - \frac{1}{2}\int (-b-a\cos(c+dx))^2\left(-3a\right.$$
$$= \frac{3a(b+a\cos(c+dx))^2\tan(c+dx)}{2d} + \frac{(b+a\cos(c+dx))^3\sec(c+dx)\tan(c+\right.}{2d}$$
$$= -\frac{5a^3\cos(c+dx)\sin(c+dx)}{2d} + \frac{3a(b+a\cos(c+dx))^2\tan(c+dx)}{2d} + \frac{(b+a\,\mathrm{c}}{}$$
$$= -\frac{15a^2b\sin(c+dx)}{2d} - \frac{5a^3\cos(c+dx)\sin(c+dx)}{2d} + \frac{3a(b+a\cos(c+dx))^2\,\mathrm{ta}}{2d}$$
$$= \frac{1}{2}a\left(a^2-6b^2\right)x - \frac{15a^2b\sin(c+dx)}{2d} - \frac{5a^3\cos(c+dx)\sin(c+dx)}{2d} + \frac{3a(b+a}{}$$
$$= \frac{1}{2}a\left(a^2-6b^2\right)x + \frac{b\left(6a^2-b^2\right)\tanh^{-1}(\sin(c+dx))}{2d} - \frac{15a^2b\sin(c+dx)}{2d} - \frac{5a^3}{}$$

Mathematica [B] time = 0.865642, size = 327, normalized size = 2.37

$$\sec^2(c+dx)\left(\left(2b^3-3a^2b\right)\sin(c+dx)+\cos(2(c+dx))\left(a\left(a^2-6b^2\right)(c+dx)+\left(b^3-6a^2b\right)\log\left(\cos\left(\tfrac{1}{2}(c+dx)\right)-\sin\right.\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*Sin[c + d*x]^2,x]
```

```
[Out] (Sec[c + d*x]^2*(a^3*c - 6*a*b^2*c + a^3*d*x - 6*a*b^2*d*x - 6*a^2*b*Log[Co
s[(c + d*x)/2] - Sin[(c + d*x)/2]] + b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*
x)/2]] + 6*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - b^3*Log[Cos[(c
+ d*x)/2] + Sin[(c + d*x)/2]] + Cos[2*(c + d*x)]*(a*(a^2 - 6*b^2)*(c + d*x)
 + (-6*a^2*b + b^3)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - b*(-6*a^2 +
b^2)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (-3*a^2*b + 2*b^3)*Sin[c +
 d*x] - (a^3*Sin[2*(c + d*x)])/2 + 6*a*b^2*Sin[2*(c + d*x)] - 3*a^2*b*Sin[3
*(c + d*x)] - (a^3*Sin[4*(c + d*x)])/4))/(4*d)
```

Maple [A] time = 0.042, size = 167, normalized size = 1.2

$$-\frac{a^3\cos(dx+c)\sin(dx+c)}{2d}+\frac{a^3x}{2}+\frac{a^3c}{2d}+3\frac{a^2b\ln(\sec(dx+c)+\tan(dx+c))}{d}-3\frac{a^2b\sin(dx+c)}{d}-3ab^2x+3\frac{ab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*sin(d*x+c)^2,x)
```

```
[Out] -1/2*a^3*cos(d*x+c)*sin(d*x+c)/d+1/2*a^3*x+1/2/d*a^3*c+3/d*a^2*b*ln(sec(d*x
+c)+tan(d*x+c))-3*a^2*b*sin(d*x+c)/d-3*a*b^2*x+3*a*b^2*tan(d*x+c)/d-3/d*a*b
^2*c+1/2/d*b^3*sin(d*x+c)^3/cos(d*x+c)^2+1/2*b^3*sin(d*x+c)/d-1/2/d*b^3*ln(
sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.49323, size = 174, normalized size = 1.26

$$\frac{(2\,dx+2\,c-\sin(2\,dx+2\,c))a^3-12\,(dx+c-\tan(dx+c))ab^2-b^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}+\log(\sin(dx+c)+1)-\log(\sin(dx\right.}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((2*d*x + 2*c - sin(2*d*x + 2*c))*a^3 - 12*(d*x + c - tan(d*x + c))*a*b
^2 - b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) + log(sin(d*x + c) + 1) - log
(sin(d*x + c) - 1)) + 6*a^2*b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1
) - 2*sin(d*x + c)))/d
```

Fricas [A] time = 1.91372, size = 359, normalized size = 2.6

$$\frac{2\left(a^3-6\,ab^2\right)dx\cos\left(dx+c\right)^2+\left(6\,a^2b-b^3\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(6\,a^2b-b^3\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx\right.\right.}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(a^3 - 6*a*b^2)*d*x*cos(d*x + c)^2 + (6*a^2*b - b^3)*cos(d*x + c)^2*
log(sin(d*x + c) + 1) - (6*a^2*b - b^3)*cos(d*x + c)^2*log(-sin(d*x + c) +
1) - 2*(a^3*cos(d*x + c)^3 + 6*a^2*b*cos(d*x + c)^2 - 6*a*b^2*cos(d*x + c)
- b^3)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*sin(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.58889, size = 467, normalized size = 3.38

$$\frac{\left(a^3-6\,ab^2\right)\left(dx+c\right)+\left(6\,a^2b-b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(6\,a^2b-b^3\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/2*((a^3 - 6*a*b^2)*(d*x + c) + (6*a^2*b - b^3)*log(abs(tan(1/2*d*x + 1/2*
c) + 1)) - (6*a^2*b - b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(a^3*tan(
1/2*d*x + 1/2*c)^7 - 6*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 6*a*b^2*tan(1/2*d*x +
 1/2*c)^7 + b^3*tan(1/2*d*x + 1/2*c)^7 - 3*a^3*tan(1/2*d*x + 1/2*c)^5 + 6*a
^2*b*tan(1/2*d*x + 1/2*c)^5 - 6*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 3*b^3*tan(1/
2*d*x + 1/2*c)^5 + 3*a^3*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*b*tan(1/2*d*x + 1/2
*c)^3 + 6*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 3*b^3*tan(1/2*d*x + 1/2*c)^3 - a^3
```

```
*tan(1/2*d*x + 1/2*c) - 6*a^2*b*tan(1/2*d*x + 1/2*c) + 6*a*b^2*tan(1/2*d*x
+ 1/2*c) + b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1)^2)/d
```

3.193 $\int \csc^2(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=133

$$-\frac{3a^2b\csc(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{3ab^2\tan(c+dx)}{d}-\frac{3ab^2\cot(c+dx)}{d}-\frac{3b^3\csc(c+dx)}{2d}$$

```
[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d + (3*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (
a^3*Cot[c + d*x])/d - (3*a*b^2*Cot[c + d*x])/d - (3*a^2*b*Csc[c + d*x])/d -
 (3*b^3*Csc[c + d*x])/(2*d) + (b^3*Csc[c + d*x]*Sec[c + d*x]^2)/(2*d) + (3*
a*b^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.272508, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3872, 2912, 3767, 8, 2621, 321, 207, 2620, 14, 288}

$$-\frac{3a^2b\csc(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}+\frac{3ab^2\tan(c+dx)}{d}-\frac{3ab^2\cot(c+dx)}{d}-\frac{3b^3\csc(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```


```
[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d + (3*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (
a^3*Cot[c + d*x])/d - (3*a*b^2*Cot[c + d*x])/d - (3*a^2*b*Csc[c + d*x])/d -
 (3*b^3*Csc[c + d*x])/(2*d) + (b^3*Csc[c + d*x]*Sec[c + d*x]^2)/(2*d) + (3*
a*b^2*Tan[c + d*x])/d
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
```

```
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^3\,dx = -\int (-b-a\cos(c+dx))^3\csc^2(c+dx)\sec^3(c+dx)\,dx$$
$$= \int \left(a^3\csc^2(c+dx)+3a^2b\csc^2(c+dx)\sec(c+dx)+3ab^2\csc^2(c+dx)\sec^2(c$$
$$= a^3\int \csc^2(c+dx)\,dx+\left(3a^2b\right)\int \csc^2(c+dx)\sec(c+dx)\,dx+\left(3ab^2\right)\int \csc^2$$
$$= -\frac{a^3\,\mathrm{Subst}(\int 1\,dx,x,\cot(c+dx))}{d}-\frac{\left(3a^2b\right)\mathrm{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,\csc(c+dx)\right.}{d}$$
$$= -\frac{a^3\cot(c+dx)}{d}-\frac{3a^2b\csc(c+dx)}{d}+\frac{b^3\csc(c+dx)\sec^2(c+dx)}{2d}-\frac{\left(3a^2b\right)\mathrm{S}}{}$$
$$= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}-\frac{3ab^2\cot(c+dx)}{d}-\frac{3a^2b\csc(c-}{d}$$
$$= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{3b^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\cot(c+dx)}{d}-\frac{3ab}{}$$

Mathematica [B] time = 0.640034, size = 406, normalized size = 3.05

$$-\frac{\csc^5\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(6a\left(a^2+2b^2\right)\cos(c+dx)+6\left(2a^2b+b^3\right)\cos(2(c+dx))+6a^2b\sin(c+dx)\log\left(\cos\left(\frac{1}{2}\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -(Csc[(c + d*x)/2]^5*Sec[(c + d*x)/2]*(12*a^2*b + 2*b^3 + 6*a*(a^2 + 2*b^2)
*Cos[c + d*x] + 6*(2*a^2*b + b^3)*Cos[2*(c + d*x)] + 2*a^3*Cos[3*(c + d*x)]
 + 12*a*b^2*Cos[3*(c + d*x)] + 6*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]]*Sin[c + d*x] + 3*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c +
d*x] - 6*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] - 3*b^
3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x] + 6*a^2*b*Log[Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 3*b^3*Log[Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 6*a^2*b*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]]*Sin[3*(c + d*x)] - 3*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*
x)/2]]*Sin[3*(c + d*x)]))/(16*d*(-1 + Cot[(c + d*x)/2]^2)^2)
```

Maple [A] time = 0.042, size = 158, normalized size = 1.2

$$-\frac{a^3\cot(dx+c)}{d}-3\frac{a^2b}{d\sin(dx+c)}+3\frac{a^2b\ln(\sec(dx+c)+\tan(dx+c))}{d}+3\frac{ab^2}{d\sin(dx+c)\cos(dx+c)}-6\frac{ab^2\cot(dx}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+b*sec(d*x+c))^3,x)
```

```
[Out] -a^3*cot(d*x+c)/d-3/d*a^2*b/sin(d*x+c)+3/d*a^2*b*ln(sec(d*x+c)+tan(d*x+c))+
3/d*a*b^2/sin(d*x+c)/cos(d*x+c)-6*a*b^2*cot(d*x+c)/d+1/2/d*b^3/sin(d*x+c)/c
os(d*x+c)^2-3/2/d*b^3/sin(d*x+c)+3/2/d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.963803, size = 188, normalized size = 1.41

$$-\frac{b^3\left(\frac{2\left(3\sin(dx+c)^2-2\right)}{\sin(dx+c)^3-\sin(dx+c)}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)+6a^2b\left(\frac{2}{\sin(dx+c)}-\log(\sin(dx+c)+1)+\log\right.}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/4*(b^3*(2*(3*sin(d*x + c)^2 - 2)/(sin(d*x + c)^3 - sin(d*x + c)) - 3*log
(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) + 6*a^2*b*(2/sin(d*x + c) - l
og(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*a*b^2*(1/tan(d*x + c) -
tan(d*x + c)) + 4*a^3/tan(d*x + c))/d
```

Fricas [A] time = 1.82114, size = 378, normalized size = 2.84

$$\frac{3\left(2a^2b+b^3\right)\cos(dx+c)^2\log(\sin(dx+c)+1)\sin(dx+c)-3\left(2a^2b+b^3\right)\cos(dx+c)^2\log(-\sin(dx+c)+1)\sin(dx}{4d\cos(dx+c)^2\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/4*(3*(2*a^2*b + b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1)*sin(d*x + c) -
3*(2*a^2*b + b^3)*cos(d*x + c)^2*log(-sin(d*x + c) + 1)*sin(d*x + c) + 12*a
*b^2*cos(d*x + c) - 4*(a^3 + 6*a*b^2)*cos(d*x + c)^3 + 2*b^3 - 6*(2*a^2*b +
 b^3)*cos(d*x + c)^2)/(d*cos(d*x + c)^2*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.301, size = 304, normalized size = 2.29

$$\frac{a^3 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 3\,a^2 b \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 3\,ab^2 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - b^3 \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 3\left(2\,a^2 b + b^3\right)\log\left(\left|\tan\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(a^3*tan(1/2*d*x + 1/2*c) - 3*a^2*b*tan(1/2*d*x + 1/2*c) + 3*a*b^2*tan(
1/2*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c) + 3*(2*a^2*b + b^3)*log(abs(tan
(1/2*d*x + 1/2*c) + 1)) - 3*(2*a^2*b + b^3)*log(abs(tan(1/2*d*x + 1/2*c) -
1)) - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)/tan(1/2*d*x + 1/2*c) - 2*(6*a*b^2*tan
(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d*x + 1/
2*c) - b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

3.194 $\int \csc^4(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=205

$$-\frac{a^2b\csc^3(c+dx)}{d}-\frac{3a^2b\csc(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}+\frac{3ab^2\tan(c+dx)}{d}$$

```
[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d + (5*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (
a^3*Cot[c + d*x])/d - (6*a*b^2*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d)
 - (a*b^2*Cot[c + d*x]^3)/d - (3*a^2*b*Csc[c + d*x])/d - (5*b^3*Csc[c + d*x
])/(2*d) - (a^2*b*Csc[c + d*x]^3)/d - (5*b^3*Csc[c + d*x]^3)/(6*d) + (b^3*C
sc[c + d*x]^3*Sec[c + d*x]^2)/(2*d) + (3*a*b^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.291242, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3872, 2912, 3767, 2621, 302, 207, 2620, 270, 288}

$$-\frac{a^2b\csc^3(c+dx)}{d}-\frac{3a^2b\csc(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot(c+dx)}{d}+\frac{3ab^2\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^3,x]
```


```
[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d + (5*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (
a^3*Cot[c + d*x])/d - (6*a*b^2*Cot[c + d*x])/d - (a^3*Cot[c + d*x]^3)/(3*d)
 - (a*b^2*Cot[c + d*x]^3)/d - (3*a^2*b*Csc[c + d*x])/d - (5*b^3*Csc[c + d*x
])/(2*d) - (a^2*b*Csc[c + d*x]^3)/d - (5*b^3*Csc[c + d*x]^3)/(6*d) + (b^3*C
sc[c + d*x]^3*Sec[c + d*x]^2)/(2*d) + (3*a*b^2*Tan[c + d*x])/d
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^3\,dx = -\int (-b-a\cos(c+dx))^3\csc^4(c+dx)\sec^3(c+dx)\,dx$$

$$= \int \left(a^3\csc^4(c+dx)+3a^2b\csc^4(c+dx)\sec(c+dx)+3ab^2\csc^4(c+dx)\sec^2(c$$

$$= a^3\int \csc^4(c+dx)\,dx+\left(3a^2b\right)\int \csc^4(c+dx)\sec(c+dx)\,dx+\left(3ab^2\right)\int \csc^4$$

$$= -\frac{a^3\operatorname{Subst}\left(\int\left(1+x^2\right)dx,x,\cot(c+dx)\right)}{d}-\frac{\left(3a^2b\right)\operatorname{Subst}\left(\int\frac{x^4}{-1+x^2}dx,x,\csc\right.}{d}$$

$$= -\frac{a^3\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}+\frac{b^3\csc^3(c+dx)\sec^2(c+dx)}{2d}-\frac{\left(3a^2b\right)\mathrm{S}}{}$$

$$= -\frac{a^3\cot(c+dx)}{d}-\frac{6ab^2\cot(c+dx)}{d}-\frac{a^3\cot^3(c+dx)}{3d}-\frac{ab^2\cot^3(c+dx)}{d}-\frac{3}{}$$

$$= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}-\frac{6ab^2\cot(c+dx)}{d}-\frac{a^3\cot^3(c+}{3d}$$

$$= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{5b^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\cot(c+dx)}{d}-\frac{6ab^2}{}$$

Mathematica [B] time = 0.915758, size = 610, normalized size = 2.98

$$-\frac{\csc^7\left(\frac{1}{2}(c+dx)\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(32a\left(a^2+3b^2\right)\cos(c+dx)+8\left(6a^2b+5b^3\right)\cos(2(c+dx))-36a^2b\cos(4(c+dx))+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -(Csc[(c + d*x)/2]^7*Sec[(c + d*x)/2]^3*(84*a^2*b + 22*b^3 + 32*a*(a^2 + 3*
b^2)*Cos[c + d*x] + 8*(6*a^2*b + 5*b^3)*Cos[2*(c + d*x)] + 4*a^3*Cos[3*(c +
 d*x)] + 48*a*b^2*Cos[3*(c + d*x)] - 36*a^2*b*Cos[4*(c + d*x)] - 30*b^3*Cos
[4*(c + d*x)] - 4*a^3*Cos[5*(c + d*x)] - 48*a*b^2*Cos[5*(c + d*x)] + 36*a^2
*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] + 30*b^3*Log[Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[c + d*x] - 36*a^2*b*Log[Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]]*Sin[c + d*x] - 30*b^3*Log[Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]]*Sin[c + d*x] + 18*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*
Sin[3*(c + d*x)] + 15*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c
 + d*x)] - 18*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c + d*x
)] - 15*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 18*
a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 15*b^3*Lo
g[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] + 18*a^2*b*Log[Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] + 15*b^3*Log[Cos[(c + d*x
)/2] + Sin[(c + d*x)/2]]*Sin[5*(c + d*x)]))/(768*d*(-1 + Cot[(c + d*x)/2]^2
)^2)
```

Maple [A] time = 0.05, size = 246, normalized size = 1.2

$$-\frac{2a^3\cot(dx+c)}{3d}-\frac{a^3\cot(dx+c)(\csc(dx+c))^2}{3d}-\frac{a^2b}{d(\sin(dx+c))^3}-3\frac{a^2b}{d\sin(dx+c)}+3\frac{a^2b\ln(\sec(dx+c)+\tan(d}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*sec(d*x+c))^3,x)
```

```
[Out] -2/3*a^3*cot(d*x+c)/d-1/3/d*a^3*cot(d*x+c)*csc(d*x+c)^2-1/d*a^2*b/sin(d*x+c
)^3-3/d*a^2*b/sin(d*x+c)+3/d*a^2*b*ln(sec(d*x+c)+tan(d*x+c))-1/d*a*b^2/sin(
d*x+c)^3/cos(d*x+c)+4/d*a*b^2/sin(d*x+c)/cos(d*x+c)-8*a*b^2*cot(d*x+c)/d-1/
3/d*b^3/sin(d*x+c)^3/cos(d*x+c)^2+5/6/d*b^3/sin(d*x+c)/cos(d*x+c)^2-5/2/d*b
^3/sin(d*x+c)+5/2/d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 1.03308, size = 257, normalized size = 1.25

$$-\frac{b^3\left(\frac{2\left(15\sin(dx+c)^4-10\sin(dx+c)^2-2\right)}{\sin(dx+c)^5-\sin(dx+c)^3}-15\log(\sin(dx+c)+1)+15\log(\sin(dx+c)-1)\right)+6a^2b\left(\frac{2\left(3\sin(dx+c)^2+1\right)}{\sin(dx+c)^3}-3\log(s\right.}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/12*(b^3*(2*(15*sin(d*x + c)^4 - 10*sin(d*x + c)^2 - 2)/(sin(d*x + c)^5 -
 sin(d*x + c)^3) - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 6
```

```
*a^2*b*(2*(3*sin(d*x + c)^2 + 1)/sin(d*x + c)^3 - 3*log(sin(d*x + c) + 1) +
 3*log(sin(d*x + c) - 1)) + 12*a*b^2*((6*tan(d*x + c)^2 + 1)/tan(d*x + c)^3
 - 3*tan(d*x + c)) + 4*(3*tan(d*x + c)^2 + 1)*a^3/tan(d*x + c)^3)/d
```

Fricas [A] time = 1.86584, size = 618, normalized size = 3.01

$$-\frac{8\left(a^3+12\,ab^2\right)\cos\left(dx+c\right)^5+6\left(6\,a^2b+5\,b^3\right)\cos\left(dx+c\right)^4+36\,ab^2\cos\left(dx+c\right)-12\left(a^3+12\,ab^2\right)\cos\left(dx+c\right)^3+6\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/12*(8*(a^3 + 12*a*b^2)*cos(d*x + c)^5 + 6*(6*a^2*b + 5*b^3)*cos(d*x + c)
^4 + 36*a*b^2*cos(d*x + c) - 12*(a^3 + 12*a*b^2)*cos(d*x + c)^3 + 6*b^3 - 8
*(6*a^2*b + 5*b^3)*cos(d*x + c)^2 - 3*((6*a^2*b + 5*b^3)*cos(d*x + c)^4 - (
6*a^2*b + 5*b^3)*cos(d*x + c)^2)*log(sin(d*x + c) + 1)*sin(d*x + c) + 3*((6
*a^2*b + 5*b^3)*cos(d*x + c)^4 - (6*a^2*b + 5*b^3)*cos(d*x + c)^2)*log(-sin
(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^4 - d*cos(d*x + c)^2)*sin(d*x
 + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.35067, size = 487, normalized size = 2.38

$$\frac{a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,a^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,a^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/24*(a^3*tan(1/2*d*x + 1/2*c)^3 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 3*a*b^2
*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 + 9*a^3*tan(1/2*d*x +
1/2*c) - 45*a^2*b*tan(1/2*d*x + 1/2*c) + 63*a*b^2*tan(1/2*d*x + 1/2*c) - 27
*b^3*tan(1/2*d*x + 1/2*c) + 12*(6*a^2*b + 5*b^3)*log(abs(tan(1/2*d*x + 1/2*
c) + 1)) - 12*(6*a^2*b + 5*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 24*(6*
a*b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2
*d*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 -
(9*a^3*tan(1/2*d*x + 1/2*c)^2 + 45*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 63*a*b^2*
tan(1/2*d*x + 1/2*c)^2 + 27*b^3*tan(1/2*d*x + 1/2*c)^2 + a^3 + 3*a^2*b + 3*
a*b^2 + b^3)/tan(1/2*d*x + 1/2*c)^3)/d
```

3.195 $\int \csc^6(c+dx)(a+b\sec(c+dx))^3\,dx$

Optimal. Leaf size=279

$$-\frac{3a^2b\csc^5(c+dx)}{5d}-\frac{a^2b\csc^3(c+dx)}{d}-\frac{3a^2b\csc(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\cot^5(c+dx)}{5d}-\frac{2a^3\cot^3(c+}{3d}$$

```
[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d + (7*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (
a^3*Cot[c + d*x])/d - (9*a*b^2*Cot[c + d*x])/d - (2*a^3*Cot[c + d*x]^3)/(3*
d) - (3*a*b^2*Cot[c + d*x]^3)/d - (a^3*Cot[c + d*x]^5)/(5*d) - (3*a*b^2*Cot
[c + d*x]^5)/(5*d) - (3*a^2*b*Csc[c + d*x])/d - (7*b^3*Csc[c + d*x])/(2*d)
- (a^2*b*Csc[c + d*x]^3)/d - (7*b^3*Csc[c + d*x]^3)/(6*d) - (3*a^2*b*Csc[c
+ d*x]^5)/(5*d) - (7*b^3*Csc[c + d*x]^5)/(10*d) + (b^3*Csc[c + d*x]^5*Sec[c
 + d*x]^2)/(2*d) + (3*a*b^2*Tan[c + d*x])/d
```

Rubi [A] time = 0.316217, antiderivative size = 279, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3872, 2912, 3767, 2621, 302, 207, 2620, 270, 288}

$$-\frac{3a^2b\csc^5(c+dx)}{5d}-\frac{a^2b\csc^3(c+dx)}{d}-\frac{3a^2b\csc(c+dx)}{d}+\frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\cot^5(c+dx)}{5d}-\frac{2a^3\cot^3(c+}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + b*Sec[c + d*x])^3,x]
```


```
[Out] (3*a^2*b*ArcTanh[Sin[c + d*x]])/d + (7*b^3*ArcTanh[Sin[c + d*x]])/(2*d) - (
a^3*Cot[c + d*x])/d - (9*a*b^2*Cot[c + d*x])/d - (2*a^3*Cot[c + d*x]^3)/(3*
d) - (3*a*b^2*Cot[c + d*x]^3)/d - (a^3*Cot[c + d*x]^5)/(5*d) - (3*a*b^2*Cot
[c + d*x]^5)/(5*d) - (3*a^2*b*Csc[c + d*x])/d - (7*b^3*Csc[c + d*x])/(2*d)
- (a^2*b*Csc[c + d*x]^3)/d - (7*b^3*Csc[c + d*x]^3)/(6*d) - (3*a^2*b*Csc[c
+ d*x]^5)/(5*d) - (7*b^3*Csc[c + d*x]^5)/(10*d) + (b^3*Csc[c + d*x]^5*Sec[c
 + d*x]^2)/(2*d) + (3*a*b^2*Tan[c + d*x])/d
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rubi steps

$$\int \csc^6(c+dx)(a+b\sec(c+dx))^3\,dx = -\int (-b-a\cos(c+dx))^3\csc^6(c+dx)\sec^3(c+dx)\,dx$$
$$= \int \left(a^3\csc^6(c+dx)+3a^2b\csc^6(c+dx)\sec(c+dx)+3ab^2\csc^6(c+dx)\sec^2(c+\right.$$
$$= a^3\int \csc^6(c+dx)\,dx+\left(3a^2b\right)\int \csc^6(c+dx)\sec(c+dx)\,dx+\left(3ab^2\right)\int \csc^6(c$$
$$= -\frac{a^3\,\text{Subst}\left(\int \left(1+2x^2+x^4\right)dx,x,\cot(c+dx)\right)}{d}-\frac{\left(3a^2b\right)\text{Subst}\left(\int \frac{x^6}{-1+x^2}\,dx,x,\right.}{d}$$
$$= -\frac{a^3\cot(c+dx)}{d}-\frac{2a^3\cot^3(c+dx)}{3d}-\frac{a^3\cot^5(c+dx)}{5d}+\frac{b^3\csc^5(c+dx)\sec^2(c+}{2d}$$
$$= -\frac{a^3\cot(c+dx)}{d}-\frac{9ab^2\cot(c+dx)}{d}-\frac{2a^3\cot^3(c+dx)}{3d}-\frac{3ab^2\cot^3(c+dx)}{d}-\frac{a^3}{}$$
$$= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}-\frac{a^3\cot(c+dx)}{d}-\frac{9ab^2\cot(c+dx)}{d}-\frac{2a^3\cot^3(c+dx}{3d}$$
$$= \frac{3a^2b\tanh^{-1}(\sin(c+dx))}{d}+\frac{7b^3\tanh^{-1}(\sin(c+dx))}{2d}-\frac{a^3\cot(c+dx)}{d}-\frac{9ab^2\cot}{}$$

Mathematica [B] time = 1.41209, size = 812, normalized size = 2.91

$$-\frac{\csc^9\left(\frac{1}{2}(c+dx)\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(16\cos(3(c+dx))a^3-48\cos(5(c+dx))a^3+16\cos(7(c+dx))a^3+1176ba^2-600b\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -(Csc[(c + d*x)/2]^9*Sec[(c + d*x)/2]^5*(1176*a^2*b + 412*b^3 + 80*a*(5*a^2
 + 18*b^2)*Cos[c + d*x] + 66*(6*a^2*b + 7*b^3)*Cos[2*(c + d*x)] + 16*a^3*Co
s[3*(c + d*x)] + 288*a*b^2*Cos[3*(c + d*x)] - 600*a^2*b*Cos[4*(c + d*x)] -
700*b^3*Cos[4*(c + d*x)] - 48*a^3*Cos[5*(c + d*x)] - 864*a*b^2*Cos[5*(c + d
*x)] + 180*a^2*b*Cos[6*(c + d*x)] + 210*b^3*Cos[6*(c + d*x)] + 16*a^3*Cos[7
*(c + d*x)] + 288*a*b^2*Cos[7*(c + d*x)] + 450*a^2*b*Log[Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2]]*Sin[c + d*x] + 525*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]]*Sin[c + d*x] - 450*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*S
in[c + d*x] - 525*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[c + d*x]
 + 90*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] + 105
*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 90*a^2*b*L
og[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 105*b^3*Log[Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[3*(c + d*x)] - 270*a^2*b*Log[Cos[(c +
d*x)/2] - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] - 315*b^3*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]]*Sin[5*(c + d*x)] + 270*a^2*b*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]]*Sin[5*(c + d*x)] + 315*b^3*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]]*Sin[5*(c + d*x)] + 90*a^2*b*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2
]]*Sin[7*(c + d*x)] + 105*b^3*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]*Sin[
7*(c + d*x)] - 90*a^2*b*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[7*(c +
 d*x)] - 105*b^3*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*Sin[7*(c + d*x)])
)/(61440*d*(-1 + Cot[(c + d*x)/2]^2)^2)
```

Maple [A] time = 0.051, size = 334, normalized size = 1.2

$$-\frac{8a^3\cot(dx+c)}{15d}-\frac{a^3\cot(dx+c)(\csc(dx+c))^4}{5d}-\frac{4a^3\cot(dx+c)(\csc(dx+c))^2}{15d}-\frac{3a^2b}{5d(\sin(dx+c))^5}-\frac{a^2b}{d(\sin(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+b*sec(d*x+c))^3,x)
```

```
[Out] -8/15*a^3*cot(d*x+c)/d-1/5/d*a^3*cot(d*x+c)*csc(d*x+c)^4-4/15/d*a^3*cot(d*x
+c)*csc(d*x+c)^2-3/5/d*a^2*b/sin(d*x+c)^5-1/d*a^2*b/sin(d*x+c)^3-3/d*a^2*b/
sin(d*x+c)+3/d*a^2*b*ln(sec(d*x+c)+tan(d*x+c))-3/5/d*a*b^2/sin(d*x+c)^5/cos
(d*x+c)-6/5/d*a*b^2/sin(d*x+c)^3/cos(d*x+c)+24/5/d*a*b^2/sin(d*x+c)/cos(d*x
+c)-48/5*a*b^2*cot(d*x+c)/d-1/5/d*b^3/sin(d*x+c)^5/cos(d*x+c)^2-7/15/d*b^3/
sin(d*x+c)^3/cos(d*x+c)^2+7/6/d*b^3/sin(d*x+c)/cos(d*x+c)^2-7/2/d*b^3/sin(d
*x+c)+7/2/d*b^3*ln(sec(d*x+c)+tan(d*x+c))
```

Maxima [A] time = 0.997119, size = 311, normalized size = 1.11

$$-\frac{b^3\left(\frac{2\left(105\sin(dx+c)^6-70\sin(dx+c)^4-14\sin(dx+c)^2-6\right)}{\sin(dx+c)^7-\sin(dx+c)^5}-105\log(\sin(dx+c)+1)+105\log(\sin(dx+c)-1)\right)+6a^2b\left(\frac{2(15\sin}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] -1/60*(b^3*(2*(105*sin(d*x + c)^6 - 70*sin(d*x + c)^4 - 14*sin(d*x + c)^2 -
 6)/(sin(d*x + c)^7 - sin(d*x + c)^5) - 105*log(sin(d*x + c) + 1) + 105*log
(sin(d*x + c) - 1)) + 6*a^2*b*(2*(15*sin(d*x + c)^4 + 5*sin(d*x + c)^2 + 3)
/sin(d*x + c)^5 - 15*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) + 36
*a*b^2*((15*tan(d*x + c)^4 + 5*tan(d*x + c)^2 + 1)/tan(d*x + c)^5 - 5*tan(d
*x + c)) + 4*(15*tan(d*x + c)^4 + 10*tan(d*x + c)^2 + 3)*a^3/tan(d*x + c)^5
)/d
```

Fricas [A] time = 1.8618, size = 857, normalized size = 3.07

$$-\frac{32\left(a^3+18ab^2\right)\cos(dx+c)^7+30\left(6a^2b+7b^3\right)\cos(dx+c)^6-80\left(a^3+18ab^2\right)\cos(dx+c)^5-70\left(6a^2b+7b^3\right)\text{co}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/60*(32*(a^3 + 18*a*b^2)*cos(d*x + c)^7 + 30*(6*a^2*b + 7*b^3)*cos(d*x +
c)^6 - 80*(a^3 + 18*a*b^2)*cos(d*x + c)^5 - 70*(6*a^2*b + 7*b^3)*cos(d*x +
c)^4 - 180*a*b^2*cos(d*x + c) + 60*(a^3 + 18*a*b^2)*cos(d*x + c)^3 - 30*b^3
 + 46*(6*a^2*b + 7*b^3)*cos(d*x + c)^2 - 15*((6*a^2*b + 7*b^3)*cos(d*x + c)
^6 - 2*(6*a^2*b + 7*b^3)*cos(d*x + c)^4 + (6*a^2*b + 7*b^3)*cos(d*x + c)^2)
*log(sin(d*x + c) + 1)*sin(d*x + c) + 15*((6*a^2*b + 7*b^3)*cos(d*x + c)^6
- 2*(6*a^2*b + 7*b^3)*cos(d*x + c)^4 + (6*a^2*b + 7*b^3)*cos(d*x + c)^2)*lo
g(-sin(d*x + c) + 1)*sin(d*x + c))/((d*cos(d*x + c)^6 - 2*d*cos(d*x + c)^4
+ d*cos(d*x + c)^2)*sin(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.33031, size = 672, normalized size = 2.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/480*(3*a^3*tan(1/2*d*x + 1/2*c)^5 - 9*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 9*a*
b^2*tan(1/2*d*x + 1/2*c)^5 - 3*b^3*tan(1/2*d*x + 1/2*c)^5 + 25*a^3*tan(1/2*
d*x + 1/2*c)^3 - 105*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 135*a*b^2*tan(1/2*d*x +
 1/2*c)^3 - 55*b^3*tan(1/2*d*x + 1/2*c)^3 + 150*a^3*tan(1/2*d*x + 1/2*c) -
990*a^2*b*tan(1/2*d*x + 1/2*c) + 1710*a*b^2*tan(1/2*d*x + 1/2*c) - 870*b^3*
tan(1/2*d*x + 1/2*c) + 240*(6*a^2*b + 7*b^3)*log(abs(tan(1/2*d*x + 1/2*c) +
 1)) - 240*(6*a^2*b + 7*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 480*(6*a*
b^2*tan(1/2*d*x + 1/2*c)^3 - b^3*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^2*tan(1/2*d
*x + 1/2*c) - b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2 - (1
50*a^3*tan(1/2*d*x + 1/2*c)^4 + 990*a^2*b*tan(1/2*d*x + 1/2*c)^4 + 1710*a*b
^2*tan(1/2*d*x + 1/2*c)^4 + 870*b^3*tan(1/2*d*x + 1/2*c)^4 + 25*a^3*tan(1/2
*d*x + 1/2*c)^2 + 105*a^2*b*tan(1/2*d*x + 1/2*c)^2 + 135*a*b^2*tan(1/2*d*x
+ 1/2*c)^2 + 55*b^3*tan(1/2*d*x + 1/2*c)^2 + 3*a^3 + 9*a^2*b + 9*a*b^2 + 3*
b^3)/tan(1/2*d*x + 1/2*c)^5)/d
```

3.196 $\int \frac{\sin^7(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=223

$$-\frac{(3a^2-b^2)\cos^5(c+dx)}{5a^3d}+\frac{b(3a^2-b^2)\cos^4(c+dx)}{4a^4d}+\frac{(-3a^2b^2+3a^4+b^4)\cos^3(c+dx)}{3a^5d}-\frac{b(-3a^2b^2+3a^4+b^4)\cos}{2a^6d}$$

```
[Out] -(((a^2 - b^2)^3*Cos[c + d*x])/(a^7*d)) - (b*(3*a^4 - 3*a^2*b^2 + b^4)*Cos[
c + d*x]^2)/(2*a^6*d) + ((3*a^4 - 3*a^2*b^2 + b^4)*Cos[c + d*x]^3)/(3*a^5*d
) + (b*(3*a^2 - b^2)*Cos[c + d*x]^4)/(4*a^4*d) - ((3*a^2 - b^2)*Cos[c + d*x
]^5)/(5*a^3*d) - (b*Cos[c + d*x]^6)/(6*a^2*d) + Cos[c + d*x]^7/(7*a*d) + (b
*(a^2 - b^2)^3*Log[b + a*Cos[c + d*x]])/(a^8*d)
```

Rubi [A] time = 0.250938, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2837, 12, 772}

$$-\frac{(3a^2-b^2)\cos^5(c+dx)}{5a^3d}+\frac{b(3a^2-b^2)\cos^4(c+dx)}{4a^4d}+\frac{(-3a^2b^2+3a^4+b^4)\cos^3(c+dx)}{3a^5d}-\frac{b(-3a^2b^2+3a^4+b^4)\cos}{2a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a + b*Sec[c + d*x]),x]
```



```
[Out] -(((a^2 - b^2)^3*Cos[c + d*x])/(a^7*d)) - (b*(3*a^4 - 3*a^2*b^2 + b^4)*Cos[
c + d*x]^2)/(2*a^6*d) + ((3*a^4 - 3*a^2*b^2 + b^4)*Cos[c + d*x]^3)/(3*a^5*d
) + (b*(3*a^2 - b^2)*Cos[c + d*x]^4)/(4*a^4*d) - ((3*a^2 - b^2)*Cos[c + d*x
]^5)/(5*a^3*d) - (b*Cos[c + d*x]^6)/(6*a^2*d) + Cos[c + d*x]^7/(7*a*d) + (b
*(a^2 - b^2)^3*Log[b + a*Cos[c + d*x]])/(a^8*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)\sin^7(c+dx)}{-b-a\cos(c+dx)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x(a^2-x^2)^3}{a(-b+x)}\,dx, x, -a\cos(c+dx)\right)}{a^7 d}$$
$$= \frac{\text{Subst}\left(\int \frac{x(a^2-x^2)^3}{-b+x}\,dx, x, -a\cos(c+dx)\right)}{a^8 d}$$
$$= \frac{\text{Subst}\left(\int \left((a^2-b^2)^3 + \frac{b(-a^2+b^2)^3}{b-x} - b\left(3a^4-3a^2b^2+b^4\right)x - \left(3a^4-3a^2b^2+b^4\right)x^2 - b\left(-3a^2 + \right.\right.\right.}{a^8 d}$$
$$= -\frac{(a^2-b^2)^3\cos(c+dx)}{a^7 d} - \frac{b\left(3a^4-3a^2b^2+b^4\right)\cos^2(c+dx)}{2a^6 d} + \frac{\left(3a^4-3a^2b^2+b^4\right)\cos^3(c+dx)}{3a^5 d}$$

Mathematica [A] time = 1.34391, size = 282, normalized size = 1.26

$$\frac{-1260a^5b^2\cos(3(c+dx)) + 84a^5b^2\cos(5(c+dx)) - 210a^4b^3\cos(4(c+dx)) + 560a^3b^4\cos(3(c+dx)) - 105a\left(-152a^4b^2 + \right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-105*a*(35*a^6 - 152*a^4*b^2 + 176*a^2*b^4 - 64*b^6)*Cos[c + d*x] - 105*(2
9*a^6*b - 40*a^4*b^3 + 16*a^2*b^5)*Cos[2*(c + d*x)] + 735*a^7*Cos[3*(c + d*
x)] - 1260*a^5*b^2*Cos[3*(c + d*x)] + 560*a^3*b^4*Cos[3*(c + d*x)] + 420*a^
6*b*Cos[4*(c + d*x)] - 210*a^4*b^3*Cos[4*(c + d*x)] - 147*a^7*Cos[5*(c + d*
x)] + 84*a^5*b^2*Cos[5*(c + d*x)] - 35*a^6*b*Cos[6*(c + d*x)] + 15*a^7*Cos[
7*(c + d*x)] + 6720*a^6*b*Log[b + a*Cos[c + d*x]] - 20160*a^4*b^3*Log[b + a
*Cos[c + d*x]] + 20160*a^2*b^5*Log[b + a*Cos[c + d*x]] - 6720*b^7*Log[b + a
*Cos[c + d*x]])/(6720*a^8*d)
```

Maple [A] time = 0.049, size = 363, normalized size = 1.6

$$\frac{(\cos(dx+c))^7}{7ad} - \frac{b(\cos(dx+c))^6}{6a^2d} - \frac{3(\cos(dx+c))^5}{5ad} + \frac{(\cos(dx+c))^5 b^2}{5da^3} + \frac{3b(\cos(dx+c))^4}{4a^2d} - \frac{(\cos(dx+c))^4 b^3}{4da^4} + \frac{(c}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/7*cos(d*x+c)^7/a/d-1/6*b*cos(d*x+c)^6/a^2/d-3/5*cos(d*x+c)^5/a/d+1/5/d/a^
3*cos(d*x+c)^5*b^2+3/4*b*cos(d*x+c)^4/a^2/d-1/4/d/a^4*cos(d*x+c)^4*b^3+cos(
d*x+c)^3/a/d-1/d/a^3*cos(d*x+c)^3*b^2+1/3/d/a^5*cos(d*x+c)^3*b^4-3/2*b*cos(
d*x+c)^2/a^2/d+3/2/d/a^4*cos(d*x+c)^2*b^3-1/2/d/a^6*cos(d*x+c)^2*b^5-cos(d*
x+c)/a/d+3/d/a^3*b^2*cos(d*x+c)-3/d/a^5*b^4*cos(d*x+c)+1/d/a^7*b^6*cos(d*x+
c)+b*ln(b+a*cos(d*x+c))/a^2/d-3/d*b^3/a^4*ln(b+a*cos(d*x+c))+3/d*b^5/a^6*ln
(b+a*cos(d*x+c))-1/d*b^7/a^8*ln(b+a*cos(d*x+c))
```

Maxima [A] time = 1.05525, size = 302, normalized size = 1.35

$$\frac{\frac{60\,a^6\cos(dx+c)^7-70\,a^5b\cos(dx+c)^6-84\left(3\,a^6-a^4b^2\right)\cos(dx+c)^5+105\left(3\,a^5b-a^3b^3\right)\cos(dx+c)^4+140\left(3\,a^6-3\,a^4b^2+a^2b^4\right)\cos(dx+c)^3-210\left(3\,a^5b-3\,a^3b^3+ab^5\right)\cdots}{a^7}\cdots}{420\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/420*((60*a^6*cos(d*x + c)^7 - 70*a^5*b*cos(d*x + c)^6 - 84*(3*a^6 - a^4*b
^2)*cos(d*x + c)^5 + 105*(3*a^5*b - a^3*b^3)*cos(d*x + c)^4 + 140*(3*a^6 -
3*a^4*b^2 + a^2*b^4)*cos(d*x + c)^3 - 210*(3*a^5*b - 3*a^3*b^3 + a*b^5)*cos
(d*x + c)^2 - 420*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cos(d*x + c))/a^7 + 4
20*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*log(a*cos(d*x + c) + b)/a^8)/d
```

Fricas [A] time = 1.9669, size = 504, normalized size = 2.26

$$\frac{60\,a^7\cos\left(dx+c\right)^7-70\,a^6b\cos\left(dx+c\right)^6-84\left(3\,a^7-a^5b^2\right)\cos\left(dx+c\right)^5+105\left(3\,a^6b-a^4b^3\right)\cos\left(dx+c\right)^4+140\left(3\,a^7\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/420*(60*a^7*cos(d*x + c)^7 - 70*a^6*b*cos(d*x + c)^6 - 84*(3*a^7 - a^5*b^
2)*cos(d*x + c)^5 + 105*(3*a^6*b - a^4*b^3)*cos(d*x + c)^4 + 140*(3*a^7 - 3
*a^5*b^2 + a^3*b^4)*cos(d*x + c)^3 - 210*(3*a^6*b - 3*a^4*b^3 + a^2*b^5)*co
s(d*x + c)^2 - 420*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c) + 420
*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*log(a*cos(d*x + c) + b))/(a^8*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28884, size = 2105, normalized size = 9.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/420*(420*(a^7*b - a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 + 3*a^3*b^5 - 3*a^2*b^6
 - a*b^7 + b^8)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^9 - a^8*b) - 420*(a^6*b - 3*a^4
```

*b^3 + 3*a^2*b^5 - b^7)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)
)/a^8 + (384*a^7 - 1089*a^6*b - 1848*a^5*b^2 + 3267*a^4*b^3 + 2240*a^3*b^4
- 3267*a^2*b^5 - 840*a*b^6 + 1089*b^7 - 2688*a^7*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) + 8463*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12096*a^5*
b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 24549*a^4*b^3*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) - 14000*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
+ 23709*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 5040*a*b^6*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) - 7623*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) +
 1) + 8064*a^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 28749*a^6*b*(cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 32088*a^5*b^2*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2 + 78687*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2 + 35280*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 72807*a^2*b
^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 12600*a*b^6*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2 + 22869*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
1)^2 - 13440*a^7*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56035*a^6*b*(c
os(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 40320*a^5*b^2*(cos(d*x + c) - 1)^
3/(cos(d*x + c) + 1)^3 - 136185*a^4*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c)
+ 1)^3 - 45920*a^3*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 122745*a
^2*b^5*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 16800*a*b^6*(cos(d*x + c
) - 1)^3/(cos(d*x + c) + 1)^3 - 38115*b^7*(cos(d*x + c) - 1)^3/(cos(d*x + c
) + 1)^3 - 56035*a^6*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 24360*a^
5*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 136185*a^4*b^3*(cos(d*x +
 c) - 1)^4/(cos(d*x + c) + 1)^4 + 32480*a^3*b^4*(cos(d*x + c) - 1)^4/(cos(d
*x + c) + 1)^4 - 122745*a^2*b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 -
 12600*a*b^6*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 38115*b^7*(cos(d*x
 + c) - 1)^4/(cos(d*x + c) + 1)^4 + 28749*a^6*b*(cos(d*x + c) - 1)^5/(cos(d
*x + c) + 1)^5 + 6720*a^5*b^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 7
8687*a^4*b^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 11760*a^3*b^4*(cos
(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 72807*a^2*b^5*(cos(d*x + c) - 1)^5/
(cos(d*x + c) + 1)^5 + 5040*a*b^6*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5
 - 22869*b^7*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 8463*a^6*b*(cos(d*
x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 840*a^5*b^2*(cos(d*x + c) - 1)^6/(cos(
d*x + c) + 1)^6 + 24549*a^4*b^3*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 +
 1680*a^3*b^4*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 23709*a^2*b^5*(co
s(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 840*a*b^6*(cos(d*x + c) - 1)^6/(co
s(d*x + c) + 1)^6 + 7623*b^7*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 10
89*a^6*b*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 3267*a^4*b^3*(cos(d*x
+ c) - 1)^7/(cos(d*x + c) + 1)^7 + 3267*a^2*b^5*(cos(d*x + c) - 1)^7/(cos(d
*x + c) + 1)^7 - 1089*b^7*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/(a^8*(
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^7))/d

3.197 $\int \frac{\sin^5(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=152

$$\frac{(2a^2-b^2)\cos^3(c+dx)}{3a^3d}-\frac{b(2a^2-b^2)\cos^2(c+dx)}{2a^4d}-\frac{(a^2-b^2)^2\cos(c+dx)}{a^5d}+\frac{b(a^2-b^2)^2\log(a\cos(c+dx)+b)}{a^6d}+\frac{b}{}$$

```
[Out] -(((a^2 - b^2)^2*Cos[c + d*x])/(a^5*d)) - (b*(2*a^2 - b^2)*Cos[c + d*x]^2)/
(2*a^4*d) + ((2*a^2 - b^2)*Cos[c + d*x]^3)/(3*a^3*d) + (b*Cos[c + d*x]^4)/(
4*a^2*d) - Cos[c + d*x]^5/(5*a*d) + (b*(a^2 - b^2)^2*Log[b + a*Cos[c + d*x]
])/(a^6*d)
```

Rubi [A] time = 0.194132, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2837, 12, 772}

$$\frac{(2a^2-b^2)\cos^3(c+dx)}{3a^3d}-\frac{b(2a^2-b^2)\cos^2(c+dx)}{2a^4d}-\frac{(a^2-b^2)^2\cos(c+dx)}{a^5d}+\frac{b(a^2-b^2)^2\log(a\cos(c+dx)+b)}{a^6d}+\frac{b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)^2*Cos[c + d*x])/(a^5*d)) - (b*(2*a^2 - b^2)*Cos[c + d*x]^2)/
(2*a^4*d) + ((2*a^2 - b^2)*Cos[c + d*x]^3)/(3*a^3*d) + (b*Cos[c + d*x]^4)/(
4*a^2*d) - Cos[c + d*x]^5/(5*a*d) + (b*(a^2 - b^2)^2*Log[b + a*Cos[c + d*x]
])/(a^6*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)\sin^5(c+dx)}{-b-a\cos(c+dx)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x(a^2-x^2)^2}{a(-b+x)}\,dx, x, -a\cos(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{x(a^2-x^2)^2}{-b+x}\,dx, x, -a\cos(c+dx)\right)}{a^6d}$$
$$= \frac{\text{Subst}\left(\int \left((a^2-b^2)^2 - \frac{b(-a^2+b^2)^2}{b-x} + b\left(-2a^2+b^2\right)x - \left(2a^2-b^2\right)x^2 + bx^3 + x^4\right)dx, x, -a\cos(c\right.}{a^6d}$$
$$= -\frac{\left(a^2-b^2\right)^2\cos(c+dx)}{a^5d} - \frac{b\left(2a^2-b^2\right)\cos^2(c+dx)}{2a^4d} + \frac{\left(2a^2-b^2\right)\cos^3(c+dx)}{3a^3d} + \frac{b\cos^4(c+dx}{4a^2d}$$

Mathematica [A] time = 0.362462, size = 172, normalized size = 1.13

$$\frac{-40a^3b^2\cos(3(c+dx)) - 60a\left(-14a^2b^2+5a^4+8b^4\right)\cos(c+dx) - 60\left(3a^4b-2a^2b^3\right)\cos(2(c+dx)) - 960a^2b^3\log(a\cos(}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-60*a*(5*a^4 - 14*a^2*b^2 + 8*b^4)*Cos[c + d*x] - 60*(3*a^4*b - 2*a^2*b^3)
*Cos[2*(c + d*x)] + 50*a^5*Cos[3*(c + d*x)] - 40*a^3*b^2*Cos[3*(c + d*x)] +
 15*a^4*b*Cos[4*(c + d*x)] - 6*a^5*Cos[5*(c + d*x)] + 480*a^4*b*Log[b + a*C
os[c + d*x]] - 960*a^2*b^3*Log[b + a*Cos[c + d*x]] + 480*b^5*Log[b + a*Cos[
c + d*x]])/(480*a^6*d)
```

Maple [A] time = 0.043, size = 216, normalized size = 1.4

$$-\frac{(\cos(dx+c))^5}{5\,ad} + \frac{b(\cos(dx+c))^4}{4\,a^2d} + \frac{2\,(\cos(dx+c))^3}{3\,ad} - \frac{(\cos(dx+c))^3\,b^2}{3\,da^3} - \frac{b(\cos(dx+c))^2}{a^2d} + \frac{(\cos(dx+c))^2\,b^3}{2\,da^4} - \text{co}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+b*sec(d*x+c)),x)
```

```
[Out] -1/5*cos(d*x+c)^5/a/d+1/4*b*cos(d*x+c)^4/a^2/d+2/3*cos(d*x+c)^3/a/d-1/3/d/a
^3*cos(d*x+c)^3*b^2-b*cos(d*x+c)^2/a^2/d+1/2/d/a^4*cos(d*x+c)^2*b^3-cos(d*x
+c)/a/d+2/d/a^3*b^2*cos(d*x+c)-1/d/a^5*b^4*cos(d*x+c)+b*ln(b+a*cos(d*x+c))/
a^2/d-2/d*b^3/a^4*ln(b+a*cos(d*x+c))+1/d*b^5/a^6*ln(b+a*cos(d*x+c))
```

Maxima [A] time = 1.06019, size = 190, normalized size = 1.25

$$-\frac{\frac{12\,a^4\cos(dx+c)^5 - 15\,a^3b\cos(dx+c)^4 - 20\left(2\,a^4-a^2b^2\right)\cos(dx+c)^3 + 30\left(2\,a^3b-ab^3\right)\cos(dx+c)^2 + 60\left(a^4-2\,a^2b^2+b^4\right)\cos(dx+c)}{a^5} - \frac{60\left(a^4b-2\,a^2b^3+b^5\right)\log(a\cos(d}{a^6}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/60*((12*a^4*cos(d*x + c)^5 - 15*a^3*b*cos(d*x + c)^4 - 20*(2*a^4 - a^2*b
^2)*cos(d*x + c)^3 + 30*(2*a^3*b - a*b^3)*cos(d*x + c)^2 + 60*(a^4 - 2*a^2*
b^2 + b^4)*cos(d*x + c))/a^5 - 60*(a^4*b - 2*a^2*b^3 + b^5)*log(a*cos(d*x +
 c) + b)/a^6)/d
```

Fricas [A] time = 1.9067, size = 327, normalized size = 2.15

$$-\frac{12\,a^5\cos\left(dx+c\right)^5-15\,a^4b\cos\left(dx+c\right)^4-20\left(2\,a^5-a^3b^2\right)\cos\left(dx+c\right)^3+30\left(2\,a^4b-a^2b^3\right)\cos\left(dx+c\right)^2+60\left(a^5-\right.}{60\,a^6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/60*(12*a^5*cos(d*x + c)^5 - 15*a^4*b*cos(d*x + c)^4 - 20*(2*a^5 - a^3*b^
2)*cos(d*x + c)^3 + 30*(2*a^4*b - a^2*b^3)*cos(d*x + c)^2 + 60*(a^5 - 2*a^3
*b^2 + a*b^4)*cos(d*x + c) - 60*(a^4*b - 2*a^2*b^3 + b^5)*log(a*cos(d*x + c
) + b))/(a^6*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31959, size = 1170, normalized size = 7.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/60*(60*(a^5*b - a^4*b^2 - 2*a^3*b^3 + 2*a^2*b^4 + a*b^5 - b^6)*log(abs(a
+ b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1)))/(a^7 - a^6*b) - 60*(a^4*b - 2*a^2*b^3 + b^5)*log(abs(-(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^6 + (64*a^5 - 137*a^4*b - 200*a^3*b
^2 + 274*a^2*b^3 + 120*a*b^4 - 137*b^5 - 320*a^5*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) + 805*a^4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 880*a^3*b^2
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1490*a^2*b^3*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - 480*a*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 685*b^
5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 640*a^5*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2 - 1970*a^4*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1
280*a^3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 3100*a^2*b^3*(cos(d
*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 720*a*b^4*(cos(d*x + c) - 1)^2/(cos(d
```

```
*x + c) + 1)^2 - 1370*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1970*
a^4*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 720*a^3*b^2*(cos(d*x + c)
 - 1)^3/(cos(d*x + c) + 1)^3 - 3100*a^2*b^3*(cos(d*x + c) - 1)^3/(cos(d*x +
 c) + 1)^3 - 480*a*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 1370*b^5
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 805*a^4*b*(cos(d*x + c) - 1)^4
/(cos(d*x + c) + 1)^4 - 120*a^3*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)
^4 + 1490*a^2*b^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 120*a*b^4*(co
s(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 685*b^5*(cos(d*x + c) - 1)^4/(cos(
d*x + c) + 1)^4 + 137*a^4*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 274
*a^2*b^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 137*b^5*(cos(d*x + c)
- 1)^5/(cos(d*x + c) + 1)^5)/(a^6*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) -
1)^5))/d
```

3.198 $\int \frac{\sin^3(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=89

$$-\frac{\left(a^2-b^2\right)\cos(c+dx)}{a^3d}+\frac{b\left(a^2-b^2\right)\log(a\cos(c+dx)+b)}{a^4d}-\frac{b\cos^2(c+dx)}{2a^2d}+\frac{\cos^3(c+dx)}{3ad}$$

```
[Out] -(((a^2 - b^2)*Cos[c + d*x])/(a^3*d)) - (b*Cos[c + d*x]^2)/(2*a^2*d) + Cos[
c + d*x]^3/(3*a*d) + (b*(a^2 - b^2)*Log[b + a*Cos[c + d*x]])/(a^4*d)
```

Rubi [A] time = 0.15573, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2837, 12, 772}

$$-\frac{\left(a^2-b^2\right)\cos(c+dx)}{a^3d}+\frac{b\left(a^2-b^2\right)\log(a\cos(c+dx)+b)}{a^4d}-\frac{b\cos^2(c+dx)}{2a^2d}+\frac{\cos^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(((a^2 - b^2)*Cos[c + d*x])/(a^3*d)) - (b*Cos[c + d*x]^2)/(2*a^2*d) + Cos[
c + d*x]^3/(3*a*d) + (b*(a^2 - b^2)*Log[b + a*Cos[c + d*x]])/(a^4*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 772

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(
p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*(a + c*x^2)^p,
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)\sin^3(c+dx)}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{x(a^2-x^2)}{a(-b+x)}\,dx, x, -a\cos(c+dx)\right)}{a^3 d}$$

$$= \frac{\text{Subst}\left(\int \frac{x(a^2-x^2)}{-b+x}\,dx, x, -a\cos(c+dx)\right)}{a^4 d}$$

$$= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{b^2}{a^2}\right)+\frac{-a^2b+b^3}{b-x}-bx-x^2\right)dx, x, -a\cos(c+dx)\right)}{a^4 d}$$

$$= -\frac{(a^2-b^2)\cos(c+dx)}{a^3 d} - \frac{b\cos^2(c+dx)}{2a^2 d} + \frac{\cos^3(c+dx)}{3ad} + \frac{b(a^2-b^2)\log(b+a\cos(c+dx))}{a^4 d}$$

Mathematica [A] time = 0.187618, size = 89, normalized size = 1.

$$\frac{(12ab^2-9a^3)\cos(c+dx) - 3a^2b\cos(2(c+dx)) + 12a^2b\log(a\cos(c+dx)+b) + a^3\cos(3(c+dx)) - 12b^3\log(a\cos(c+}{12a^4 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((-9*a^3 + 12*a*b^2)*Cos[c + d*x] - 3*a^2*b*Cos[2*(c + d*x)] + a^3*Cos[3*(c
 + d*x)] + 12*a^2*b*Log[b + a*Cos[c + d*x]] - 12*b^3*Log[b + a*Cos[c + d*x]
])/(12*a^4*d)
```

Maple [A] time = 0.041, size = 106, normalized size = 1.2

$$\frac{(\cos(dx+c))^3}{3ad} - \frac{b(\cos(dx+c))^2}{2a^2d} - \frac{\cos(dx+c)}{ad} + \frac{b^2\cos(dx+c)}{da^3} + \frac{b\ln(b+a\cos(dx+c))}{a^2d} - \frac{b^3\ln(b+a\cos(dx+c))}{da^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/3*cos(d*x+c)^3/a/d-1/2*b*cos(d*x+c)^2/a^2/d-cos(d*x+c)/a/d+1/d/a^3*b^2*co
s(d*x+c)+b*ln(b+a*cos(d*x+c))/a^2/d-1/d*b^3/a^4*ln(b+a*cos(d*x+c))
```

Maxima [A] time = 0.97901, size = 108, normalized size = 1.21

$$\frac{\frac{2a^2\cos(dx+c)^3-3ab\cos(dx+c)^2-6(a^2-b^2)\cos(dx+c)}{a^3}+\frac{6(a^2b-b^3)\log(a\cos(dx+c)+b)}{a^4}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*((2*a^2*cos(d*x + c)^3 - 3*a*b*cos(d*x + c)^2 - 6*(a^2 - b^2)*cos(d*x +
 c))/a^3 + 6*(a^2*b - b^3)*log(a*cos(d*x + c) + b)/a^4)/d
```

Fricas [A] time = 1.80522, size = 181, normalized size = 2.03

$$\frac{2\,a^3\cos{(dx+c)}^3-3\,a^2b\cos{(dx+c)}^2-6\left(a^3-ab^2\right)\cos{(dx+c)}+6\left(a^2b-b^3\right)\log{(a\cos{(dx+c)}+b)}}{6\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^3*cos(d*x + c)^3 - 3*a^2*b*cos(d*x + c)^2 - 6*(a^3 - a*b^2)*cos(d*
x + c) + 6*(a^2*b - b^3)*log(a*cos(d*x + c) + b))/(a^4*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30075, size = 138, normalized size = 1.55

$$\frac{\left(a^2b-b^3\right)\log{(|-a\cos{(dx+c)}-b|)}}{a^4d}+\frac{2\,a^2d^2\cos{(dx+c)}^3-3\,abd^2\cos{(dx+c)}^2-6\,a^2d^2\cos{(dx+c)}+6\,b^2d^2\cos{(d}}{6\,a^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (a^2*b - b^3)*log(abs(-a*cos(d*x + c) - b))/(a^4*d) + 1/6*(2*a^2*d^2*cos(d*
x + c)^3 - 3*a*b*d^2*cos(d*x + c)^2 - 6*a^2*d^2*cos(d*x + c) + 6*b^2*d^2*co
s(d*x + c))/(a^3*d^3)
```

3.199 $\int \frac{\sin(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=34

$$\frac{b\log(a\cos(c+dx)+b)}{a^2d}-\frac{\cos(c+dx)}{ad}$$

```
[Out] -(Cos[c + d*x]/(a*d)) + (b*Log[b + a*Cos[c + d*x]])/(a^2*d)
```

Rubi [A] time = 0.0766032, antiderivative size = 34, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3872, 2833, 12, 43}

$$\frac{b\log(a\cos(c+dx)+b)}{a^2d}-\frac{\cos(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(Cos[c + d*x]/(a*d)) + (b*Log[b + a*Cos[c + d*x]])/(a^2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\sin(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)\sin(c+dx)}{-b-a\cos(c+dx)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x}{a(-b+x)}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x}{-b+x}\,dx, x, -a\cos(c+dx)\right)}{a^2d}$$
$$= \frac{\text{Subst}\left(\int \left(1-\frac{b}{b-x}\right)dx, x, -a\cos(c+dx)\right)}{a^2d}$$
$$= -\frac{\cos(c+dx)}{ad} + \frac{b\log(b+a\cos(c+dx))}{a^2d}$$

Mathematica [A] time = 0.0180372, size = 30, normalized size = 0.88

$$\frac{b\log(a\cos(c+dx)+b) - a\cos(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-(a*Cos[c + d*x]) + b*Log[b + a*Cos[c + d*x]])/(a^2*d)
```

Maple [A] time = 0.027, size = 53, normalized size = 1.6

$$\frac{b\ln(a+b\sec(dx+c))}{da^2} - \frac{1}{ad\sec(dx+c)} - \frac{b\ln(\sec(dx+c))}{da^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*b/a^2*ln(a+b*sec(d*x+c))-1/d/a/sec(d*x+c)-1/d*b/a^2*ln(sec(d*x+c))
```

Maxima [A] time = 1.02355, size = 45, normalized size = 1.32

$$-\frac{\frac{\cos(dx+c)}{a} - \frac{b\log(a\cos(dx+c)+b)}{a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -(cos(d*x + c)/a - b*log(a*cos(d*x + c) + b)/a^2)/d
```

Fricas [A] time = 1.70301, size = 74, normalized size = 2.18

$$-\frac{a\cos(dx+c) - b\log(a\cos(dx+c)+b)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -(a*cos(d*x + c) - b*log(a*cos(d*x + c) + b))/(a^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.30374, size = 51, normalized size = 1.5

$$-\frac{\cos(dx+c)}{ad}+\frac{b\log(|-a\cos(dx+c)-b|)}{a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -cos(d*x + c)/(a*d) + b*log(abs(-a*cos(d*x + c) - b))/(a^2*d)
```

3.200 $\int \frac{\csc(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{b\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}-\frac{\log(\cos(c+dx)+1)}{2d(a-b)}$$

```
[Out] Log[1 - Cos[c + d*x]]/(2*(a + b)*d) - Log[1 + Cos[c + d*x]]/(2*(a - b)*d) +
 (b*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)*d)
```

Rubi [A] time = 0.105345, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3872, 2721, 801}

$$\frac{b\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)}+\frac{\log(1-\cos(c+dx))}{2d(a+b)}-\frac{\log(\cos(c+dx)+1)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] Log[1 - Cos[c + d*x]]/(2*(a + b)*d) - Log[1 + Cos[c + d*x]]/(2*(a - b)*d) +
 (b*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cot(c+dx)}{-b-a\cos(c+dx)}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x}{(-b+x)\left(a^2-x^2\right)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{2(a-b)(a-x)} - \frac{b}{(a-b)(a+b)(b-x)} + \frac{1}{2(a+b)(a+x)}\right)dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{\log(1-\cos(c+dx))}{2(a+b)d} - \frac{\log(1+\cos(c+dx))}{2(a-b)d} + \frac{b\log(b+a\cos(c+dx))}{\left(a^2-b^2\right)d}$$

Mathematica [A] time = 0.0929826, size = 63, normalized size = 0.85

$$\frac{(a-b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) - (a+b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + b\log(a\cos(c+dx)+b)}{d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-((a + b)*Log[Cos[(c + d*x)/2]]) + b*Log[b + a*Cos[c + d*x]] + (a - b)*Log
[Sin[(c + d*x)/2]])/((a - b)*(a + b)*d)
```

Maple [A] time = 0.048, size = 75, normalized size = 1.

$$\frac{b\ln(b+a\cos(dx+c))}{d(a-b)(a+b)} - \frac{\ln(\cos(dx+c)+1)}{d(2a-2b)} + \frac{\ln(-1+\cos(dx+c))}{d(2a+2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*b/(a-b)/(a+b)*ln(b+a*cos(d*x+c))-1/d/(2*a-2*b)*ln(cos(d*x+c)+1)+1/d/(2*
a+2*b)*ln(-1+cos(d*x+c))
```

Maxima [A] time = 0.981529, size = 86, normalized size = 1.16

$$\frac{\frac{2b\log(a\cos(dx+c)+b)}{a^2-b^2} - \frac{\log(\cos(dx+c)+1)}{a-b} + \frac{\log(\cos(dx+c)-1)}{a+b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*log(a*cos(d*x + c) + b)/(a^2 - b^2) - log(cos(d*x + c) + 1)/(a - b
) + log(cos(d*x + c) - 1)/(a + b))/d
```

Fricas [A] time = 1.86108, size = 173, normalized size = 2.34

$$\frac{2\,b\log\left(a\cos\left(dx+c\right)+b\right)-\left(a+b\right)\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)+\left(a-b\right)\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\left(a^2-b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*log(a*cos(d*x + c) + b) - (a + b)*log(1/2*cos(d*x + c) + 1/2) + (a
 - b)*log(-1/2*cos(d*x + c) + 1/2))/((a^2 - b^2)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.34327, size = 135, normalized size = 1.82

$$\frac{\frac{2\,b\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^2-b^2}+\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*b*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^2 - b^2) + log(abs(-cos(d*x + c) + 1)
/abs(cos(d*x + c) + 1))/(a + b))/d
```

3.201 $\int \frac{\csc^3(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=116

$$\frac{a^2 b\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^2}+\frac{\csc^2(c+dx)(b-a\cos(c+dx))}{2d\left(a^2-b^2\right)}+\frac{a\log(1-\cos(c+dx))}{4d(a+b)^2}-\frac{a\log(\cos(c+dx)+1)}{4d(a-b)^2}$$

```
[Out] ((b - a*Cos[c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)*d) + (a*Log[1 - Cos[c
+ d*x]])/(4*(a + b)^2*d) - (a*Log[1 + Cos[c + d*x]])/(4*(a - b)^2*d) + (a^2
*b*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^2*d)
```

Rubi [A] time = 0.21299, antiderivative size = 116, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2837, 12, 823, 801}

$$\frac{a^2 b\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^2}+\frac{\csc^2(c+dx)(b-a\cos(c+dx))}{2d\left(a^2-b^2\right)}+\frac{a\log(1-\cos(c+dx))}{4d(a+b)^2}-\frac{a\log(\cos(c+dx)+1)}{4d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((b - a*Cos[c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)*d) + (a*Log[1 - Cos[c
+ d*x]])/(4*(a + b)^2*d) - (a*Log[1 + Cos[c + d*x]])/(4*(a - b)^2*d) + (a^2
*b*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
```

```
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cot(c+dx)\csc^2(c+dx)}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{a^3\,\text{Subst}\left(\int \frac{x}{a(-b+x)\left(a^2-x^2\right)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a^2\,\text{Subst}\left(\int \frac{x}{(-b+x)\left(a^2-x^2\right)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{(b-a\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right)d} + \frac{\text{Subst}\left(\int \frac{a^2b+a^2x}{(-b+x)\left(a^2-x^2\right)}\,dx, x, -a\cos(c+dx)\right)}{2\left(a^2-b^2\right)d}$$

$$= \frac{(b-a\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right)d} + \frac{\text{Subst}\left(\int \left(\frac{a(a+b)}{2(a-b)(a-x)} - \frac{2a^2b}{(a-b)(a+b)(b-x)} + \frac{a(a-b)}{2(a+b)(a+x)}\right) dx, x,$$

$$= \frac{(b-a\cos(c+dx))\csc^2(c+dx)}{2\left(a^2-b^2\right)d} + \frac{a\log(1-\cos(c+dx))}{4(a+b)^2d} - \frac{a\log(1+\cos(c+dx))}{4(a-b)^2d} + \frac{a^2b\log}{}$$

Mathematica [A] time = 0.595094, size = 123, normalized size = 1.06

$$\frac{-(a-b)^2(a+b)\csc^2\left(\frac{1}{2}(c+dx)\right) + (a-b)(a+b)^2\sec^2\left(\frac{1}{2}(c+dx)\right) - 4a\left((a-b)^2\left(-\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right) + (a+b)^2}{8d(a-b)^2(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-((a - b)^2*(a + b)*Csc[(c + d*x)/2]^2) - 4*a*((a + b)^2*Log[Cos[(c + d*x)
/2]] - 2*a*b*Log[b + a*Cos[c + d*x]] - (a - b)^2*Log[Sin[(c + d*x)/2]]) + (
a - b)*(a + b)^2*Sec[(c + d*x)/2]^2)/(8*(a - b)^2*(a + b)^2*d)
```

Maple [A] time = 0.069, size = 121, normalized size = 1.

$$\frac{a^2b\ln(b+a\cos(dx+c))}{d(a+b)^2(a-b)^2} + \frac{1}{d(4a-4b)(\cos(dx+c)+1)} - \frac{a\ln(\cos(dx+c)+1)}{4(a-b)^2d} + \frac{1}{d(4a+4b)(-1+\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*b*a^2/(a+b)^2/(a-b)^2*ln(b+a*cos(d*x+c))+1/d/(4*a-4*b)/(cos(d*x+c)+1)-1
/4*a*ln(cos(d*x+c)+1)/(a-b)^2/d+1/d/(4*a+4*b)/(-1+cos(d*x+c))+1/4/d*a/(a+b)
```

```
^2*ln(-1+cos(d*x+c))
```

Maxima [A] time = 1.00064, size = 178, normalized size = 1.53

$$\frac{\frac{4\,a^2 b\log(a\cos(dx+c)+b)}{a^4-2\,a^2b^2+b^4}-\frac{a\log(\cos(dx+c)+1)}{a^2-2\,ab+b^2}+\frac{a\log(\cos(dx+c)-1)}{a^2+2\,ab+b^2}+\frac{2\,(a\cos(dx+c)-b)}{(a^2-b^2)\cos(dx+c)^2-a^2+b^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*a^2*b*log(a*cos(d*x + c) + b)/(a^4 - 2*a^2*b^2 + b^4) - a*log(cos(d*
x + c) + 1)/(a^2 - 2*a*b + b^2) + a*log(cos(d*x + c) - 1)/(a^2 + 2*a*b + b^
2) + 2*(a*cos(d*x + c) - b)/((a^2 - b^2)*cos(d*x + c)^2 - a^2 + b^2))/d
```

Fricas [A] time = 2.17163, size = 508, normalized size = 4.38

$$-\frac{2\,a^2b-2\,b^3-2\left(a^3-ab^2\right)\cos\left(dx+c\right)-4\left(a^2b\cos\left(dx+c\right)^2-a^2b\right)\log\left(a\cos\left(dx+c\right)+b\right)-\left(a^3+2\,a^2b+ab^2-\left(a^3+\right.\right.}{4\left(\left(a^4-2\,a^2b^2+b^4\right)d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] -1/4*(2*a^2*b - 2*b^3 - 2*(a^3 - a*b^2)*cos(d*x + c) - 4*(a^2*b*cos(d*x + c
)^2 - a^2*b)*log(a*cos(d*x + c) + b) - (a^3 + 2*a^2*b + a*b^2 - (a^3 + 2*a^
2*b + a*b^2)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + (a^3 - 2*a^2*b +
 a*b^2 - (a^3 - 2*a^2*b + a*b^2)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/
2))/((a^4 - 2*a^2*b^2 + b^4)*d*cos(d*x + c)^2 - (a^4 - 2*a^2*b^2 + b^4)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc^3\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**3/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.32966, size = 273, normalized size = 2.35

$$\frac{\frac{8\,a^2b\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^4-2\,a^2b^2+b^4}+\frac{2\,a\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2+2\,ab+b^2}+\frac{\left(a+b-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\left(a^2+2\,ab+b^2\right)(\cos(dx+c)-1)}-\frac{\cos(dx+c)-1}{(a-b)(\cos(dx+c)+1)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/8*(8*a^2*b*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^4 - 2*a^2*b^2 + b^4) + 2*a*log(ab
s(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^2 + 2*a*b + b^2) + (a + b -
2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/((a^2 + 2*a*b
 + b^2)*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/((a - b)*(cos(d*x + c) + 1
)))/d
```

3.202 $\int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=179

$$\frac{a^4b\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^3}+\frac{\csc^4(c+dx)(b-a\cos(c+dx))}{4d\left(a^2-b^2\right)}+\frac{\csc^2(c+dx)\left(4a^2b-a\left(3a^2+b^2\right)\cos(c+dx)\right)}{8d\left(a^2-b^2\right)^2}+\frac{a(3a+b}{}$$

```
[Out] ((4*a^2*b - a*(3*a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x]^2)/(8*(a^2 - b^2)^2*
d) + ((b - a*Cos[c + d*x])*Csc[c + d*x]^4)/(4*(a^2 - b^2)*d) + (a*(3*a + b)
*Log[1 - Cos[c + d*x]])/(16*(a + b)^3*d) - (a*(3*a - b)*Log[1 + Cos[c + d*x
]])/(16*(a - b)^3*d) + (a^4*b*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^3*d)
```

Rubi [A] time = 0.30139, antiderivative size = 179, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2837, 12, 823, 801}

$$\frac{a^4b\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^3}+\frac{\csc^4(c+dx)(b-a\cos(c+dx))}{4d\left(a^2-b^2\right)}+\frac{\csc^2(c+dx)\left(4a^2b-a\left(3a^2+b^2\right)\cos(c+dx)\right)}{8d\left(a^2-b^2\right)^2}+\frac{a(3a+b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((4*a^2*b - a*(3*a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x]^2)/(8*(a^2 - b^2)^2*
d) + ((b - a*Cos[c + d*x])*Csc[c + d*x]^4)/(4*(a^2 - b^2)*d) + (a*(3*a + b)
*Log[1 - Cos[c + d*x]])/(16*(a + b)^3*d) - (a*(3*a - b)*Log[1 + Cos[c + d*x
]])/(16*(a - b)^3*d) + (a^4*b*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^3*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 823

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> -Simp[((d + e*x)^(m + 1)*(f*a*c*e - a*g*c*d + c*(c*d*f + a
*e*g)*x)*(a + c*x^2)^(p + 1))/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[1/
(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp[f
*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f + a
*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*
```

```
d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2
*m, 2*p])
```

Rule 801

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[((d + e*x)^m*(f + g*x))/(a + c*x^2), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\csc^5(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cot(c+dx)\csc^4(c+dx)}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{a^5\,\mathrm{Subst}\left(\int \frac{x}{a(-b+x)\left(a^2-x^2\right)^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a^4\,\mathrm{Subst}\left(\int \frac{x}{(-b+x)\left(a^2-x^2\right)^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{(b-a\cos(c+dx))\csc^4(c+dx)}{4\left(a^2-b^2\right)d} + \frac{a^2\,\mathrm{Subst}\left(\int \frac{a^2b+3a^2x}{(-b+x)\left(a^2-x^2\right)^2}\,dx, x, -a\cos(c+dx)\right)}{4\left(a^2-b^2\right)d}$$

$$= \frac{\left(4a^2b-a\left(3a^2+b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{8\left(a^2-b^2\right)^2 d} + \frac{(b-a\cos(c+dx))\csc^4(c+dx)}{4\left(a^2-b^2\right)d} + \frac{\mathrm{Subst}\left(\right.}{}$$

$$= \frac{\left(4a^2b-a\left(3a^2+b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{8\left(a^2-b^2\right)^2 d} + \frac{(b-a\cos(c+dx))\csc^4(c+dx)}{4\left(a^2-b^2\right)d} + \frac{\mathrm{Subst}\left(\right.}{}$$

$$= \frac{\left(4a^2b-a\left(3a^2+b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{8\left(a^2-b^2\right)^2 d} + \frac{(b-a\cos(c+dx))\csc^4(c+dx)}{4\left(a^2-b^2\right)d} + \frac{a(3a+}{}$$

Mathematica [A] time = 5.16327, size = 207, normalized size = 1.16

$$\frac{-2(a-b)^3\left(3a^2+4ab+b^2\right)\csc^2\left(\frac{1}{2}(c+dx)\right)+2(a+b)^3\left(3a^2-4ab+b^2\right)\sec^2\left(\frac{1}{2}(c+dx)\right)+8a\left(8a^3b\log(a\cos(c+d\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*(a - b)^3*(3*a^2 + 4*a*b + b^2)*Csc[(c + d*x)/2]^2 - (a - b)^3*(a + b)^
2*Csc[(c + d*x)/2]^4 + 8*a*(-((3*a - b)*(a + b)^3*Log[Cos[(c + d*x)/2]]) +
8*a^3*b*Log[b + a*Cos[c + d*x]] + (a - b)^3*(3*a + b)*Log[Sin[(c + d*x)/2]]
) + 2*(a + b)^3*(3*a^2 - 4*a*b + b^2)*Sec[(c + d*x)/2]^2 + (a - b)^2*(a + b
)^3*Sec[(c + d*x)/2]^4)/(64*(a - b)^3*(a + b)^3*d)
```

Maple [A] time = 0.067, size = 259, normalized size = 1.5

$$\frac{a^4 b\ln(b+a\cos(dx+c))}{d(a+b)^3(a-b)^3} + \frac{1}{2d(8a-8b)(\cos(dx+c)+1)^2} + \frac{3a}{16d(a-b)^2(\cos(dx+c)+1)} - \frac{b}{16d(a-b)^2(\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*b*a^4/(a+b)^3/(a-b)^3*ln(b+a*cos(d*x+c))+1/2/d/(8*a-8*b)/(cos(d*x+c)+1)
^2+3/16/d/(a-b)^2/(cos(d*x+c)+1)*a-1/16/d/(a-b)^2/(cos(d*x+c)+1)*b-3/16/d*a
^2/(a-b)^3*ln(cos(d*x+c)+1)+1/16/d*a/(a-b)^3*ln(cos(d*x+c)+1)*b-1/2/d/(8*a+
8*b)/(-1+cos(d*x+c))^2+3/16/d/(a+b)^2/(-1+cos(d*x+c))*a+1/16/d/(a+b)^2/(-1+
cos(d*x+c))*b+3/16/d/(a+b)^3*a^2*ln(-1+cos(d*x+c))+1/16/d/(a+b)^3*a*ln(-1+c
os(d*x+c))*b
```

Maxima [A] time = 1.01762, size = 362, normalized size = 2.02

$$\frac{\frac{16\,a^4b\log(a\cos(dx+c)+b)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}-\frac{\left(3\,a^2-ab\right)\log(\cos(dx+c)+1)}{a^3-3\,a^2b+3\,ab^2-b^3}+\frac{\left(3\,a^2+ab\right)\log(\cos(dx+c)-1)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\left(4\,a^2b\cos(dx+c)^2-\left(3\,a^3+ab^2\right)\cos(dx+c)^3-6\,a^2b+2\,b^3+\left(5\,a^3-ab\right.}{\left(a^4-2\,a^2b^2+b^4\right)\cos(dx+c)^4+a^4-2\,a^2b^2+b^4-2\left(a^4-2\,a^2b^2+b^4\right)c}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/16*(16*a^4*b*log(a*cos(d*x + c) + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
- (3*a^2 - a*b)*log(cos(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (3*
a^2 + a*b)*log(cos(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(4*a^2
*b*cos(d*x + c)^2 - (3*a^3 + a*b^2)*cos(d*x + c)^3 - 6*a^2*b + 2*b^3 + (5*a
^3 - a*b^2)*cos(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^4 + a^4 - 2
*a^2*b^2 + b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos(d*x + c)^2))/d
```

Fricas [B] time = 3.13934, size = 1033, normalized size = 5.77

$$\frac{12\,a^4b-16\,a^2b^3+4\,b^5+2\left(3\,a^5-2\,a^3b^2-ab^4\right)\cos\left(dx+c\right)^3-8\left(a^4b-a^2b^3\right)\cos\left(dx+c\right)^2-2\left(5\,a^5-6\,a^3b^2+ab^4\right)\cos\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/16*(12*a^4*b - 16*a^2*b^3 + 4*b^5 + 2*(3*a^5 - 2*a^3*b^2 - a*b^4)*cos(d*x
 + c)^3 - 8*(a^4*b - a^2*b^3)*cos(d*x + c)^2 - 2*(5*a^5 - 6*a^3*b^2 + a*b^4
)*cos(d*x + c) + 16*(a^4*b*cos(d*x + c)^4 - 2*a^4*b*cos(d*x + c)^2 + a^4*b)
*log(a*cos(d*x + c) + b) - (3*a^5 + 8*a^4*b + 6*a^3*b^2 - a*b^4 + (3*a^5 +
8*a^4*b + 6*a^3*b^2 - a*b^4)*cos(d*x + c)^4 - 2*(3*a^5 + 8*a^4*b + 6*a^3*b^
2 - a*b^4)*cos(d*x + c)^2)*log(1/2*cos(d*x + c) + 1/2) + (3*a^5 - 8*a^4*b +
 6*a^3*b^2 - a*b^4 + (3*a^5 - 8*a^4*b + 6*a^3*b^2 - a*b^4)*cos(d*x + c)^4 -
 2*(3*a^5 - 8*a^4*b + 6*a^3*b^2 - a*b^4)*cos(d*x + c)^2)*log(-1/2*cos(d*x +
 c) + 1/2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^4 - 2*(a^6
- 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^2 + (a^6 - 3*a^4*b^2 + 3*a^2*
b^4 - b^6)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc^5\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**5/(a + b*sec(c + d*x)), x)
```

Giac [B] time = 1.44221, size = 566, normalized size = 3.16

$$\frac{\frac{64\,a^4 b \log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6}+\frac{4\left(3\,a^2+ab\right)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{\frac{8\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^2-2\,ab+b^2}\cdots}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/64*(64*a^4*b*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b
*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6
) + 4*(3*a^2 + a*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^3
+ 3*a^2*b + 3*a*b^2 + b^3) - (8*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4
*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x
+ c) + 1)^2 + b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(a^2 - 2*a*b + b
^2) - (a^2 + 2*a*b + b^2 - 8*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12
*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b^2*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + 18*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 6*a*b*(c
os(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^2/((a^3 + 3*a^2
*b + 3*a*b^2 + b^3)*(cos(d*x + c) - 1)^2))/d
```

3.203 $\int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=230

$$\frac{\sin^3(c+dx)\left(8b\left(a^2-b^2\right)-a\left(5a^2-6b^2\right)\cos(c+dx)\right)}{24a^4d}+\frac{\sin(c+dx)\left(16b\left(a^2-b^2\right)^2-a\left(-14a^2b^2+5a^4+8b^4\right)\cos(c+d\right.}{16a^6d}$$

```
[Out] ((5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*x)/(16*a^7) - (2*(a - b)^(5/2)*
b*(a + b)^(5/2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^7*d
) + ((16*b*(a^2 - b^2)^2 - a*(5*a^4 - 14*a^2*b^2 + 8*b^4)*Cos[c + d*x])*Sin
[c + d*x])/(16*a^6*d) + ((8*b*(a^2 - b^2) - a*(5*a^2 - 6*b^2)*Cos[c + d*x])
*Sin[c + d*x]^3)/(24*a^4*d) + ((6*b - 5*a*Cos[c + d*x])*Sin[c + d*x]^5)/(30
*a^2*d)
```

Rubi [A] time = 0.607589, antiderivative size = 230, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2865, 2735, 2659, 208}

$$\frac{\sin^3(c+dx)\left(8b\left(a^2-b^2\right)-a\left(5a^2-6b^2\right)\cos(c+dx)\right)}{24a^4d}+\frac{\sin(c+dx)\left(16b\left(a^2-b^2\right)^2-a\left(-14a^2b^2+5a^4+8b^4\right)\cos(c+d\right.}{16a^6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```


```
[Out] ((5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*x)/(16*a^7) - (2*(a - b)^(5/2)*
b*(a + b)^(5/2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^7*d
) + ((16*b*(a^2 - b^2)^2 - a*(5*a^4 - 14*a^2*b^2 + 8*b^4)*Cos[c + d*x])*Sin
[c + d*x])/(16*a^6*d) + ((8*b*(a^2 - b^2) - a*(5*a^2 - 6*b^2)*Cos[c + d*x])
*Sin[c + d*x]^3)/(24*a^4*d) + ((6*b - 5*a*Cos[c + d*x])*Sin[c + d*x]^5)/(30
*a^2*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
```

```
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)\sin^6(c+dx)}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{(6b-5a\cos(c+dx))\sin^5(c+dx)}{30a^2d} - \frac{\int \frac{\left(-ab+\left(5a^2-6b^2\right)\cos(c+dx)\right)\sin^4(c+dx)}{-b-a\cos(c+dx)}\,dx}{6a^2}$$

$$= \frac{\left(8b\left(a^2-b^2\right)-a\left(5a^2-6b^2\right)\cos(c+dx)\right)\sin^3(c+dx)}{24a^4d} + \frac{(6b-5a\cos(c+dx))\sin^5(c+dx)}{30a^2d}$$

$$= \frac{\left(16b\left(a^2-b^2\right)^2-a\left(5a^4-14a^2b^2+8b^4\right)\cos(c+dx)\right)\sin(c+dx)}{16a^6d} + \frac{\left(8b\left(a^2-b^2\right)-a\left(5a^2-\right.\right.}{}$$

$$= \frac{\left(5a^6-30a^4b^2+40a^2b^4-16b^6\right)x}{16a^7} + \frac{\left(16b\left(a^2-b^2\right)^2-a\left(5a^4-14a^2b^2+8b^4\right)\cos(c+dx)\right)\text{s}}{16a^6d}$$

$$= \frac{\left(5a^6-30a^4b^2+40a^2b^4-16b^6\right)x}{16a^7} + \frac{\left(16b\left(a^2-b^2\right)^2-a\left(5a^4-14a^2b^2+8b^4\right)\cos(c+dx)\right)\text{s}}{16a^6d}$$

$$= \frac{\left(5a^6-30a^4b^2+40a^2b^4-16b^6\right)x}{16a^7} - \frac{2(a-b)^{5/2}b(a+b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^7d} + \frac{(16b}{}$$

Mathematica [A] time = 2.38069, size = 268, normalized size = 1.17

$$-30a^4b^2\sin(4(c+dx)) + 80a^3b^3\sin(3(c+dx)) + 120ab\left(-18a^2b^2+11a^4+8b^4\right)\sin(c+dx) - 15\left(-32a^4b^2+16a^2b^4+\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

```
[Out] (300*a^6*c - 1800*a^4*b^2*c + 2400*a^2*b^4*c - 960*b^6*c + 300*a^6*d*x - 18
00*a^4*b^2*d*x + 2400*a^2*b^4*d*x - 960*b^6*d*x + 1920*b*(a^2 - b^2)^(5/2)*
ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] + 120*a*b*(11*a^4 - 18
*a^2*b^2 + 8*b^4)*Sin[c + d*x] - 15*(15*a^6 - 32*a^4*b^2 + 16*a^2*b^4)*Sin[
2*(c + d*x)] - 140*a^5*b*Sin[3*(c + d*x)] + 80*a^3*b^3*Sin[3*(c + d*x)] + 4
5*a^6*Sin[4*(c + d*x)] - 30*a^4*b^2*Sin[4*(c + d*x)] + 12*a^5*b*Sin[5*(c +
d*x)] - 5*a^6*Sin[6*(c + d*x)])/(960*a^7*d)
```

Maple [B] time = 0.074, size = 1566, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+b*sec(d*x+c)),x)
```

```
[Out] 5/8/d/a*arctan(tan(1/2*d*x+1/2*c))-2/d*b/a/((a+b)*(a-b))^(1/2)*arctanh((a-b
)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^
6*tan(1/2*d*x+1/2*c)*b-6/d*b^5/a^5/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/
2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+2/d*b^7/a^7/((a+b)*(a-b))^(1/2)*arctanh((
a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-4/d/a^4/(1+tan(1/2*d*x+1/2*c)^
2)^6*tan(1/2*d*x+1/2*c)*b^3+2/d/a^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+
1/2*c)*b^5+7/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)*b^2-48/d
/a^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*b^3-11/2/d/a^3/(1+tan(
1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7*b^2+1/d/a^5/(1+tan(1/2*d*x+1/2*c)^
2)^6*tan(1/2*d*x+1/2*c)^11*b^4+10/d/a^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*
d*x+1/2*c)^9*b^5-68/3/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9
*b^3+172/5/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7*b+2/d/a^6/
(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11*b^5+38/3/d/a^2/(1+tan(1/2*
d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*b-29/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^
6*tan(1/2*d*x+1/2*c)^9*b^2+3/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1
/2*c)^9*b^4+20/d/a^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*b^5+38
/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*b+29/4/d/a^3/(1+ta
n(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*b^2-48/d/a^4/(1+tan(1/2*d*x+1/2*
c)^2)^6*tan(1/2*d*x+1/2*c)^7*b^3+2/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2
*d*x+1/2*c)^7*b^4+20/d/a^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7*
b^5+11/2/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*b^2-2/d/a^5/
(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*b^4+172/5/d/a^2/(1+tan(1/2*
d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*b+6/d*b^3/a^3/((a+b)*(a-b))^(1/2)*arct
anh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-1/d/a^5/(1+tan(1/2*d*x+1/
2*c)^2)^6*tan(1/2*d*x+1/2*c)*b^4+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2
*d*x+1/2*c)^11*b-7/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11
*b^2-4/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11*b^3-68/3/d/a^
4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*b^3-3/d/a^5/(1+tan(1/2*d*
x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*b^4+10/d/a^6/(1+tan(1/2*d*x+1/2*c)^2)^6*
tan(1/2*d*x+1/2*c)^3*b^5-33/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/
2*c)^5+33/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7+5/8/d/a/(1+
tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11+85/24/d/a/(1+tan(1/2*d*x+1/2*
c)^2)^6*tan(1/2*d*x+1/2*c)^9-85/24/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d
*x+1/2*c)^3-5/8/d/a/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)-15/4/d/a^
3*arctan(tan(1/2*d*x+1/2*c))*b^2+5/d/a^5*arctan(tan(1/2*d*x+1/2*c))*b^4-2/d
/a^7*arctan(tan(1/2*d*x+1/2*c))*b^6
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.40288, size = 1296, normalized size = 5.63

$$\left[\frac{15\left(5a^6 - 30a^4b^2 + 40a^2b^4 - 16b^6\right)dx + 120\left(a^4b - 2a^2b^3 + b^5\right)\sqrt{a^2-b^2}\log\left(\frac{2ab\cos(dx+c) - \left(a^2-2b^2\right)\cos(dx+c)^2 - 2\sqrt{a^2-b^2}(b}{a^2\cos(dx+c)^2 + 2ab\cos(d}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/240*(15*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*d*x + 120*(a^4*b - 2*
a^2*b^3 + b^5)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(
d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 -
b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (40*a^6*cos(d*x + c
)^5 - 48*a^5*b*cos(d*x + c)^4 - 368*a^5*b + 560*a^3*b^3 - 240*a*b^5 - 10*(1
3*a^6 - 6*a^4*b^2)*cos(d*x + c)^3 + 16*(11*a^5*b - 5*a^3*b^3)*cos(d*x + c)^
2 + 15*(11*a^6 - 18*a^4*b^2 + 8*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/(a^7*d
), 1/240*(15*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*d*x - 240*(a^4*b -
2*a^2*b^3 + b^5)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c)
+ a)/((a^2 - b^2)*sin(d*x + c))) - (40*a^6*cos(d*x + c)^5 - 48*a^5*b*cos(d*
x + c)^4 - 368*a^5*b + 560*a^3*b^3 - 240*a*b^5 - 10*(13*a^6 - 6*a^4*b^2)*co
s(d*x + c)^3 + 16*(11*a^5*b - 5*a^3*b^3)*cos(d*x + c)^2 + 15*(11*a^6 - 18*a
^4*b^2 + 8*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/(a^7*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25831, size = 1054, normalized size = 4.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/240*(15*(5*a^6 - 30*a^4*b^2 + 40*a^2*b^4 - 16*b^6)*(d*x + c)/a^7 - 480*(a
^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-
2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt
(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^7) + 2*(75*a^5*tan(1/2*d*x + 1/2*c)^11 +
 240*a^4*b*tan(1/2*d*x + 1/2*c)^11 - 210*a^3*b^2*tan(1/2*d*x + 1/2*c)^11 -
480*a^2*b^3*tan(1/2*d*x + 1/2*c)^11 + 120*a*b^4*tan(1/2*d*x + 1/2*c)^11 + 2
40*b^5*tan(1/2*d*x + 1/2*c)^11 + 425*a^5*tan(1/2*d*x + 1/2*c)^9 + 1520*a^4*
b*tan(1/2*d*x + 1/2*c)^9 - 870*a^3*b^2*tan(1/2*d*x + 1/2*c)^9 - 2720*a^2*b^
3*tan(1/2*d*x + 1/2*c)^9 + 360*a*b^4*tan(1/2*d*x + 1/2*c)^9 + 1200*b^5*tan(
1/2*d*x + 1/2*c)^9 + 990*a^5*tan(1/2*d*x + 1/2*c)^7 + 4128*a^4*b*tan(1/2*d*
x + 1/2*c)^7 - 660*a^3*b^2*tan(1/2*d*x + 1/2*c)^7 - 5760*a^2*b^3*tan(1/2*d*
```

x + 1/2*c)^7 + 240*a*b^4*tan(1/2*d*x + 1/2*c)^7 + 2400*b^5*tan(1/2*d*x + 1/2*c)^7 - 990*a^5*tan(1/2*d*x + 1/2*c)^5 + 4128*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 660*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 5760*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 240*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 2400*b^5*tan(1/2*d*x + 1/2*c)^5 - 425*a^5*tan(1/2*d*x + 1/2*c)^3 + 1520*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 870*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 2720*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 360*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 1200*b^5*tan(1/2*d*x + 1/2*c)^3 - 75*a^5*tan(1/2*d*x + 1/2*c) + 240*a^4*b*tan(1/2*d*x + 1/2*c) + 210*a^3*b^2*tan(1/2*d*x + 1/2*c) - 480*a^2*b^3*tan(1/2*d*x + 1/2*c) - 120*a*b^4*tan(1/2*d*x + 1/2*c) + 240*b^5*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^6*a^6))/d

3.204 $\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{\sin(c+dx)\left(8b\left(a^2-b^2\right)-a\left(3a^2-4b^2\right)\cos(c+dx)\right)}{8a^4d}+\frac{x\left(-12a^2b^2+3a^4+8b^4\right)}{8a^5}+\frac{\sin^3(c+dx)(4b-3a\cos(c+dx))}{12a^2d}$$

```
[Out] ((3*a^4 - 12*a^2*b^2 + 8*b^4)*x)/(8*a^5) - (2*(a - b)^(3/2)*b*(a + b)^(3/2)
*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*d) + ((8*b*(a^2
- b^2) - a*(3*a^2 - 4*b^2)*Cos[c + d*x])*Sin[c + d*x])/(8*a^4*d) + ((4*b -
3*a*Cos[c + d*x])*Sin[c + d*x]^3)/(12*a^2*d)
```

Rubi [A] time = 0.380718, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2865, 2735, 2659, 208}

$$\frac{\sin(c+dx)\left(8b\left(a^2-b^2\right)-a\left(3a^2-4b^2\right)\cos(c+dx)\right)}{8a^4d}+\frac{x\left(-12a^2b^2+3a^4+8b^4\right)}{8a^5}+\frac{\sin^3(c+dx)(4b-3a\cos(c+dx))}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```


```
[Out] ((3*a^4 - 12*a^2*b^2 + 8*b^4)*x)/(8*a^5) - (2*(a - b)^(3/2)*b*(a + b)^(3/2)
*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*d) + ((8*b*(a^2
- b^2) - a*(3*a^2 - 4*b^2)*Cos[c + d*x])*Sin[c + d*x])/(8*a^4*d) + ((4*b -
3*a*Cos[c + d*x])*Sin[c + d*x]^3)/(12*a^2*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)\sin^4(c+dx)}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{(4b-3a\cos(c+dx))\sin^3(c+dx)}{12a^2d} - \frac{\int \frac{\left(-ab+\left(3a^2-4b^2\right)\cos(c+dx)\right)\sin^2(c+dx)}{-b-a\cos(c+dx)}\,dx}{4a^2}$$

$$= \frac{\left(8b\left(a^2-b^2\right)-a\left(3a^2-4b^2\right)\cos(c+dx)\right)\sin(c+dx)}{8a^4d} + \frac{(4b-3a\cos(c+dx))\sin^3(c+dx)}{12a^2d} - \int$$

$$= \frac{\left(3a^4-12a^2b^2+8b^4\right)x}{8a^5} + \frac{\left(8b\left(a^2-b^2\right)-a\left(3a^2-4b^2\right)\cos(c+dx)\right)\sin(c+dx)}{8a^4d} + \frac{(4b-3a\cos}{}$$

$$= \frac{\left(3a^4-12a^2b^2+8b^4\right)x}{8a^5} + \frac{\left(8b\left(a^2-b^2\right)-a\left(3a^2-4b^2\right)\cos(c+dx)\right)\sin(c+dx)}{8a^4d} + \frac{(4b-3a\cos}{}$$

$$= \frac{\left(3a^4-12a^2b^2+8b^4\right)x}{8a^5} - \frac{2(a-b)^{3/2}b(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d} + \frac{\left(8b\left(a^2-b^2\right)-a\left(\right.\right.}{}$$

Mathematica [A] time = 0.812052, size = 172, normalized size = 1.07

$$\frac{24ab\left(5a^2-4b^2\right)\sin(c+dx)-24\left(a^4-a^2b^2\right)\sin(2(c+dx))+192b\left(a^2-b^2\right)^{3/2}\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)-144a^2b^2c-14}{96a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] (36*a^4*c - 144*a^2*b^2*c + 96*b^4*c + 36*a^4*d*x - 144*a^2*b^2*d*x + 96*b^
4*d*x + 192*b*(a^2 - b^2)^(3/2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^
2 - b^2]] + 24*a*b*(5*a^2 - 4*b^2)*Sin[c + d*x] - 24*(a^4 - a^2*b^2)*Sin[2*
(c + d*x)] - 8*a^3*b*Sin[3*(c + d*x)] + 3*a^4*Sin[4*(c + d*x)])/(96*a^5*d)
```

Maple [B] time = 0.065, size = 769, normalized size = 4.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*sec(d*x+c)),x)
```

```
[Out] 3/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7+2/d/a^2/(1+tan(1/2*
d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*b-1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*t
an(1/2*d*x+1/2*c)^7*b^2-2/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*
c)^7*b^3+26/3/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*b-1/d/a
^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*b^2-6/d/a^4/(1+tan(1/2*d
*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*b^3+11/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4
*tan(1/2*d*x+1/2*c)^5-11/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c
)^3+1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*b^2+26/3/d/a^2/
(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*b-6/d/a^4/(1+tan(1/2*d*x+1/
2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*b^3+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1
/2*d*x+1/2*c)*b-2/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*b^3-3
/4/d/a/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)+1/d/a^3/(1+tan(1/2*d*x
+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*b^2-3/d/a^3*arctan(tan(1/2*d*x+1/2*c))*b^2+
2/d/a^5*arctan(tan(1/2*d*x+1/2*c))*b^4+3/4/d/a*arctan(tan(1/2*d*x+1/2*c))-2
/d*b/a/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(
1/2))+4/d*b^3/a^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+
b)*(a-b))^(1/2))-2/d*b^5/a^5/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+
1/2*c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.48273, size = 902, normalized size = 5.6

$$\left[\frac{3\left(3\,a^4-12\,a^2b^2+8\,b^4\right)dx-12\left(a^2b-b^3\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right.}{24\,a^5d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(3*a^4 - 12*a^2*b^2 + 8*b^4)*d*x - 12*(a^2*b - b^3)*sqrt(a^2 - b^2
)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2
)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*
a*b*cos(d*x + c) + b^2)) + (6*a^4*cos(d*x + c)^3 - 8*a^3*b*cos(d*x + c)^2 +
 32*a^3*b - 24*a*b^3 - 3*(5*a^4 - 4*a^2*b^2)*cos(d*x + c))*sin(d*x + c))/(a
^5*d), 1/24*(3*(3*a^4 - 12*a^2*b^2 + 8*b^4)*d*x - 24*(a^2*b - b^3)*sqrt(-a^
2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x
 + c))) + (6*a^4*cos(d*x + c)^3 - 8*a^3*b*cos(d*x + c)^2 + 32*a^3*b - 24*a*
b^3 - 3*(5*a^4 - 4*a^2*b^2)*cos(d*x + c))*sin(d*x + c))/(a^5*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

Giac [B] time = 1.33762, size = 549, normalized size = 3.41

$$\frac{3\left(3a^4-12a^2b^2+8b^4\right)(dx+c)}{a^5}-\frac{48\left(a^4b-2a^2b^3+b^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,a^5}+\frac{2\left(9a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+24a^2b\,t\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(3*(3*a^4 - 12*a^2*b^2 + 8*b^4)*(d*x + c)/a^5 - 48*(a^4*b - 2*a^2*b^3
+ b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1
/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b
^2)*a^5) + 2*(9*a^3*tan(1/2*d*x + 1/2*c)^7 + 24*a^2*b*tan(1/2*d*x + 1/2*c)^
7 - 12*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 24*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*a^
3*tan(1/2*d*x + 1/2*c)^5 + 104*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 12*a*b^2*tan(
1/2*d*x + 1/2*c)^5 - 72*b^3*tan(1/2*d*x + 1/2*c)^5 - 33*a^3*tan(1/2*d*x + 1
/2*c)^3 + 104*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 12*a*b^2*tan(1/2*d*x + 1/2*c)^
3 - 72*b^3*tan(1/2*d*x + 1/2*c)^3 - 9*a^3*tan(1/2*d*x + 1/2*c) + 24*a^2*b*t
an(1/2*d*x + 1/2*c) + 12*a*b^2*tan(1/2*d*x + 1/2*c) - 24*b^3*tan(1/2*d*x +
1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^4))/d
```

3.205 $\int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=100

$$\frac{x\left(a^2-2b^2\right)}{2a^3}+\frac{\sin(c+dx)(2b-a\cos(c+dx))}{2a^2d}-\frac{2b\sqrt{a-b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d}$$

```
[Out] ((a^2 - 2*b^2)*x)/(2*a^3) - (2*Sqrt[a - b]*b*Sqrt[a + b]*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*d) + ((2*b - a*Cos[c + d*x])*Sin[c
+ d*x])/(2*a^2*d)
```

Rubi [A] time = 0.207073, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2865, 2735, 2659, 208}

$$\frac{x\left(a^2-2b^2\right)}{2a^3}+\frac{\sin(c+dx)(2b-a\cos(c+dx))}{2a^2d}-\frac{2b\sqrt{a-b}\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((a^2 - 2*b^2)*x)/(2*a^3) - (2*Sqrt[a - b]*b*Sqrt[a + b]*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*d) + ((2*b - a*Cos[c + d*x])*Sin[c
+ d*x])/(2*a^2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx=-\int \frac{\cos(c+dx)\sin^2(c+dx)}{-b-a\cos(c+dx)}\,dx$$

$$=\frac{(2b-a\cos(c+dx))\sin(c+dx)}{2a^2d}-\frac{\int \frac{-ab+\left(a^2-2b^2\right)\cos(c+dx)}{-b-a\cos(c+dx)}\,dx}{2a^2}$$

$$=\frac{\left(a^2-2b^2\right)x}{2a^3}+\frac{(2b-a\cos(c+dx))\sin(c+dx)}{2a^2d}+\frac{\left(b\left(a^2-b^2\right)\right)\int \frac{1}{-b-a\cos(c+dx)}\,dx}{a^3}$$

$$=\frac{\left(a^2-2b^2\right)x}{2a^3}+\frac{(2b-a\cos(c+dx))\sin(c+dx)}{2a^2d}+\frac{\left(2b\left(a^2-b^2\right)\right)\text{Subst}\left(\int \frac{1}{-a-b+(a-b)x^2}\,dx,x,\text{ta}\right.}{a^3d}$$

$$=\frac{\left(a^2-2b^2\right)x}{2a^3}-\frac{2\sqrt{a-b}b\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d}+\frac{(2b-a\cos(c+dx))\sin(c+dx)}{2a^2d}$$

Mathematica [A] time = 0.301632, size = 96, normalized size = 0.96

$$\frac{2\left(a^2-2b^2\right)(c+dx)+8b\sqrt{a^2-b^2}\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)+a^2(-\sin(2(c+dx)))+4ab\sin(c+dx)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(a^2 - 2*b^2)*(c + d*x) + 8*b*Sqrt[a^2 - b^2]*ArcTanh[((-a + b)*Tan[(c +
 d*x)/2])/Sqrt[a^2 - b^2]] + 4*a*b*Sin[c + d*x] - a^2*Sin[2*(c + d*x)])/(4*
a^3*d)
```

Maple [B] time = 0.058, size = 269, normalized size = 2.7

$$\frac{1}{ad}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-2}+2\frac{(\tan(1/2\,dx+c/2))^3b}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)^2}+2\frac{\tan(1/2\,dx+c/2)\,b}{da^2\left(1+(\tan(1/2\,dx+c/2))^2\right)^2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+2/d/a^2/(1+tan(1/2*d*
x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*b+2/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan
(1/2*d*x+1/2*c)*b-1/d/a/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)-2/d/a
^3*arctan(tan(1/2*d*x+1/2*c))*b^2+1/d/a*arctan(tan(1/2*d*x+1/2*c))-2/d*b/a/
```

```
((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+2
/d*b^3/a^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b
))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.27683, size = 599, normalized size = 5.99

$$\left[\frac{\left(a^2-2b^2\right)dx+\sqrt{a^2-b^2}b\log\left(\frac{2ab\cos(dx+c)-\left(a^2-2b^2\right)\cos(dx+c)^2-2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2a^2-b^2}{a^2\cos(dx+c)^2+2ab\cos(dx+c)+b^2}\right)-\left(a^2\cos(dx+c)-\right.}{2a^3d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*((a^2 - 2*b^2)*d*x + sqrt(a^2 - b^2)*b*log((2*a*b*cos(d*x + c) - (a^2
- 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x +
c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (a^2*c
os(d*x + c) - 2*a*b)*sin(d*x + c))/(a^3*d), 1/2*((a^2 - 2*b^2)*d*x - 2*sqrt
(-a^2 + b^2)*b*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*s
in(d*x + c))) - (a^2*cos(d*x + c) - 2*a*b)*sin(d*x + c))/(a^3*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sin(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

Giac [B] time = 1.33578, size = 250, normalized size = 2.5

$$\frac{\frac{\left(a^2-2b^2\right)(dx+c)}{a^3}-\frac{4\left(a^2b-b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}a^3}+\frac{2\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-a\tan\left(\right.\right.}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((a^2 - 2*b^2)*(d*x + c)/a^3 - 4*(a^2*b - b^3)*(pi*floor(1/2*(d*x + c)/
pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x
 + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^3) + 2*(a*tan(1/2*d*x + 1
/2*c)^3 + 2*b*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) + 2*b*tan(1/2
*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2))/d
```

3.206 $\int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=84

$$\frac{\csc(c+dx)(b-a\cos(c+dx))}{d\left(a^2-b^2\right)}-\frac{2ab\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}$$

```
[Out] (-2*a*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)
*(a + b)^(3/2)*d) + ((b - a*Cos[c + d*x])*Csc[c + d*x])/((a^2 - b^2)*d)
```

Rubi [A] time = 0.148712, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2866, 12, 2659, 208}

$$\frac{\csc(c+dx)(b-a\cos(c+dx))}{d\left(a^2-b^2\right)}-\frac{2ab\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*a*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)
*(a + b)^(3/2)*d) + ((b - a*Cos[c + d*x])*Csc[c + d*x])/((a^2 - b^2)*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx &= -\int \frac{\cot(c+dx)\csc(c+dx)}{-b-a\cos(c+dx)}\,dx \\
&= \frac{(b-a\cos(c+dx))\csc(c+dx)}{\left(a^2-b^2\right)d} + \frac{\int \frac{ab}{-b-a\cos(c+dx)}\,dx}{a^2-b^2} \\
&= \frac{(b-a\cos(c+dx))\csc(c+dx)}{\left(a^2-b^2\right)d} + \frac{(ab)\int \frac{1}{-b-a\cos(c+dx)}\,dx}{a^2-b^2} \\
&= \frac{(b-a\cos(c+dx))\csc(c+dx)}{\left(a^2-b^2\right)d} + \frac{(2ab)\,\mathrm{Subst}\left(\int \frac{1}{-a-b+(a-b)x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{\left(a^2-b^2\right)d} \\
&= -\frac{2ab\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d} + \frac{(b-a\cos(c+dx))\csc(c+dx)}{\left(a^2-b^2\right)d}
\end{aligned}$$

Mathematica [A] time = 0.195303, size = 118, normalized size = 1.4

$$\frac{\csc\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{a^2-b^2}(b-a\cos(c+dx))+2ab\sin(c+dx)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)\right)}{2d(a-b)(a+b)\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Sec[c + d*x]),x]
```

```
[Out] (Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(Sqrt[a^2 - b^2]*(b - a*Cos[c + d*x]) +
2*a*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*Sin[c + d*x]))/(
2*(a - b)*(a + b)*Sqrt[a^2 - b^2]*d)
```

Maple [A] time = 0.058, size = 96, normalized size = 1.1

$$\frac{1}{d}\left(\frac{1}{2a-2b}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2\frac{ab}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}\mathrm{Artanh}\left(\frac{(a-b)\tan(1/2\,dx+c/2)}{\sqrt{(a+b)(a-b)}}\right)-\frac{1}{2a+2b}\left(\tan\left(\frac{dx}{2}+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*(1/2/(a-b)*tan(1/2*d*x+1/2*c)-2/(a-b)/(a+b)*a*b/((a+b)*(a-b))^(1/2)*arc
tanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-1/2/(a+b)/tan(1/2*d*x+1/
2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.86099, size = 680, normalized size = 8.1

$$\left[-\frac{\sqrt{a^2-b^2}ab\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)\sin(dx+c)-2\,a^2b+2\,b^3+2\left(a^3\right.}{2\left(a^4-2\,a^2b^2+b^4\right)d\sin(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(a^2 - b^2)*a*b*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
+ c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)
/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*a^2*b +
2*b^3 + 2*(a^3 - a*b^2)*cos(d*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*sin(d*x +
c)), -(sqrt(-a^2 + b^2)*a*b*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/(
(a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - a^2*b + b^3 + (a^3 - a*b^2)*cos(d
*x + c))/((a^4 - 2*a^2*b^2 + b^4)*d*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

Giac [A] time = 1.37457, size = 174, normalized size = 2.07

$$-\frac{\frac{4\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)ab}{\left(a^2-b^2\right)\sqrt{-a^2+b^2}}-\frac{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a-b}+\frac{1}{(a+b)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/2*(4*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a*b/((a^2 - b
^2)*sqrt(-a^2 + b^2)) - tan(1/2*d*x + 1/2*c)/(a - b) + 1/((a + b)*tan(1/2*d
*x + 1/2*c)))/d
```

3.207 $\int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=140

$$\frac{\csc^3(c+dx)(b-a\cos(c+dx))}{3d(a^2-b^2)}+\frac{\csc(c+dx)\left(3a^2b-a\left(2a^2+b^2\right)\cos(c+dx)\right)}{3d(a^2-b^2)^2}-\frac{2a^3b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}$$

```
[Out] (-2*a^3*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/
2)*(a + b)^(5/2)*d) + ((3*a^2*b - a*(2*a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x
])/(3*(a^2 - b^2)^2*d) + ((b - a*Cos[c + d*x])*Csc[c + d*x]^3)/(3*(a^2 - b^
2)*d)
```

Rubi [A] time = 0.306471, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2866, 12, 2659, 208}

$$\frac{\csc^3(c+dx)(b-a\cos(c+dx))}{3d(a^2-b^2)}+\frac{\csc(c+dx)\left(3a^2b-a\left(2a^2+b^2\right)\cos(c+dx)\right)}{3d(a^2-b^2)^2}-\frac{2a^3b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```


```
[Out] (-2*a^3*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/
2)*(a + b)^(5/2)*d) + ((3*a^2*b - a*(2*a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x
])/(3*(a^2 - b^2)^2*d) + ((b - a*Cos[c + d*x])*Csc[c + d*x]^3)/(3*(a^2 - b^
2)*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cot(c+dx)\csc^3(c+dx)}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{(b-a\cos(c+dx))\csc^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{\int \frac{\left(ab-2a^2\cos(c+dx)\right)\csc^2(c+dx)}{-b-a\cos(c+dx)}\,dx}{3\left(a^2-b^2\right)}$$

$$= \frac{\left(3a^2b-a\left(2a^2+b^2\right)\cos(c+dx)\right)\csc(c+dx)}{3\left(a^2-b^2\right)^2 d} + \frac{(b-a\cos(c+dx))\csc^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{\int \frac{3a^3b}{-b-a\cos(c+}}{3\left(a^2-b\right.}$$

$$= \frac{\left(3a^2b-a\left(2a^2+b^2\right)\cos(c+dx)\right)\csc(c+dx)}{3\left(a^2-b^2\right)^2 d} + \frac{(b-a\cos(c+dx))\csc^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{\left(a^3b\right)\int \frac{}{-b-}}{\left(a^2-\right.}$$

$$= \frac{\left(3a^2b-a\left(2a^2+b^2\right)\cos(c+dx)\right)\csc(c+dx)}{3\left(a^2-b^2\right)^2 d} + \frac{(b-a\cos(c+dx))\csc^3(c+dx)}{3\left(a^2-b^2\right)d} + \frac{\left(2a^3b\right)\mathrm{Subs}}{}$$

$$= -\frac{2a^3b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{\left(3a^2b-a\left(2a^2+b^2\right)\cos(c+dx)\right)\csc(c+dx)}{3\left(a^2-b^2\right)^2 d} + \frac{(b-a\cos(}{3}$$

Mathematica [A] time = 0.891265, size = 162, normalized size = 1.16

$$\frac{\sqrt{a^2-b^2}\csc^3(c+dx)\left(\left(3ab^2-6a^3\right)\cos(c+dx)-6a^2b\cos(2(c+dx))+10a^2b+2a^3\cos(3(c+dx))+ab^2\cos(3(c+dx))\right.}{12d(a-b)^2(a+b)^2\sqrt{a^2-b^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Sec[c + d*x]),x]
```

```
[Out] (24*a^3*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]] + Sqrt[a^2 -
 b^2]*(10*a^2*b - 4*b^3 + (-6*a^3 + 3*a*b^2)*Cos[c + d*x] - 6*a^2*b*Cos[2*(
c + d*x)] + 2*a^3*Cos[3*(c + d*x)] + a*b^2*Cos[3*(c + d*x)])*Csc[c + d*x]^3
)/(12*(a - b)^2*(a + b)^2*Sqrt[a^2 - b^2]*d)
```

Maple [A] time = 0.069, size = 165, normalized size = 1.2

$$\frac{1}{d}\left(\frac{1}{8\,(a-b)^2}\left(\frac{a}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{b}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+3\,a\tan\left(1/2\,dx+c/2\right)-b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\,\frac{a^3 b}{(a-b)^2\,(a+b)^2\,\surd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*(1/8/(a-b)^2*(1/3*tan(1/2*d*x+1/2*c)^3*a-1/3*b*tan(1/2*d*x+1/2*c)^3+3*a
*tan(1/2*d*x+1/2*c)-b*tan(1/2*d*x+1/2*c))-2/(a-b)^2/(a+b)^2*a^3*b/((a+b)*(a
-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-1/24/(a+b)
/tan(1/2*d*x+1/2*c)^3-1/8*(3*a+b)/(a+b)^2/tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.92126, size = 1214, normalized size = 8.67

$$\left[-\frac{8\,a^{4}b-10\,a^{2}b^{3}+2\,b^{5}+2\left(2\,a^{5}-a^{3}b^{2}-ab^{4}\right)\cos\left(dx+c\right)^{3}-3\left(a^{3}b\cos\left(dx+c\right)^{2}-a^{3}b\right)\sqrt{a^{2}-b^{2}}\log\left(\frac{2\,ab\cos(dx+c)-(a}{}\right.}{6\left(\left(a^{6}-3\,a^{4}b^{2}+3\,a^{2}b^{4}-b^{6}\right)d\cos\left(dx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [-1/6*(8*a^4*b - 10*a^2*b^3 + 2*b^5 + 2*(2*a^5 - a^3*b^2 - a*b^4)*cos(d*x +
 c)^3 - 3*(a^3*b*cos(d*x + c)^2 - a^3*b)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x
 + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) +
a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b
^2))*sin(d*x + c) - 6*(a^4*b - a^2*b^3)*cos(d*x + c)^2 - 6*(a^5 - a^3*b^2)*
cos(d*x + c))/(((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*cos(d*x + c)^2 - (a^6
 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d)*sin(d*x + c)), -1/3*(4*a^4*b - 5*a^2*b^3
 + b^5 + (2*a^5 - a^3*b^2 - a*b^4)*cos(d*x + c)^3 + 3*(a^3*b*cos(d*x + c)^2
 - a^3*b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((
a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - 3*(a^4*b - a^2*b^3)*cos(d*x + c)^2
 - 3*(a^5 - a^3*b^2)*cos(d*x + c))/(((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d*
cos(d*x + c)^2 - (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d)*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^{4}\left(c+dx\right)}{a+b\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(csc(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

Giac [B] time = 1.40393, size = 363, normalized size = 2.59

$$\frac{\frac{48\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)a^3b}{\left(a^4-2a^2b^2+b^4\right)\sqrt{-a^2+b^2}}+\frac{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+9a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right.}{a^3-3a^2b+3ab^2-b^3}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(48*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1
/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^3*b/((a^4 -
2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + (a^2*tan(1/2*d*x + 1/2*c)^3 - 2*a*b*ta
n(1/2*d*x + 1/2*c)^3 + b^2*tan(1/2*d*x + 1/2*c)^3 + 9*a^2*tan(1/2*d*x + 1/2
*c) - 12*a*b*tan(1/2*d*x + 1/2*c) + 3*b^2*tan(1/2*d*x + 1/2*c))/(a^3 - 3*a^
2*b + 3*a*b^2 - b^3) - (9*a*tan(1/2*d*x + 1/2*c)^2 + 3*b*tan(1/2*d*x + 1/2*
c)^2 + a + b)/((a^2 + 2*a*b + b^2)*tan(1/2*d*x + 1/2*c)^3))/d
```

3.208 $\int \frac{\csc^6(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=201

$$\frac{\csc^5(c+dx)(b-a\cos(c+dx))}{5d\left(a^2-b^2\right)}+\frac{\csc^3(c+dx)\left(5a^2b-a\left(4a^2+b^2\right)\cos(c+dx)\right)}{15d\left(a^2-b^2\right)^2}+\frac{\csc(c+dx)\left(15a^4b-a\left(9a^2b^2+8a\right.\right.}{15d\left(a^2-b^2\right)^3}$$

```
[Out] (-2*a^5*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/
2)*(a + b)^(7/2)*d) + ((15*a^4*b - a*(8*a^4 + 9*a^2*b^2 - 2*b^4)*Cos[c + d*
x])*Csc[c + d*x])/(15*(a^2 - b^2)^3*d) + ((5*a^2*b - a*(4*a^2 + b^2)*Cos[c
+ d*x])*Csc[c + d*x]^3)/(15*(a^2 - b^2)^2*d) + ((b - a*Cos[c + d*x])*Csc[c
+ d*x]^5)/(5*(a^2 - b^2)*d)
```

Rubi [A] time = 0.519973, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2866, 12, 2659, 208}

$$\frac{\csc^5(c+dx)(b-a\cos(c+dx))}{5d\left(a^2-b^2\right)}+\frac{\csc^3(c+dx)\left(5a^2b-a\left(4a^2+b^2\right)\cos(c+dx)\right)}{15d\left(a^2-b^2\right)^2}+\frac{\csc(c+dx)\left(15a^4b-a\left(9a^2b^2+8a\right.\right.}{15d\left(a^2-b^2\right)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```


```
[Out] (-2*a^5*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/
2)*(a + b)^(7/2)*d) + ((15*a^4*b - a*(8*a^4 + 9*a^2*b^2 - 2*b^4)*Cos[c + d*
x])*Csc[c + d*x])/(15*(a^2 - b^2)^3*d) + ((5*a^2*b - a*(4*a^2 + b^2)*Cos[c
+ d*x])*Csc[c + d*x]^3)/(15*(a^2 - b^2)^2*d) + ((b - a*Cos[c + d*x])*Csc[c
+ d*x]^5)/(5*(a^2 - b^2)*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\csc^6(c+dx)}{a+b\sec(c+dx)}\,dx = -\int \frac{\cot(c+dx)\csc^5(c+dx)}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{(b-a\cos(c+dx))\csc^5(c+dx)}{5\left(a^2-b^2\right)d} + \frac{\int \frac{\left(ab-4a^2\cos(c+dx)\right)\csc^4(c+dx)}{-b-a\cos(c+dx)}\,dx}{5\left(a^2-b^2\right)}$$

$$= \frac{\left(5a^2b-a\left(4a^2+b^2\right)\cos(c+dx)\right)\csc^3(c+dx)}{15\left(a^2-b^2\right)^2 d} + \frac{(b-a\cos(c+dx))\csc^5(c+dx)}{5\left(a^2-b^2\right)d} + \frac{\int \frac{\left(ab\left(7a^2-2b\right.\right.}{}}{}$$

$$= \frac{\left(15a^4b-a\left(8a^4+9a^2b^2-2b^4\right)\cos(c+dx)\right)\csc(c+dx)}{15\left(a^2-b^2\right)^3 d} + \frac{\left(5a^2b-a\left(4a^2+b^2\right)\cos(c+dx)\right)\mathrm{cs}}{15\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(15a^4b-a\left(8a^4+9a^2b^2-2b^4\right)\cos(c+dx)\right)\csc(c+dx)}{15\left(a^2-b^2\right)^3 d} + \frac{\left(5a^2b-a\left(4a^2+b^2\right)\cos(c+dx)\right)\mathrm{cs}}{15\left(a^2-b^2\right)^2 d}$$

$$= \frac{\left(15a^4b-a\left(8a^4+9a^2b^2-2b^4\right)\cos(c+dx)\right)\csc(c+dx)}{15\left(a^2-b^2\right)^3 d} + \frac{\left(5a^2b-a\left(4a^2+b^2\right)\cos(c+dx)\right)\mathrm{cs}}{15\left(a^2-b^2\right)^2 d}$$

$$= -\frac{2a^5b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} + \frac{\left(15a^4b-a\left(8a^4+9a^2b^2-2b^4\right)\cos(c+dx)\right)\csc(c+dx)}{15\left(a^2-b^2\right)^3 d} + \cdot$$

Mathematica [A] time = 1.24476, size = 277, normalized size = 1.38

$$\frac{\sec(c+dx)(a\cos(c+dx)+b)\left(\frac{2\left(64a^2-43ab+9b^2\right)\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^3} - \frac{2\left(64a^2+43ab+9b^2\right)\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^3} + \frac{960a^5b\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}} + 96\cdots\right.}{480d(a+b\sec(c+\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*((960*a^5*b*ArcTanh[((-a + b)*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2) - (2*(64*a^2 + 43*a*b + 9*b^2)*
Cot[(c + d*x)/2])/(a + b)^3 + (8*(19*a - 9*b)*Csc[c + d*x]^3*Sin[(c + d*x)/
2]^4)/(a - b)^2 + (96*Csc[c + d*x]^5*Sin[(c + d*x)/2]^6)/(a - b) - ((19*a +
 9*b)*Csc[(c + d*x)/2]^4*Sin[c + d*x])/(2*(a + b)^2) - (3*Csc[(c + d*x)/2]^
6*Sin[c + d*x])/(2*(a + b)) + (2*(64*a^2 - 43*a*b + 9*b^2)*Tan[(c + d*x)/2]
```

```
)/(a - b)^3))/(480*d*(a + b*Sec[c + d*x]))
```

Maple [A] time = 0.074, size = 282, normalized size = 1.4

$$\frac{1}{d}\left(\frac{1}{32\,(a-b)^3}\left(\frac{a^2}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5-\frac{2\,ab}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{b^2}{5}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^5+\frac{5\,a^2}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3-\frac{8\,ab}{3}\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*(1/32/(a-b)^3*(1/5*tan(1/2*d*x+1/2*c)^5*a^2-2/5*tan(1/2*d*x+1/2*c)^5*a*
b+1/5*b^2*tan(1/2*d*x+1/2*c)^5+5/3*tan(1/2*d*x+1/2*c)^3*a^2-8/3*tan(1/2*d*x
+1/2*c)^3*a*b+tan(1/2*d*x+1/2*c)^3*b^2+10*a^2*tan(1/2*d*x+1/2*c)-8*tan(1/2*
d*x+1/2*c)*a*b+2*b^2*tan(1/2*d*x+1/2*c))-2/(a-b)^3/(a+b)^3*b*a^5/((a+b)*(a-
b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-1/160/(a+b)
/tan(1/2*d*x+1/2*c)^5-1/96*(5*a+3*b)/(a+b)^2/tan(1/2*d*x+1/2*c)^3-1/32/(a+b
)^3*(10*a^2+8*a*b+2*b^2)/tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.09674, size = 1917, normalized size = 9.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/30*(46*a^6*b - 68*a^4*b^3 + 28*a^2*b^5 - 6*b^7 - 2*(8*a^7 + a^5*b^2 - 11
*a^3*b^4 + 2*a*b^6)*cos(d*x + c)^5 + 30*(a^6*b - a^4*b^3)*cos(d*x + c)^4 +
10*(4*a^7 - a^5*b^2 - 4*a^3*b^4 + a*b^6)*cos(d*x + c)^3 - 15*(a^5*b*cos(d*x
 + c)^4 - 2*a^5*b*cos(d*x + c)^2 + a^5*b)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*
x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) +
 a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) +
b^2))*sin(d*x + c) - 10*(7*a^6*b - 8*a^4*b^3 + a^2*b^5)*cos(d*x + c)^2 - 30
*(a^7 - a^5*b^2)*cos(d*x + c))/(((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 +
 b^8)*d*cos(d*x + c)^4 - 2*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*
d*cos(d*x + c)^2 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d)*sin(d
*x + c)), 1/15*(23*a^6*b - 34*a^4*b^3 + 14*a^2*b^5 - 3*b^7 - (8*a^7 + a^5*b
^2 - 11*a^3*b^4 + 2*a*b^6)*cos(d*x + c)^5 + 15*(a^6*b - a^4*b^3)*cos(d*x +
c)^4 + 5*(4*a^7 - a^5*b^2 - 4*a^3*b^4 + a*b^6)*cos(d*x + c)^3 - 15*(a^5*b*c
os(d*x + c)^4 - 2*a^5*b*cos(d*x + c)^2 + a^5*b)*sqrt(-a^2 + b^2)*arctan(-sq
rt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c
```

```
) - 5*(7*a^6*b - 8*a^4*b^3 + a^2*b^5)*cos(d*x + c)^2 - 15*(a^7 - a^5*b^2)*c
os(d*x + c))/(((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d*cos(d*x +
c)^4 - 2*(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d*cos(d*x + c)^2 +
 (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d)*sin(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a+b*sec(d*x+c)),x)

[Out] Timed out
```

Giac [B] time = 1.32361, size = 730, normalized size = 3.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*sec(d*x+c)),x, algorithm="giac")

[Out] -1/480*(960*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*
tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*a^5*b/((a
^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) - (3*a^4*tan(1/2*d*x +
1/2*c)^5 - 12*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 18*a^2*b^2*tan(1/2*d*x + 1/2*c
)^5 - 12*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*b^4*tan(1/2*d*x + 1/2*c)^5 + 25*a
^4*tan(1/2*d*x + 1/2*c)^3 - 90*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 120*a^2*b^2*t
an(1/2*d*x + 1/2*c)^3 - 70*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 15*b^4*tan(1/2*d*
x + 1/2*c)^3 + 150*a^4*tan(1/2*d*x + 1/2*c) - 420*a^3*b*tan(1/2*d*x + 1/2*c
) + 420*a^2*b^2*tan(1/2*d*x + 1/2*c) - 180*a*b^3*tan(1/2*d*x + 1/2*c) + 30*
b^4*tan(1/2*d*x + 1/2*c))/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^
4 - b^5) + (150*a^2*tan(1/2*d*x + 1/2*c)^4 + 120*a*b*tan(1/2*d*x + 1/2*c)^4
 + 30*b^2*tan(1/2*d*x + 1/2*c)^4 + 25*a^2*tan(1/2*d*x + 1/2*c)^2 + 40*a*b*t
an(1/2*d*x + 1/2*c)^2 + 15*b^2*tan(1/2*d*x + 1/2*c)^2 + 3*a^2 + 6*a*b + 3*b
^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(1/2*d*x + 1/2*c)^5))/d
```

3.209 $\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=267

$$-\frac{3\left(a^2-b^2\right)\cos^5(c+dx)}{5a^4d}+\frac{b\left(3a^2-2b^2\right)\cos^4(c+dx)}{2a^5d}+\frac{\left(-9a^2b^2+3a^4+5b^4\right)\cos^3(c+dx)}{3a^6d}-\frac{3b\left(a^2-b^2\right)^2\cos^2(c+$$

```
[Out] -(((a^2 - 7*b^2)*(a^2 - b^2)^2*Cos[c + d*x])/(a^8*d)) - (3*b*(a^2 - b^2)^2*
Cos[c + d*x]^2)/(a^7*d) + ((3*a^4 - 9*a^2*b^2 + 5*b^4)*Cos[c + d*x]^3)/(3*a
^6*d) + (b*(3*a^2 - 2*b^2)*Cos[c + d*x]^4)/(2*a^5*d) - (3*(a^2 - b^2)*Cos[c
 + d*x]^5)/(5*a^4*d) - (b*Cos[c + d*x]^6)/(3*a^3*d) + Cos[c + d*x]^7/(7*a^2
*d) + (b^2*(a^2 - b^2)^3)/(a^9*d*(b + a*Cos[c + d*x])) + (2*b*(a^2 - 4*b^2)
*(a^2 - b^2)^2*Log[b + a*Cos[c + d*x]])/(a^9*d)
```

Rubi [A] time = 0.372333, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2837, 12, 948}

$$-\frac{3\left(a^2-b^2\right)\cos^5(c+dx)}{5a^4d}+\frac{b\left(3a^2-2b^2\right)\cos^4(c+dx)}{2a^5d}+\frac{\left(-9a^2b^2+3a^4+5b^4\right)\cos^3(c+dx)}{3a^6d}-\frac{3b\left(a^2-b^2\right)^2\cos^2(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(((a^2 - 7*b^2)*(a^2 - b^2)^2*Cos[c + d*x])/(a^8*d)) - (3*b*(a^2 - b^2)^2*
Cos[c + d*x]^2)/(a^7*d) + ((3*a^4 - 9*a^2*b^2 + 5*b^4)*Cos[c + d*x]^3)/(3*a
^6*d) + (b*(3*a^2 - 2*b^2)*Cos[c + d*x]^4)/(2*a^5*d) - (3*(a^2 - b^2)*Cos[c
 + d*x]^5)/(5*a^4*d) - (b*Cos[c + d*x]^6)/(3*a^3*d) + Cos[c + d*x]^7/(7*a^2
*d) + (b^2*(a^2 - b^2)^3)/(a^9*d*(b + a*Cos[c + d*x])) + (2*b*(a^2 - 4*b^2)
*(a^2 - b^2)^2*Log[b + a*Cos[c + d*x]])/(a^9*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
```

```
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin^7(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{x^2(a^2-x^2)^3}{a^2(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^7 d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^2(a^2-x^2)^3}{(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^9 d}$$

$$= \frac{\text{Subst}\left(\int \left((a^2-7b^2)(a^2-b^2)^2 - \frac{b^2(-a^2+b^2)^3}{(b-x)^2} + \frac{2b(-a^2+b^2)^2(-a^2+4b^2)}{b-x} - 6b(-a^2+b^2)^2 x - (3a^4\right.\right.}{a^9}$$

$$= -\frac{(a^2-7b^2)(a^2-b^2)^2\cos(c+dx)}{a^8 d} - \frac{3b(a^2-b^2)^2\cos^2(c+dx)}{a^7 d} + \frac{(3a^4-9a^2b^2+5b^4)\cos^3(}{3a^6 d}$$

Mathematica [A] time = 3.65723, size = 417, normalized size = 1.56

$$\frac{-1848a^6b^2\cos(4(c+dx)) + 112a^6b^2\cos(6(c+dx)) + 8400a^5b^3\cos(3(c+dx)) - 336a^5b^3\cos(5(c+dx)) + 1120a^4b^4\cos(4}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-3675*a^8 + 61320*a^6*b^2 - 132720*a^4*b^4 + 87360*a^2*b^6 - 13440*b^8 - 1
40*(21*a^8 - 228*a^6*b^2 + 400*a^4*b^4 - 192*a^2*b^6)*Cos[2*(c + d*x)] - 37
80*a^7*b*Cos[3*(c + d*x)] + 8400*a^5*b^3*Cos[3*(c + d*x)] - 4480*a^3*b^5*Co
s[3*(c + d*x)] + 588*a^8*Cos[4*(c + d*x)] - 1848*a^6*b^2*Cos[4*(c + d*x)] +
 1120*a^4*b^4*Cos[4*(c + d*x)] + 476*a^7*b*Cos[5*(c + d*x)] - 336*a^5*b^3*C
os[5*(c + d*x)] - 132*a^8*Cos[6*(c + d*x)] + 112*a^6*b^2*Cos[6*(c + d*x)] -
 40*a^7*b*Cos[7*(c + d*x)] + 15*a^8*Cos[8*(c + d*x)] + 26880*a^6*b^2*Log[b
+ a*Cos[c + d*x]] - 161280*a^4*b^4*Log[b + a*Cos[c + d*x]] + 241920*a^2*b^6
*Log[b + a*Cos[c + d*x]] - 107520*b^8*Log[b + a*Cos[c + d*x]] + 1680*a*b*Co
s[c + d*x]*(-8*a^6 + 67*a^4*b^2 - 116*a^2*b^4 + 56*b^6 + 16*(a^2 - 4*b^2)*(
a^2 - b^2)^2*Log[b + a*Cos[c + d*x]]))/(13440*a^9*d*(b + a*Cos[c + d*x]))
```

Maple [A] time = 0.067, size = 456, normalized size = 1.7

$$\frac{(\cos(dx+c))^7}{7a^2d} - \frac{b(\cos(dx+c))^6}{3a^3d} - \frac{3(\cos(dx+c))^5}{5a^2d} + \frac{3(\cos(dx+c))^5b^2}{5da^4} + \frac{3b(\cos(dx+c))^4}{2a^3d} - \frac{(\cos(dx+c))^4b^3}{da^5} +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/7*cos(d*x+c)^7/a^2/d-1/3*b*cos(d*x+c)^6/a^3/d-3/5*cos(d*x+c)^5/a^2/d+3/5/
d/a^4*cos(d*x+c)^5*b^2+3/2*b*cos(d*x+c)^4/a^3/d-1/d/a^5*cos(d*x+c)^4*b^3+co
```

```
s(d*x+c)^3/a^2/d-3/d/a^4*cos(d*x+c)^3*b^2+5/3/d/a^6*cos(d*x+c)^3*b^4-3*b*co
s(d*x+c)^2/a^3/d+6/d/a^5*cos(d*x+c)^2*b^3-3/d/a^7*cos(d*x+c)^2*b^5-cos(d*x+
c)/a^2/d+9/d/a^4*b^2*cos(d*x+c)-15/d/a^6*b^4*cos(d*x+c)+7/d/a^8*b^6*cos(d*x
+c)+2*b*ln(b+a*cos(d*x+c))/a^3/d-12/d/a^5*b^3*ln(b+a*cos(d*x+c))+18/d/a^7*b
^5*ln(b+a*cos(d*x+c))-8/d/a^9*b^7*ln(b+a*cos(d*x+c))+b^2/a^3/d/(b+a*cos(d*x
+c))-3/d*b^4/a^5/(b+a*cos(d*x+c))+3/d*b^6/a^7/(b+a*cos(d*x+c))-1/d*b^8/a^9/
(b+a*cos(d*x+c))
```

Maxima [A] time = 1.03138, size = 366, normalized size = 1.37

$$\frac{\frac{210\left(a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8\right)}{a^{10}\cos(dx+c)+a^9b}+\frac{30\,a^6\cos(dx+c)^7-70\,a^5b\cos(dx+c)^6-126\left(a^6-a^4b^2\right)\cos(dx+c)^5+105\left(3\,a^5b-2\,a^3b^3\right)\cos(dx+c)^4+70\left(3\,a^6-9\,a^4b^2+5\,a^2b^4\right)\cdots}{a^8}}{210\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/210*(210*(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)/(a^10*cos(d*x + c) + a^9
*b) + (30*a^6*cos(d*x + c)^7 - 70*a^5*b*cos(d*x + c)^6 - 126*(a^6 - a^4*b^2
)*cos(d*x + c)^5 + 105*(3*a^5*b - 2*a^3*b^3)*cos(d*x + c)^4 + 70*(3*a^6 - 9
*a^4*b^2 + 5*a^2*b^4)*cos(d*x + c)^3 - 630*(a^5*b - 2*a^3*b^3 + a*b^5)*cos(
d*x + c)^2 - 210*(a^6 - 9*a^4*b^2 + 15*a^2*b^4 - 7*b^6)*cos(d*x + c))/a^8 +
 420*(a^6*b - 6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*log(a*cos(d*x + c) + b)/a^9)/d
```

Fricas [A] time = 2.41865, size = 803, normalized size = 3.01

$$\frac{120\,a^8\cos\left(dx+c\right)^8-160\,a^7b\cos\left(dx+c\right)^7+1715\,a^6b^2-4725\,a^4b^4+3780\,a^2b^6-840\,b^8-56\left(9\,a^8-4\,a^6b^2\right)\cos\left(dx\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/840*(120*a^8*cos(d*x + c)^8 - 160*a^7*b*cos(d*x + c)^7 + 1715*a^6*b^2 - 4
725*a^4*b^4 + 3780*a^2*b^6 - 840*b^8 - 56*(9*a^8 - 4*a^6*b^2)*cos(d*x + c)^
6 + 84*(9*a^7*b - 4*a^5*b^3)*cos(d*x + c)^5 + 140*(6*a^8 - 9*a^6*b^2 + 4*a^
4*b^4)*cos(d*x + c)^4 - 280*(6*a^7*b - 9*a^5*b^3 + 4*a^3*b^5)*cos(d*x + c)^
3 - 840*(a^8 - 6*a^6*b^2 + 9*a^4*b^4 - 4*a^2*b^6)*cos(d*x + c)^2 + 35*(a^7*
b + 153*a^5*b^3 - 324*a^3*b^5 + 168*a*b^7)*cos(d*x + c) + 1680*(a^6*b^2 - 6
*a^4*b^4 + 9*a^2*b^6 - 4*b^8 + (a^7*b - 6*a^5*b^3 + 9*a^3*b^5 - 4*a*b^7)*co
s(d*x + c))*log(a*cos(d*x + c) + b))/(a^10*d*cos(d*x + c) + a^9*b*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.39638, size = 2512, normalized size = 9.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/210*(420*(a^7*b - a^6*b^2 - 6*a^5*b^3 + 6*a^4*b^4 + 9*a^3*b^5 - 9*a^2*b^6
 - 4*a*b^7 + 4*b^8)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^10 - a^9*b) - 420*(a^6*b -
6*a^4*b^3 + 9*a^2*b^5 - 4*b^7)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 1))/a^9 - 420*(a^7*b - 7*a^5*b^3 - 4*a^4*b^4 + 11*a^3*b^5 + 8*a^2*b^6
- 5*a*b^7 - 4*b^8 + a^7*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a^6*b^2*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 6*a^5*b^3*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) + 6*a^4*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^3*b^5*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 9*a^2*b^6*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - 4*a*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*b^8*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1))/((a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*a^9) + (192*a^7 - 1089*a^
6*b - 2772*a^5*b^2 + 6534*a^4*b^3 + 5600*a^3*b^4 - 9801*a^2*b^5 - 2940*a*b^
6 + 4356*b^7 - 1344*a^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8463*a^6*b*
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 18144*a^5*b^2*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - 49098*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 35
000*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 71127*a^2*b^5*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 17640*a*b^6*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) - 30492*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4032*a^7*(cos(d*x
 + c) - 1)^2/(cos(d*x + c) + 1)^2 - 28749*a^6*b*(cos(d*x + c) - 1)^2/(cos(d
*x + c) + 1)^2 - 48132*a^5*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
157374*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 88200*a^3*b^4*(c
os(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 218421*a^2*b^5*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 - 44100*a*b^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
1)^2 + 91476*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6720*a^7*(cos(
d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56035*a^6*b*(cos(d*x + c) - 1)^3/(co
s(d*x + c) + 1)^3 + 60480*a^5*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3
 - 272370*a^4*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 114800*a^3*b^
4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 368235*a^2*b^5*(cos(d*x + c)
- 1)^3/(cos(d*x + c) + 1)^3 + 58800*a*b^6*(cos(d*x + c) - 1)^3/(cos(d*x + c
) + 1)^3 - 152460*b^7*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 56035*a^6
*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 36540*a^5*b^2*(cos(d*x + c)
- 1)^4/(cos(d*x + c) + 1)^4 + 272370*a^4*b^3*(cos(d*x + c) - 1)^4/(cos(d*x
+ c) + 1)^4 + 81200*a^3*b^4*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 368
235*a^2*b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 44100*a*b^6*(cos(d*
x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 152460*b^7*(cos(d*x + c) - 1)^4/(cos(d
*x + c) + 1)^4 + 28749*a^6*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 10
080*a^5*b^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 157374*a^4*b^3*(cos
(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 29400*a^3*b^4*(cos(d*x + c) - 1)^5/
(cos(d*x + c) + 1)^5 + 218421*a^2*b^5*(cos(d*x + c) - 1)^5/(cos(d*x + c) +
1)^5 + 17640*a*b^6*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 91476*b^7*(c
os(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 8463*a^6*b*(cos(d*x + c) - 1)^6/(
cos(d*x + c) + 1)^6 - 1260*a^5*b^2*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^
6 + 49098*a^4*b^3*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 4200*a^3*b^4*
(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 71127*a^2*b^5*(cos(d*x + c) - 1
)^6/(cos(d*x + c) + 1)^6 - 2940*a*b^6*(cos(d*x + c) - 1)^6/(cos(d*x + c) +
1)^6 + 30492*b^7*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 + 1089*a^6*b*(co
```

```
s(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 6534*a^4*b^3*(cos(d*x + c) - 1)^7/
(cos(d*x + c) + 1)^7 + 9801*a^2*b^5*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)
^7 - 4356*b^7*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7)/(a^9*((cos(d*x + c
) - 1)/(cos(d*x + c) + 1) - 1)^7))/d
```

3.210 $\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=194

$$\frac{(2a^2-3b^2)\cos^3(c+dx)}{3a^4d}-\frac{2b(a^2-b^2)\cos^2(c+dx)}{a^5d}-\frac{(-6a^2b^2+a^4+5b^4)\cos(c+dx)}{a^6d}+\frac{b^2(a^2-b^2)^2}{a^7d(a\cos(c+dx)+b)}+\frac{2b(-$$

```
[Out] -(((a^4 - 6*a^2*b^2 + 5*b^4)*Cos[c + d*x])/(a^6*d)) - (2*b*(a^2 - b^2)*Cos[
c + d*x]^2)/(a^5*d) + ((2*a^2 - 3*b^2)*Cos[c + d*x]^3)/(3*a^4*d) + (b*Cos[c
 + d*x]^4)/(2*a^3*d) - Cos[c + d*x]^5/(5*a^2*d) + (b^2*(a^2 - b^2)^2)/(a^7*
d*(b + a*Cos[c + d*x])) + (2*b*(a^4 - 4*a^2*b^2 + 3*b^4)*Log[b + a*Cos[c +
d*x]])/(a^7*d)
```

Rubi [A] time = 0.299352, antiderivative size = 194, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2837, 12, 948}

$$\frac{(2a^2-3b^2)\cos^3(c+dx)}{3a^4d}-\frac{2b(a^2-b^2)\cos^2(c+dx)}{a^5d}-\frac{(-6a^2b^2+a^4+5b^4)\cos(c+dx)}{a^6d}+\frac{b^2(a^2-b^2)^2}{a^7d(a\cos(c+dx)+b)}+\frac{2b(-$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(((a^4 - 6*a^2*b^2 + 5*b^4)*Cos[c + d*x])/(a^6*d)) - (2*b*(a^2 - b^2)*Cos[
c + d*x]^2)/(a^5*d) + ((2*a^2 - 3*b^2)*Cos[c + d*x]^3)/(3*a^4*d) + (b*Cos[c
 + d*x]^4)/(2*a^3*d) - Cos[c + d*x]^5/(5*a^2*d) + (b^2*(a^2 - b^2)^2)/(a^7*
d*(b + a*Cos[c + d*x])) + (2*b*(a^4 - 4*a^2*b^2 + 3*b^4)*Log[b + a*Cos[c +
d*x]])/(a^7*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
```

```
& EqQ[d, 0]))
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin^5(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^2\left(a^2-x^2\right)^2}{a^2(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2\left(a^2-x^2\right)^2}{(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^7d}$$
$$= \frac{\text{Subst}\left(\int \left(a^4\left(1+\frac{-6a^2b^2+5b^4}{a^4}\right)+\frac{b^2\left(a^2-b^2\right)^2}{(b-x)^2}-\frac{2b\left(a^4-4a^2b^2+3b^4\right)}{b-x}+4b\left(-a^2+b^2\right)x-\left(2a^2-3b\right.\right.\right.}{a^7d}$$
$$= -\frac{\left(a^4-6a^2b^2+5b^4\right)\cos(c+dx)}{a^6d}-\frac{2b\left(a^2-b^2\right)\cos^2(c+dx)}{a^5d}+\frac{\left(2a^2-3b^2\right)\cos^3(c+dx)}{3a^4d}$$

Mathematica [A] time = 1.07203, size = 280, normalized size = 1.44

$$\frac{-30a^4b^2\cos(4(c+dx))+120a^3b^3\cos(3(c+dx))-5\left(-168a^4b^2+144a^2b^4+25a^6\right)\cos(2(c+dx))+960a^4b^2\log(a\cos}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-150*a^6 + 1740*a^4*b^2 - 2160*a^2*b^4 + 480*b^6 - 5*(25*a^6 - 168*a^4*b^2
 + 144*a^2*b^4)*Cos[2*(c + d*x)] - 115*a^5*b*Cos[3*(c + d*x)] + 120*a^3*b^3
*Cos[3*(c + d*x)] + 22*a^6*Cos[4*(c + d*x)] - 30*a^4*b^2*Cos[4*(c + d*x)] +
 9*a^5*b*Cos[5*(c + d*x)] - 3*a^6*Cos[6*(c + d*x)] + 960*a^4*b^2*Log[b + a*
Cos[c + d*x]] - 3840*a^2*b^4*Log[b + a*Cos[c + d*x]] + 2880*b^6*Log[b + a*C
os[c + d*x]] + 120*a*b*Cos[c + d*x]*(-4*a^4 + 23*a^2*b^2 - 20*b^4 + 8*(a^4
- 4*a^2*b^2 + 3*b^4)*Log[b + a*Cos[c + d*x]]))/(480*a^7*d*(b + a*Cos[c + d*
x]))
```

Maple [A] time = 0.061, size = 285, normalized size = 1.5

$$-\frac{(\cos(dx+c))^5}{5a^2d}+\frac{b(\cos(dx+c))^4}{2a^3d}+\frac{2(\cos(dx+c))^3}{3a^2d}-\frac{(\cos(dx+c))^3b^2}{da^4}-2\frac{b(\cos(dx+c))^2}{a^3d}+2\frac{(\cos(dx+c))^2b}{da^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/5*cos(d*x+c)^5/a^2/d+1/2*b*cos(d*x+c)^4/a^3/d+2/3*cos(d*x+c)^3/a^2/d-1/d
/a^4*cos(d*x+c)^3*b^2-2*b*cos(d*x+c)^2/a^3/d+2/d/a^5*cos(d*x+c)^2*b^3-cos(d
*x+c)/a^2/d+6/d/a^4*b^2*cos(d*x+c)-5/d/a^6*b^4*cos(d*x+c)+2*b*ln(b+a*cos(d*
x+c))/a^3/d-8/d/a^5*b^3*ln(b+a*cos(d*x+c))+6/d/a^7*b^5*ln(b+a*cos(d*x+c))+b
^2/a^3/d/(b+a*cos(d*x+c))-2/d*b^4/a^5/(b+a*cos(d*x+c))+1/d*b^6/a^7/(b+a*cos
(d*x+c))
```

Maxima [A] time = 1.04436, size = 248, normalized size = 1.28

$$\frac{\frac{30\left(a^4b^2-2\,a^2b^4+b^6\right)}{a^8\cos(dx+c)+a^7b}-\frac{6\,a^4\cos(dx+c)^5-15\,a^3b\cos(dx+c)^4-10\left(2\,a^4-3\,a^2b^2\right)\cos(dx+c)^3+60\left(a^3b-ab^3\right)\cos(dx+c)^2+30\left(a^4-6\,a^2b^2+5\,b^4\right)\cos(dx+c)}{a^6}+\frac{60\left(a^4b-\right.}{}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/30*(30*(a^4*b^2 - 2*a^2*b^4 + b^6)/(a^8*cos(d*x + c) + a^7*b) - (6*a^4*co
s(d*x + c)^5 - 15*a^3*b*cos(d*x + c)^4 - 10*(2*a^4 - 3*a^2*b^2)*cos(d*x + c
)^3 + 60*(a^3*b - a*b^3)*cos(d*x + c)^2 + 30*(a^4 - 6*a^2*b^2 + 5*b^4)*cos(
d*x + c))/a^6 + 60*(a^4*b - 4*a^2*b^3 + 3*b^5)*log(a*cos(d*x + c) + b)/a^7)
/d
```

Fricas [A] time = 2.13182, size = 563, normalized size = 2.9

$$-\frac{48\,a^6\cos\left(dx+c\right)^6-72\,a^5b\cos\left(dx+c\right)^5-435\,a^4b^2+720\,a^2b^4-240\,b^6-40\left(4\,a^6-3\,a^4b^2\right)\cos\left(dx+c\right)^4+80\left(4\,a^5b-\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/240*(48*a^6*cos(d*x + c)^6 - 72*a^5*b*cos(d*x + c)^5 - 435*a^4*b^2 + 720
*a^2*b^4 - 240*b^6 - 40*(4*a^6 - 3*a^4*b^2)*cos(d*x + c)^4 + 80*(4*a^5*b -
3*a^3*b^3)*cos(d*x + c)^3 + 240*(a^6 - 4*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^
2 + 15*(3*a^5*b - 80*a^3*b^3 + 80*a*b^5)*cos(d*x + c) - 480*(a^4*b^2 - 4*a^
2*b^4 + 3*b^6 + (a^5*b - 4*a^3*b^3 + 3*a*b^5)*cos(d*x + c))*log(a*cos(d*x +
 c) + b))/(a^8*d*cos(d*x + c) + a^7*b*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.37862, size = 1488, normalized size = 7.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/30*(60*(a^5*b - a^4*b^2 - 4*a^3*b^3 + 4*a^2*b^4 + 3*a*b^5 - 3*b^6)*log(ab
s(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1)))/(a^8 - a^7*b) - 60*(a^4*b - 4*a^2*b^3 + 3*b^5)*log(abs(-
(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^7 - 60*(a^5*b - 5*a^3*b^3 - 3
*a^2*b^4 + 4*a*b^5 + 3*b^6 + a^5*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) -
a^4*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*a^3*b^3*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 4*a^2*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*
a*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 3*b^6*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1))/((a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1))*a^7) + (32*a^5 - 137*a^4*b - 300*a^3*b^2
+ 548*a^2*b^3 + 300*a*b^4 - 411*b^5 - 160*a^5*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) + 805*a^4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1320*a^3*b^2*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2980*a^2*b^3*(cos(d*x + c) - 1)/(cos
(d*x + c) + 1) - 1200*a*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2055*b^
5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 320*a^5*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2 - 1970*a^4*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1
920*a^3*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 6200*a^2*b^3*(cos(d
*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1800*a*b^4*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2 - 4110*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1970
*a^4*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 1080*a^3*b^2*(cos(d*x +
c) - 1)^3/(cos(d*x + c) + 1)^3 - 6200*a^2*b^3*(cos(d*x + c) - 1)^3/(cos(d*x
 + c) + 1)^3 - 1200*a*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 4110*
b^5*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 805*a^4*b*(cos(d*x + c) - 1
)^4/(cos(d*x + c) + 1)^4 - 180*a^3*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) +
 1)^4 + 2980*a^2*b^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 300*a*b^4*
(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 2055*b^5*(cos(d*x + c) - 1)^4/(
cos(d*x + c) + 1)^4 + 137*a^4*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 -
 548*a^2*b^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 411*b^5*(cos(d*x +
 c) - 1)^5/(cos(d*x + c) + 1)^5)/(a^7*((cos(d*x + c) - 1)/(cos(d*x + c) + 1
) - 1)^5))/d
```

3.211 $\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=119

$$-\frac{\left(a^2-3b^2\right)\cos(c+dx)}{a^4d}+\frac{b^2\left(a^2-b^2\right)}{a^5d(a\cos(c+dx)+b)}+\frac{2b\left(a^2-2b^2\right)\log(a\cos(c+dx)+b)}{a^5d}-\frac{b\cos^2(c+dx)}{a^3d}+\frac{\cos^3(c+dx)}{3a^2d}$$

```
[Out] -(((a^2 - 3*b^2)*Cos[c + d*x])/(a^4*d)) - (b*Cos[c + d*x]^2)/(a^3*d) + Cos[
c + d*x]^3/(3*a^2*d) + (b^2*(a^2 - b^2))/(a^5*d*(b + a*Cos[c + d*x])) + (2*
b*(a^2 - 2*b^2)*Log[b + a*Cos[c + d*x]])/(a^5*d)
```

Rubi [A] time = 0.228387, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2837, 12, 894}

$$-\frac{\left(a^2-3b^2\right)\cos(c+dx)}{a^4d}+\frac{b^2\left(a^2-b^2\right)}{a^5d(a\cos(c+dx)+b)}+\frac{2b\left(a^2-2b^2\right)\log(a\cos(c+dx)+b)}{a^5d}-\frac{b\cos^2(c+dx)}{a^3d}+\frac{\cos^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] -(((a^2 - 3*b^2)*Cos[c + d*x])/(a^4*d)) - (b*Cos[c + d*x]^2)/(a^3*d) + Cos[
c + d*x]^3/(3*a^2*d) + (b^2*(a^2 - b^2))/(a^5*d*(b + a*Cos[c + d*x])) + (2*
b*(a^2 - 2*b^2)*Log[b + a*Cos[c + d*x]])/(a^5*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin^3(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^2(a^2-x^2)}{a^2(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2(a^2-x^2)}{(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^5d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{3b^2}{a^2}\right)-\frac{b^2(-a^2+b^2)}{(b-x)^2}+\frac{2b(-a^2+2b^2)}{b-x}-2bx-x^2\right)dx, x, -a\cos(c+dx)\right)}{a^5d}$$
$$= -\frac{(a^2-3b^2)\cos(c+dx)}{a^4d}-\frac{b\cos^2(c+dx)}{a^3d}+\frac{\cos^3(c+dx)}{3a^2d}+\frac{b^2(a^2-b^2)}{a^5d(b+a\cos(c+dx))}+\frac{2b(}{}$$

Mathematica [A] time = 0.426407, size = 167, normalized size = 1.4

$$\frac{-8(a^4-3a^2b^2)\cos(2(c+dx))+48a^2b^2\log(a\cos(c+dx)+b)+24ab\cos(c+dx)\left(2(a^2-2b^2)\log(a\cos(c+dx)+b)\right.}{24a^5d(a\cos(c+d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-9*a^4 + 60*a^2*b^2 - 24*b^4 - 8*(a^4 - 3*a^2*b^2)*Cos[2*(c + d*x)] - 4*a^
3*b*Cos[3*(c + d*x)] + a^4*Cos[4*(c + d*x)] + 48*a^2*b^2*Log[b + a*Cos[c +
d*x]] - 96*b^4*Log[b + a*Cos[c + d*x]] + 24*a*b*Cos[c + d*x]*(-a^2 + 3*b^2
+ 2*(a^2 - 2*b^2)*Log[b + a*Cos[c + d*x]]))/(24*a^5*d*(b + a*Cos[c + d*x]))
```

Maple [A] time = 0.063, size = 153, normalized size = 1.3

$$\frac{(\cos(dx+c))^3}{3a^2d}-\frac{b(\cos(dx+c))^2}{a^3d}-\frac{\cos(dx+c)}{a^2d}+3\frac{b^2\cos(dx+c)}{da^4}+2\frac{b\ln(b+a\cos(dx+c))}{a^3d}-4\frac{b^3\ln(b+a\cos(}{da^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/3*cos(d*x+c)^3/a^2/d-b*cos(d*x+c)^2/a^3/d-cos(d*x+c)/a^2/d+3/d/a^4*b^2*co
s(d*x+c)+2*b*ln(b+a*cos(d*x+c))/a^3/d-4/d/a^5*b^3*ln(b+a*cos(d*x+c))+b^2/a^
3/d/(b+a*cos(d*x+c))-1/d*b^4/a^5/(b+a*cos(d*x+c))
```

Maxima [A] time = 1.07083, size = 151, normalized size = 1.27

$$\frac{\frac{3(a^2b^2-b^4)}{a^6\cos(dx+c)+a^5b}+\frac{a^2\cos(dx+c)^3-3ab\cos(dx+c)^2-3(a^2-3b^2)\cos(dx+c)}{a^4}+\frac{6(a^2b-2b^3)\log(a\cos(dx+c)+b)}{a^5}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/3*(3*(a^2*b^2 - b^4)/(a^6*cos(d*x + c) + a^5*b) + (a^2*cos(d*x + c)^3 - 3
*a*b*cos(d*x + c)^2 - 3*(a^2 - 3*b^2)*cos(d*x + c))/a^4 + 6*(a^2*b - 2*b^3)
*log(a*cos(d*x + c) + b)/a^5)/d
```

Fricas [A] time = 1.84538, size = 346, normalized size = 2.91

$$\frac{2\,a^4\cos\left(dx+c\right)^4 - 4\,a^3b\cos\left(dx+c\right)^3 + 9\,a^2b^2 - 6\,b^4 - 6\left(a^4 - 2\,a^2b^2\right)\cos\left(dx+c\right)^2 - 3\left(a^3b - 6\,ab^3\right)\cos\left(dx+c\right) + 12\left(a\right.}{6\left(a^6d\cos\left(dx+c\right) + a^5bd\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/6*(2*a^4*cos(d*x + c)^4 - 4*a^3*b*cos(d*x + c)^3 + 9*a^2*b^2 - 6*b^4 - 6*
(a^4 - 2*a^2*b^2)*cos(d*x + c)^2 - 3*(a^3*b - 6*a*b^3)*cos(d*x + c) + 12*(a
^2*b^2 - 2*b^4 + (a^3*b - 2*a*b^3)*cos(d*x + c))*log(a*cos(d*x + c) + b))/(
a^6*d*cos(d*x + c) + a^5*b*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.34274, size = 188, normalized size = 1.58

$$\frac{2\left(a^2b - 2\,b^3\right)\log\left(\left|-a\cos\left(dx+c\right) - b\right|\right)}{a^5d} + \frac{a^2b^2 - b^4}{\left(a\cos\left(dx+c\right) + b\right)a^5d} + \frac{a^4d^5\cos\left(dx+c\right)^3 - 3\,a^3bd^5\cos\left(dx+c\right)^2 - 3\,a^4d^5\cos}{3\,a^6d^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 2*(a^2*b - 2*b^3)*log(abs(-a*cos(d*x + c) - b))/(a^5*d) + (a^2*b^2 - b^4)/(
(a*cos(d*x + c) + b)*a^5*d) + 1/3*(a^4*d^5*cos(d*x + c)^3 - 3*a^3*b*d^5*cos
(d*x + c)^2 - 3*a^4*d^5*cos(d*x + c) + 9*a^2*b^2*d^5*cos(d*x + c))/(a^6*d^6
)
```

3.212 $\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2} dx$

Optimal. Leaf size=57

$$\frac{b^2}{a^3d(a\cos(c+dx)+b)}+\frac{2b\log(a\cos(c+dx)+b)}{a^3d}-\frac{\cos(c+dx)}{a^2d}$$

```
[Out] -(Cos[c + d*x]/(a^2*d)) + b^2/(a^3*d*(b + a*Cos[c + d*x])) + (2*b*Log[b + a
*Cos[c + d*x]])/(a^3*d)
```

Rubi [A] time = 0.112259, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3872, 2833, 12, 43}

$$\frac{b^2}{a^3d(a\cos(c+dx)+b)}+\frac{2b\log(a\cos(c+dx)+b)}{a^3d}-\frac{\cos(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(Cos[c + d*x]/(a^2*d)) + b^2/(a^3*d*(b + a*Cos[c + d*x])) + (2*b*Log[b + a
*Cos[c + d*x]])/(a^3*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{a^2(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(-b+x)^2}\,dx, x, -a\cos(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \left(1+\frac{b^2}{(b-x)^2}-\frac{2b}{b-x}\right)dx, x, -a\cos(c+dx)\right)}{a^3d}$$
$$= -\frac{\cos(c+dx)}{a^2d} + \frac{b^2}{a^3d(b+a\cos(c+dx))} + \frac{2b\log(b+a\cos(c+dx))}{a^3d}$$

Mathematica [A] time = 0.132602, size = 76, normalized size = 1.33

$$\frac{-a^2\cos^2(c+dx)+b^2(2\log(a\cos(c+dx)+b)+1)+ab\cos(c+dx)(2\log(a\cos(c+dx)+b)-1)}{a^3d(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-(a^2*Cos[c + d*x]^2) + a*b*Cos[c + d*x]*(-1 + 2*Log[b + a*Cos[c + d*x]])
+ b^2*(1 + 2*Log[b + a*Cos[c + d*x]]))/(a^3*d*(b + a*Cos[c + d*x]))
```

Maple [A] time = 0.036, size = 75, normalized size = 1.3

$$-\frac{b}{da^2(a+b\sec(dx+c))}+2\frac{b\ln(a+b\sec(dx+c))}{da^3}-\frac{1}{da^2\sec(dx+c)}-2\frac{b\ln(\sec(dx+c))}{da^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -1/d*b/a^2/(a+b*sec(d*x+c))+2/d/a^3*b*ln(a+b*sec(d*x+c))-1/d/a^2/sec(d*x+c)
-2/d/a^3*b*ln(sec(d*x+c))
```

Maxima [A] time = 1.00148, size = 74, normalized size = 1.3

$$\frac{\frac{b^2}{a^4\cos(dx+c)+a^3b}-\frac{\cos(dx+c)}{a^2}+\frac{2\,b\log(a\cos(dx+c)+b)}{a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] (b^2/(a^4*cos(d*x + c) + a^3*b) - cos(d*x + c)/a^2 + 2*b*log(a*cos(d*x + c)
 + b)/a^3)/d
```

Fricas [A] time = 1.75957, size = 178, normalized size = 3.12

$$-\frac{a^2\cos(dx+c)^2+ab\cos(dx+c)-b^2-2\left(ab\cos(dx+c)+b^2\right)\log(a\cos(dx+c)+b)}{a^4d\cos(dx+c)+a^3bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -(a^2*cos(d*x + c)^2 + a*b*cos(d*x + c) - b^2 - 2*(a*b*cos(d*x + c) + b^2)*
log(a*cos(d*x + c) + b))/(a^4*d*cos(d*x + c) + a^3*b*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

Giac [A] time = 1.2496, size = 82, normalized size = 1.44

$$-\frac{\cos(dx+c)}{a^2d}+\frac{2\,b\log(|-a\cos(dx+c)-b|)}{a^3d}+\frac{b^2}{(a\cos(dx+c)+b)a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -cos(d*x + c)/(a^2*d) + 2*b*log(abs(-a*cos(d*x + c) - b))/(a^3*d) + b^2/((a
*cos(d*x + c) + b)*a^3*d)
```

3.213 $\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=109

$$\frac{b^2}{ad\left(a^2-b^2\right)\left(a\cos(c+dx)+b\right)}+\frac{2ab\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^2}+\frac{\log(1-\cos(c+dx))}{2d(a+b)^2}-\frac{\log(\cos(c+dx)+1)}{2d(a-b)^2}$$

```
[Out] b^2/(a*(a^2 - b^2)*d*(b + a*Cos[c + d*x])) + Log[1 - Cos[c + d*x]]/(2*(a +
b)^2*d) - Log[1 + Cos[c + d*x]]/(2*(a - b)^2*d) + (2*a*b*Log[b + a*Cos[c +
d*x]])/((a^2 - b^2)^2*d)
```

Rubi [A] time = 0.226432, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3872, 2837, 12, 1629}

$$\frac{b^2}{ad\left(a^2-b^2\right)\left(a\cos(c+dx)+b\right)}+\frac{2ab\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^2}+\frac{\log(1-\cos(c+dx))}{2d(a+b)^2}-\frac{\log(\cos(c+dx)+1)}{2d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] b^2/(a*(a^2 - b^2)*d*(b + a*Cos[c + d*x])) + Log[1 - Cos[c + d*x]]/(2*(a +
b)^2*d) - Log[1 + Cos[c + d*x]]/(2*(a - b)^2*d) + (2*a*b*Log[b + a*Cos[c +
d*x]])/((a^2 - b^2)^2*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos(c+dx)\cot(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$
$$= \frac{a\,\text{Subst}\left(\int \frac{x^2}{a^2(-b+x)^2(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{(-b+x)^2(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a}{2(a-b)^2(a-x)} + \frac{b^2}{(a-b)(a+b)(b-x)^2} - \frac{2a^2b}{(a-b)^2(a+b)^2(b-x)} + \frac{a}{2(a+b)^2(a+x)}\right)dx, x, -a\cos(c+d\right.}{ad}$$
$$= \frac{b^2}{a\left(a^2-b^2\right)d(b+a\cos(c+dx))} + \frac{\log(1-\cos(c+dx))}{2(a+b)^2d} - \frac{\log(1+\cos(c+dx))}{2(a-b)^2d} + \frac{2ab\log}{}$$

Mathematica [A] time = 0.281333, size = 165, normalized size = 1.51

$$\frac{b\left(2a^2b\log(a\cos(c+dx)+b)+(a-b)\left(a(a-b)\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+b(a+b)\right)-a(a+b)^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)-}{ad(a-b)^2(a+b)^2(a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-(a^2*Cos[c + d*x]*((a + b)^2*Log[Cos[(c + d*x)/2]] - 2*a*b*Log[b + a*Cos[
c + d*x]] - (a - b)^2*Log[Sin[(c + d*x)/2]])) + b*(-(a*(a + b)^2*Log[Cos[(c
 + d*x)/2]]) + 2*a^2*b*Log[b + a*Cos[c + d*x]] + (a - b)*(b*(a + b) + a*(a
- b)*Log[Sin[(c + d*x)/2]])))/(a*(a - b)^2*(a + b)^2*d*(b + a*Cos[c + d*x])
)
```

Maple [A] time = 0.067, size = 106, normalized size = 1.

$$\frac{b^2}{d(a+b)(a-b)a(b+a\cos(dx+c))} + 2\frac{ab\ln(b+a\cos(dx+c))}{d(a+b)^2(a-b)^2} - \frac{\ln(\cos(dx+c)+1)}{2(a-b)^2d} + \frac{\ln(-1+\cos(dx+c))}{2d(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*b^2/(a+b)/(a-b)/a/(b+a*cos(d*x+c))+2/d*a*b/(a+b)^2/(a-b)^2*ln(b+a*cos(d
*x+c))-1/2*ln(cos(d*x+c)+1)/(a-b)^2/d+1/2/d/(a+b)^2*ln(-1+cos(d*x+c))
```

Maxima [A] time = 1.0816, size = 166, normalized size = 1.52

$$\frac{\frac{4ab\log(a\cos(dx+c)+b)}{a^4-2a^2b^2+b^4} + \frac{2b^2}{a^3b-ab^3+\left(a^4-a^2b^2\right)\cos(dx+c)} - \frac{\log(\cos(dx+c)+1)}{a^2-2ab+b^2} + \frac{\log(\cos(dx+c)-1)}{a^2+2ab+b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*b*log(a*cos(d*x + c) + b)/(a^4 - 2*a^2*b^2 + b^4) + 2*b^2/(a^3*b -
 a*b^3 + (a^4 - a^2*b^2)*cos(d*x + c)) - log(cos(d*x + c) + 1)/(a^2 - 2*a*b
 + b^2) + log(cos(d*x + c) - 1)/(a^2 + 2*a*b + b^2))/d
```

Fricas [A] time = 2.19723, size = 486, normalized size = 4.46

$$\frac{2\,a^2b^2 - 2\,b^4 + 4\left(a^3b\cos\left(dx+c\right) + a^2b^2\right)\log\left(a\cos\left(dx+c\right)+b\right) - \left(a^3b + 2\,a^2b^2 + ab^3 + \left(a^4 + 2\,a^3b + a^2b^2\right)\cos\left(dx+c\right.\right.}{2\left(\left(a^6 - 2\,a^4b^2 + a^2b^4\right)d\cos\left(dx+c\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*a^2*b^2 - 2*b^4 + 4*(a^3*b*cos(d*x + c) + a^2*b^2)*log(a*cos(d*x + c
) + b) - (a^3*b + 2*a^2*b^2 + a*b^3 + (a^4 + 2*a^3*b + a^2*b^2)*cos(d*x + c
))*log(1/2*cos(d*x + c) + 1/2) + (a^3*b - 2*a^2*b^2 + a*b^3 + (a^4 - 2*a^3*
b + a^2*b^2)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^6 - 2*a^4*b^2
+ a^2*b^4)*d*cos(d*x + c) + (a^5*b - 2*a^3*b^3 + a*b^5)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

Giac [B] time = 1.32882, size = 288, normalized size = 2.64

$$\frac{\frac{4\,ab\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^4-2\,a^2b^2+b^4} + \frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2+2\,ab+b^2} - \frac{4\left(ab+b^2+\frac{ab(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\left(a^3+a^2b-ab^2-b^3\right)\left(a+b+\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a*b*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^4 - 2*a^2*b^2 + b^4) + log(abs(-cos
(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^2 + 2*a*b + b^2) - 4*(a*b + b^2 +
a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((a^3 + a^2*b - a*b^2 - b^3)*(a
+ b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1))))/d
```

3.214 $\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=168

$$\frac{ab^2}{d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}+\frac{2ab\left(a^2+b^2\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^3}+\frac{\csc^2(c+dx)\left(2ab-\left(a^2+b^2\right)\cos(c+dx)\right)}{2d\left(a^2-b^2\right)^2}+$$

```
[Out] (a*b^2)/((a^2 - b^2)^2*d*(b + a*Cos[c + d*x])) + ((2*a*b - (a^2 + b^2)*Cos[
c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)^2*d) + ((a - b)*Log[1 - Cos[c + d*
x]])/(4*(a + b)^3*d) - ((a + b)*Log[1 + Cos[c + d*x]])/(4*(a - b)^3*d) + (2
*a*b*(a^2 + b^2)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^3*d)
```

Rubi [A] time = 0.432896, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2837, 12, 1647, 1629}

$$\frac{ab^2}{d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}+\frac{2ab\left(a^2+b^2\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^3}+\frac{\csc^2(c+dx)\left(2ab-\left(a^2+b^2\right)\cos(c+dx)\right)}{2d\left(a^2-b^2\right)^2}+$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] (a*b^2)/((a^2 - b^2)^2*d*(b + a*Cos[c + d*x])) + ((2*a*b - (a^2 + b^2)*Cos[
c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)^2*d) + ((a - b)*Log[1 - Cos[c + d*
x]])/(4*(a + b)^3*d) - ((a + b)*Log[1 + Cos[c + d*x]])/(4*(a - b)^3*d) + (2
*a*b*(a^2 + b^2)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^3*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
```

```
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)\csc(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \frac{a^3\,\text{Subst}\left(\int \frac{x^2}{a^2(-b+x)^2\left(a^2-x^2\right)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a\,\text{Subst}\left(\int \frac{x^2}{(-b+x)^2\left(a^2-x^2\right)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{\left(2ab-\left(a^2+b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^2 d} + \frac{\text{Subst}\left(\int \frac{-\frac{a^2b^2\left(a^2+b^2\right)}{\left(a^2-b^2\right)^2}+\frac{2a^2bx}{a^2-b^2}+\frac{a^2\left(a^2+b^2\right)x^2}{\left(a^2-b^2\right)^2}}{(-b+x)^2\left(a^2-x^2\right)}\,dx, x, -a\,c\right.}{2ad}$$

$$= \frac{\left(2ab-\left(a^2+b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^2 d} + \frac{\text{Subst}\left(\int\left(\frac{a(a+b)}{2(a-b)^3(a-x)}+\frac{2a^2b^2}{(a-b)^2(a+b)^2(b-x)^2}-\frac{4a}{(a-b)}\right.\right.}{2a}$$

$$= \frac{ab^2}{\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))} + \frac{\left(2ab-\left(a^2+b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^2 d} + \frac{(a-b)\log(1}{4(a}$$

Mathematica [A] time = 1.30005, size = 224, normalized size = 1.33

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)\left(\frac{16ab\left(a^2+b^2\right)(a\cos(c+dx)+b)\log(a\cos(c+dx)+b)}{\left(a^2-b^2\right)^3}+\frac{8ab^2}{(a-b)^2(a+b)^2}+\frac{4(a+b)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b)}{(b-a)^3}-\right.}{8d(a+b\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*((8*a*b^2)/((a - b)^2*(a + b)^2) - ((b + a*Cos[c + d*
x])*Csc[(c + d*x)/2]^2)/(a + b)^2 + (4*(a + b)*(b + a*Cos[c + d*x])*Log[Cos
[(c + d*x)/2]])/(-a + b)^3 + (16*a*b*(a^2 + b^2)*(b + a*Cos[c + d*x])*Log[b
 + a*Cos[c + d*x]])/(a^2 - b^2)^3 + (4*(a - b)*(b + a*Cos[c + d*x])*Log[Sin
[(c + d*x)/2]])/(a + b)^3 + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a -
b)^2)*Sec[c + d*x]^2)/(8*d*(a + b*Sec[c + d*x])^2)
```

Maple [A] time = 0.078, size = 224, normalized size = 1.3

$$\frac{ab^2}{d\,(a+b)^2\,(a-b)^2\,(b+a\cos(dx+c))}+2\,\frac{a^3b\ln(b+a\cos(dx+c))}{d\,(a+b)^3\,(a-b)^3}+2\,\frac{ab^3\ln(b+a\cos(dx+c))}{d\,(a+b)^3\,(a-b)^3}+\frac{1}{4\,d\,(a-b)^2\,(\cos(dx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*b^2/(a+b)^2*a/(a-b)^2/(b+a*cos(d*x+c))+2/d*a^3*b/(a+b)^3/(a-b)^3*ln(b+a
*cos(d*x+c))+2/d*a*b^3/(a+b)^3/(a-b)^3*ln(b+a*cos(d*x+c))+1/4/d/(a-b)^2/(co
s(d*x+c)+1)-1/4/d/(a-b)^3*ln(cos(d*x+c)+1)*a-1/4/d/(a-b)^3*ln(cos(d*x+c)+1)
*b+1/4/d/(a+b)^2/(-1+cos(d*x+c))+1/4/d/(a+b)^3*ln(-1+cos(d*x+c))*a-1/4/d/(a
+b)^3*ln(-1+cos(d*x+c))*b
```

Maxima [A] time = 1.10275, size = 370, normalized size = 2.2

$$\frac{\frac{8\left(a^3b+ab^3\right)\log(a\cos(dx+c)+b)}{a^6-3a^4b^2+3a^2b^4-b^6}-\frac{(a+b)\log(\cos(dx+c)+1)}{a^3-3a^2b+3ab^2-b^3}+\frac{(a-b)\log(\cos(dx+c)-1)}{a^3+3a^2b+3ab^2+b^3}+\frac{2\left(4ab^2-\left(a^3+3ab^2\right)\cos(dx+c)^2+\left(a^2b-\right.\right.}{a^4b-2a^2b^3+b^5-\left(a^5-2a^3b^2+ab^4\right)\cos(dx+c)^3-\left(a^4b-2a^2b^3+b^5\right)}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/4*(8*(a^3*b + a*b^3)*log(a*cos(d*x + c) + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6) - (a + b)*log(cos(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) + (
a - b)*log(cos(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(4*a*b^2 -
 (a^3 + 3*a*b^2)*cos(d*x + c)^2 + (a^2*b - b^3)*cos(d*x + c))/(a^4*b - 2*a^
2*b^3 + b^5 - (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c)^3 - (a^4*b - 2*a^2*b^3
 + b^5)*cos(d*x + c)^2 + (a^5 - 2*a^3*b^2 + a*b^4)*cos(d*x + c)))/d
```

Fricas [B] time = 2.81496, size = 1378, normalized size = 8.2

$$-\frac{8a^3b^2-8ab^4-2\left(a^5+2a^3b^2-3ab^4\right)\cos(dx+c)^2+2\left(a^4b-2a^2b^3+b^5\right)\cos(dx+c)+8\left(a^3b^2+ab^4-\left(a^4b+a^2b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] -1/4*(8*a^3*b^2 - 8*a*b^4 - 2*(a^5 + 2*a^3*b^2 - 3*a*b^4)*cos(d*x + c)^2 +
2*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c) + 8*(a^3*b^2 + a*b^4 - (a^4*b + a^
2*b^3)*cos(d*x + c)^3 - (a^3*b^2 + a*b^4)*cos(d*x + c)^2 + (a^4*b + a^2*b^3
)*cos(d*x + c))*log(a*cos(d*x + c) + b) - (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 +
4*a*b^4 + b^5 - (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(d*x + c
)^3 - (a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*cos(d*x + c)^2 + (a^5
 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(d*x + c))*log(1/2*cos(d*x +
 c) + 1/2) + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5 - (a^5 - 4*a^4*
b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*cos(d*x + c)^3 - (a^4*b - 4*a^3*b^2 + 6*
a^2*b^3 - 4*a*b^4 + b^5)*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^
2*b^3 + a*b^4)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^7 - 3*a^5*b^
2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c)^3 + (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 -
b^7)*d*cos(d*x + c)^2 - (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c
) - (a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**3/(a + b*sec(c + d*x))**2, x)
```

Giac [B] time = 1.43979, size = 616, normalized size = 3.67

$$\frac{\frac{2(a-b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3+3a^2b+3ab^2+b^3}+\frac{16\left(a^3b+ab^3\right)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^6-3a^4b^2+3a^2b^4-b^6}+\frac{a^3-a^2b-ab^2+b^3-\frac{8a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{8ab^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a^3(\cos(dx+c)-}{(\cos(dx+c)+1}}{\left(a^4-2a^2b^2+b^4\right)\left(\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-}{\cos(dx+c)+}\right.}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/8*(2*(a - b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^3 + 3*a
^2*b + 3*a*b^2 + b^3) + 16*(a^3*b + a*b^3)*log(abs(-a - b - a*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^6 -
 3*a^4*b^2 + 3*a^2*b^4 - b^6) + (a^3 - a^2*b - a*b^2 + b^3 - 8*a^2*b*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1) + 8*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) - a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3*a^2*b*(cos(d*x + c
) - 1)^2/(cos(d*x + c) + 1)^2 - 3*a*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 - b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((a^4 - 2*a^2*b^2 +
 b^4)*(a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x +
c) - 1)^2/(cos(d*x + c) + 1)^2)) - (cos(d*x + c) - 1)/((a^2 - 2*a*b + b^2)*
(cos(d*x + c) + 1)))/d
```

3.215 $\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=259

$$\frac{a^3b^2}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}+\frac{\left(3a^2-4ab-b^2\right)\log(1-\cos(c+dx))}{16d(a+b)^4}-\frac{\left(3a^2+4ab-b^2\right)\log(\cos(c+dx)+1)}{16d(a-b)^4}+\frac{2a}{}$$

```
[Out] (a^3*b^2)/((a^2 - b^2)^3*d*(b + a*Cos[c + d*x])) + ((8*a*b*(a^2 + b^2) - (3
*a^4 + 12*a^2*b^2 + b^4)*Cos[c + d*x])*Csc[c + d*x]^2)/(8*(a^2 - b^2)^3*d)
+ ((2*a*b - (a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x]^4)/(4*(a^2 - b^2)^2*d) +
 ((3*a^2 - 4*a*b - b^2)*Log[1 - Cos[c + d*x]])/(16*(a + b)^4*d) - ((3*a^2 +
 4*a*b - b^2)*Log[1 + Cos[c + d*x]])/(16*(a - b)^4*d) + (2*a^3*b*(a^2 + 2*b
^2)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d)
```

Rubi [A] time = 0.741104, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2837, 12, 1647, 1629}

$$\frac{a^3b^2}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}+\frac{\left(3a^2-4ab-b^2\right)\log(1-\cos(c+dx))}{16d(a+b)^4}-\frac{\left(3a^2+4ab-b^2\right)\log(\cos(c+dx)+1)}{16d(a-b)^4}+\frac{2a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (a^3*b^2)/((a^2 - b^2)^3*d*(b + a*Cos[c + d*x])) + ((8*a*b*(a^2 + b^2) - (3
*a^4 + 12*a^2*b^2 + b^4)*Cos[c + d*x])*Csc[c + d*x]^2)/(8*(a^2 - b^2)^3*d)
+ ((2*a*b - (a^2 + b^2)*Cos[c + d*x])*Csc[c + d*x]^4)/(4*(a^2 - b^2)^2*d) +
 ((3*a^2 - 4*a*b - b^2)*Log[1 - Cos[c + d*x]])/(16*(a + b)^4*d) - ((3*a^2 +
 4*a*b - b^2)*Log[1 + Cos[c + d*x]])/(16*(a - b)^4*d) + (2*a^3*b*(a^2 + 2*b
^2)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
```

```
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)\csc^3(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \frac{a^5\,\text{Subst}\left(\int \frac{x^2}{a^2(-b+x)^2\left(a^2-x^2\right)^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a^3\,\text{Subst}\left(\int \frac{x^2}{(-b+x)^2\left(a^2-x^2\right)^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{\left(2ab-\left(a^2+b^2\right)\cos(c+dx)\right)\csc^4(c+dx)}{4\left(a^2-b^2\right)^2 d} + \frac{a\,\text{Subst}\left(\int \frac{-\frac{a^2b^2\left(a^2+b^2\right)}{\left(a^2-b^2\right)^2}+\frac{2a^2b\left(a^2-3b^2\right)x}{\left(a^2-b^2\right)^2}+\frac{3a^2\left(a^2+b^2\right)x^2}{\left(a^2-b^2\right)^2}}{(-b+x)^2\left(a^2-x^2\right)^2}\,dx$$

$$= \frac{\left(8ab\left(a^2+b^2\right)-\left(3a^4+12a^2b^2+b^4\right)\cos(c+dx)\right)\csc^2(c+dx)}{8\left(a^2-b^2\right)^3 d} + \frac{\left(2ab-\left(a^2+b^2\right)\cos(c+d\right.}{4\left(a^2-b^2\right)^2}$$

$$= \frac{\left(8ab\left(a^2+b^2\right)-\left(3a^4+12a^2b^2+b^4\right)\cos(c+dx)\right)\csc^2(c+dx)}{8\left(a^2-b^2\right)^3 d} + \frac{\left(2ab-\left(a^2+b^2\right)\cos(c+d\right.}{4\left(a^2-b^2\right)^2}$$

$$= \frac{a^3b^2}{\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))} + \frac{\left(8ab\left(a^2+b^2\right)-\left(3a^4+12a^2b^2+b^4\right)\cos(c+dx)\right)\csc^2(c+}{8\left(a^2-b^2\right)^3 d}$$

Mathematica [A] time = 1.38311, size = 320, normalized size = 1.24

$$\sec^2(c+dx)(a\cos(c+dx)+b)\left(\frac{8\left(-3a^2-4ab+b^2\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)(a\cos(c+dx)+b)}{(a-b)^4} + \frac{128a^3b\left(a^2+2b^2\right)(a\cos(c+dx)+b)\log(a\cos(c+dx)+b)}{\left(a^2-b^2\right)^4} + \frac{8(3}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*((64*a^3*b^2)/((a - b)^3*(a + b)^3) + (2*(-3*a + b)*(
b + a*Cos[c + d*x])*Csc[(c + d*x)/2]^2)/(a + b)^3 - ((b + a*Cos[c + d*x])*C
sc[(c + d*x)/2]^4)/(a + b)^2 + (8*(-3*a^2 - 4*a*b + b^2)*(b + a*Cos[c + d*x
```

```
])*Log[Cos[(c + d*x)/2]])/(a - b)^4 + (128*a^3*b*(a^2 + 2*b^2)*(b + a*Cos[c
 + d*x])*Log[b + a*Cos[c + d*x]])/(a^2 - b^2)^4 + (8*(3*a^2 - 4*a*b - b^2)*
(b + a*Cos[c + d*x])*Log[Sin[(c + d*x)/2]])/(a + b)^4 + (2*(3*a + b)*(b + a
*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a - b)^3 + ((b + a*Cos[c + d*x])*Sec[(c
 + d*x)/2]^4)/(a - b)^2)*Sec[c + d*x]^2)/(64*d*(a + b*Sec[c + d*x])^2)
```

Maple [A] time = 0.086, size = 368, normalized size = 1.4

$$\frac{a^3b^2}{d(a+b)^3(a-b)^3(b+a\cos(dx+c))}+2\frac{ba^5\ln(b+a\cos(dx+c))}{d(a+b)^4(a-b)^4}+4\frac{a^3b^3\ln(b+a\cos(dx+c))}{d(a+b)^4(a-b)^4}+\frac{1}{16d(a-b)^2(\cos \ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*a^3*b^2/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))+2/d*b*a^5/(a+b)^4/(a-b)^4*ln(b
+a*cos(d*x+c))+4/d*b^3*a^3/(a+b)^4/(a-b)^4*ln(b+a*cos(d*x+c))+1/16/d/(a-b)^
2/(cos(d*x+c)+1)^2+3/16/d/(a-b)^3/(cos(d*x+c)+1)*a+1/16/d/(a-b)^3/(cos(d*x+
c)+1)*b-3/16/d/(a-b)^4*ln(cos(d*x+c)+1)*a^2-1/4/d/(a-b)^4*ln(cos(d*x+c)+1)*
a*b+1/16/d/(a-b)^4*ln(cos(d*x+c)+1)*b^2-1/16/d/(a+b)^2/(-1+cos(d*x+c))^2+3/
16/d/(a+b)^3/(-1+cos(d*x+c))*a-1/16/d/(a+b)^3/(-1+cos(d*x+c))*b+3/16/d/(a+b
)^4*ln(-1+cos(d*x+c))*a^2-1/4/d/(a+b)^4*ln(-1+cos(d*x+c))*a*b-1/16/d/(a+b)^
4*ln(-1+cos(d*x+c))*b^2
```

Maxima [B] time = 1.04761, size = 690, normalized size = 2.66

$$\frac{\frac{32\left(a^5b+2a^3b^3\right)\log(a\cos(dx+c)+b)}{a^8-4a^6b^2+6a^4b^4-4a^2b^6+b^8}-\frac{\left(3a^2+4ab-b^2\right)\log(\cos(dx+c)+1)}{a^4-4a^3b+6a^2b^2-4ab^3+b^4}+\frac{\left(3a^2-4ab-b^2\right)\log(\cos(dx+c)-1)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}+\frac{2\left(20a^3 \ldots\right.}{a^6b-3a^4b^3+3a^2b^5-b^7+\left(a^7-3a^5b^2+3a^3 \ldots\right.}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] 1/16*(32*(a^5*b + 2*a^3*b^3)*log(a*cos(d*x + c) + b)/(a^8 - 4*a^6*b^2 + 6*a
^4*b^4 - 4*a^2*b^6 + b^8) - (3*a^2 + 4*a*b - b^2)*log(cos(d*x + c) + 1)/(a^
4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + (3*a^2 - 4*a*b - b^2)*log(cos(d*
x + c) - 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 2*(20*a^3*b^2 + 4
*a*b^4 + (3*a^5 + 20*a^3*b^2 + a*b^4)*cos(d*x + c)^4 - (5*a^4*b - 4*a^2*b^3
 - b^5)*cos(d*x + c)^3 - (5*a^5 + 36*a^3*b^2 + 7*a*b^4)*cos(d*x + c)^2 + (7
*a^4*b - 8*a^2*b^3 + b^5)*cos(d*x + c))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^
7 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c)^5 + (a^6*b - 3*a^4*b
^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)^4 - 2*(a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b
^6)*cos(d*x + c)^3 - 2*(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*cos(d*x + c)^2
 + (a^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*cos(d*x + c)))/d
```

Fricas [B] time = 4.19684, size = 2653, normalized size = 10.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/16*(40*a^5*b^2 - 32*a^3*b^4 - 8*a*b^6 + 2*(3*a^7 + 17*a^5*b^2 - 19*a^3*b^
4 - a*b^6)*cos(d*x + c)^4 - 2*(5*a^6*b - 9*a^4*b^3 + 3*a^2*b^5 + b^7)*cos(d
*x + c)^3 - 2*(5*a^7 + 31*a^5*b^2 - 29*a^3*b^4 - 7*a*b^6)*cos(d*x + c)^2 +
2*(7*a^6*b - 15*a^4*b^3 + 9*a^2*b^5 - b^7)*cos(d*x + c) + 32*(a^5*b^2 + 2*a
^3*b^4 + (a^6*b + 2*a^4*b^3)*cos(d*x + c)^5 + (a^5*b^2 + 2*a^3*b^4)*cos(d*x
 + c)^4 - 2*(a^6*b + 2*a^4*b^3)*cos(d*x + c)^3 - 2*(a^5*b^2 + 2*a^3*b^4)*co
s(d*x + c)^2 + (a^6*b + 2*a^4*b^3)*cos(d*x + c))*log(a*cos(d*x + c) + b) -
(3*a^6*b + 16*a^5*b^2 + 33*a^4*b^3 + 32*a^3*b^4 + 13*a^2*b^5 - b^7 + (3*a^7
 + 16*a^6*b + 33*a^5*b^2 + 32*a^4*b^3 + 13*a^3*b^4 - a*b^6)*cos(d*x + c)^5
+ (3*a^6*b + 16*a^5*b^2 + 33*a^4*b^3 + 32*a^3*b^4 + 13*a^2*b^5 - b^7)*cos(d
*x + c)^4 - 2*(3*a^7 + 16*a^6*b + 33*a^5*b^2 + 32*a^4*b^3 + 13*a^3*b^4 - a*
b^6)*cos(d*x + c)^3 - 2*(3*a^6*b + 16*a^5*b^2 + 33*a^4*b^3 + 32*a^3*b^4 + 1
3*a^2*b^5 - b^7)*cos(d*x + c)^2 + (3*a^7 + 16*a^6*b + 33*a^5*b^2 + 32*a^4*b
^3 + 13*a^3*b^4 - a*b^6)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + (3*a^6
*b - 16*a^5*b^2 + 33*a^4*b^3 - 32*a^3*b^4 + 13*a^2*b^5 - b^7 + (3*a^7 - 16*
a^6*b + 33*a^5*b^2 - 32*a^4*b^3 + 13*a^3*b^4 - a*b^6)*cos(d*x + c)^5 + (3*a
^6*b - 16*a^5*b^2 + 33*a^4*b^3 - 32*a^3*b^4 + 13*a^2*b^5 - b^7)*cos(d*x + c
)^4 - 2*(3*a^7 - 16*a^6*b + 33*a^5*b^2 - 32*a^4*b^3 + 13*a^3*b^4 - a*b^6)*c
os(d*x + c)^3 - 2*(3*a^6*b - 16*a^5*b^2 + 33*a^4*b^3 - 32*a^3*b^4 + 13*a^2*
b^5 - b^7)*cos(d*x + c)^2 + (3*a^7 - 16*a^6*b + 33*a^5*b^2 - 32*a^4*b^3 + 1
3*a^3*b^4 - a*b^6)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^9 - 4*a^
7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^5 + (a^8*b - 4*a^6*b^
3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^4 - 2*(a^9 - 4*a^7*b^2 + 6*
a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3 - 2*(a^8*b - 4*a^6*b^3 + 6*a^
4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^2 + (a^9 - 4*a^7*b^2 + 6*a^5*b^4 -
4*a^3*b^6 + a*b^8)*d*cos(d*x + c) + (a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*
b^7 + b^9)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.44167, size = 959, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/64*(4*(3*a^2 - 4*a*b - b^2)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) +
 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 128*(a^5*b + 2*a^3*b^3)*
log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) -
 (8*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 8*a*b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1) - a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 2*a*b*(c
```

```
os(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b^2*(cos(d*x + c) - 1)^2/(cos(d*x
 + c) + 1)^2)/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) - (a^2 + 2*a*b +
b^2 - 8*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 8*a*b*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 18*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 2
4*a*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*b^2*(cos(d*x + c) - 1)^
2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)^2/((a^4 + 4*a^3*b + 6*a^2*b^2 +
4*a*b^3 + b^4)*(cos(d*x + c) - 1)^2) - 128*(a^6*b + a^4*b^3 + 2*a^3*b^4 + a
^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a^5*b^2*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) + 2*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a^3*b
^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4
*a^2*b^6 + b^8)*(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d
*x + c) - 1)/(cos(d*x + c) + 1))))/d
```

3.216 $\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=473

$$\frac{b\left(-170a^2b^2+61a^4+105b^4\right)\sin(c+dx)}{15a^7d}+\frac{\left(-20a^2b^2+5a^4+14b^4\right)\sin(c+dx)\cos^4(c+dx)}{10a^3b^2d(a\cos(c+dx)+b)}-\frac{\left(-61a^2b^2+16a^4+42b^4\right)}{24a^4}$$

```
[Out] ((5*a^6 - 90*a^4*b^2 + 200*a^2*b^4 - 112*b^6)*x)/(16*a^8) - (2*(a - b)^(3/2
)*b*(a + b)^(3/2)*(2*a^2 - 7*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sq
rt[a + b]])/(a^8*d) + (b*(61*a^4 - 170*a^2*b^2 + 105*b^4)*Sin[c + d*x])/(15
*a^7*d) - ((27*a^4 - 86*a^2*b^2 + 56*b^4)*Cos[c + d*x]*Sin[c + d*x])/(16*a^
6*d) + ((15*a^4 - 52*a^2*b^2 + 35*b^4)*Cos[c + d*x]^2*Sin[c + d*x])/(15*a^5
*b*d) - ((16*a^4 - 61*a^2*b^2 + 42*b^4)*Cos[c + d*x]^3*Sin[c + d*x])/(24*a^
4*b^2*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(3*b*d*(b + a*Cos[c + d*x])) + (a*
Cos[c + d*x]^4*Sin[c + d*x])/(6*b^2*d*(b + a*Cos[c + d*x])) + ((5*a^4 - 20*
a^2*b^2 + 14*b^4)*Cos[c + d*x]^4*Sin[c + d*x])/(10*a^3*b^2*d*(b + a*Cos[c +
 d*x])) + (7*b*Cos[c + d*x]^5*Sin[c + d*x])/(30*a^2*d*(b + a*Cos[c + d*x]))
 - (Cos[c + d*x]^6*Sin[c + d*x])/(6*a*d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 1.71296, antiderivative size = 473, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2896, 3047, 3049, 3023, 2735, 2659, 208}

$$\frac{b\left(-170a^2b^2+61a^4+105b^4\right)\sin(c+dx)}{15a^7d}+\frac{\left(-20a^2b^2+5a^4+14b^4\right)\sin(c+dx)\cos^4(c+dx)}{10a^3b^2d(a\cos(c+dx)+b)}-\frac{\left(-61a^2b^2+16a^4+42b^4\right)}{24a^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((5*a^6 - 90*a^4*b^2 + 200*a^2*b^4 - 112*b^6)*x)/(16*a^8) - (2*(a - b)^(3/2
)*b*(a + b)^(3/2)*(2*a^2 - 7*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sq
rt[a + b]])/(a^8*d) + (b*(61*a^4 - 170*a^2*b^2 + 105*b^4)*Sin[c + d*x])/(15
*a^7*d) - ((27*a^4 - 86*a^2*b^2 + 56*b^4)*Cos[c + d*x]*Sin[c + d*x])/(16*a^
6*d) + ((15*a^4 - 52*a^2*b^2 + 35*b^4)*Cos[c + d*x]^2*Sin[c + d*x])/(15*a^5
*b*d) - ((16*a^4 - 61*a^2*b^2 + 42*b^4)*Cos[c + d*x]^3*Sin[c + d*x])/(24*a^
4*b^2*d) - (Cos[c + d*x]^3*Sin[c + d*x])/(3*b*d*(b + a*Cos[c + d*x])) + (a*
Cos[c + d*x]^4*Sin[c + d*x])/(6*b^2*d*(b + a*Cos[c + d*x])) + ((5*a^4 - 20*
a^2*b^2 + 14*b^4)*Cos[c + d*x]^4*Sin[c + d*x])/(10*a^3*b^2*d*(b + a*Cos[c +
 d*x])) + (7*b*Cos[c + d*x]^5*Sin[c + d*x])/(30*a^2*d*(b + a*Cos[c + d*x]))
 - (Cos[c + d*x]^6*Sin[c + d*x])/(6*a*d*(b + a*Cos[c + d*x]))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
```

```
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin^6(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{3bd(b+a\cos(c+dx))} + \frac{a\cos^4(c+dx)\sin(c+dx)}{6b^2d(b+a\cos(c+dx))} + \frac{7b\cos^5(c+dx)\sin(c+dx)}{30a^2d(b+a\cos(c+dx))} - \frac{\cos^6}{6ac}$$

$$= -\frac{\cos^3(c+dx)\sin(c+dx)}{3bd(b+a\cos(c+dx))} + \frac{a\cos^4(c+dx)\sin(c+dx)}{6b^2d(b+a\cos(c+dx))} + \frac{\left(5a^4-20a^2b^2+14b^4\right)\cos^4(c+dx}{10a^3b^2d(b+a\cos(c+d}$$

$$= -\frac{\left(16a^4-61a^2b^2+42b^4\right)\cos^3(c+dx)\sin(c+dx)}{24a^4b^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{3bd(b+a\cos(c+dx))} + \frac{a\cos^4(c+d}{6b^2d(b+a}$$

$$= \frac{\left(15a^4-52a^2b^2+35b^4\right)\cos^2(c+dx)\sin(c+dx)}{15a^5bd} - \frac{\left(16a^4-61a^2b^2+42b^4\right)\cos^3(c+dx)\sin(}{24a^4b^2d}$$

$$= -\frac{\left(27a^4-86a^2b^2+56b^4\right)\cos(c+dx)\sin(c+dx)}{16a^6d} + \frac{\left(15a^4-52a^2b^2+35b^4\right)\cos^2(c+dx)\sin}{15a^5bd}$$

$$= \frac{b\left(61a^4-170a^2b^2+105b^4\right)\sin(c+dx)}{15a^7d} - \frac{\left(27a^4-86a^2b^2+56b^4\right)\cos(c+dx)\sin(c+dx)}{16a^6d} +$$

$$= \frac{\left(5a^6-90a^4b^2+200a^2b^4-112b^6\right)x}{16a^8} + \frac{b\left(61a^4-170a^2b^2+105b^4\right)\sin(c+dx)}{15a^7d} - \frac{\left(27a^4-86\right.}{}$$

$$= \frac{\left(5a^6-90a^4b^2+200a^2b^4-112b^6\right)x}{16a^8} + \frac{b\left(61a^4-170a^2b^2+105b^4\right)\sin(c+dx)}{15a^7d} - \frac{\left(27a^4-86\right.}{}$$

$$= \frac{\left(5a^6-90a^4b^2+200a^2b^4-112b^6\right)x}{16a^8} - \frac{2(a-b)^{3/2}b(a+b)^{3/2}\left(2a^2-7b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c\right.}{\sqrt{a+b}}\right.}{a^8d}$$

Mathematica [A] time = 6.90907, size = 402, normalized size = 0.85

$$\frac{790a^5b^2\sin(3(c+dx))-42a^5b^2\sin(5(c+dx))-5440a^4b^3\sin(2(c+dx))+140a^4b^3\sin(4(c+dx))-560a^3b^4\sin(3(c+dx))+3360a^2b^5\sin(2(c+dx))-15a\left(-576a^4b^2+1488a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (3840*b*(2*a^2 - 7*b^2)*(a^2 - b^2)^(3/2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2
])/Sqrt[a^2 - b^2]] + (600*a^6*b*c - 10800*a^4*b^3*c + 24000*a^2*b^5*c - 13
```

```
440*b^7*c + 600*a^6*b*d*x - 10800*a^4*b^3*d*x + 24000*a^2*b^5*d*x - 13440*b
^7*d*x + 120*a*(5*a^6 - 90*a^4*b^2 + 200*a^2*b^4 - 112*b^6)*(c + d*x)*Cos[c
 + d*x] - 15*a*(15*a^6 - 576*a^4*b^2 + 1488*a^2*b^4 - 896*b^6)*Sin[c + d*x]
 + 1910*a^6*b*Sin[2*(c + d*x)] - 5440*a^4*b^3*Sin[2*(c + d*x)] + 3360*a^2*b
^5*Sin[2*(c + d*x)] - 180*a^7*Sin[3*(c + d*x)] + 790*a^5*b^2*Sin[3*(c + d*x
)] - 560*a^3*b^4*Sin[3*(c + d*x)] - 166*a^6*b*Sin[4*(c + d*x)] + 140*a^4*b^
3*Sin[4*(c + d*x)] + 40*a^7*Sin[5*(c + d*x)] - 42*a^5*b^2*Sin[5*(c + d*x)]
+ 14*a^6*b*Sin[6*(c + d*x)] - 5*a^7*Sin[7*(c + d*x)])/(b + a*Cos[c + d*x]))
/(1920*a^8*d)
```

Maple [B] time = 0.091, size = 1735, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+b*sec(d*x+c))^2,x)
```

```
[Out] -32/d*b^5/a^6/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(
a-b))^(1/2))+4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11*b+344
/5/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7*b+60/d/a^7/(1+tan(
1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*b^5-16/d/a^5/(1+tan(1/2*d*x+1/2*c)
^2)^6*tan(1/2*d*x+1/2*c)*b^3+4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x
+1/2*c)*b-272/3/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*b^3-3
3/2/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7*b^2-192/d/a^5/(1+
tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7*b^3+10/d/a^6/(1+tan(1/2*d*x+1/
2*c)^2)^6*tan(1/2*d*x+1/2*c)^7*b^4+14/d*b^7/a^8/((a+b)*(a-b))^(1/2)*arctanh
((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+21/4/d/a^4/(1+tan(1/2*d*x+1/
2*c)^2)^6*tan(1/2*d*x+1/2*c)*b^2+120/d/a^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1
/2*d*x+1/2*c)^7*b^5-21/4/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c
)^11*b^2+33/2/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*b^2-10/
d/a^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*b^4+344/5/d/a^3/(1+ta
n(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*b+60/d/a^7/(1+tan(1/2*d*x+1/2*c)
^2)^6*tan(1/2*d*x+1/2*c)^9*b^5-15/d/a^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*
d*x+1/2*c)^3*b^4-4/d*b/a^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/
2*c)/((a+b)*(a-b))^(1/2))+76/3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x
+1/2*c)^3*b+12/d/a^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)*b^5-272/
3/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*b^3+22/d*b^3/a^4/((
a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-5/d
/a^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)*b^4+87/4/d/a^4/(1+tan(1/
2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*b^2-16/d/a^5/(1+tan(1/2*d*x+1/2*c)^2
)^6*tan(1/2*d*x+1/2*c)^11*b^3+120/d/a^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*
d*x+1/2*c)^5*b^5-87/4/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9
*b^2+12/d/a^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11*b^5+76/3/d/a
^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*b-2/d*b^2/a^3*tan(1/2*d*
x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)+4/d*b^4/a^5*ta
n(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-2/d*b^
6/a^7*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b
)+15/d/a^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9*b^4-192/d/a^5/(1
+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*b^3+33/4/d/a^2/(1+tan(1/2*d*x
+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7-33/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan
(1/2*d*x+1/2*c)^5-85/24/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)
^3-5/8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)-14/d/a^8*arctan(
tan(1/2*d*x+1/2*c))*b^6-45/4/d/a^4*arctan(tan(1/2*d*x+1/2*c))*b^2+25/d/a^6*
arctan(tan(1/2*d*x+1/2*c))*b^4+5/8/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2
*d*x+1/2*c)^11+85/24/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9+
5/d/a^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11*b^4+5/8/d/a^2*arct
```

```
an(tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.53648, size = 1871, normalized size = 3.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/240*(15*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*d*x*cos(d*x + c)
+ 15*(5*a^6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*d*x + 120*(2*a^4*b^2 -
9*a^2*b^4 + 7*b^6 + (2*a^5*b - 9*a^3*b^3 + 7*a*b^5)*cos(d*x + c))*sqrt(a^2
- b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2
- b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2
 + 2*a*b*cos(d*x + c) + b^2)) - (40*a^7*cos(d*x + c)^6 - 56*a^6*b*cos(d*x +
 c)^5 - 976*a^5*b^2 + 2720*a^3*b^4 - 1680*a*b^6 - 2*(65*a^7 - 42*a^5*b^2)*c
os(d*x + c)^4 + 2*(111*a^6*b - 70*a^4*b^3)*cos(d*x + c)^3 + (165*a^7 - 458*
a^5*b^2 + 280*a^3*b^4)*cos(d*x + c)^2 - (571*a^6*b - 1430*a^4*b^3 + 840*a^2
*b^5)*cos(d*x + c))*sin(d*x + c))/(a^9*d*cos(d*x + c) + a^8*b*d), 1/240*(15
*(5*a^7 - 90*a^5*b^2 + 200*a^3*b^4 - 112*a*b^6)*d*x*cos(d*x + c) + 15*(5*a^
6*b - 90*a^4*b^3 + 200*a^2*b^5 - 112*b^7)*d*x - 240*(2*a^4*b^2 - 9*a^2*b^4
+ 7*b^6 + (2*a^5*b - 9*a^3*b^3 + 7*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*ar
ctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (
40*a^7*cos(d*x + c)^6 - 56*a^6*b*cos(d*x + c)^5 - 976*a^5*b^2 + 2720*a^3*b^
4 - 1680*a*b^6 - 2*(65*a^7 - 42*a^5*b^2)*cos(d*x + c)^4 + 2*(111*a^6*b - 70
*a^4*b^3)*cos(d*x + c)^3 + (165*a^7 - 458*a^5*b^2 + 280*a^3*b^4)*cos(d*x +
c)^2 - (571*a^6*b - 1430*a^4*b^3 + 840*a^2*b^5)*cos(d*x + c))*sin(d*x + c))
/(a^9*d*cos(d*x + c) + a^8*b*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38277, size = 1175, normalized size = 2.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/240*(15*(5*a^6 - 90*a^4*b^2 + 200*a^2*b^4 - 112*b^6)*(d*x + c)/a^8 - 480*
(2*a^6*b - 11*a^4*b^3 + 16*a^2*b^5 - 7*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/
2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*
c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^8) - 480*(a^4*b^2*tan(1/2*d*x +
1/2*c) - 2*a^2*b^4*tan(1/2*d*x + 1/2*c) + b^6*tan(1/2*d*x + 1/2*c))/((a*tan
(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)*a^7) + 2*(75*a^5*ta
n(1/2*d*x + 1/2*c)^11 + 480*a^4*b*tan(1/2*d*x + 1/2*c)^11 - 630*a^3*b^2*tan
(1/2*d*x + 1/2*c)^11 - 1920*a^2*b^3*tan(1/2*d*x + 1/2*c)^11 + 600*a*b^4*tan
(1/2*d*x + 1/2*c)^11 + 1440*b^5*tan(1/2*d*x + 1/2*c)^11 + 425*a^5*tan(1/2*d
*x + 1/2*c)^9 + 3040*a^4*b*tan(1/2*d*x + 1/2*c)^9 - 2610*a^3*b^2*tan(1/2*d*
x + 1/2*c)^9 - 10880*a^2*b^3*tan(1/2*d*x + 1/2*c)^9 + 1800*a*b^4*tan(1/2*d*
x + 1/2*c)^9 + 7200*b^5*tan(1/2*d*x + 1/2*c)^9 + 990*a^5*tan(1/2*d*x + 1/2*
c)^7 + 8256*a^4*b*tan(1/2*d*x + 1/2*c)^7 - 1980*a^3*b^2*tan(1/2*d*x + 1/2*c
)^7 - 23040*a^2*b^3*tan(1/2*d*x + 1/2*c)^7 + 1200*a*b^4*tan(1/2*d*x + 1/2*c
)^7 + 14400*b^5*tan(1/2*d*x + 1/2*c)^7 - 990*a^5*tan(1/2*d*x + 1/2*c)^5 + 8
256*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 1980*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 23
040*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 1200*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 14
400*b^5*tan(1/2*d*x + 1/2*c)^5 - 425*a^5*tan(1/2*d*x + 1/2*c)^3 + 3040*a^4*
b*tan(1/2*d*x + 1/2*c)^3 + 2610*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 10880*a^2*
b^3*tan(1/2*d*x + 1/2*c)^3 - 1800*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 7200*b^5*t
an(1/2*d*x + 1/2*c)^3 - 75*a^5*tan(1/2*d*x + 1/2*c) + 480*a^4*b*tan(1/2*d*x
 + 1/2*c) + 630*a^3*b^2*tan(1/2*d*x + 1/2*c) - 1920*a^2*b^3*tan(1/2*d*x + 1
/2*c) - 600*a*b^4*tan(1/2*d*x + 1/2*c) + 1440*b^5*tan(1/2*d*x + 1/2*c))/((t
an(1/2*d*x + 1/2*c)^2 + 1)^6*a^7))/d
```

3.217 $\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=261

$$\frac{b\left(11a^2-15b^2\right)\sin(c+dx)}{3a^5d}-\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^3(c+dx)}{a^2bd(a\cos(c+dx)+b)}+\frac{\left(3a^2-5b^2\right)\sin(c+dx)\cos^2(c+dx)}{3a^3bd}-\frac{\left(13a^2-20b^2\right)s}{}$$

```
[Out] ((3*a^4 - 36*a^2*b^2 + 40*b^4)*x)/(8*a^6) - (2*Sqrt[a - b]*b*Sqrt[a + b]*(2
*a^2 - 5*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^6*d)
+ (b*(11*a^2 - 15*b^2)*Sin[c + d*x])/(3*a^5*d) - ((13*a^2 - 20*b^2)*Cos[c +
 d*x]*Sin[c + d*x])/(8*a^4*d) + ((3*a^2 - 5*b^2)*Cos[c + d*x]^2*Sin[c + d*x
])/(3*a^3*b*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*a^2*d) - ((a^2 - b^2)*Cos
[c + d*x]^3*Sin[c + d*x])/(a^2*b*d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 0.825278, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2892, 3049, 3023, 2735, 2659, 208}

$$\frac{b\left(11a^2-15b^2\right)\sin(c+dx)}{3a^5d}-\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^3(c+dx)}{a^2bd(a\cos(c+dx)+b)}+\frac{\left(3a^2-5b^2\right)\sin(c+dx)\cos^2(c+dx)}{3a^3bd}-\frac{\left(13a^2-20b^2\right)s}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] ((3*a^4 - 36*a^2*b^2 + 40*b^4)*x)/(8*a^6) - (2*Sqrt[a - b]*b*Sqrt[a + b]*(2
*a^2 - 5*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^6*d)
+ (b*(11*a^2 - 15*b^2)*Sin[c + d*x])/(3*a^5*d) - ((13*a^2 - 20*b^2)*Cos[c +
 d*x]*Sin[c + d*x])/(8*a^4*d) + ((3*a^2 - 5*b^2)*Cos[c + d*x]^2*Sin[c + d*x
])/(3*a^3*b*d) + (Cos[c + d*x]^3*Sin[c + d*x])/(4*a^2*d) - ((a^2 - b^2)*Cos
[c + d*x]^3*Sin[c + d*x])/(a^2*b*d*(b + a*Cos[c + d*x]))
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2892

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a*b^2*(m + 1)*(m + n + 4)), Int[(a + b*Sin[e + f*x])^
(m + 1)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m +
n + 4) + a*b*(m + 1)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 3)*
(m + n + 4))*Sin[e + f*x]^2, x], x], x] - Simp[(Cos[e + f*x]*(a + b*Sin[e +
 f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/(b^2*d*f*(m + n + 4)), x]) /; Free
Q[{a, b, d, e, f, n}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ
[m, -1] &&  !LtQ[n, -1] && NeQ[m + n + 4, 0]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin^4(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d} - \frac{\left(a^2-b^2\right)\cos^3(c+dx)\sin(c+dx)}{a^2bd(b+a\cos(c+dx))} - \frac{\int \frac{\cos^2(c+dx)\left(-8a^2+15b^2-ab\cos(c+\right.}{-b-a\cos(}}{4a^2}$$

$$= \frac{\left(3a^2-5b^2\right)\cos^2(c+dx)\sin(c+dx)}{3a^3bd} + \frac{\cos^3(c+dx)\sin(c+dx)}{4a^2d} - \frac{\left(a^2-b^2\right)\cos^3(c+dx)\sin}{a^2bd(b+a\cos(c+d}$$

$$= -\frac{\left(13a^2-20b^2\right)\cos(c+dx)\sin(c+dx)}{8a^4d} + \frac{\left(3a^2-5b^2\right)\cos^2(c+dx)\sin(c+dx)}{3a^3bd} + \frac{\cos^3(c+d}{4}$$

$$= \frac{b\left(11a^2-15b^2\right)\sin(c+dx)}{3a^5d} - \frac{\left(13a^2-20b^2\right)\cos(c+dx)\sin(c+dx)}{8a^4d} + \frac{\left(3a^2-5b^2\right)\cos^2(c+}{3a^3bd}$$

$$= \frac{\left(3a^4-36a^2b^2+40b^4\right)x}{8a^6} + \frac{b\left(11a^2-15b^2\right)\sin(c+dx)}{3a^5d} - \frac{\left(13a^2-20b^2\right)\cos(c+dx)\sin(c+}{8a^4d}$$

$$= \frac{\left(3a^4-36a^2b^2+40b^4\right)x}{8a^6} + \frac{b\left(11a^2-15b^2\right)\sin(c+dx)}{3a^5d} - \frac{\left(13a^2-20b^2\right)\cos(c+dx)\sin(c+}{8a^4d}$$

$$= \frac{\left(3a^4-36a^2b^2+40b^4\right)x}{8a^6} - \frac{2\sqrt{a-b}b\sqrt{a+b}\left(2a^2-5b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^6d} + \frac{b\left(11a^2\right.}{}$$

Mathematica [A] time = 3.09277, size = 282, normalized size = 1.08

$$\frac{\frac{40a^3b^2\sin(3(c+dx))-240a^2b^3\sin(2(c+dx))-24a\left(-31a^2b^2+a^4+40b^4\right)\sin(c+dx)+24a\left(-36a^2b^2+3a^4+40b^4\right)(c+dx)\cos(c+dx)-864a^2b^3c-864a^2b^3dx+176a^4b\sin(2(}{a\cos(c+dx)+b}}{192a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((384*b*(2*a^4 - 7*a^2*b^2 + 5*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqr
t[a^2 - b^2]])/Sqrt[a^2 - b^2] + (72*a^4*b*c - 864*a^2*b^3*c + 960*b^5*c +
72*a^4*b*d*x - 864*a^2*b^3*d*x + 960*b^5*d*x + 24*a*(3*a^4 - 36*a^2*b^2 + 4
0*b^4)*(c + d*x)*Cos[c + d*x] - 24*a*(a^4 - 31*a^2*b^2 + 40*b^4)*Sin[c + d*
x] + 176*a^4*b*Sin[2*(c + d*x)] - 240*a^2*b^3*Sin[2*(c + d*x)] - 21*a^5*Sin
[3*(c + d*x)] + 40*a^3*b^2*Sin[3*(c + d*x)] - 10*a^4*b*Sin[4*(c + d*x)] + 3
*a^5*Sin[5*(c + d*x)])/(b + a*Cos[c + d*x]))/(192*a^6*d)
```

Maple [B] time = 0.081, size = 883, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 3/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7+4/d/a^3/(1+tan(1/
2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*b-3/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^4
```

```
*tan(1/2*d*x+1/2*c)^7*b^2-8/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/
2*c)^7*b^3+52/3/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*b-3/d
/a^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*b^2-24/d/a^5/(1+tan(1/
2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*b^3+11/4/d/a^2/(1+tan(1/2*d*x+1/2*c)
^2)^4*tan(1/2*d*x+1/2*c)^5-11/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*
x+1/2*c)^3+3/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*b^2+52/3
/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*b-24/d/a^5/(1+tan(1/
2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*b^3+4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)
^4*tan(1/2*d*x+1/2*c)*b-8/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*
c)*b^3-3/4/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)+3/d/a^4/(1+t
an(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*b^2-9/d/a^4*arctan(tan(1/2*d*x+1/
2*c))*b^2+10/d/a^6*arctan(tan(1/2*d*x+1/2*c))*b^4+3/4/d/a^2*arctan(tan(1/2*
d*x+1/2*c))-2/d*b^2/a^3*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*
d*x+1/2*c)^2*b-a-b)+2/d*b^4/a^5*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-
tan(1/2*d*x+1/2*c)^2*b-a-b)-4/d*b/a^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan
(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+14/d*b^3/a^4/((a+b)*(a-b))^(1/2)*arcta
nh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-10/d*b^5/a^6/((a+b)*(a-b))
^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [A] time = 2.18792, size = 1347, normalized size = 5.16

$$\left[\frac{3\left(3\,a^5-36\,a^3b^2+40\,ab^4\right)dx\cos\left(dx+c\right)+3\left(3\,a^4b-36\,a^2b^3+40\,b^5\right)dx-12\left(2\,a^2b^2-5\,b^4+\left(2\,a^3b-5\,ab^3\right)\cos\left(dx\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="fricas")

[Out] [1/24*(3*(3*a^5 - 36*a^3*b^2 + 40*a*b^4)*d*x*cos(d*x + c) + 3*(3*a^4*b - 36
*a^2*b^3 + 40*b^5)*d*x - 12*(2*a^2*b^2 - 5*b^4 + (2*a^3*b - 5*a*b^3)*cos(d*
x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c
)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a
^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (6*a^5*cos(d*x + c)^4 - 10
*a^4*b*cos(d*x + c)^3 + 88*a^3*b^2 - 120*a*b^4 - 5*(3*a^5 - 4*a^3*b^2)*cos(
d*x + c)^2 + (49*a^4*b - 60*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/(a^7*d*cos
(d*x + c) + a^6*b*d), 1/24*(3*(3*a^5 - 36*a^3*b^2 + 40*a*b^4)*d*x*cos(d*x +
 c) + 3*(3*a^4*b - 36*a^2*b^3 + 40*b^5)*d*x - 24*(2*a^2*b^2 - 5*b^4 + (2*a^
3*b - 5*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*c
os(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (6*a^5*cos(d*x + c)^4 - 10*a
^4*b*cos(d*x + c)^3 + 88*a^3*b^2 - 120*a*b^4 - 5*(3*a^5 - 4*a^3*b^2)*cos(d*
x + c)^2 + (49*a^4*b - 60*a^2*b^3)*cos(d*x + c))*sin(d*x + c))/(a^7*d*cos(d
*x + c) + a^6*b*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

Giac [A] time = 1.28715, size = 651, normalized size = 2.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/24*(3*(3*a^4 - 36*a^2*b^2 + 40*b^4)*(d*x + c)/a^6 - 48*(2*a^4*b - 7*a^2*b
^3 + 5*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*
tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^
2 + b^2)*a^6) - 48*(a^2*b^2*tan(1/2*d*x + 1/2*c) - b^4*tan(1/2*d*x + 1/2*c)
)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)*a^5) + 2*(
9*a^3*tan(1/2*d*x + 1/2*c)^7 + 48*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 36*a*b^2*t
an(1/2*d*x + 1/2*c)^7 - 96*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*a^3*tan(1/2*d*x
+ 1/2*c)^5 + 208*a^2*b*tan(1/2*d*x + 1/2*c)^5 - 36*a*b^2*tan(1/2*d*x + 1/2*
c)^5 - 288*b^3*tan(1/2*d*x + 1/2*c)^5 - 33*a^3*tan(1/2*d*x + 1/2*c)^3 + 208
*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 36*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 288*b^3*t
an(1/2*d*x + 1/2*c)^3 - 9*a^3*tan(1/2*d*x + 1/2*c) + 48*a^2*b*tan(1/2*d*x +
 1/2*c) + 36*a*b^2*tan(1/2*d*x + 1/2*c) - 96*b^3*tan(1/2*d*x + 1/2*c))/((ta
n(1/2*d*x + 1/2*c)^2 + 1)^4*a^5))/d
```

3.218 $\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=152

$$-\frac{2b\left(2a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(a^2-6b^2\right)}{2a^4}+\frac{3b\sin(c+dx)}{a^3d}-\frac{3\sin(c+dx)\cos(c+dx)}{2a^2d}+\frac{\sin(c+dx)\,\cdots}{ad(a\cos(c\,\cdots}$$

```
[Out] ((a^2 - 6*b^2)*x)/(2*a^4) - (2*b*(2*a^2 - 3*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(
c + d*x)/2])/Sqrt[a + b]])/(a^4*Sqrt[a - b]*Sqrt[a + b]*d) + (3*b*Sin[c + d
*x])/(a^3*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) + (Cos[c + d*x]^2*Si
n[c + d*x])/(a*d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 0.570967, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2889, 3048, 3050, 3023, 2735, 2659, 208}

$$-\frac{2b\left(2a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}+\frac{x\left(a^2-6b^2\right)}{2a^4}+\frac{3b\sin(c+dx)}{a^3d}-\frac{3\sin(c+dx)\cos(c+dx)}{2a^2d}+\frac{\sin(c+dx)\,\cdots}{ad(a\cos(c\,\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((a^2 - 6*b^2)*x)/(2*a^4) - (2*b*(2*a^2 - 3*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(
c + d*x)/2])/Sqrt[a + b]])/(a^4*Sqrt[a - b]*Sqrt[a + b]*d) + (3*b*Sin[c + d
*x])/(a^3*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) + (Cos[c + d*x]^2*Si
n[c + d*x])/(a*d*(b + a*Cos[c + d*x]))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
```

```
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3050

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :
> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1)
)/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n + 2)), Int[(a + b*Sin[e + f*x])^
(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 2) + C*(b*c*m + a*d*(n +
 1)) + (A*b*d*(m + n + 2) - C*(a*c - b*d*(m + n + 1)))*Sin[e + f*x] + C*(a*
d*m - b*c*(m + 1))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A
, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
&& GtQ[m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])
))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sin^2(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \int \frac{\cos^2(c+dx)\left(1-\cos^2(c+dx)\right)}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \frac{\cos^2(c+dx)\sin(c+dx)}{ad(b+a\cos(c+dx))} - \frac{\int \frac{\cos(c+dx)\left(2\left(a^2-b^2\right)-3\left(a^2-b^2\right)\cos^2(c+dx)\right)}{-b-a\cos(c+dx)}dx}{a\left(a^2-b^2\right)}$$

$$= -\frac{3\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\sin(c+dx)}{ad(b+a\cos(c+dx))} + \frac{\int \frac{3b\left(a^2-b^2\right)-a\left(a^2-b^2\right)\cos(c+dx)-6b\left(a^2-\right.}{-b-a\cos(c+dx)}}{2a^2\left(a^2-b^2\right)}$$

$$= \frac{3b\sin(c+dx)}{a^3d} - \frac{3\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\sin(c+dx)}{ad(b+a\cos(c+dx))} - \frac{\int \frac{-3ab\left(a^2-b^2\right)+\left(a^2-\right.}{-b-a}}{2a^3}$$

$$= \frac{\left(a^2-6b^2\right)x}{2a^4} + \frac{3b\sin(c+dx)}{a^3d} - \frac{3\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\sin(c+dx)}{ad(b+a\cos(c+dx))} + \ldots$$

$$= \frac{\left(a^2-6b^2\right)x}{2a^4} + \frac{3b\sin(c+dx)}{a^3d} - \frac{3\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{\cos^2(c+dx)\sin(c+dx)}{ad(b+a\cos(c+dx))} + \ldots$$

$$= \frac{\left(a^2-6b^2\right)x}{2a^4} - \frac{2b\left(2a^2-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}d} + \frac{3b\sin(c+dx)}{a^3d} - \frac{3\cos(c+dx)}{2a^2}\ldots$$

Mathematica [A] time = 1.1514, size = 178, normalized size = 1.17

$$\frac{\frac{-a\left(a^2-24b^2\right)\sin(c+dx)+4a\left(a^2-6b^2\right)(c+dx)\cos(c+dx)+6a^2b\sin(2(c+dx))+4a^2bc+4a^2bdx+a^3(-\sin(3(c+dx)))-24b^3c-24b^3dx}{a\cos(c+dx)+b} + \frac{16b\left(2a^2-3b^2\right)\tanh^{-1}\left(\frac{(b-a}{}\right.}{\sqrt{a^2-b^2}}}{8a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((16*b*(2*a^2 - 3*b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
)/Sqrt[a^2 - b^2] + (4*a^2*b*c - 24*b^3*c + 4*a^2*b*d*x - 24*b^3*d*x + 4*a*
(a^2 - 6*b^2)*(c + d*x)*Cos[c + d*x] - a*(a^2 - 24*b^2)*Sin[c + d*x] + 6*a^
2*b*Sin[2*(c + d*x)] - a^3*Sin[3*(c + d*x)])/(b + a*Cos[c + d*x]))/(8*a^4*d
)
```

Maple [B] time = 0.072, size = 325, normalized size = 2.1

$$\frac{1}{da^2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(1+\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\right)^{-2} + 4\frac{b\left(\tan\left(1/2\,dx+c/2\right)\right)^3}{da^3\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^2\right)^2} + 4\frac{b\tan\left(1/2\,dx+c/2\right)}{da^3\left(1+\left(\tan\left(1/2\,dx+c/2\right)\right)^2\right)}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+4/d/a^3/(1+tan(1/2*
d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*b+4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*t
```

```
an(1/2*d*x+1/2*c)*b-1/d/a^2/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)-6
/d/a^4*arctan(tan(1/2*d*x+1/2*c))*b^2+1/d/a^2*arctan(tan(1/2*d*x+1/2*c))-2/
d*b^2/a^3*tan(1/2*d*x+1/2*c)/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b
-a-b)-4/d*b/a^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)
*(a-b))^(1/2))+6/d*b^3/a^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/
2*c)/((a+b)*(a-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.03398, size = 1204, normalized size = 7.92

$$\left[\frac{\left(a^5 - 7\,a^3b^2 + 6\,ab^4\right)dx\cos\left(dx + c\right) + \left(a^4b - 7\,a^2b^3 + 6\,b^5\right)dx - \left(2\,a^2b^2 - 3\,b^4 + \left(2\,a^3b - 3\,ab^3\right)\cos\left(dx + c\right)\right)\sqrt{a^2 - b^2}\,\mathrm{lo}}{2\left(\left(a^{7}\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*((a^5 - 7*a^3*b^2 + 6*a*b^4)*d*x*cos(d*x + c) + (a^4*b - 7*a^2*b^3 + 6
*b^5)*d*x - (2*a^2*b^2 - 3*b^4 + (2*a^3*b - 3*a*b^3)*cos(d*x + c))*sqrt(a^2
 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2
 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^
2 + 2*a*b*cos(d*x + c) + b^2)) + (6*a^3*b^2 - 6*a*b^4 - (a^5 - a^3*b^2)*cos
(d*x + c)^2 + 3*(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*x + c))/((a^7 - a^5*b
^2)*d*cos(d*x + c) + (a^6*b - a^4*b^3)*d), 1/2*((a^5 - 7*a^3*b^2 + 6*a*b^4)
*d*x*cos(d*x + c) + (a^4*b - 7*a^2*b^3 + 6*b^5)*d*x - 2*(2*a^2*b^2 - 3*b^4
+ (2*a^3*b - 3*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^
2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (6*a^3*b^2 - 6*a*b^4
- (a^5 - a^3*b^2)*cos(d*x + c)^2 + 3*(a^4*b - a^2*b^3)*cos(d*x + c))*sin(d*
x + c))/((a^7 - a^5*b^2)*d*cos(d*x + c) + (a^6*b - a^4*b^3)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sin(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

Giac [A] time = 1.30816, size = 324, normalized size = 2.13

$$-\frac{\frac{4\,b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)a^3}-\frac{(a^2-6\,b^2)(dx+c)}{a^4}+\frac{4\left(2\,a^2b-3\,b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right.}{\sqrt{-a^2+b^2}}\right.\right.}{\sqrt{-a^2+b^2}\,a^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*b^2*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x
 + 1/2*c)^2 - a - b)*a^3) - (a^2 - 6*b^2)*(d*x + c)/a^4 + 4*(2*a^2*b - 3*b^
3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d
*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*
a^4) - 2*(a*tan(1/2*d*x + 1/2*c)^3 + 4*b*tan(1/2*d*x + 1/2*c)^3 - a*tan(1/2
*d*x + 1/2*c) + 4*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a
^3))/d
```

3.219 $\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2} dx$

Optimal. Leaf size=203

$$\frac{ab^2\sin(c+dx)}{d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\sin(c+dx)}{2d(a+b)^2(1-\cos(c}$$

```
[Out] (-4*a^2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/
2)*(a + b)^(5/2)*d) - (2*b^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
+ b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - Sin[c + d*x]/(2*(a + b)^2*d*(1 - C
os[c + d*x])) + Sin[c + d*x]/(2*(a - b)^2*d*(1 + Cos[c + d*x])) + (a*b^2*Si
n[c + d*x])/((a^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 0.435704, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2731, 2648, 2664, 12, 2659, 208}

$$\frac{ab^2\sin(c+dx)}{d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}-\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{\sin(c+dx)}{2d(a+b)^2(1-\cos(c}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```


```
[Out] (-4*a^2*b*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/
2)*(a + b)^(5/2)*d) - (2*b^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
+ b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - Sin[c + d*x]/(2*(a + b)^2*d*(1 - C
os[c + d*x])) + Sin[c + d*x]/(2*(a - b)^2*d*(1 + Cos[c + d*x])) + (a*b^2*Si
n[c + d*x])/((a^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2731

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*tan[(e_.) + (f_.)*(x_)]^(p_
), x_Symbol] :> Int[ExpandIntegrand[(Sin[e + f*x]^p*(a + b*Sin[e + f*x])^m)
/(1 - Sin[e + f*x]^2)^(p/2), x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 -
b^2, 0] && IntegersQ[m, p/2]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
```

```
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \int \left(-\frac{1}{2(a+b)^2(-1+\cos(c+dx))} + \frac{1}{2(a-b)^2(1+\cos(c+dx))} - \frac{b^2}{\left(-a^2+b^2\right)(b+a\cos(c\right.$$

$$= \frac{\int \frac{1}{1+\cos(c+dx)}\,dx}{2(a-b)^2} - \frac{\int \frac{1}{-1+\cos(c+dx)}\,dx}{2(a+b)^2} - \frac{\left(2a^2b\right)\int \frac{1}{b+a\cos(c+dx)}\,dx}{\left(a^2-b^2\right)^2} + \frac{b^2\int \frac{1}{(b+a\cos(c+dx))^2}\,dx}{a^2-b^2}$$

$$= -\frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+dx))} + \frac{ab^2\sin(c+dx)}{\left(a^2-b^2\right)^2 d(b+a\cos(c+\right.}$$

$$= -\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+\right.}$$

$$= -\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^2d(1+\cos(c+\right.}$$

$$= -\frac{4a^2b\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{\sin(c+dx)}{2(a+b)^2d(1-\cos(c\right.}$$

Mathematica [A] time = 0.823409, size = 128, normalized size = 0.63

$$\frac{\frac{4b\left(2a^2+b^2\right)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{\frac{2ab^2\sin(c+dx)}{(a+b)^2(a\cos(c+dx)+b)}+\tan\left(\frac{1}{2}(c+dx)\right)}{(a-b)^2} - \frac{\cot\left(\frac{1}{2}(c+dx)\right)}{(a+b)^2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((4*b*(2*a^2 + b^2)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(5/2) - Cot[(c + d*x)/2]/(a + b)^2 + ((2*a*b^2*Sin[c + d*x])/((a
 + b)^2*(b + a*Cos[c + d*x])) + Tan[(c + d*x)/2])/(a - b)^2)/(2*d)
```

Maple [A] time = 0.082, size = 162, normalized size = 0.8

$$\frac{1}{d}\left(\frac{1}{2a^2-4ab+2b^2}\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2\frac{b}{(a-b)^2(a+b)^2}\left(-\frac{\tan(1/2\,dx+c/2)\,ab}{(\tan(1/2\,dx+c/2))^2\,a-(\tan(1/2\,dx+c/2))^2\,b-a-b}-\frac{2}{\sqrt{(a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(1/2/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)+2*b/(a-b)^2/(a+b)^2*(-tan(1/2*d
*x+1/2*c)*a*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)-(2*a^2+b^
2)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)
))-1/2/(a+b)^2/tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 1.95161, size = 1164, normalized size = 5.73

$$\left[\frac{6\,a^3b^2-6\,ab^4+\left(2\,a^2b^2+b^4+\left(2\,a^3b+ab^3\right)\cos(dx+c)\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-\left(a^2-2\,b^2\right)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}(b\cos(dx+c)+a}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right.}{2\left(\left(a^7-3\,a^5b^2+3\,a^3b^4-ab^6\right)d\cos(dx+c)+\left(a^6b-\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(6*a^3*b^2 - 6*a*b^4 + (2*a^2*b^2 + b^4 + (2*a^3*b + a*b^3)*cos(d*x +
c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2
- 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*c
os(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 2*(a^5 + a^3*b^2
- 2*a*b^4)*cos(d*x + c)^2 + 2*(a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c))/(((a^
7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3*
a^2*b^5 - b^7)*d)*sin(d*x + c)), (3*a^3*b^2 - 3*a*b^4 - (2*a^2*b^2 + b^4 +
(2*a^3*b + a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(
b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) - (a^5 + a^3*b
^2 - 2*a*b^4)*cos(d*x + c)^2 + (a^4*b - 2*a^2*b^3 + b^5)*cos(d*x + c))/(((a
```

```
^7 - 3*a^5*b^2 + 3*a^3*b^4 - a*b^6)*d*cos(d*x + c) + (a^6*b - 3*a^4*b^3 + 3
*a^2*b^5 - b^7)*d)*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

Giac [A] time = 1.3782, size = 390, normalized size = 1.92

$$\frac{\frac{4\left(2a^2b+b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{-a^2+b^2}}+\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^2-2ab+b^2}-\frac{a^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-3a^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+7ab^2\mathrm{ta}}{\left(a^4-2a^2b^2+b^4\right)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-b\tan\left(\frac{1}{2}\right.\right.}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*(2*a^2*b + b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + a
rctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/
((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + tan(1/2*d*x + 1/2*c)/(a^2 - 2*
a*b + b^2) - (a^3*tan(1/2*d*x + 1/2*c)^2 - 3*a^2*b*tan(1/2*d*x + 1/2*c)^2 +
 7*a*b^2*tan(1/2*d*x + 1/2*c)^2 - b^3*tan(1/2*d*x + 1/2*c)^2 - a^3 + a^2*b
+ a*b^2 - b^3)/((a^4 - 2*a^2*b^2 + b^4)*(a*tan(1/2*d*x + 1/2*c)^3 - b*tan(1
/2*d*x + 1/2*c)^3 - a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))))/d
```

3.220 $\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=343

$$\frac{a^3b^2\sin(c+dx)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{4a^2b\left(a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{2a^2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{}{12d(a+}$$

```
[Out] (-2*a^2*b^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(
7/2)*(a + b)^(7/2)*d) - (4*a^2*b*(a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c +
d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - Sin[c + d*x]/(12*(
a + b)^2*d*(1 - Cos[c + d*x])^2) - ((a - b)*Sin[c + d*x])/(4*(a + b)^3*d*(1
 - Cos[c + d*x])) - Sin[c + d*x]/(12*(a + b)^2*d*(1 - Cos[c + d*x])) + Sin[
c + d*x]/(12*(a - b)^2*d*(1 + Cos[c + d*x])^2) + Sin[c + d*x]/(12*(a - b)^2
*d*(1 + Cos[c + d*x])) + ((a + b)*Sin[c + d*x])/(4*(a - b)^3*d*(1 + Cos[c +
 d*x])) + (a^3*b^2*Sin[c + d*x])/((a^2 - b^2)^3*d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 0.547166, antiderivative size = 343, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2897, 2650, 2648, 2664, 12, 2659, 208}

$$\frac{a^3b^2\sin(c+dx)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{4a^2b\left(a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{2a^2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{}{12d(a+}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-2*a^2*b^3*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(
7/2)*(a + b)^(7/2)*d) - (4*a^2*b*(a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c +
d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - Sin[c + d*x]/(12*(
a + b)^2*d*(1 - Cos[c + d*x])^2) - ((a - b)*Sin[c + d*x])/(4*(a + b)^3*d*(1
 - Cos[c + d*x])) - Sin[c + d*x]/(12*(a + b)^2*d*(1 - Cos[c + d*x])) + Sin[
c + d*x]/(12*(a - b)^2*d*(1 + Cos[c + d*x])^2) + Sin[c + d*x]/(12*(a - b)^2
*d*(1 + Cos[c + d*x])) + ((a + b)*Sin[c + d*x])/(4*(a - b)^3*d*(1 + Cos[c +
 d*x])) + (a^3*b^2*Sin[c + d*x])/((a^2 - b^2)^3*d*(b + a*Cos[c + d*x]))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cot^2(c+dx)\csc^2(c+dx)}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \int \left(\frac{1}{4(a-b)^2(-1-\cos(c+dx))^2} + \frac{-a-b}{4(a-b)^3(-1-\cos(c+dx))} + \frac{1}{4(a+b)^2(1-\cos(c+dx))}\right.$$

$$= \frac{\int \frac{1}{(-1-\cos(c+dx))^2}\,dx}{4(a-b)^2} + \frac{(a-b)\int \frac{1}{1-\cos(c+dx)}\,dx}{4(a+b)^3} + \frac{\int \frac{1}{(1-\cos(c+dx))^2}\,dx}{4(a+b)^2} - \frac{(a+b)\int \frac{1}{-1-\cos(c+dx)}\,dx}{4(a-b)^3}$$

$$= -\frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))^2} - \frac{(a-b)\sin(c+dx)}{4(a+b)^3d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{12(a-b)^2d(1+\cos(c+dx)}$$

$$= -\frac{4a^2b\left(a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))^2} - \frac{(a-b)\sin}{4(a+b)^3d(1-}$$

$$= -\frac{4a^2b\left(a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin(c+dx)}{12(a+b)^2d(1-\cos(c+dx))^2} - \frac{(a-b)\sin}{4(a+b)^3d(1-}$$

$$= -\frac{2a^2b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{4a^2b\left(a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin}{12(a+b)^2d(}$$

Mathematica [A] time = 1.08097, size = 281, normalized size = 0.82

$$\frac{\sec^2(c+dx)(a\cos(c+dx)+b)\left(\frac{24a^3b^2\sin(c+dx)}{(a-b)^3(a+b)^3} + \frac{48a^2b\left(2a^2+3b^2\right)(a\cos(c+dx)+b)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}} + \frac{4(2a+b)\tan\left(\frac{1}{2}(c+dx)\right)(a\cos(c}{(a-b)^3}\right.}{24d(a+b\sec(}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*((48*a^2*b*(2*a^2 + 3*b^2)*ArcTanh[((-
a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x]))/(a^2 - b^2)
^(7/2) - (4*(2*a - b)*(b + a*Cos[c + d*x])*Cot[(c + d*x)/2])/(a + b)^3 - ((
b + a*Cos[c + d*x])*Cot[(c + d*x)/2]*Csc[(c + d*x)/2]^2)/(a + b)^2 + (24*a^
3*b^2*Sin[c + d*x])/((a - b)^3*(a + b)^3) + (4*(2*a + b)*(b + a*Cos[c + d*x
])*Tan[(c + d*x)/2])/(a - b)^3 + ((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2])/(a - b)^2))/(24*d*(a + b*Sec[c + d*x])^2)
```

Maple [A] time = 0.092, size = 242, normalized size = 0.7

$$\frac{1}{d}\left(\frac{1}{\left(8a^2-16ab+8b^2\right)(a-b)}\left(\frac{a}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 - \frac{b}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + 3a\tan(1/2\,dx+c/2) + b\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right) + 2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d*(1/8/(a^2-2*a*b+b^2)/(a-b)*(1/3*tan(1/2*d*x+1/2*c)^3*a-1/3*b*tan(1/2*d*
x+1/2*c)^3+3*a*tan(1/2*d*x+1/2*c)+b*tan(1/2*d*x+1/2*c))+2*a^2*b/(a+b)^3/(a-
b)^3*(-tan(1/2*d*x+1/2*c)*a*b/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*
b-a-b)-(2*a^2+3*b^2)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/(
(a+b)*(a-b))^(1/2)))-1/24/(a+b)^2/tan(1/2*d*x+1/2*c)^3-1/8*(3*a-b)/(a+b)^3/
tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.25787, size = 2268, normalized size = 6.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [-1/6*(22*a^5*b^2 - 14*a^3*b^4 - 8*a*b^6 + 2*(2*a^7 + 10*a^5*b^2 - 11*a^3*b
^4 - a*b^6)*cos(d*x + c)^4 - 2*(4*a^6*b - 7*a^4*b^3 + 2*a^2*b^5 + b^7)*cos(
d*x + c)^3 - 3*(2*a^4*b^2 + 3*a^2*b^4 - (2*a^5*b + 3*a^3*b^3)*cos(d*x + c)^
3 - (2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c)^2 + (2*a^5*b + 3*a^3*b^3)*cos(d*x
+ c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^
2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2
*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) - 6*(a^7 + 6*a^5*
b^2 - 5*a^3*b^4 - 2*a*b^6)*cos(d*x + c)^2 + 10*(a^6*b - 2*a^4*b^3 + a^2*b^5
)*cos(d*x + c))/(((a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d
*x + c)^3 + (a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c
)^2 - (a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c) - (a
^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^7 + b^9)*d)*sin(d*x + c)), -1/3*(11*
a^5*b^2 - 7*a^3*b^4 - 4*a*b^6 + (2*a^7 + 10*a^5*b^2 - 11*a^3*b^4 - a*b^6)*c
os(d*x + c)^4 - (4*a^6*b - 7*a^4*b^3 + 2*a^2*b^5 + b^7)*cos(d*x + c)^3 - 3*
(2*a^4*b^2 + 3*a^2*b^4 - (2*a^5*b + 3*a^3*b^3)*cos(d*x + c)^3 - (2*a^4*b^2
+ 3*a^2*b^4)*cos(d*x + c)^2 + (2*a^5*b + 3*a^3*b^3)*cos(d*x + c))*sqrt(-a^2
 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x
+ c)))*sin(d*x + c) - 3*(a^7 + 6*a^5*b^2 - 5*a^3*b^4 - 2*a*b^6)*cos(d*x + c
)^2 + 5*(a^6*b - 2*a^4*b^3 + a^2*b^5)*cos(d*x + c))/(((a^9 - 4*a^7*b^2 + 6*
a^5*b^4 - 4*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3 + (a^8*b - 4*a^6*b^3 + 6*a^4*
b^5 - 4*a^2*b^7 + b^9)*d*cos(d*x + c)^2 - (a^9 - 4*a^7*b^2 + 6*a^5*b^4 - 4*
a^3*b^6 + a*b^8)*d*cos(d*x + c) - (a^8*b - 4*a^6*b^3 + 6*a^4*b^5 - 4*a^2*b^
7 + b^9)*d)*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(csc(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

Giac [A] time = 1.37596, size = 617, normalized size = 1.8

$$-\frac{\frac{48\,a^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)}-\frac{48\left(2\,a^4b+3\,a^2b^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^6-3\,a^4b^2+3\,a^2b^4-b^6\right)\sqrt{-a^2+b^2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] -1/24*(48*a^3*b^2*tan(1/2*d*x + 1/2*c)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)
*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - 48*(2*a^4
*b + 3*a^2*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((
a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^6 -
 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) - (a^4*tan(1/2*d*x + 1/2*c)
^3 - 4*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 6*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*
a*b^3*tan(1/2*d*x + 1/2*c)^3 + b^4*tan(1/2*d*x + 1/2*c)^3 + 9*a^4*tan(1/2*d
*x + 1/2*c) - 24*a^3*b*tan(1/2*d*x + 1/2*c) + 18*a^2*b^2*tan(1/2*d*x + 1/2*
c) - 3*b^4*tan(1/2*d*x + 1/2*c))/(a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 +
 15*a^2*b^4 - 6*a*b^5 + b^6) + (9*a*tan(1/2*d*x + 1/2*c)^2 - 3*b*tan(1/2*d*
x + 1/2*c)^2 + a + b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(1/2*d*x + 1/2*c)
^3))/d
```

3.221 $\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=329

$$-\frac{3\left(a^2-2b^2\right)\cos^5(c+dx)}{5a^5d}+\frac{b\left(9a^2-10b^2\right)\cos^4(c+dx)}{4a^6d}+\frac{\left(-6a^2b^2+a^4+5b^4\right)\cos^3(c+dx)}{a^7d}-\frac{3b\left(-10a^2b^2+3a^4+\right.}{2a^8d}$$

```
[Out] -(((a^6 - 18*a^4*b^2 + 45*a^2*b^4 - 28*b^6)*Cos[c + d*x])/(a^9*d)) - (3*b*(
3*a^4 - 10*a^2*b^2 + 7*b^4)*Cos[c + d*x]^2)/(2*a^8*d) + ((a^4 - 6*a^2*b^2 +
 5*b^4)*Cos[c + d*x]^3)/(a^7*d) + (b*(9*a^2 - 10*b^2)*Cos[c + d*x]^4)/(4*a^
6*d) - (3*(a^2 - 2*b^2)*Cos[c + d*x]^5)/(5*a^5*d) - (b*Cos[c + d*x]^6)/(2*a
^4*d) + Cos[c + d*x]^7/(7*a^3*d) - (b^3*(a^2 - b^2)^3)/(2*a^10*d*(b + a*Cos
[c + d*x])^2) + (3*b^2*(a^2 - 3*b^2)*(a^2 - b^2)^2)/(a^10*d*(b + a*Cos[c +
d*x])) + (3*b*(a^2 - b^2)*(a^4 - 9*a^2*b^2 + 12*b^4)*Log[b + a*Cos[c + d*x]
])/(a^10*d)
```

Rubi [A] time = 0.502169, antiderivative size = 329, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2837, 12, 948}

$$-\frac{3\left(a^2-2b^2\right)\cos^5(c+dx)}{5a^5d}+\frac{b\left(9a^2-10b^2\right)\cos^4(c+dx)}{4a^6d}+\frac{\left(-6a^2b^2+a^4+5b^4\right)\cos^3(c+dx)}{a^7d}-\frac{3b\left(-10a^2b^2+3a^4+\right.}{2a^8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -(((a^6 - 18*a^4*b^2 + 45*a^2*b^4 - 28*b^6)*Cos[c + d*x])/(a^9*d)) - (3*b*(
3*a^4 - 10*a^2*b^2 + 7*b^4)*Cos[c + d*x]^2)/(2*a^8*d) + ((a^4 - 6*a^2*b^2 +
 5*b^4)*Cos[c + d*x]^3)/(a^7*d) + (b*(9*a^2 - 10*b^2)*Cos[c + d*x]^4)/(4*a^
6*d) - (3*(a^2 - 2*b^2)*Cos[c + d*x]^5)/(5*a^5*d) - (b*Cos[c + d*x]^6)/(2*a
^4*d) + Cos[c + d*x]^7/(7*a^3*d) - (b^3*(a^2 - b^2)^3)/(2*a^10*d*(b + a*Cos
[c + d*x])^2) + (3*b^2*(a^2 - 3*b^2)*(a^2 - b^2)^2)/(a^10*d*(b + a*Cos[c +
d*x])) + (3*b*(a^2 - b^2)*(a^4 - 9*a^2*b^2 + 12*b^4)*Log[b + a*Cos[c + d*x]
])/(a^10*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^7(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{x^3\left(a^2-x^2\right)^3}{a^3(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^7 d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3\left(a^2-x^2\right)^3}{(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^{10} d}$$

$$= \frac{\text{Subst}\left(\int \left(a^6\left(1+\frac{-18a^4b^2+45a^2b^4-28b^6}{a^6}\right)+\frac{b^3\left(-a^2+b^2\right)^3}{(b-x)^3}+\frac{3b^2\left(a^2-3b^2\right)\left(a^2-b^2\right)^2}{(b-x)^2}+\frac{3b\left(-a^6+10a^4b^2-21a^2b^4+\right.}{b-x}\right.\right.}{}$$

$$= -\frac{\left(a^6-18a^4b^2+45a^2b^4-28b^6\right)\cos(c+dx)}{a^9 d} - \frac{3b\left(3a^4-10a^2b^2+7b^4\right)\cos^2(c+dx)}{2a^8 d} + \frac{\left(a^4-\right.}{}$$

Mathematica [A] time = 4.69105, size = 550, normalized size = 1.67

$$\frac{17528a^7b^2\cos(3(c+dx)) - 840a^7b^2\cos(5(c+dx)) + 48a^7b^2\cos(7(c+dx)) + 4872a^6b^3\cos(4(c+dx)) - 168a^6b^3\cos(6(c}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-7945*a^8*b + 164080*a^6*b^3 - 502320*a^4*b^5 + 425600*a^2*b^7 - 76160*b^9
 - 784*a^9*Cos[3*(c + d*x)] + 17528*a^7*b^2*Cos[3*(c + d*x)] - 43680*a^5*b^
4*Cos[3*(c + d*x)] + 26880*a^3*b^6*Cos[3*(c + d*x)] - 1456*a^8*b*Cos[4*(c +
 d*x)] + 4872*a^6*b^3*Cos[4*(c + d*x)] - 3360*a^4*b^5*Cos[4*(c + d*x)] + 15
2*a^9*Cos[5*(c + d*x)] - 840*a^7*b^2*Cos[5*(c + d*x)] + 672*a^5*b^4*Cos[5*(
c + d*x)] + 174*a^8*b*Cos[6*(c + d*x)] - 168*a^6*b^3*Cos[6*(c + d*x)] - 39*
a^9*Cos[7*(c + d*x)] + 48*a^7*b^2*Cos[7*(c + d*x)] - 15*a^8*b*Cos[8*(c + d*
x)] + 5*a^9*Cos[9*(c + d*x)] + 13440*a^8*b*Log[b + a*Cos[c + d*x]] - 107520
*a^6*b^3*Log[b + a*Cos[c + d*x]] + 13440*a^4*b^5*Log[b + a*Cos[c + d*x]] +
403200*a^2*b^7*Log[b + a*Cos[c + d*x]] - 322560*b^9*Log[b + a*Cos[c + d*x]]
 + 70*a^2*b*Cos[2*(c + d*x)]*(-137*a^6 + 1896*a^4*b^2 - 4656*a^2*b^4 + 2912
*b^6 + 192*(a^6 - 10*a^4*b^2 + 21*a^2*b^4 - 12*b^6)*Log[b + a*Cos[c + d*x]]
) - 70*a*Cos[c + d*x]*(49*a^8 - 1472*a^6*b^2 + 3216*a^4*b^4 + 576*a^2*b^6 -
 2432*b^8 - 768*b^2*(a^6 - 10*a^4*b^2 + 21*a^2*b^4 - 12*b^6)*Log[b + a*Cos[
c + d*x]]))/(8960*a^10*d*(b + a*Cos[c + d*x])^2)
```

Maple [A] time = 0.072, size = 549, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -5/2/d/a^6*cos(d*x+c)^4*b^3-15/d/a^6*b^4/(b+a*cos(d*x+c))-6/d/a^5*cos(d*x+c
)^3*b^2+6/5/d/a^5*cos(d*x+c)^5*b^2+18/d/a^5*cos(d*x+c)*b^2-9/d/a^10*b^8/(b+
a*cos(d*x+c))-45/d/a^7*b^4*cos(d*x+c)+5/d/a^7*cos(d*x+c)^3*b^4+15/d/a^6*cos
(d*x+c)^2*b^3-21/2/d/a^8*cos(d*x+c)^2*b^5-30/d/a^6*b^3*ln(b+a*cos(d*x+c))+6
3/d/a^8*b^5*ln(b+a*cos(d*x+c))-36/d/a^10*b^7*ln(b+a*cos(d*x+c))+28/d/a^9*b^
6*cos(d*x+c)+3/2/d*b^5/a^6/(b+a*cos(d*x+c))^2-3/2/d*b^7/a^8/(b+a*cos(d*x+c)
)^2+1/2/d*b^9/a^10/(b+a*cos(d*x+c))^2+21/d/a^8*b^6/(b+a*cos(d*x+c))+3*b*ln(
b+a*cos(d*x+c))/a^4/d-1/2*b*cos(d*x+c)^6/a^4/d+9/4*b*cos(d*x+c)^4/a^4/d-9/2
*b*cos(d*x+c)^2/a^4/d-1/2*b^3/a^4/d/(b+a*cos(d*x+c))^2+3*b^2/a^4/d/(b+a*cos
(d*x+c))-3/5*cos(d*x+c)^5/a^3/d+1/7*cos(d*x+c)^7/a^3/d-cos(d*x+c)/a^3/d+cos
(d*x+c)^3/a^3/d
```

Maxima [A] time = 0.998666, size = 440, normalized size = 1.34

$$\frac{70\left(5a^6b^3-27a^4b^5+39a^2b^7-17b^9+6\left(a^7b^2-5a^5b^4+7a^3b^6-3ab^8\right)\cos(dx+c)\right)}{a^{12}\cos(dx+c)^2+2a^{11}b\cos(dx+c)+a^{10}b^2}+\frac{20a^6\cos(dx+c)^7-70a^5b\cos(dx+c)^6-84\left(a^6-2a^4b^2\right)\cos(dx+c)^5+35\left(9a^5b\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/140*(70*(5*a^6*b^3 - 27*a^4*b^5 + 39*a^2*b^7 - 17*b^9 + 6*(a^7*b^2 - 5*a^
5*b^4 + 7*a^3*b^6 - 3*a*b^8)*cos(d*x + c))/(a^12*cos(d*x + c)^2 + 2*a^11*b*
cos(d*x + c) + a^10*b^2) + (20*a^6*cos(d*x + c)^7 - 70*a^5*b*cos(d*x + c)^6
 - 84*(a^6 - 2*a^4*b^2)*cos(d*x + c)^5 + 35*(9*a^5*b - 10*a^3*b^3)*cos(d*x
+ c)^4 + 140*(a^6 - 6*a^4*b^2 + 5*a^2*b^4)*cos(d*x + c)^3 - 210*(3*a^5*b -
10*a^3*b^3 + 7*a*b^5)*cos(d*x + c)^2 - 140*(a^6 - 18*a^4*b^2 + 45*a^2*b^4 -
 28*b^6)*cos(d*x + c))/a^9 + 420*(a^6*b - 10*a^4*b^3 + 21*a^2*b^5 - 12*b^7)
*log(a*cos(d*x + c) + b)/a^10)/d
```

Fricas [A] time = 3.07578, size = 1062, normalized size = 3.23

$$\frac{80a^9\cos(dx+c)^9-120a^8b\cos(dx+c)^8+2275a^6b^3-11235a^4b^5+13860a^2b^7-4760b^9-48\left(7a^9-4a^7b^2\right)\cos(\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/560*(80*a^9*cos(d*x + c)^9 - 120*a^8*b*cos(d*x + c)^8 + 2275*a^6*b^3 - 11
235*a^4*b^5 + 13860*a^2*b^7 - 4760*b^9 - 48*(7*a^9 - 4*a^7*b^2)*cos(d*x + c
)^7 + 84*(7*a^8*b - 4*a^6*b^3)*cos(d*x + c)^6 + 56*(10*a^9 - 21*a^7*b^2 + 1
2*a^5*b^4)*cos(d*x + c)^5 - 140*(10*a^8*b - 21*a^6*b^3 + 12*a^4*b^5)*cos(d*
x + c)^4 - 560*(a^9 - 10*a^7*b^2 + 21*a^5*b^4 - 12*a^3*b^6)*cos(d*x + c)^3
- 35*(7*a^8*b - 399*a^6*b^3 + 1116*a^4*b^5 - 728*a^2*b^7)*cos(d*x + c)^2 +
70*(41*a^7*b^2 - 81*a^5*b^4 - 108*a^3*b^6 + 152*a*b^8)*cos(d*x + c) + 1680*
(a^6*b^3 - 10*a^4*b^5 + 21*a^2*b^7 - 12*b^9 + (a^8*b - 10*a^6*b^3 + 21*a^4*
b^5 - 12*a^2*b^7)*cos(d*x + c)^2 + 2*(a^7*b^2 - 10*a^5*b^4 + 21*a^3*b^6 - 1
2*a*b^8)*cos(d*x + c))*log(a*cos(d*x + c) + b))/(a^12*d*cos(d*x + c)^2 + 2*
```

```
a^11*b*d*cos(d*x + c) + a^10*b^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.57516, size = 2903, normalized size = 8.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/140*(420*(a^7*b - a^6*b^2 - 10*a^5*b^3 + 10*a^4*b^4 + 21*a^3*b^5 - 21*a^2
*b^6 - 12*a*b^7 + 12*b^8)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^11 - a^10*b) - 420*(a
^6*b - 10*a^4*b^3 + 21*a^2*b^5 - 12*b^7)*log(abs(-(cos(d*x + c) - 1)/(cos(d
*x + c) + 1) + 1))/a^10 - 70*(9*a^8*b + 6*a^7*b^2 - 105*a^6*b^3 - 148*a^5*b
^4 + 187*a^4*b^5 + 390*a^3*b^6 + 17*a^2*b^7 - 248*a*b^8 - 108*b^9 + 18*a^8*
b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*a^7*b^2*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) - 202*a^6*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 56*a^
5*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 566*a^4*b^5*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) - 76*a^3*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 5
98*a^2*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 32*a*b^8*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) + 216*b^9*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*
a^8*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 18*a^7*b^2*(cos(d*x + c)
- 1)^2/(cos(d*x + c) + 1)^2 - 81*a^6*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 + 180*a^5*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 99*a^4*b^
5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 378*a^3*b^6*(cos(d*x + c) - 1
)^2/(cos(d*x + c) + 1)^2 + 81*a^2*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
1)^2 + 216*a*b^8*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 108*b^9*(cos(d
*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((a + b + a*(cos(d*x + c) - 1)/(cos(d*
x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2*a^10) + (128*a^7 -
 1089*a^6*b - 3696*a^5*b^2 + 10890*a^4*b^3 + 11200*a^3*b^4 - 22869*a^2*b^5
- 7840*a*b^6 + 13068*b^7 - 896*a^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
8463*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 24192*a^5*b^2*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) - 81830*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) - 70000*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 165963*a^2*b
^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 47040*a*b^6*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1) - 91476*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2688*
a^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 28749*a^6*b*(cos(d*x + c) -
 1)^2/(cos(d*x + c) + 1)^2 - 64176*a^5*b^2*(cos(d*x + c) - 1)^2/(cos(d*x +
c) + 1)^2 + 262290*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 1764
00*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 509649*a^2*b^5*(cos(
d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 117600*a*b^6*(cos(d*x + c) - 1)^2/(c
os(d*x + c) + 1)^2 + 274428*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 -
 4480*a^7*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 56035*a^6*b*(cos(d*x
```

```
+ c) - 1)^3/(cos(d*x + c) + 1)^3 + 80640*a^5*b^2*(cos(d*x + c) - 1)^3/(cos(
d*x + c) + 1)^3 - 453950*a^4*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3
- 229600*a^3*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 859215*a^2*b^5
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 156800*a*b^6*(cos(d*x + c) - 1
)^3/(cos(d*x + c) + 1)^3 - 457380*b^7*(cos(d*x + c) - 1)^3/(cos(d*x + c) +
1)^3 - 56035*a^6*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 48720*a^5*b^
2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 453950*a^4*b^3*(cos(d*x + c)
- 1)^4/(cos(d*x + c) + 1)^4 + 162400*a^3*b^4*(cos(d*x + c) - 1)^4/(cos(d*x
+ c) + 1)^4 - 859215*a^2*b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 11
7600*a*b^6*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 457380*b^7*(cos(d*x
+ c) - 1)^4/(cos(d*x + c) + 1)^4 + 28749*a^6*b*(cos(d*x + c) - 1)^5/(cos(d*
x + c) + 1)^5 + 13440*a^5*b^2*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 2
62290*a^4*b^3*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 58800*a^3*b^4*(co
s(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 509649*a^2*b^5*(cos(d*x + c) - 1)^
5/(cos(d*x + c) + 1)^5 + 47040*a*b^6*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1
)^5 - 274428*b^7*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 8463*a^6*b*(co
s(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 1680*a^5*b^2*(cos(d*x + c) - 1)^6/
(cos(d*x + c) + 1)^6 + 81830*a^4*b^3*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1
)^6 + 8400*a^3*b^4*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 165963*a^2*b
^5*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6 - 7840*a*b^6*(cos(d*x + c) - 1
)^6/(cos(d*x + c) + 1)^6 + 91476*b^7*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1
)^6 + 1089*a^6*b*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 - 10890*a^4*b^3*
(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1)^7 + 22869*a^2*b^5*(cos(d*x + c) - 1
)^7/(cos(d*x + c) + 1)^7 - 13068*b^7*(cos(d*x + c) - 1)^7/(cos(d*x + c) + 1
)^7)/(a^10*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^7))/d
```

3.222 $\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=239

$$\frac{2\left(a^2-3b^2\right)\cos^3(c+dx)}{3a^5d}-\frac{b\left(3a^2-5b^2\right)\cos^2(c+dx)}{a^6d}-\frac{\left(-12a^2b^2+a^4+15b^4\right)\cos(c+dx)}{a^7d}+\frac{b^2\left(-10a^2b^2+3a^4+7b^4\right)}{a^8d(a\cos(c+dx)+b)}$$

```
[Out] -(((a^4 - 12*a^2*b^2 + 15*b^4)*Cos[c + d*x])/(a^7*d)) - (b*(3*a^2 - 5*b^2)*
Cos[c + d*x]^2)/(a^6*d) + (2*(a^2 - 3*b^2)*Cos[c + d*x]^3)/(3*a^5*d) + (3*b
*Cos[c + d*x]^4)/(4*a^4*d) - Cos[c + d*x]^5/(5*a^3*d) - (b^3*(a^2 - b^2)^2)
/(2*a^8*d*(b + a*Cos[c + d*x])^2) + (b^2*(3*a^4 - 10*a^2*b^2 + 7*b^4))/(a^8
*d*(b + a*Cos[c + d*x])) + (b*(3*a^4 - 20*a^2*b^2 + 21*b^4)*Log[b + a*Cos[c
 + d*x]])/(a^8*d)
```

Rubi [A] time = 0.364761, antiderivative size = 239, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2837, 12, 948}

$$\frac{2\left(a^2-3b^2\right)\cos^3(c+dx)}{3a^5d}-\frac{b\left(3a^2-5b^2\right)\cos^2(c+dx)}{a^6d}-\frac{\left(-12a^2b^2+a^4+15b^4\right)\cos(c+dx)}{a^7d}+\frac{b^2\left(-10a^2b^2+3a^4+7b^4\right)}{a^8d(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -(((a^4 - 12*a^2*b^2 + 15*b^4)*Cos[c + d*x])/(a^7*d)) - (b*(3*a^2 - 5*b^2)*
Cos[c + d*x]^2)/(a^6*d) + (2*(a^2 - 3*b^2)*Cos[c + d*x]^3)/(3*a^5*d) + (3*b
*Cos[c + d*x]^4)/(4*a^4*d) - Cos[c + d*x]^5/(5*a^3*d) - (b^3*(a^2 - b^2)^2)
/(2*a^8*d*(b + a*Cos[c + d*x])^2) + (b^2*(3*a^4 - 10*a^2*b^2 + 7*b^4))/(a^8
*d*(b + a*Cos[c + d*x])) + (b*(3*a^4 - 20*a^2*b^2 + 21*b^4)*Log[b + a*Cos[c
 + d*x]])/(a^8*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 948

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
```

```
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IGtQ[p, 0] && (IGtQ[m, 0] || (EqQ[m, -2] && EqQ[p, 1] &
& EqQ[d, 0]))
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^5(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{x^3(a^2-x^2)^2}{a^3(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^5 d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3(a^2-x^2)^2}{(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^8 d}$$

$$= \frac{\text{Subst}\left(\int \left(a^4\left(1+\frac{3b^2(-4a^2+5b^2)}{a^4}\right) - \frac{b^3(-a^2+b^2)^2}{(b-x)^3} + \frac{3a^4b^2-10a^2b^4+7b^6}{(b-x)^2} + \frac{-3a^4b+20a^2b^3-21b^5}{b-x} + 2b\left(-\right.\right.\right.}{a^8 d}$$

$$= -\frac{(a^4-12a^2b^2+15b^4)\cos(c+dx)}{a^7 d} - \frac{b(3a^2-5b^2)\cos^2(c+dx)}{a^6 d} + \frac{2(a^2-3b^2)\cos^3(c+}{3a^5 d}$$

Mathematica [A] time = 2.91796, size = 388, normalized size = 1.62

$$\frac{2780a^5b^2\cos(3(c+dx)) - 84a^5b^2\cos(5(c+dx)) + 420a^4b^3\cos(4(c+dx)) - 3360a^3b^4\cos(3(c+dx)) - 13440a^4b^3\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-1740*a^6*b + 26160*a^4*b^3 - 46080*a^2*b^5 + 12480*b^7 - 206*a^7*Cos[3*(c
 + d*x)] + 2780*a^5*b^2*Cos[3*(c + d*x)] - 3360*a^3*b^4*Cos[3*(c + d*x)] -
274*a^6*b*Cos[4*(c + d*x)] + 420*a^4*b^3*Cos[4*(c + d*x)] + 38*a^7*Cos[5*(c
 + d*x)] - 84*a^5*b^2*Cos[5*(c + d*x)] + 21*a^6*b*Cos[6*(c + d*x)] - 6*a^7*
Cos[7*(c + d*x)] + 2880*a^6*b*Log[b + a*Cos[c + d*x]] - 13440*a^4*b^3*Log[b
 + a*Cos[c + d*x]] - 18240*a^2*b^5*Log[b + a*Cos[c + d*x]] + 40320*b^7*Log[
b + a*Cos[c + d*x]] + 5*a^2*b*Cos[2*(c + d*x)]*(-407*a^4 + 3888*a^2*b^2 - 4
800*b^4 + 192*(3*a^4 - 20*a^2*b^2 + 21*b^4)*Log[b + a*Cos[c + d*x]]) - 10*a
*Cos[c + d*x]*(85*a^6 - 1728*a^4*b^2 + 1584*a^2*b^4 + 1536*b^6 - 384*b^2*(3
*a^4 - 20*a^2*b^2 + 21*b^4)*Log[b + a*Cos[c + d*x]]))/(1920*a^8*d*(b + a*Co
s[c + d*x])^2)
```

Maple [A] time = 0.071, size = 355, normalized size = 1.5

$$-\frac{(\cos(dx+c))^5}{5a^3d} + \frac{3b(\cos(dx+c))^4}{4a^4d} + \frac{2(\cos(dx+c))^3}{3a^3d} - 2\frac{(\cos(dx+c))^3b^2}{da^5} - 3\frac{b(\cos(dx+c))^2}{a^4d} + 5\frac{(\cos(dx+c)}{da^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -1/5*cos(d*x+c)^5/a^3/d+3/4*b*cos(d*x+c)^4/a^4/d+2/3*cos(d*x+c)^3/a^3/d-2/d
/a^5*cos(d*x+c)^3*b^2-3*b*cos(d*x+c)^2/a^4/d+5/d/a^6*cos(d*x+c)^2*b^3-cos(d
```

```
*x+c)/a^3/d+12/d/a^5*cos(d*x+c)*b^2-15/d/a^7*b^4*cos(d*x+c)+3*b*ln(b+a*cos(
d*x+c))/a^4/d-20/d/a^6*b^3*ln(b+a*cos(d*x+c))+21/d/a^8*b^5*ln(b+a*cos(d*x+c
))-1/2*b^3/a^4/d/(b+a*cos(d*x+c))^2+1/d*b^5/a^6/(b+a*cos(d*x+c))^2-1/2/d*b^
7/a^8/(b+a*cos(d*x+c))^2+3*b^2/a^4/d/(b+a*cos(d*x+c))-10/d/a^6*b^4/(b+a*cos
(d*x+c))+7/d/a^8*b^6/(b+a*cos(d*x+c))
```

Maxima [A] time = 0.973615, size = 316, normalized size = 1.32

$$\frac{\frac{30\left(5a^4b^3-18a^2b^5+13b^7+2\left(3a^5b^2-10a^3b^4+7ab^6\right)\cos(dx+c)\right)}{a^{10}\cos(dx+c)^2+2a^9b\cos(dx+c)+a^8b^2}-\frac{12a^4\cos(dx+c)^5-45a^3b\cos(dx+c)^4-40\left(a^4-3a^2b^2\right)\cos(dx+c)^3+60\left(3a^3b-5ab^3\right)\cos(dx+c)}{a^7}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/60*(30*(5*a^4*b^3 - 18*a^2*b^5 + 13*b^7 + 2*(3*a^5*b^2 - 10*a^3*b^4 + 7*a
*b^6)*cos(d*x + c))/(a^10*cos(d*x + c)^2 + 2*a^9*b*cos(d*x + c) + a^8*b^2)
- (12*a^4*cos(d*x + c)^5 - 45*a^3*b*cos(d*x + c)^4 - 40*(a^4 - 3*a^2*b^2)*c
os(d*x + c)^3 + 60*(3*a^3*b - 5*a*b^3)*cos(d*x + c)^2 + 60*(a^4 - 12*a^2*b^
2 + 15*b^4)*cos(d*x + c))/a^7 + 60*(3*a^4*b - 20*a^2*b^3 + 21*b^5)*log(a*co
s(d*x + c) + b)/a^8)/d
```

Fricas [A] time = 2.39359, size = 792, normalized size = 3.31

$$-\frac{96a^7\cos(dx+c)^7-168a^6b\cos(dx+c)^6-1785a^4b^3+5520a^2b^5-3120b^7-16\left(20a^7-21a^5b^2\right)\cos(dx+c)^5+40\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/480*(96*a^7*cos(d*x + c)^7 - 168*a^6*b*cos(d*x + c)^6 - 1785*a^4*b^3 + 5
520*a^2*b^5 - 3120*b^7 - 16*(20*a^7 - 21*a^5*b^2)*cos(d*x + c)^5 + 40*(20*a
^6*b - 21*a^4*b^3)*cos(d*x + c)^4 + 160*(3*a^7 - 20*a^5*b^2 + 21*a^3*b^4)*c
os(d*x + c)^3 + 15*(25*a^6*b - 592*a^4*b^3 + 800*a^2*b^5)*cos(d*x + c)^2 -
30*(71*a^5*b^2 - 48*a^3*b^4 - 128*a*b^6)*cos(d*x + c) - 480*(3*a^4*b^3 - 20
*a^2*b^5 + 21*b^7 + (3*a^6*b - 20*a^4*b^3 + 21*a^2*b^5)*cos(d*x + c)^2 + 2*
(3*a^5*b^2 - 20*a^3*b^4 + 21*a*b^6)*cos(d*x + c))*log(a*cos(d*x + c) + b))/
(a^10*d*cos(d*x + c)^2 + 2*a^9*b*d*cos(d*x + c) + a^8*b^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.39382, size = 1805, normalized size = 7.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(60*(3*a^5*b - 3*a^4*b^2 - 20*a^3*b^3 + 20*a^2*b^4 + 21*a*b^5 - 21*b^6
)*log(abs(a + b + a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1)))/(a^9 - a^8*b) - 60*(3*a^4*b - 20*a^2*b^3 + 21*b^
5)*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1))/a^8 - 30*(9*a^6*b +
 6*a^5*b^2 - 75*a^4*b^3 - 108*a^3*b^4 + 51*a^2*b^5 + 150*a*b^6 + 63*b^7 + 1
8*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*a^5*b^2*(cos(d*x + c) -
1)/(cos(d*x + c) + 1) - 142*a^4*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) +
 36*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 250*a^2*b^5*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 24*a*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 - 126*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 9*a^6*b*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2 - 18*a^5*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) +
 1)^2 - 51*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 120*a^3*b^4*
(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 3*a^2*b^5*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2 - 126*a*b^6*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
+ 63*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/((a + b + a*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2*a^8
) + (64*a^5 - 411*a^4*b - 1200*a^3*b^2 + 2740*a^2*b^3 + 1800*a*b^4 - 2877*b
^5 - 320*a^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2415*a^4*b*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + 5280*a^3*b^2*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) - 14900*a^2*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 7200*a*b^4*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 14385*b^5*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) + 640*a^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 5910*a^4*b*
(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 7680*a^3*b^2*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 + 31000*a^2*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 + 10800*a*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 28770*b^5*
(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 5910*a^4*b*(cos(d*x + c) - 1)^3
/(cos(d*x + c) + 1)^3 + 4320*a^3*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1
)^3 - 31000*a^2*b^3*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 7200*a*b^4*
(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 28770*b^5*(cos(d*x + c) - 1)^3/
(cos(d*x + c) + 1)^3 - 2415*a^4*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4
 - 720*a^3*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 14900*a^2*b^3*(c
os(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 1800*a*b^4*(cos(d*x + c) - 1)^4/(
cos(d*x + c) + 1)^4 - 14385*b^5*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 +
 411*a^4*b*(cos(d*x + c) - 1)^5/(cos(d*x + c) + 1)^5 - 2740*a^2*b^3*(cos(d*
x + c) - 1)^5/(cos(d*x + c) + 1)^5 + 2877*b^5*(cos(d*x + c) - 1)^5/(cos(d*x
 + c) + 1)^5)/(a^8*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5))/d
```

3.223 $\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=158

$$-\frac{b^3\left(a^2-b^2\right)}{2a^6d(a\cos(c+dx)+b)^2}+\frac{b^2\left(3a^2-5b^2\right)}{a^6d(a\cos(c+dx)+b)}-\frac{\left(a^2-6b^2\right)\cos(c+dx)}{a^5d}+\frac{b\left(3a^2-10b^2\right)\log(a\cos(c+dx)+b)}{a^6d}-3$$

```
[Out] -(((a^2 - 6*b^2)*Cos[c + d*x])/(a^5*d)) - (3*b*Cos[c + d*x]^2)/(2*a^4*d) +
Cos[c + d*x]^3/(3*a^3*d) - (b^3*(a^2 - b^2))/(2*a^6*d*(b + a*Cos[c + d*x])^
2) + (b^2*(3*a^2 - 5*b^2))/(a^6*d*(b + a*Cos[c + d*x])) + (b*(3*a^2 - 10*b^
2)*Log[b + a*Cos[c + d*x]])/(a^6*d)
```

Rubi [A] time = 0.271682, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2837, 12, 894}

$$-\frac{b^3\left(a^2-b^2\right)}{2a^6d(a\cos(c+dx)+b)^2}+\frac{b^2\left(3a^2-5b^2\right)}{a^6d(a\cos(c+dx)+b)}-\frac{\left(a^2-6b^2\right)\cos(c+dx)}{a^5d}+\frac{b\left(3a^2-10b^2\right)\log(a\cos(c+dx)+b)}{a^6d}-3$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -(((a^2 - 6*b^2)*Cos[c + d*x])/(a^5*d)) - (3*b*Cos[c + d*x]^2)/(2*a^4*d) +
Cos[c + d*x]^3/(3*a^3*d) - (b^3*(a^2 - b^2))/(2*a^6*d*(b + a*Cos[c + d*x])^
2) + (b^2*(3*a^2 - 5*b^2))/(a^6*d*(b + a*Cos[c + d*x])) + (b*(3*a^2 - 10*b^
2)*Log[b + a*Cos[c + d*x]])/(a^6*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 894

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^3(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^3(a^2-x^2)}{a^3(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^3d}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3(a^2-x^2)}{(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^6d}$$
$$= \frac{\text{Subst}\left(\int \left(a^2\left(1-\frac{6b^2}{a^2}\right)+\frac{-a^2b^3+b^5}{(b-x)^3}+\frac{3a^2b^2-5b^4}{(b-x)^2}+\frac{-3a^2b+10b^3}{b-x}-3bx-x^2\right)dx, x, -a\cos(c+d\right.}{a^6d}$$
$$= -\frac{(a^2-6b^2)\cos(c+dx)}{a^5d}-\frac{3b\cos^2(c+dx)}{2a^4d}+\frac{\cos^3(c+dx)}{3a^3d}-\frac{b^3(a^2-b^2)}{2a^6d(b+a\cos(c+dx))^2}+$$

Mathematica [A] time = 0.921472, size = 208, normalized size = 1.32

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b)\left(9a^4(2a\cos(c+dx)+b)-(a\cos(c+dx)+b)^2\left(72a\left(a^2-8b^2\right)\cos(c+dx)+\frac{6(-48a^2b^2+}{a\cos(c}\right.\right.}{96a^6d(a+b\,\mathrm{se}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*(9*a^4*(b + 2*a*Cos[c + d*x]) - (b + a*Cos[c + d*x])^
2*(72*a*(a^2 - 8*b^2)*Cos[c + d*x] + (-9*a^4*b + 48*a^2*b^3 - 48*b^5)/(b +
a*Cos[c + d*x])^2 + (6*(3*a^4 - 48*a^2*b^2 + 80*b^4))/(b + a*Cos[c + d*x])
+ 72*a^2*b*Cos[2*(c + d*x)] - 8*a^3*Cos[3*(c + d*x)] + 96*(-3*a^2*b + 10*b^
3)*Log[b + a*Cos[c + d*x]]))*Sec[c + d*x]^3)/(96*a^6*d*(a + b*Sec[c + d*x])
^3)
```

Maple [A] time = 0.062, size = 200, normalized size = 1.3

$$\frac{(\cos(dx+c))^3}{3a^3d}-\frac{3b(\cos(dx+c))^2}{2a^4d}-\frac{\cos(dx+c)}{a^3d}+6\frac{\cos(dx+c)b^2}{da^5}+3\frac{b\ln(b+a\cos(dx+c))}{a^4d}-10\frac{b^3\ln(b+a\,\mathrm{cc}}{da^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/3*cos(d*x+c)^3/a^3/d-3/2*b*cos(d*x+c)^2/a^4/d-cos(d*x+c)/a^3/d+6/d/a^5*co
s(d*x+c)*b^2+3*b*ln(b+a*cos(d*x+c))/a^4/d-10/d/a^6*b^3*ln(b+a*cos(d*x+c))-1
/2*b^3/a^4/d/(b+a*cos(d*x+c))^2+1/2/d*b^5/a^6/(b+a*cos(d*x+c))^2+3*b^2/a^4/
d/(b+a*cos(d*x+c))-5/d/a^6*b^4/(b+a*cos(d*x+c))
```

Maxima [A] time = 0.968214, size = 208, normalized size = 1.32

$$\frac{\frac{3\left(5a^2b^3-9b^5+2\left(3a^3b^2-5ab^4\right)\cos(dx+c)\right)}{a^8\cos(dx+c)^2+2a^7b\cos(dx+c)+a^6b^2}+\frac{2a^2\cos(dx+c)^3-9ab\cos(dx+c)^2-6\left(a^2-6b^2\right)\cos(dx+c)}{a^5}+\frac{6\left(3a^2b-10b^3\right)\log(a\cos(dx+c)+b)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(5*a^2*b^3 - 9*b^5 + 2*(3*a^3*b^2 - 5*a*b^4)*cos(d*x + c))/(a^8*cos(
d*x + c)^2 + 2*a^7*b*cos(d*x + c) + a^6*b^2) + (2*a^2*cos(d*x + c)^3 - 9*a*
b*cos(d*x + c)^2 - 6*(a^2 - 6*b^2)*cos(d*x + c))/a^5 + 6*(3*a^2*b - 10*b^3)
*log(a*cos(d*x + c) + b)/a^6)/d
```

Fricas [A] time = 2.15444, size = 524, normalized size = 3.32

$$\frac{4\,a^5\cos\left(dx+c\right)^5 - 10\,a^4b\cos\left(dx+c\right)^4 + 39\,a^2b^3 - 54\,b^5 - 4\left(3\,a^5 - 10\,a^3b^2\right)\cos\left(dx+c\right)^3 - 3\left(5\,a^4b - 42\,a^2b^3\right)\cos\left(dx+\right.}{12\left(a^8d\cos\left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/12*(4*a^5*cos(d*x + c)^5 - 10*a^4*b*cos(d*x + c)^4 + 39*a^2*b^3 - 54*b^5
- 4*(3*a^5 - 10*a^3*b^2)*cos(d*x + c)^3 - 3*(5*a^4*b - 42*a^2*b^3)*cos(d*x
+ c)^2 + 6*(7*a^3*b^2 + 2*a*b^4)*cos(d*x + c) + 12*(3*a^2*b^3 - 10*b^5 + (3
*a^4*b - 10*a^2*b^3)*cos(d*x + c)^2 + 2*(3*a^3*b^2 - 10*a*b^4)*cos(d*x + c)
)*log(a*cos(d*x + c) + b))/(a^8*d*cos(d*x + c)^2 + 2*a^7*b*d*cos(d*x + c) +
 a^6*b^2*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.44863, size = 230, normalized size = 1.46

$$\frac{\left(3\,a^2b - 10\,b^3\right)\log\left(\left|-a\cos\left(dx+c\right) - b\right|\right)}{a^6d} + \frac{5\,a^2b^3 - 9\,b^5 + \frac{2\left(3\,a^3b^2d - 5\,ab^4d\right)\cos(dx+c)}{d}}{2\left(a\cos\left(dx+c\right)+b\right)^2a^6d} + \frac{2\,a^6d^8\cos\left(dx+c\right)^3 - 9\,a^5bd^8\cos\left(d\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] (3*a^2*b - 10*b^3)*log(abs(-a*cos(d*x + c) - b))/(a^6*d) + 1/2*(5*a^2*b^3 -
 9*b^5 + 2*(3*a^3*b^2*d - 5*a*b^4*d)*cos(d*x + c)/d)/((a*cos(d*x + c) + b)^
2*a^6*d) + 1/6*(2*a^6*d^8*cos(d*x + c)^3 - 9*a^5*b*d^8*cos(d*x + c)^2 - 6*a
^6*d^8*cos(d*x + c) + 36*a^4*b^2*d^8*cos(d*x + c))/(a^9*d^9)
```

3.224 $\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^3} \, dx$

Optimal. Leaf size=83

$$-\frac{b^3}{2a^4d(a\cos(c+dx)+b)^2}+\frac{3b^2}{a^4d(a\cos(c+dx)+b)}+\frac{3b\log(a\cos(c+dx)+b)}{a^4d}-\frac{\cos(c+dx)}{a^3d}$$

```
[Out] -(Cos[c + d*x]/(a^3*d)) - b^3/(2*a^4*d*(b + a*Cos[c + d*x])^2) + (3*b^2)/(a
^4*d*(b + a*Cos[c + d*x])) + (3*b*Log[b + a*Cos[c + d*x]])/(a^4*d)
```

Rubi [A] time = 0.133559, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3872, 2833, 12, 43}

$$-\frac{b^3}{2a^4d(a\cos(c+dx)+b)^2}+\frac{3b^2}{a^4d(a\cos(c+dx)+b)}+\frac{3b\log(a\cos(c+dx)+b)}{a^4d}-\frac{\cos(c+dx)}{a^3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -(Cos[c + d*x]/(a^3*d)) - b^3/(2*a^4*d*(b + a*Cos[c + d*x])^2) + (3*b^2)/(a
^4*d*(b + a*Cos[c + d*x])) + (3*b*Log[b + a*Cos[c + d*x]])/(a^4*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2833

```
Int[cos[(e_.) + (f_.)*(x_)]*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((
c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b*f), Sub
st[Int[(a + x)^m*(c + (d*x)/b)^n, x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b
, c, d, e, f, m, n}, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\sin(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{a^3(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{ad}$$
$$= \frac{\text{Subst}\left(\int \frac{x^3}{(-b+x)^3}\,dx, x, -a\cos(c+dx)\right)}{a^4d}$$
$$= \frac{\text{Subst}\left(\int \left(1-\frac{b^3}{(b-x)^3}+\frac{3b^2}{(b-x)^2}-\frac{3b}{b-x}\right)dx, x, -a\cos(c+dx)\right)}{a^4d}$$
$$= -\frac{\cos(c+dx)}{a^3d} - \frac{b^3}{2a^4d(b+a\cos(c+dx))^2} + \frac{3b^2}{a^4d(b+a\cos(c+dx))} + \frac{3b\log(b+a\cos(c+dx))}{a^4d}$$

Mathematica [A] time = 0.399499, size = 111, normalized size = 1.34

$$\frac{2a^2b\cos^2(c+dx)(3\log(a\cos(c+dx)+b)-2)-2a^3\cos^3(c+dx)+b^3(6\log(a\cos(c+dx)+b)+5)+4ab^2\cos(c+dx)(3\log(a\cos(c+dx)+b)+1)}{2a^4d(a\cos(c+dx)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-2*a^3*Cos[c + d*x]^3 + 2*a^2*b*Cos[c + d*x]^2*(-2 + 3*Log[b + a*Cos[c + d
*x]]) + 4*a*b^2*Cos[c + d*x]*(1 + 3*Log[b + a*Cos[c + d*x]]) + b^3*(5 + 6*L
og[b + a*Cos[c + d*x]]))/(2*a^4*d*(b + a*Cos[c + d*x])^2)
```

Maple [A] time = 0.033, size = 96, normalized size = 1.2

$$-\frac{b}{2\,da^2\,(a+b\sec(dx+c))^2}+3\,\frac{b\ln(a+b\sec(dx+c))}{da^4}-2\,\frac{b}{da^3\,(a+b\sec(dx+c))}-\frac{1}{da^3\sec(dx+c)}-3\,\frac{b\ln(\sec(dx+c))}{da^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -1/2/d*b/a^2/(a+b*sec(d*x+c))^2+3/d/a^4*b*ln(a+b*sec(d*x+c))-2/d/a^3*b/(a+b
*sec(d*x+c))-1/d/a^3/sec(d*x+c)-3/d/a^4*b*ln(sec(d*x+c))
```

Maxima [A] time = 0.961331, size = 117, normalized size = 1.41

$$\frac{\frac{6\,ab^2\cos(dx+c)+5\,b^3}{a^6\cos(dx+c)^2+2\,a^5b\cos(dx+c)+a^4b^2}-\frac{2\cos(dx+c)}{a^3}+\frac{6\,b\log(a\cos(dx+c)+b)}{a^4}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*((6*a*b^2*cos(d*x + c) + 5*b^3)/(a^6*cos(d*x + c)^2 + 2*a^5*b*cos(d*x +
 c) + a^4*b^2) - 2*cos(d*x + c)/a^3 + 6*b*log(a*cos(d*x + c) + b)/a^4)/d
```

Fricas [A] time = 1.88912, size = 304, normalized size = 3.66

$$-\frac{2\,a^3\cos\left(dx+c\right)^3+4\,a^2b\cos\left(dx+c\right)^2-4\,ab^2\cos\left(dx+c\right)-5\,b^3-6\left(a^2b\cos\left(dx+c\right)^2+2\,ab^2\cos\left(dx+c\right)+b^3\right)\log}{2\left(a^6d\cos\left(dx+c\right)^2+2\,a^5bd\cos\left(dx+c\right)+a^4b^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/2*(2*a^3*cos(d*x + c)^3 + 4*a^2*b*cos(d*x + c)^2 - 4*a*b^2*cos(d*x + c)
- 5*b^3 - 6*(a^2*b*cos(d*x + c)^2 + 2*a*b^2*cos(d*x + c) + b^3)*log(a*cos(d
*x + c) + b))/(a^6*d*cos(d*x + c)^2 + 2*a^5*b*d*cos(d*x + c) + a^4*b^2*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sin\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*sec(c + d*x))**3, x)
```

Giac [A] time = 1.33973, size = 104, normalized size = 1.25

$$-\frac{\cos\left(dx+c\right)}{a^3d}+\frac{3\,b\log\left(|-a\cos\left(dx+c\right)-b|\right)}{a^4d}+\frac{6\,ab^2\cos\left(dx+c\right)+5\,b^3}{2\left(a\cos\left(dx+c\right)+b\right)^2a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -cos(d*x + c)/(a^3*d) + 3*b*log(abs(-a*cos(d*x + c) - b))/(a^4*d) + 1/2*(6*
a*b^2*cos(d*x + c) + 5*b^3)/((a*cos(d*x + c) + b)^2*a^4*d)
```

3.225 $\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=163

$$-\frac{b^3}{2a^2d\left(a^2-b^2\right)(a\cos(c+dx)+b)^2}+\frac{b^2\left(3a^2-b^2\right)}{a^2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}+\frac{b\left(3a^2+b^2\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^3}+\frac{\log(1-}{2d}$$

```
[Out] -b^3/(2*a^2*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) + (b^2*(3*a^2 - b^2))/(a^
2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])) + Log[1 - Cos[c + d*x]]/(2*(a + b)^
3*d) - Log[1 + Cos[c + d*x]]/(2*(a - b)^3*d) + (b*(3*a^2 + b^2)*Log[b + a*C
os[c + d*x]])/((a^2 - b^2)^3*d)
```

Rubi [A] time = 0.31935, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3872, 2837, 12, 1629}

$$-\frac{b^3}{2a^2d\left(a^2-b^2\right)(a\cos(c+dx)+b)^2}+\frac{b^2\left(3a^2-b^2\right)}{a^2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}+\frac{b\left(3a^2+b^2\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^3}+\frac{\log(1-}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```



```
[Out] -b^3/(2*a^2*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) + (b^2*(3*a^2 - b^2))/(a^
2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])) + Log[1 - Cos[c + d*x]]/(2*(a + b)^
3*d) - Log[1 + Cos[c + d*x]]/(2*(a - b)^3*d) + (b*(3*a^2 + b^2)*Log[b + a*C
os[c + d*x]])/((a^2 - b^2)^3*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cos^2(c+dx)\cot(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$

$$= \frac{a\,\text{Subst}\left(\int \frac{x^3}{a^3(-b+x)^3(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^3}{(-b+x)^3(a^2-x^2)}\,dx, x, -a\cos(c+dx)\right)}{a^2 d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{2(a-b)^3(a-x)} - \frac{b^3}{(a-b)(a+b)(b-x)^3} + \frac{3a^2b^2-b^4}{(a-b)^2(a+b)^2(b-x)^2} - \frac{a^2b(3a^2+b^2)}{(a-b)^3(a+b)^3(b-x)} + \frac{a^2}{2(a+b)^3(a+x)}\right) d$$

$$= -\frac{b^3}{2a^2(a^2-b^2)\,d(b+a\cos(c+dx))^2} + \frac{b^2(3a^2-b^2)}{a^2(a^2-b^2)^2\,d(b+a\cos(c+dx))} + \frac{\log(1-\cos(c}{2(a+b)^3}$$

Mathematica [A] time = 0.554559, size = 203, normalized size = 1.25

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b)\left(-\frac{2b^2(b^2-3a^2)(a\cos(c+dx)+b)}{a^2(a-b)^2(a+b)^2} + \frac{2b(3a^2+b^2)(a\cos(c+dx)+b)^2\log(a\cos(c+dx)+b)}{(a^2-b^2)^3} + \frac{b^3}{a^2(b^2-a^2)} + \frac{2\log(\cos(}{}\right.}{2d(a+b\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*(b^3/(a^2*(-a^2 + b^2)) - (2*b^2*(-3*a^2 + b^2)*(b +
a*Cos[c + d*x]))/(a^2*(a - b)^2*(a + b)^2) + (2*(b + a*Cos[c + d*x])^2*Log[
Cos[(c + d*x)/2]])/(-a + b)^3 + (2*b*(3*a^2 + b^2)*(b + a*Cos[c + d*x])^2*L
og[b + a*Cos[c + d*x]])/(a^2 - b^2)^3 + (2*(b + a*Cos[c + d*x])^2*Log[Sin[(
c + d*x)/2]])/(a + b)^3)*Sec[c + d*x]^3)/(2*d*(a + b*Sec[c + d*x])^3)
```

Maple [A] time = 0.071, size = 206, normalized size = 1.3

$$-\frac{b^3}{2\,da^2\,(a+b)\,(a-b)\,(b+a\cos(dx+c))^2} + 3\,\frac{b\ln(b+a\cos(dx+c))\,a^2}{d\,(a+b)^3\,(a-b)^3} + \frac{b^3\ln(b+a\cos(dx+c))}{d\,(a+b)^3\,(a-b)^3} + 3\,\frac{}{d\,(a+b)^2\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -1/2/d/a^2*b^3/(a+b)/(a-b)/(b+a*cos(d*x+c))^2+3/d*b/(a+b)^3/(a-b)^3*ln(b+a*
cos(d*x+c))*a^2+1/d*b^3/(a+b)^3/(a-b)^3*ln(b+a*cos(d*x+c))+3/d*b^2/(a+b)^2/
(a-b)^2/(b+a*cos(d*x+c))-1/d*b^4/(a+b)^2/(a-b)^2/a^2/(b+a*cos(d*x+c))-1/2*l
n(cos(d*x+c)+1)/(a-b)^3/d+1/2/d/(a+b)^3*ln(-1+cos(d*x+c))
```

Maxima [A] time = 0.979614, size = 325, normalized size = 1.99

$$\frac{\frac{2(3a^2b+b^3)\log(a\cos(dx+c)+b)}{a^6-3a^4b^2+3a^2b^4-b^6} + \frac{5a^2b^3-b^5+2(3a^3b^2-ab^4)\cos(dx+c)}{a^6b^2-2a^4b^4+a^2b^6+(a^8-2a^6b^2+a^4b^4)\cos(dx+c)^2+2(a^7b-2a^5b^3+a^3b^5)\cos(dx+c)} - \frac{\log(\cos(dx+c)+1)}{a^3-3a^2b+3ab^2-b^3} + \frac{\log(\cos(}{a^3+3a^2b}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(3*a^2*b + b^3)*log(a*cos(d*x + c) + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6) + (5*a^2*b^3 - b^5 + 2*(3*a^3*b^2 - a*b^4)*cos(d*x + c))/(a^6*b^2 -
 2*a^4*b^4 + a^2*b^6 + (a^8 - 2*a^6*b^2 + a^4*b^4)*cos(d*x + c)^2 + 2*(a^7*
b - 2*a^5*b^3 + a^3*b^5)*cos(d*x + c)) - log(cos(d*x + c) + 1)/(a^3 - 3*a^2
*b + 3*a*b^2 - b^3) + log(cos(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
)/d
```

Fricas [B] time = 2.60537, size = 1021, normalized size = 6.26

$$\frac{5\,a^4b^3 - 6\,a^2b^5 + b^7 + 2\left(3\,a^5b^2 - 4\,a^3b^4 + ab^6\right)\cos\left(dx + c\right) + 2\left(3\,a^4b^3 + a^2b^5 + \left(3\,a^6b + a^4b^3\right)\cos\left(dx + c\right)^2 + 2\left(3\,a^5b^2 - \cdots\right.\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/2*(5*a^4*b^3 - 6*a^2*b^5 + b^7 + 2*(3*a^5*b^2 - 4*a^3*b^4 + a*b^6)*cos(d*
x + c) + 2*(3*a^4*b^3 + a^2*b^5 + (3*a^6*b + a^4*b^3)*cos(d*x + c)^2 + 2*(3
*a^5*b^2 + a^3*b^4)*cos(d*x + c))*log(a*cos(d*x + c) + b) - (a^5*b^2 + 3*a^
4*b^3 + 3*a^3*b^4 + a^2*b^5 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*cos(d*x
 + c)^2 + 2*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)*cos(d*x + c))*log(1/2
*cos(d*x + c) + 1/2) + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5 + (a^7 -
3*a^6*b + 3*a^5*b^2 - a^4*b^3)*cos(d*x + c)^2 + 2*(a^6*b - 3*a^5*b^2 + 3*a^
4*b^3 - a^3*b^4)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^10 - 3*a^8
*b^2 + 3*a^6*b^4 - a^4*b^6)*d*cos(d*x + c)^2 + 2*(a^9*b - 3*a^7*b^3 + 3*a^5
*b^5 - a^3*b^7)*d*cos(d*x + c) + (a^8*b^2 - 3*a^6*b^4 + 3*a^4*b^6 - a^2*b^8
)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)/(a + b*sec(c + d*x))**3, x)
```

Giac [B] time = 1.50738, size = 610, normalized size = 3.74

$$\frac{\frac{2\left(3\,a^2b+b^3\right)\log\left(\left|-a-b-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right|\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6} + \frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{9\,a^3b+15\,a^2b^2+3\,ab^3-3\,b^4+\frac{18\,a^3b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{6\,a^2b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{10\,ab^3(\cos \cdots}{\cos(dx \cdots}}{\left(a^5+a^4b-2\,a^3b^2-2\,a^2 \cdots\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*(3*a^2*b + b^3)*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6) + log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^3 + 3*a^2*
b + 3*a*b^2 + b^3) - (9*a^3*b + 15*a^2*b^2 + 3*a*b^3 - 3*b^4 + 18*a^3*b*(co
s(d*x + c) - 1)/(cos(d*x + c) + 1) + 6*a^2*b^2*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) - 10*a*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2*b^4*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 9*a^3*b*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 - 9*a^2*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 3*a*b^3*(cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x
 + c) + 1)^2)/((a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5)*(a + b +
 a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x +
c) + 1))^2))/d
```

3.226 $\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3} dx$

Optimal. Leaf size=229

$$-\frac{b^3}{2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}+\frac{b^2\left(3a^2+b^2\right)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}+\frac{b\left(8a^2b^2+3a^4+b^4\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^4}+\frac{\csc}{}$$

```
[Out] -b^3/(2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) + (b^2*(3*a^2 + b^2))/((a^2
 - b^2)^3*d*(b + a*Cos[c + d*x])) + ((b*(3*a^2 + b^2) - a*(a^2 + 3*b^2)*Cos
[c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)^3*d) + ((a - 2*b)*Log[1 - Cos[c +
 d*x]])/(4*(a + b)^4*d) - ((a + 2*b)*Log[1 + Cos[c + d*x]])/(4*(a - b)^4*d)
 + (b*(3*a^4 + 8*a^2*b^2 + b^4)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d)
```

Rubi [A] time = 0.512592, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3872, 2721, 1647, 1629}

$$-\frac{b^3}{2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}+\frac{b^2\left(3a^2+b^2\right)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}+\frac{b\left(8a^2b^2+3a^4+b^4\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^4}+\frac{\csc}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -b^3/(2*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) + (b^2*(3*a^2 + b^2))/((a^2
 - b^2)^3*d*(b + a*Cos[c + d*x])) + ((b*(3*a^2 + b^2) - a*(a^2 + 3*b^2)*Cos
[c + d*x])*Csc[c + d*x]^2)/(2*(a^2 - b^2)^3*d) + ((a - 2*b)*Log[1 - Cos[c +
 d*x]])/(4*(a + b)^4*d) - ((a + 2*b)*Log[1 + Cos[c + d*x]])/(4*(a - b)^4*d)
 + (b*(3*a^4 + 8*a^2*b^2 + b^4)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^4*d)
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2721

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Dist[1/f, Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)/
2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^
2, 0] && IntegerQ[(p + 1)/2]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cot^3(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{x^3}{(-b+x)^3(a^2-x^2)^2}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{\left(b\left(3a^2+b^2\right)-a\left(a^2+3b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} + \frac{\text{Subst}\left(\int \frac{\frac{a^4b^3(a^2+3b^2)}{(a^2-b^2)^3} - \frac{a^2b^2(3a^4+3a^2b^2}{(a^2-b^2)^3}}{(-b-}\right.}{}$$

$$= \frac{\left(b\left(3a^2+b^2\right)-a\left(a^2+3b^2\right)\cos(c+dx)\right)\csc^2(c+dx)}{2\left(a^2-b^2\right)^3 d} + \frac{\text{Subst}\left(\int\left(\frac{a^2(a+2b)}{2(a-b)^4(a-x)} - \frac{2a^2b^3}{(a^2-b^2)^2(}\right.\right.}{}$$

$$= -\frac{b^3}{2\left(a^2-b^2\right)^2 d(b+a\cos(c+dx))^2} + \frac{b^2\left(3a^2+b^2\right)}{\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))} + \frac{\left(b\left(3a^2+b^2\right)-a\right.}{}$$

Mathematica [C] time = 6.3066, size = 332, normalized size = 1.45

$$-\frac{2i\left(8a^2b^3+3a^4b+b^5\right)(c+dx)}{d(a-b)^4(a+b)^4} - \frac{b^2\left(3a^2+b^2\right)}{d(b-a)^3(a+b)^3(a\cos(c+dx)+b)} + \frac{\left(8a^2b^3+3a^4b+b^5\right)\log(a\cos(c+dx)+b)}{d\left(b^2-a^2\right)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((-2*I)*(3*a^4*b + 8*a^2*b^3 + b^5)*(c + d*x))/((a - b)^4*(a + b)^4*d) - ((
I/2)*(-a - 2*b)*ArcTan[Tan[c + d*x]])/((-a + b)^4*d) - ((I/2)*(a - 2*b)*Arc
Tan[Tan[c + d*x]])/((a + b)^4*d) - b^3/(2*(-a + b)^2*(a + b)^2*d*(b + a*Cos
[c + d*x])^2) - (b^2*(3*a^2 + b^2))/((-a + b)^3*(a + b)^3*d*(b + a*Cos[c +
d*x])) - Csc[(c + d*x)/2]^2/(8*(a + b)^3*d) + ((-a - 2*b)*Log[Cos[(c + d*x)
/2]^2])/(4*(-a + b)^4*d) + ((3*a^4*b + 8*a^2*b^3 + b^5)*Log[b + a*Cos[c + d
*x]])/((-a^2 + b^2)^4*d) + ((a - 2*b)*Log[Sin[(c + d*x)/2]^2])/(4*(a + b)^4
*d) - Sec[(c + d*x)/2]^2/(8*(-a + b)^3*d)
```

Maple [A] time = 0.087, size = 322, normalized size = 1.4

$$-\frac{b^3}{2d(a+b)^2(a-b)^2(b+a\cos(dx+c))^2} + 3\frac{b\ln(b+a\cos(dx+c))a^4}{d(a+b)^4(a-b)^4} + 8\frac{b^3\ln(b+a\cos(dx+c))a^2}{d(a+b)^4(a-b)^4} + \frac{b^5\ln(b+}{d(a+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -1/2/d*b^3/(a+b)^2/(a-b)^2/(b+a*cos(d*x+c))^2+3/d*b/(a+b)^4/(a-b)^4*ln(b+a*
cos(d*x+c))*a^4+8/d*b^3/(a+b)^4/(a-b)^4*ln(b+a*cos(d*x+c))*a^2+1/d*b^5/(a+b
)^4/(a-b)^4*ln(b+a*cos(d*x+c))+3/d*b^2/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))*a^2
+1/d*b^4/(a+b)^3/(a-b)^3/(b+a*cos(d*x+c))+1/4/d/(a-b)^3/(cos(d*x+c)+1)-1/4/
d/(a-b)^4*ln(cos(d*x+c)+1)*a-1/2/d/(a-b)^4*ln(cos(d*x+c)+1)*b+1/4/d/(a+b)^3
/(-1+cos(d*x+c))+1/4/d/(a+b)^4*ln(-1+cos(d*x+c))*a-1/2/d/(a+b)^4*ln(-1+cos(
d*x+c))*b
```

Maxima [A] time = 1.04154, size = 587, normalized size = 2.56

$$\frac{\frac{4\left(3\,a^4b+8\,a^2b^3+b^5\right)\log(a\cos(dx+c)+b)}{a^8-4\,a^6b^2+6\,a^4b^4-4\,a^2b^6+b^8}-\frac{(a+2\,b)\log(\cos(dx+c)+1)}{a^4-4\,a^3b+6\,a^2b^2-4\,ab^3+b^4}+\frac{(a-2\,b)\log(\cos(dx+c)-1)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{2\left(8\,a^2b^3+4\,b^5 \cdots\right.}{a^6b^2-3\,a^4b^4+3\,a^2b^6-b^8-\left(a^8-3\,a^6b^2+3\,a^4b^4-a^2b^6\right)\cos \cdots}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*(3*a^4*b + 8*a^2*b^3 + b^5)*log(a*cos(d*x + c) + b)/(a^8 - 4*a^6*b^2
 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) - (a + 2*b)*log(cos(d*x + c) + 1)/(a^4 - 4*
a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4) + (a - 2*b)*log(cos(d*x + c) - 1)/(a^4 +
 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 2*(8*a^2*b^3 + 4*b^5 - (a^5 + 9*a^3
*b^2 + 2*a*b^4)*cos(d*x + c)^3 + (a^4*b - 10*a^2*b^3 - 3*b^5)*cos(d*x + c)^
2 + (11*a^3*b^2 + a*b^4)*cos(d*x + c))/(a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b
^8 - (a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*cos(d*x + c)^4 - 2*(a^7*b - 3*
a^5*b^3 + 3*a^3*b^5 - a*b^7)*cos(d*x + c)^3 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4
- 4*a^2*b^6 + b^8)*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^
7)*cos(d*x + c)))/d
```

Fricas [B] time = 4.01581, size = 2344, normalized size = 10.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] -1/4*(16*a^4*b^3 - 8*a^2*b^5 - 8*b^7 - 2*(a^7 + 8*a^5*b^2 - 7*a^3*b^4 - 2*a
*b^6)*cos(d*x + c)^3 + 2*(a^6*b - 11*a^4*b^3 + 7*a^2*b^5 + 3*b^7)*cos(d*x +
 c)^2 + 2*(11*a^5*b^2 - 10*a^3*b^4 - a*b^6)*cos(d*x + c) + 4*(3*a^4*b^3 + 8
*a^2*b^5 + b^7 - (3*a^6*b + 8*a^4*b^3 + a^2*b^5)*cos(d*x + c)^4 - 2*(3*a^5*
b^2 + 8*a^3*b^4 + a*b^6)*cos(d*x + c)^3 + (3*a^6*b + 5*a^4*b^3 - 7*a^2*b^5
- b^7)*cos(d*x + c)^2 + 2*(3*a^5*b^2 + 8*a^3*b^4 + a*b^6)*cos(d*x + c))*log
(a*cos(d*x + c) + b) - (a^5*b^2 + 6*a^4*b^3 + 14*a^3*b^4 + 16*a^2*b^5 + 9*a
*b^6 + 2*b^7 - (a^7 + 6*a^6*b + 14*a^5*b^2 + 16*a^4*b^3 + 9*a^3*b^4 + 2*a^2
*b^5)*cos(d*x + c)^4 - 2*(a^6*b + 6*a^5*b^2 + 14*a^4*b^3 + 16*a^3*b^4 + 9*a
^2*b^5 + 2*a*b^6)*cos(d*x + c)^3 + (a^7 + 6*a^6*b + 13*a^5*b^2 + 10*a^4*b^3
 - 5*a^3*b^4 - 14*a^2*b^5 - 9*a*b^6 - 2*b^7)*cos(d*x + c)^2 + 2*(a^6*b + 6*
a^5*b^2 + 14*a^4*b^3 + 16*a^3*b^4 + 9*a^2*b^5 + 2*a*b^6)*cos(d*x + c))*log(
1/2*cos(d*x + c) + 1/2) + (a^5*b^2 - 6*a^4*b^3 + 14*a^3*b^4 - 16*a^2*b^5 +
9*a*b^6 - 2*b^7 - (a^7 - 6*a^6*b + 14*a^5*b^2 - 16*a^4*b^3 + 9*a^3*b^4 - 2*
a^2*b^5)*cos(d*x + c)^4 - 2*(a^6*b - 6*a^5*b^2 + 14*a^4*b^3 - 16*a^3*b^4 +
9*a^2*b^5 - 2*a*b^6)*cos(d*x + c)^3 + (a^7 - 6*a^6*b + 13*a^5*b^2 - 10*a^4*
b^3 - 5*a^3*b^4 + 14*a^2*b^5 - 9*a*b^6 + 2*b^7)*cos(d*x + c)^2 + 2*(a^6*b -
 6*a^5*b^2 + 14*a^4*b^3 - 16*a^3*b^4 + 9*a^2*b^5 - 2*a*b^6)*cos(d*x + c))*l
```

```
og(-1/2*cos(d*x + c) + 1/2))/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a
^2*b^8)*d*cos(d*x + c)^4 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a
*b^9)*d*cos(d*x + c)^3 - (a^10 - 5*a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^
2*b^8 - b^10)*d*cos(d*x + c)^2 - 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b
^7 + a*b^9)*d*cos(d*x + c) - (a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 +
 b^10)*d)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^3(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**3/(a + b*sec(c + d*x))**3, x)
```

Giac [B] time = 1.45744, size = 1080, normalized size = 4.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(2*(a - 2*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a^4 + 4
*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 8*(3*a^4*b + 8*a^2*b^3 + b^5)*log(abs
(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(c
os(d*x + c) + 1)))/(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8) + (a + b
 - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1))*(cos(d*x + c) + 1)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b
^4)*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/((a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*(cos(d*x + c) + 1)) - 4*(9*a^6*b + 6*a^5*b^2 + 9*a^4*b^3 + 28*a^3*b^4 +
11*a^2*b^5 - 2*a*b^6 + 3*b^7 + 18*a^6*b*(cos(d*x + c) - 1)/(cos(d*x + c) +
1) - 12*a^5*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 26*a^4*b^3*(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) + 4*a^3*b^4*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) - 38*a^2*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 8*a*b^6*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) - 6*b^7*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
+ 9*a^6*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 18*a^5*b^2*(cos(d*x +
 c) - 1)^2/(cos(d*x + c) + 1)^2 + 33*a^4*b^3*(cos(d*x + c) - 1)^2/(cos(d*x
+ c) + 1)^2 - 48*a^3*b^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 27*a^2
*b^5*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 6*a*b^6*(cos(d*x + c) - 1)
^2/(cos(d*x + c) + 1)^2 + 3*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/
((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a + b + a*(cos(d*x + c) -
 1)/(cos(d*x + c) + 1) - b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2))/d
```

3.227 $\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^3} dx$

Optimal. Leaf size=313

$$\frac{3a^2b^2\left(a^2+b^2\right)}{d\left(a^2-b^2\right)^4(a\cos(c+dx)+b)}-\frac{a^2b^3}{2d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)^2}+\frac{3a^2b\left(5a^2b^2+a^4+2b^4\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^5}+\cdots$$

```
[Out] -(a^2*b^3)/(2*(a^2 - b^2)^3*d*(b + a*Cos[c + d*x])^2) + (3*a^2*b^2*(a^2 + b
^2))/((a^2 - b^2)^4*d*(b + a*Cos[c + d*x])) + ((4*b*(3*a^4 + 8*a^2*b^2 + b^
4) - 3*a*(a^4 + 10*a^2*b^2 + 5*b^4)*Cos[c + d*x])*Csc[c + d*x]^2)/(8*(a^2 -
 b^2)^4*d) + ((b*(3*a^2 + b^2) - a*(a^2 + 3*b^2)*Cos[c + d*x])*Csc[c + d*x]
^4)/(4*(a^2 - b^2)^3*d) + (3*a*(a - 3*b)*Log[1 - Cos[c + d*x]])/(16*(a + b)
^5*d) - (3*a*(a + 3*b)*Log[1 + Cos[c + d*x]])/(16*(a - b)^5*d) + (3*a^2*b*(
a^4 + 5*a^2*b^2 + 2*b^4)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^5*d)
```

Rubi [A] time = 1.00667, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2837, 12, 1647, 1629}

$$\frac{3a^2b^2\left(a^2+b^2\right)}{d\left(a^2-b^2\right)^4(a\cos(c+dx)+b)}-\frac{a^2b^3}{2d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)^2}+\frac{3a^2b\left(5a^2b^2+a^4+2b^4\right)\log(a\cos(c+dx)+b)}{d\left(a^2-b^2\right)^5}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```


```
[Out] -(a^2*b^3)/(2*(a^2 - b^2)^3*d*(b + a*Cos[c + d*x])^2) + (3*a^2*b^2*(a^2 + b
^2))/((a^2 - b^2)^4*d*(b + a*Cos[c + d*x])) + ((4*b*(3*a^4 + 8*a^2*b^2 + b^
4) - 3*a*(a^4 + 10*a^2*b^2 + 5*b^4)*Cos[c + d*x])*Csc[c + d*x]^2)/(8*(a^2 -
 b^2)^4*d) + ((b*(3*a^2 + b^2) - a*(a^2 + 3*b^2)*Cos[c + d*x])*Csc[c + d*x]
^4)/(4*(a^2 - b^2)^3*d) + (3*a*(a - 3*b)*Log[1 - Cos[c + d*x]])/(16*(a + b)
^5*d) - (3*a*(a + 3*b)*Log[1 + Cos[c + d*x]])/(16*(a - b)^5*d) + (3*a^2*b*(
a^4 + 5*a^2*b^2 + 2*b^4)*Log[b + a*Cos[c + d*x]])/((a^2 - b^2)^5*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2837

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_
.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_.), x_Symbol] :> Dist[1/(b^p*
f), Subst[Int[(a + x)^m*(c + (d*x)/b)^n*(b^2 - x^2)^((p - 1)/2), x], x, b*S
in[e + f*x]], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IntegerQ[(p - 1)/
2] && NeQ[a^2 - b^2, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1647

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[((a*g - c*f*x)*(a + c
*x^2)^(p + 1))/(2*a*c*(p + 1)), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1629

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{\csc^5(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cot^3(c+dx)\csc^2(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$

$$= \frac{a^5\,\text{Subst}\left(\int \frac{x^3}{a^3(-b+x)^3\left(a^2-x^2\right)^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{a^2\,\text{Subst}\left(\int \frac{x^3}{(-b+x)^3\left(a^2-x^2\right)^3}\,dx, x, -a\cos(c+dx)\right)}{d}$$

$$= \frac{\left(b\left(3a^2+b^2\right)-a\left(a^2+3b^2\right)\cos(c+dx)\right)\csc^4(c+dx)}{4\left(a^2-b^2\right)^3 d} + \frac{\text{Subst}\left(\int \frac{\frac{a^4b^3\left(a^2+3b^2\right)}{\left(a^2-b^2\right)^3}-\frac{a^2b^2\left(3a^4-3a^2b^2\cdots\right.}{\left(a^2-b^2\right)^3}}{(-b\cdots}\right.}{\cdots}$$

$$= \frac{\left(4b\left(3a^4+8a^2b^2+b^4\right)-3a\left(a^4+10a^2b^2+5b^4\right)\cos(c+dx)\right)\csc^2(c+dx)}{8\left(a^2-b^2\right)^4 d} + \frac{\left(b\left(3a^2+b^2\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(4b\left(3a^4+8a^2b^2+b^4\right)-3a\left(a^4+10a^2b^2+5b^4\right)\cos(c+dx)\right)\csc^2(c+dx)}{8\left(a^2-b^2\right)^4 d} + \frac{\left(b\left(3a^2+b^2\cdots\right.\right.}{\cdots}$$

$$= -\frac{a^2b^3}{2\left(a^2-b^2\right)^3 d(b+a\cos(c+dx))^2} + \frac{3a^2b^2\left(a^2+b^2\right)}{\left(a^2-b^2\right)^4 d(b+a\cos(c+dx))} + \frac{\left(4b\left(3a^4+8a^2b^2\cdots\right.\right.}{\cdots}$$

Mathematica [C] time = 4.50544, size = 496, normalized size = 1.58

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b)\left(-\frac{384ia^2b\left(5a^2b^2+a^4+2b^4\right)(c+dx)(a\cos(c+dx)+b)^2}{(a-b)^5(a+b)^5}+\frac{192a^2b^2(a-ib)(a+ib)(a\cos(c+dx)+b)}{(a-b)^4(a+b)^4}+\frac{192a^2b\left(5a^2b^2+a^4\cdots\right.}{\cdots}\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*((32*a^2*b^3)/((-a + b)^3*(a + b)^3) + (192*a^2*(a -
I*b)*(a + I*b)*b^2*(b + a*Cos[c + d*x]))/((a - b)^4*(a + b)^4) - ((384*I)*a
```

```
^2*b*(a^4 + 5*a^2*b^2 + 2*b^4)*(c + d*x)*(b + a*Cos[c + d*x])^2)/((a - b)^5
*(a + b)^5) - ((24*I)*a*(a - 3*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x])
^2)/(a + b)^5 + ((24*I)*a*(a + 3*b)*ArcTan[Tan[c + d*x]]*(b + a*Cos[c + d*x
])^2)/(a - b)^5 + (6*(-a + b)*(b + a*Cos[c + d*x])^2*Csc[(c + d*x)/2]^2)/(a
 + b)^4 - ((b + a*Cos[c + d*x])^2*Csc[(c + d*x)/2]^4)/(a + b)^3 - (12*a*(a
+ 3*b)*(b + a*Cos[c + d*x])^2*Log[Cos[(c + d*x)/2]^2])/(a - b)^5 + (192*a^2
*b*(a^4 + 5*a^2*b^2 + 2*b^4)*(b + a*Cos[c + d*x])^2*Log[b + a*Cos[c + d*x]]
)/(a^2 - b^2)^5 + (12*a*(a - 3*b)*(b + a*Cos[c + d*x])^2*Log[Sin[(c + d*x)/
2]^2])/(a + b)^5 + (6*(a + b)*(b + a*Cos[c + d*x])^2*Sec[(c + d*x)/2]^2)/(a
 - b)^4 + ((b + a*Cos[c + d*x])^2*Sec[(c + d*x)/2]^4)/(a - b)^3)*Sec[c + d*
x]^3)/(64*d*(a + b*Sec[c + d*x])^3)
```

Maple [A] time = 0.093, size = 427, normalized size = 1.4

$$-\frac{a^2b^3}{2\,d\,(a+b)^3\,(a-b)^3\,(b+a\cos(dx+c))^2}+3\,\frac{a^4b^2}{d\,(a+b)^4\,(a-b)^4\,(b+a\cos(dx+c))}+3\,\frac{a^2b^4}{d\,(a+b)^4\,(a-b)^4\,(b+a\cos(dx\cdot}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -1/2/d*b^3/(a+b)^3*a^2/(a-b)^3/(b+a*cos(d*x+c))^2+3/d*a^4*b^2/(a+b)^4/(a-b)
^4/(b+a*cos(d*x+c))+3/d*a^2*b^4/(a+b)^4/(a-b)^4/(b+a*cos(d*x+c))+3/d*b*a^6/
(a+b)^5/(a-b)^5*ln(b+a*cos(d*x+c))+15/d*b^3*a^4/(a+b)^5/(a-b)^5*ln(b+a*cos(
d*x+c))+6/d*b^5*a^2/(a+b)^5/(a-b)^5*ln(b+a*cos(d*x+c))+1/16/d/(a-b)^3/(cos(
d*x+c)+1)^2+3/16/d/(a-b)^4/(cos(d*x+c)+1)*a+3/16/d/(a-b)^4/(cos(d*x+c)+1)*b
-3/16/d*a^2/(a-b)^5*ln(cos(d*x+c)+1)-9/16/d*a/(a-b)^5*ln(cos(d*x+c)+1)*b-1/
16/d/(a+b)^3/(-1+cos(d*x+c))^2+3/16/d/(a+b)^4/(-1+cos(d*x+c))*a-3/16/d/(a+b
)^4/(-1+cos(d*x+c))*b+3/16/d*a^2/(a+b)^5*ln(-1+cos(d*x+c))-9/16/d*a/(a+b)^5
*ln(-1+cos(d*x+c))*b
```

Maxima [B] time = 1.04808, size = 954, normalized size = 3.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] 1/16*(48*(a^6*b + 5*a^4*b^3 + 2*a^2*b^5)*log(a*cos(d*x + c) + b)/(a^10 - 5*
a^8*b^2 + 10*a^6*b^4 - 10*a^4*b^6 + 5*a^2*b^8 - b^10) - 3*(a^2 + 3*a*b)*log
(cos(d*x + c) + 1)/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5
) + 3*(a^2 - 3*a*b)*log(cos(d*x + c) - 1)/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*
a^2*b^3 + 5*a*b^4 + b^5) + 2*(38*a^4*b^3 + 56*a^2*b^5 + 2*b^7 + 3*(a^7 + 18
*a^5*b^2 + 13*a^3*b^4)*cos(d*x + c)^5 - 6*(a^6*b - 8*a^4*b^3 - 9*a^2*b^5)*c
os(d*x + c)^4 - (5*a^7 + 103*a^5*b^2 + 91*a^3*b^4 - 7*a*b^6)*cos(d*x + c)^3
 + 4*(2*a^6*b - 23*a^4*b^3 - 26*a^2*b^5 - b^7)*cos(d*x + c)^2 + (55*a^5*b^2
 + 46*a^3*b^4 - 5*a*b^6)*cos(d*x + c))/(a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4
*a^2*b^8 + b^10 + (a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*cos(
d*x + c)^6 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*cos(d*x
+ c)^5 - (2*a^10 - 9*a^8*b^2 + 16*a^6*b^4 - 14*a^4*b^6 + 6*a^2*b^8 - b^10)*
cos(d*x + c)^4 - 4*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*cos(
d*x + c)^3 + (a^10 - 6*a^8*b^2 + 14*a^6*b^4 - 16*a^4*b^6 + 9*a^2*b^8 - 2*b^
10)*cos(d*x + c)^2 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*
```

```
cos(d*x + c)))/d
```

Fricas [B] time = 5.87997, size = 3970, normalized size = 12.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] 1/16*(76*a^6*b^3 + 36*a^4*b^5 - 108*a^2*b^7 - 4*b^9 + 6*(a^9 + 17*a^7*b^2 -
 5*a^5*b^4 - 13*a^3*b^6)*cos(d*x + c)^5 - 12*(a^8*b - 9*a^6*b^3 - a^4*b^5 +
 9*a^2*b^7)*cos(d*x + c)^4 - 2*(5*a^9 + 98*a^7*b^2 - 12*a^5*b^4 - 98*a^3*b^
6 + 7*a*b^8)*cos(d*x + c)^3 + 8*(2*a^8*b - 25*a^6*b^3 - 3*a^4*b^5 + 25*a^2*
b^7 + b^9)*cos(d*x + c)^2 + 2*(55*a^7*b^2 - 9*a^5*b^4 - 51*a^3*b^6 + 5*a*b^
8)*cos(d*x + c) + 48*(a^6*b^3 + 5*a^4*b^5 + 2*a^2*b^7 + (a^8*b + 5*a^6*b^3
+ 2*a^4*b^5)*cos(d*x + c)^6 + 2*(a^7*b^2 + 5*a^5*b^4 + 2*a^3*b^6)*cos(d*x +
 c)^5 - (2*a^8*b + 9*a^6*b^3 - a^4*b^5 - 2*a^2*b^7)*cos(d*x + c)^4 - 4*(a^7
*b^2 + 5*a^5*b^4 + 2*a^3*b^6)*cos(d*x + c)^3 + (a^8*b + 3*a^6*b^3 - 8*a^4*b
^5 - 4*a^2*b^7)*cos(d*x + c)^2 + 2*(a^7*b^2 + 5*a^5*b^4 + 2*a^3*b^6)*cos(d*
x + c))*log(a*cos(d*x + c) + b) - 3*(a^7*b^2 + 8*a^6*b^3 + 25*a^5*b^4 + 40*
a^4*b^5 + 35*a^3*b^6 + 16*a^2*b^7 + 3*a*b^8 + (a^9 + 8*a^8*b + 25*a^7*b^2 +
 40*a^6*b^3 + 35*a^5*b^4 + 16*a^4*b^5 + 3*a^3*b^6)*cos(d*x + c)^6 + 2*(a^8*
b + 8*a^7*b^2 + 25*a^6*b^3 + 40*a^5*b^4 + 35*a^4*b^5 + 16*a^3*b^6 + 3*a^2*b
^7)*cos(d*x + c)^5 - (2*a^9 + 16*a^8*b + 49*a^7*b^2 + 72*a^6*b^3 + 45*a^5*b
^4 - 8*a^4*b^5 - 29*a^3*b^6 - 16*a^2*b^7 - 3*a*b^8)*cos(d*x + c)^4 - 4*(a^8
*b + 8*a^7*b^2 + 25*a^6*b^3 + 40*a^5*b^4 + 35*a^4*b^5 + 16*a^3*b^6 + 3*a^2*
b^7)*cos(d*x + c)^3 + (a^9 + 8*a^8*b + 23*a^7*b^2 + 24*a^6*b^3 - 15*a^5*b^4
 - 64*a^4*b^5 - 67*a^3*b^6 - 32*a^2*b^7 - 6*a*b^8)*cos(d*x + c)^2 + 2*(a^8*
b + 8*a^7*b^2 + 25*a^6*b^3 + 40*a^5*b^4 + 35*a^4*b^5 + 16*a^3*b^6 + 3*a^2*b
^7)*cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 3*(a^7*b^2 - 8*a^6*b^3 + 25
*a^5*b^4 - 40*a^4*b^5 + 35*a^3*b^6 - 16*a^2*b^7 + 3*a*b^8 + (a^9 - 8*a^8*b
+ 25*a^7*b^2 - 40*a^6*b^3 + 35*a^5*b^4 - 16*a^4*b^5 + 3*a^3*b^6)*cos(d*x +
c)^6 + 2*(a^8*b - 8*a^7*b^2 + 25*a^6*b^3 - 40*a^5*b^4 + 35*a^4*b^5 - 16*a^3
*b^6 + 3*a^2*b^7)*cos(d*x + c)^5 - (2*a^9 - 16*a^8*b + 49*a^7*b^2 - 72*a^6*
b^3 + 45*a^5*b^4 + 8*a^4*b^5 - 29*a^3*b^6 + 16*a^2*b^7 - 3*a*b^8)*cos(d*x +
 c)^4 - 4*(a^8*b - 8*a^7*b^2 + 25*a^6*b^3 - 40*a^5*b^4 + 35*a^4*b^5 - 16*a^
3*b^6 + 3*a^2*b^7)*cos(d*x + c)^3 + (a^9 - 8*a^8*b + 23*a^7*b^2 - 24*a^6*b^
3 - 15*a^5*b^4 + 64*a^4*b^5 - 67*a^3*b^6 + 32*a^2*b^7 - 6*a*b^8)*cos(d*x +
c)^2 + 2*(a^8*b - 8*a^7*b^2 + 25*a^6*b^3 - 40*a^5*b^4 + 35*a^4*b^5 - 16*a^3
*b^6 + 3*a^2*b^7)*cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2))/((a^12 - 5*a^
10*b^2 + 10*a^8*b^4 - 10*a^6*b^6 + 5*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^6 +
 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*co
s(d*x + c)^5 - (2*a^12 - 11*a^10*b^2 + 25*a^8*b^4 - 30*a^6*b^6 + 20*a^4*b^8
 - 7*a^2*b^10 + b^12)*d*cos(d*x + c)^4 - 4*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5
 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x + c)^3 + (a^12 - 7*a^10*b^2 +
 20*a^8*b^4 - 30*a^6*b^6 + 25*a^4*b^8 - 11*a^2*b^10 + 2*b^12)*d*cos(d*x + c
)^2 + 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)
*d*cos(d*x + c) + (a^10*b^2 - 5*a^8*b^4 + 10*a^6*b^6 - 10*a^4*b^8 + 5*a^2*b
^10 - b^12)*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.60742, size = 2094, normalized size = 6.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/64*(12*(a^2 - 3*a*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/(a
^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5) + 192*(a^6*b + 5*a^
4*b^3 + 2*a^2*b^5)*log(abs(-a - b - a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 + b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/(a^10 - 5*a^8*b^2 + 10*a^6*b^4
 - 10*a^4*b^6 + 5*a^2*b^8 - b^10) - (8*a^3*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) - 12*a^2*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 4*b^3*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) - a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
 3*a^2*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 3*a*b^2*(cos(d*x + c)
- 1)^2/(cos(d*x + c) + 1)^2 + b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
)/(a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6) -
(a^8 - 2*a^7*b - 2*a^6*b^2 + 6*a^5*b^3 - 6*a^3*b^5 + 2*a^2*b^6 + 2*a*b^7 -
b^8 - 6*a^8*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 20*a^7*b*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - 12*a^6*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
- 28*a^5*b^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 40*a^4*b^4*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 4*a^3*b^5*(cos(d*x + c) - 1)/(cos(d*x + c) + 1
) - 20*a^2*b^6*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 12*a*b^7*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 2*b^8*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) -
6*a^8*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 163*a^7*b*(cos(d*x + c) -
 1)^2/(cos(d*x + c) + 1)^2 - 257*a^6*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c)
 + 1)^2 + 339*a^5*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 203*a^4*b
^4*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 223*a^3*b^5*(cos(d*x + c) -
1)^2/(cos(d*x + c) + 1)^2 + 309*a^2*b^6*(cos(d*x + c) - 1)^2/(cos(d*x + c)
+ 1)^2 - 23*a*b^7*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 7*b^8*(cos(d*
x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 10*a^8*(cos(d*x + c) - 1)^3/(cos(d*x +
 c) + 1)^3 + 186*a^7*b*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 274*a^6*
b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 890*a^5*b^3*(cos(d*x + c) -
 1)^3/(cos(d*x + c) + 1)^3 - 894*a^4*b^4*(cos(d*x + c) - 1)^3/(cos(d*x + c)
 + 1)^3 + 478*a^3*b^5*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 374*a^2*b
^6*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 18*a*b^7*(cos(d*x + c) - 1)^
3/(cos(d*x + c) + 1)^3 - 4*b^8*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 +
9*a^8*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 45*a^7*b*(cos(d*x + c) -
1)^4/(cos(d*x + c) + 1)^4 + 45*a^6*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) +
 1)^4 - 63*a^5*b^3*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 117*a^4*b^4*
(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 - 9*a^3*b^5*(cos(d*x + c) - 1)^4/
(cos(d*x + c) + 1)^4 + 63*a^2*b^6*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4
 + 27*a*b^7*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/((a^9 - a^8*b - 4*a^
7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 - 6*a^4*b^5 - 4*a^3*b^6 + 4*a^2*b^7 + a*b^8 -
 b^9)*(a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + b*(cos(d*x + c) - 1)/(cos(
d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - b*(cos(d*x +
c) - 1)^2/(cos(d*x + c) + 1)^2)^2))/d
```

3.228 $\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=539

$$\frac{b\left(-985a^2b^2+213a^4+840b^4\right)\sin(c+dx)}{30a^8d}+\frac{\left(-60a^2b^2+9a^4+56b^4\right)\sin(c+dx)\cos^5(c+dx)}{60a^3b^2d(a\cos(c+dx)+b)^2}+\frac{\left(-110a^2b^2+15a^4+\right.}{20a^4b^2d}$$

```
[Out] ((5*a^6 - 180*a^4*b^2 + 600*a^2*b^4 - 448*b^6)*x)/(16*a^9) - (Sqrt[a - b]*b
*Sqrt[a + b]*(6*a^4 - 47*a^2*b^2 + 56*b^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*
x)/2])/Sqrt[a + b]])/(a^9*d) + (b*(213*a^4 - 985*a^2*b^2 + 840*b^4)*Sin[c +
 d*x])/(30*a^8*d) - ((43*a^4 - 244*a^2*b^2 + 224*b^4)*Cos[c + d*x]*Sin[c +
d*x])/(16*a^7*d) + ((45*a^4 - 291*a^2*b^2 + 280*b^4)*Cos[c + d*x]^2*Sin[c +
 d*x])/(30*a^6*b*d) - ((24*a^4 - 169*a^2*b^2 + 168*b^4)*Cos[c + d*x]^3*Sin[
c + d*x])/(24*a^5*b^2*d) - (Cos[c + d*x]^4*Sin[c + d*x])/(4*b*d*(b + a*Cos[
c + d*x])^2) + (a*Cos[c + d*x]^5*Sin[c + d*x])/(10*b^2*d*(b + a*Cos[c + d*x
])^2) + ((9*a^4 - 60*a^2*b^2 + 56*b^4)*Cos[c + d*x]^5*Sin[c + d*x])/(60*a^3
*b^2*d*(b + a*Cos[c + d*x])^2) + (4*b*Cos[c + d*x]^6*Sin[c + d*x])/(15*a^2*
d*(b + a*Cos[c + d*x])^2) - (Cos[c + d*x]^7*Sin[c + d*x])/(6*a*d*(b + a*Cos
[c + d*x])^2) + ((15*a^4 - 110*a^2*b^2 + 112*b^4)*Cos[c + d*x]^4*Sin[c + d*
x])/(20*a^4*b^2*d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 2.43567, antiderivative size = 539, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2896, 3047, 3049, 3023, 2735, 2659, 208}

$$\frac{b\left(-985a^2b^2+213a^4+840b^4\right)\sin(c+dx)}{30a^8d}+\frac{\left(-60a^2b^2+9a^4+56b^4\right)\sin(c+dx)\cos^5(c+dx)}{60a^3b^2d(a\cos(c+dx)+b)^2}+\frac{\left(-110a^2b^2+15a^4+\right.}{20a^4b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((5*a^6 - 180*a^4*b^2 + 600*a^2*b^4 - 448*b^6)*x)/(16*a^9) - (Sqrt[a - b]*b
*Sqrt[a + b]*(6*a^4 - 47*a^2*b^2 + 56*b^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*
x)/2])/Sqrt[a + b]])/(a^9*d) + (b*(213*a^4 - 985*a^2*b^2 + 840*b^4)*Sin[c +
 d*x])/(30*a^8*d) - ((43*a^4 - 244*a^2*b^2 + 224*b^4)*Cos[c + d*x]*Sin[c +
d*x])/(16*a^7*d) + ((45*a^4 - 291*a^2*b^2 + 280*b^4)*Cos[c + d*x]^2*Sin[c +
 d*x])/(30*a^6*b*d) - ((24*a^4 - 169*a^2*b^2 + 168*b^4)*Cos[c + d*x]^3*Sin[
c + d*x])/(24*a^5*b^2*d) - (Cos[c + d*x]^4*Sin[c + d*x])/(4*b*d*(b + a*Cos[
c + d*x])^2) + (a*Cos[c + d*x]^5*Sin[c + d*x])/(10*b^2*d*(b + a*Cos[c + d*x
])^2) + ((9*a^4 - 60*a^2*b^2 + 56*b^4)*Cos[c + d*x]^5*Sin[c + d*x])/(60*a^3
*b^2*d*(b + a*Cos[c + d*x])^2) + (4*b*Cos[c + d*x]^6*Sin[c + d*x])/(15*a^2*
d*(b + a*Cos[c + d*x])^2) - (Cos[c + d*x]^7*Sin[c + d*x])/(6*a*d*(b + a*Cos
[c + d*x])^2) + ((15*a^4 - 110*a^2*b^2 + 112*b^4)*Cos[c + d*x]^4*Sin[c + d*
x])/(20*a^4*b^2*d*(b + a*Cos[c + d*x]))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2896

```
Int[cos[(e_.) + (f_.)*(x_)]^6*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[(Cos[e + f*x]*(d*Sin
[e + f*x])^(n + 1)*(a + b*Sin[e + f*x])^(m + 1))/(a*d*f*(n + 1)), x] + (Dis
t[1/(a^2*b^2*d^2*(n + 1)*(n + 2)*(m + n + 5)*(m + n + 6)), Int[(d*Sin[e + f
*x])^(n + 2)*(a + b*Sin[e + f*x])^m*Simp[a^4*(n + 1)*(n + 2)*(n + 3)*(n + 5
) - a^2*b^2*(n + 2)*(2*n + 1)*(m + n + 5)*(m + n + 6) + b^4*(m + n + 2)*(m
+ n + 3)*(m + n + 5)*(m + n + 6) + a*b*m*(a^2*(n + 1)*(n + 2) - b^2*(m + n
+ 5)*(m + n + 6))*Sin[e + f*x] - (a^4*(n + 1)*(n + 2)*(4 + n)*(n + 5) + b^4
*(m + n + 2)*(m + n + 4)*(m + n + 5)*(m + n + 6) - a^2*b^2*(n + 1)*(n + 2)*
(m + n + 5)*(2*n + 2*m + 13))*Sin[e + f*x]^2, x], x], x] - Simp[(b*(m + n +
 2)*Cos[e + f*x]*(d*Sin[e + f*x])^(n + 2)*(a + b*Sin[e + f*x])^(m + 1))/(a^
2*d^2*f*(n + 1)*(n + 2)), x] - Simp[(a*(n + 5)*Cos[e + f*x]*(d*Sin[e + f*x]
)^(n + 3)*(a + b*Sin[e + f*x])^(m + 1))/(b^2*d^3*f*(m + n + 5)*(m + n + 6))
, x] + Simp[(Cos[e + f*x]*(d*Sin[e + f*x])^(n + 4)*(a + b*Sin[e + f*x])^(m
+ 1))/(b*d^4*f*(m + n + 6)), x]) /; FreeQ[{a, b, d, e, f, m, n}, x] && NeQ[
a^2 - b^2, 0] && IntegersQ[2*m, 2*n] && NeQ[n, -1] && NeQ[n, -2] && NeQ[m +
 n + 5, 0] && NeQ[m + n + 6, 0] &&  !IGtQ[m, 0]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> -Simp[((c^2*C - B*c*d + A*d^2)*Cos[e + f*x]
*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d
^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 1)
*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d)*
(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1)
- a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] +
 b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0
] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^6(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{4bd(b+a\cos(c+dx))^2} + \frac{a\cos^5(c+dx)\sin(c+dx)}{10b^2d(b+a\cos(c+dx))^2} + \frac{4b\cos^6(c+dx)\sin(c+dx)}{15a^2d(b+a\cos(c+dx))^2} - \cdots$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{4bd(b+a\cos(c+dx))^2} + \frac{a\cos^5(c+dx)\sin(c+dx)}{10b^2d(b+a\cos(c+dx))^2} + \frac{\left(9a^4-60a^2b^2+56b^4\right)\cos^5(c+\cdots}{60a^3b^2d(b+a\cos(c-\cdots}$$

$$= -\frac{\cos^4(c+dx)\sin(c+dx)}{4bd(b+a\cos(c+dx))^2} + \frac{a\cos^5(c+dx)\sin(c+dx)}{10b^2d(b+a\cos(c+dx))^2} + \frac{\left(9a^4-60a^2b^2+56b^4\right)\cos^5(c+\cdots}{60a^3b^2d(b+a\cos(c-\cdots}$$

$$= -\frac{\left(24a^4-169a^2b^2+168b^4\right)\cos^3(c+dx)\sin(c+dx)}{24a^5b^2d} - \frac{\cos^4(c+dx)\sin(c+dx)}{4bd(b+a\cos(c+dx))^2} + \frac{a\cos^5\cdots}{10b^2d\cdots}$$

$$= \frac{\left(45a^4-291a^2b^2+280b^4\right)\cos^2(c+dx)\sin(c+dx)}{30a^6bd} - \frac{\left(24a^4-169a^2b^2+168b^4\right)\cos^3(c+\cdots}{24a^5b^2d}$$

$$= -\frac{\left(43a^4-244a^2b^2+224b^4\right)\cos(c+dx)\sin(c+dx)}{16a^7d} + \frac{\left(45a^4-291a^2b^2+280b^4\right)\cos^2(c+\cdots}{30a^6bd}$$

$$= \frac{b\left(213a^4-985a^2b^2+840b^4\right)\sin(c+dx)}{30a^8d} - \frac{\left(43a^4-244a^2b^2+224b^4\right)\cos(c+dx)\sin(c+\cdots}{16a^7d}$$

$$= \frac{\left(5a^6-180a^4b^2+600a^2b^4-448b^6\right)x}{16a^9} + \frac{b\left(213a^4-985a^2b^2+840b^4\right)\sin(c+dx)}{30a^8d} - \frac{\left(43a\cdots\right.}{}$$

$$= \frac{\left(5a^6-180a^4b^2+600a^2b^4-448b^6\right)x}{16a^9} + \frac{b\left(213a^4-985a^2b^2+840b^4\right)\sin(c+dx)}{30a^8d} - \frac{\left(43a\cdots\right.}{}$$

$$= \frac{\left(5a^6-180a^4b^2+600a^2b^4-448b^6\right)x}{16a^9} - \frac{\sqrt{a-b}b\sqrt{a+b}\left(6a^4-47a^2b^2+56b^4\right)\tanh^{-1}\left(\cdots\right.}{a^9d}$$

Mathematica [A] time = 12.1435, size = 599, normalized size = 1.11

$$2\left(a^2-b^2\right)^{5/2}\left(24600a^6b^2\sin(2(c+dx))+1164a^6b^2\sin(4(c+dx))-56a^6b^2\sin(6(c+dx))+16160a^5b^3\sin(c+dx)-\cdots\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-7680*b*(-a^2 + b^2)^3*(6*a^4 - 47*a^2*b^2 + 56*b^4)*ArcTanh[((-a + b)*Tan
[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2 + 2*(a^2 - b^2)^(5/2
)*(600*a^8*c - 20400*a^6*b^2*c + 28800*a^4*b^4*c + 90240*a^2*b^6*c - 107520
*b^8*c + 600*a^8*d*x - 20400*a^6*b^2*d*x + 28800*a^4*b^4*d*x + 90240*a^2*b^
6*d*x - 107520*b^8*d*x + 480*a*b*(5*a^6 - 180*a^4*b^2 + 600*a^2*b^4 - 448*b
^6)*(c + d*x)*Cos[c + d*x] + 120*a^2*(5*a^6 - 180*a^4*b^2 + 600*a^2*b^4 - 4
48*b^6)*(c + d*x)*Cos[2*(c + d*x)] + 2640*a^7*b*Sin[c + d*x] + 16160*a^5*b^
3*Sin[c + d*x] - 117120*a^3*b^5*Sin[c + d*x] + 107520*a*b^7*Sin[c + d*x] -
405*a^8*Sin[2*(c + d*x)] + 24600*a^6*b^2*Sin[2*(c + d*x)] - 99040*a^4*b^4*S
in[2*(c + d*x)] + 80640*a^2*b^6*Sin[2*(c + d*x)] + 2436*a^7*b*Sin[3*(c + d*
x)] - 10880*a^5*b^3*Sin[3*(c + d*x)] + 8960*a^3*b^5*Sin[3*(c + d*x)] - 140*
a^8*Sin[4*(c + d*x)] + 1164*a^6*b^2*Sin[4*(c + d*x)] - 1120*a^4*b^4*Sin[4*(
c + d*x)] - 188*a^7*b*Sin[5*(c + d*x)] + 224*a^5*b^3*Sin[5*(c + d*x)] + 35*
a^8*Sin[6*(c + d*x)] - 56*a^6*b^2*Sin[6*(c + d*x)] + 16*a^7*b*Sin[7*(c + d*
x)] - 5*a^8*Sin[8*(c + d*x)]))/(7680*a^9*(a - b)^2*(a + b)^2*Sqrt[a^2 - b^2
]*d*(b + a*Cos[c + d*x])^2)
```

Maple [B] time = 0.096, size = 2251, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -45/2/d/a^5*arctan(tan(1/2*d*x+1/2*c))*b^2+75/d/a^7*arctan(tan(1/2*d*x+1/2*
c))*b^4+5/8/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11+85/24/d/
a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^9+33/4/d/a^3/(1+tan(1/2*d
*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7-33/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*t
an(1/2*d*x+1/2*c)^5-85/24/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*
c)^3-5/8/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)-56/d/a^9*arcta
n(tan(1/2*d*x+1/2*c))*b^6+15/d*b^6/a^7/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+
1/2*c)^2*b-a-b)^2*tan(1/2*d*x+1/2*c)-680/3/d/a^6/(1+tan(1/2*d*x+1/2*c)^2)^6
*tan(1/2*d*x+1/2*c)^9*b^3-21/2/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x
+1/2*c)^11*b^2+6/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11*b-3
3/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7*b^2+87/2/d/a^5/(1+t
an(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*b^2-680/3/d/a^6/(1+tan(1/2*d*x+
1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*b^3+420/d/a^8/(1+tan(1/2*d*x+1/2*c)^2)^6*t
an(1/2*d*x+1/2*c)^5*b^5+516/5/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+
1/2*c)^5*b-480/d/a^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*b^3+21
/d*b^4/a^5/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*tan(1/2*d*
x+1/2*c)^3+5/8/d/a^3*arctan(tan(1/2*d*x+1/2*c))+38/d/a^4/(1+tan(1/2*d*x+1/2
*c)^2)^6*tan(1/2*d*x+1/2*c)^3*b+210/d/a^8/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/
2*d*x+1/2*c)^9*b^5-480/d/a^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^
7*b^3+5/d*b^3/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*tan
(1/2*d*x+1/2*c)^3-19/d*b^5/a^6/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2
*b-a-b)^2*tan(1/2*d*x+1/2*c)^3-15/d*b^6/a^7/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2
*d*x+1/2*c)^2*b-a-b)^2*tan(1/2*d*x+1/2*c)^3+5/d*b^3/a^4/(tan(1/2*d*x+1/2*c)
^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*tan(1/2*d*x+1/2*c)-19/d*b^5/a^6/(tan(1/2
*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*tan(1/2*d*x+1/2*c)+14/d*b^7/a
^8/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*tan(1/2*d*x+1/2*c)
+6/d*b^2/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*tan(1/2*
d*x+1/2*c)-21/d*b^4/a^5/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)
```

```
^2*tan(1/2*d*x+1/2*c)+45/d/a^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c
)^9*b^4-40/d/a^6/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11*b^3+15/d/
a^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11*b^4+42/d/a^8/(1+tan(1/
2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^11*b^5+38/d/a^4/(1+tan(1/2*d*x+1/2*c)^
2)^6*tan(1/2*d*x+1/2*c)^9*b-87/2/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d
*x+1/2*c)^9*b^2-45/d/a^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*b^
4+210/d/a^8/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^3*b^5+6/d/a^4/(1+
tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)*b-40/d/a^6/(1+tan(1/2*d*x+1/2*c)
^2)^6*tan(1/2*d*x+1/2*c)*b^3+516/5/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2
*d*x+1/2*c)^7*b+30/d/a^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^7*b^
4+33/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^5*b^2+21/2/d/a^5/(
1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)*b^2-15/d/a^7/(1+tan(1/2*d*x+1/
2*c)^2)^6*tan(1/2*d*x+1/2*c)*b^4-30/d/a^7/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/
2*d*x+1/2*c)^5*b^4+420/d/a^8/(1+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)^
7*b^5-6/d*b/a^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)
*(a-b))^(1/2))+53/d*b^3/a^5/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1
/2*c)/((a+b)*(a-b))^(1/2))-103/d*b^5/a^7/((a+b)*(a-b))^(1/2)*arctanh((a-b)*
tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+56/d*b^7/a^9/((a+b)*(a-b))^(1/2)*ar
ctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-6/d*b^2/a^3/(tan(1/2*d*
x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*tan(1/2*d*x+1/2*c)^3+42/d/a^8/(1
+tan(1/2*d*x+1/2*c)^2)^6*tan(1/2*d*x+1/2*c)*b^5+14/d*b^7/a^8/(tan(1/2*d*x+1
/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*tan(1/2*d*x+1/2*c)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.95001, size = 2515, normalized size = 4.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/240*(15*(5*a^8 - 180*a^6*b^2 + 600*a^4*b^4 - 448*a^2*b^6)*d*x*cos(d*x +
c)^2 + 30*(5*a^7*b - 180*a^5*b^3 + 600*a^3*b^5 - 448*a*b^7)*d*x*cos(d*x + c
) + 15*(5*a^6*b^2 - 180*a^4*b^4 + 600*a^2*b^6 - 448*b^8)*d*x + 60*(6*a^4*b^
3 - 47*a^2*b^5 + 56*b^7 + (6*a^6*b - 47*a^4*b^3 + 56*a^2*b^5)*cos(d*x + c)^
2 + 2*(6*a^5*b^2 - 47*a^3*b^4 + 56*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*log
((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*
cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*c
os(d*x + c) + b^2)) - (40*a^8*cos(d*x + c)^7 - 64*a^7*b*cos(d*x + c)^6 - 17
04*a^5*b^3 + 7880*a^3*b^5 - 6720*a*b^7 - 2*(65*a^8 - 56*a^6*b^2)*cos(d*x +
c)^5 + 4*(67*a^7*b - 56*a^5*b^3)*cos(d*x + c)^4 + (165*a^8 - 694*a^6*b^2 +
560*a^4*b^4)*cos(d*x + c)^3 - 2*(387*a^7*b - 1444*a^5*b^3 + 1120*a^3*b^5)*c
os(d*x + c)^2 - (2763*a^6*b^2 - 12100*a^4*b^4 + 10080*a^2*b^6)*cos(d*x + c)
)*sin(d*x + c))/(a^11*d*cos(d*x + c)^2 + 2*a^10*b*d*cos(d*x + c) + a^9*b^2*
d), 1/240*(15*(5*a^8 - 180*a^6*b^2 + 600*a^4*b^4 - 448*a^2*b^6)*d*x*cos(d*x
```

```
 + c)^2 + 30*(5*a^7*b - 180*a^5*b^3 + 600*a^3*b^5 - 448*a*b^7)*d*x*cos(d*x
+ c) + 15*(5*a^6*b^2 - 180*a^4*b^4 + 600*a^2*b^6 - 448*b^8)*d*x - 120*(6*a^
4*b^3 - 47*a^2*b^5 + 56*b^7 + (6*a^6*b - 47*a^4*b^3 + 56*a^2*b^5)*cos(d*x +
 c)^2 + 2*(6*a^5*b^2 - 47*a^3*b^4 + 56*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2
)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))
 - (40*a^8*cos(d*x + c)^7 - 64*a^7*b*cos(d*x + c)^6 - 1704*a^5*b^3 + 7880*a
^3*b^5 - 6720*a*b^7 - 2*(65*a^8 - 56*a^6*b^2)*cos(d*x + c)^5 + 4*(67*a^7*b
- 56*a^5*b^3)*cos(d*x + c)^4 + (165*a^8 - 694*a^6*b^2 + 560*a^4*b^4)*cos(d*
x + c)^3 - 2*(387*a^7*b - 1444*a^5*b^3 + 1120*a^3*b^5)*cos(d*x + c)^2 - (27
63*a^6*b^2 - 12100*a^4*b^4 + 10080*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/(a^
11*d*cos(d*x + c)^2 + 2*a^10*b*d*cos(d*x + c) + a^9*b^2*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.72462, size = 1391, normalized size = 2.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/240*(15*(5*a^6 - 180*a^4*b^2 + 600*a^2*b^4 - 448*b^6)*(d*x + c)/a^9 - 240
*(6*a^6*b - 53*a^4*b^3 + 103*a^2*b^5 - 56*b^7)*(pi*floor(1/2*(d*x + c)/pi +
 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1
/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^9) - 240*(6*a^5*b^2*tan(1/2*d
*x + 1/2*c)^3 - 5*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 21*a^3*b^4*tan(1/2*d*x +
 1/2*c)^3 + 19*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 15*a*b^6*tan(1/2*d*x + 1/2*
c)^3 - 14*b^7*tan(1/2*d*x + 1/2*c)^3 - 6*a^5*b^2*tan(1/2*d*x + 1/2*c) - 5*a
^4*b^3*tan(1/2*d*x + 1/2*c) + 21*a^3*b^4*tan(1/2*d*x + 1/2*c) + 19*a^2*b^5*
tan(1/2*d*x + 1/2*c) - 15*a*b^6*tan(1/2*d*x + 1/2*c) - 14*b^7*tan(1/2*d*x +
 1/2*c))/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2*a
^8) + 2*(75*a^5*tan(1/2*d*x + 1/2*c)^11 + 720*a^4*b*tan(1/2*d*x + 1/2*c)^11
 - 1260*a^3*b^2*tan(1/2*d*x + 1/2*c)^11 - 4800*a^2*b^3*tan(1/2*d*x + 1/2*c)
^11 + 1800*a*b^4*tan(1/2*d*x + 1/2*c)^11 + 5040*b^5*tan(1/2*d*x + 1/2*c)^11
 + 425*a^5*tan(1/2*d*x + 1/2*c)^9 + 4560*a^4*b*tan(1/2*d*x + 1/2*c)^9 - 522
0*a^3*b^2*tan(1/2*d*x + 1/2*c)^9 - 27200*a^2*b^3*tan(1/2*d*x + 1/2*c)^9 + 5
400*a*b^4*tan(1/2*d*x + 1/2*c)^9 + 25200*b^5*tan(1/2*d*x + 1/2*c)^9 + 990*a
^5*tan(1/2*d*x + 1/2*c)^7 + 12384*a^4*b*tan(1/2*d*x + 1/2*c)^7 - 3960*a^3*b
^2*tan(1/2*d*x + 1/2*c)^7 - 57600*a^2*b^3*tan(1/2*d*x + 1/2*c)^7 + 3600*a*b
^4*tan(1/2*d*x + 1/2*c)^7 + 50400*b^5*tan(1/2*d*x + 1/2*c)^7 - 990*a^5*tan(
1/2*d*x + 1/2*c)^5 + 12384*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 3960*a^3*b^2*tan(
1/2*d*x + 1/2*c)^5 - 57600*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 3600*a*b^4*tan(
1/2*d*x + 1/2*c)^5 + 50400*b^5*tan(1/2*d*x + 1/2*c)^5 - 425*a^5*tan(1/2*d*x
 + 1/2*c)^3 + 4560*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 5220*a^3*b^2*tan(1/2*d*x
+ 1/2*c)^3 - 27200*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 5400*a*b^4*tan(1/2*d*x
```

```
+ 1/2*c)^3 + 25200*b^5*tan(1/2*d*x + 1/2*c)^3 - 75*a^5*tan(1/2*d*x + 1/2*c)
 + 720*a^4*b*tan(1/2*d*x + 1/2*c) + 1260*a^3*b^2*tan(1/2*d*x + 1/2*c) - 480
0*a^2*b^3*tan(1/2*d*x + 1/2*c) - 1800*a*b^4*tan(1/2*d*x + 1/2*c) + 5040*b^5
*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^6*a^8))/d
```

3.229 $\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=333

$$\frac{b\left(13a^2-30b^2\right)\sin(c+dx)}{2a^6d}+\frac{\left(2a^2-7b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2b^2d(a\cos(c+dx)+b)}-\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2bd(a\cos(c+dx)+b)^2}-\frac{\left(4a^2-15b^2\right)\mathrm{si}}{}$$

```
[Out] (3*(a^4 - 24*a^2*b^2 + 40*b^4)*x)/(8*a^7) - (3*b*(2*a^4 - 11*a^2*b^2 + 10*b
^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^7*Sqrt[a - b]*S
qrt[a + b]*d) + (b*(13*a^2 - 30*b^2)*Sin[c + d*x])/(2*a^6*d) - (3*(7*a^2 -
20*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*a^5*d) + ((3*a^2 - 10*b^2)*Cos[c + d*
x]^2*Sin[c + d*x])/(2*a^4*b*d) - ((4*a^2 - 15*b^2)*Cos[c + d*x]^3*Sin[c + d
*x])/(4*a^3*b^2*d) - ((a^2 - b^2)*Cos[c + d*x]^4*Sin[c + d*x])/(2*a^2*b*d*(
b + a*Cos[c + d*x])^2) + ((2*a^2 - 7*b^2)*Cos[c + d*x]^4*Sin[c + d*x])/(2*a
^2*b^2*d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 1.13839, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3872, 2891, 3049, 3023, 2735, 2659, 208}

$$\frac{b\left(13a^2-30b^2\right)\sin(c+dx)}{2a^6d}+\frac{\left(2a^2-7b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2b^2d(a\cos(c+dx)+b)}-\frac{\left(a^2-b^2\right)\sin(c+dx)\cos^4(c+dx)}{2a^2bd(a\cos(c+dx)+b)^2}-\frac{\left(4a^2-15b^2\right)\mathrm{si}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```


```
[Out] (3*(a^4 - 24*a^2*b^2 + 40*b^4)*x)/(8*a^7) - (3*b*(2*a^4 - 11*a^2*b^2 + 10*b
^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^7*Sqrt[a - b]*S
qrt[a + b]*d) + (b*(13*a^2 - 30*b^2)*Sin[c + d*x])/(2*a^6*d) - (3*(7*a^2 -
20*b^2)*Cos[c + d*x]*Sin[c + d*x])/(8*a^5*d) + ((3*a^2 - 10*b^2)*Cos[c + d*
x]^2*Sin[c + d*x])/(2*a^4*b*d) - ((4*a^2 - 15*b^2)*Cos[c + d*x]^3*Sin[c + d
*x])/(4*a^3*b^2*d) - ((a^2 - b^2)*Cos[c + d*x]^4*Sin[c + d*x])/(2*a^2*b*d*(
b + a*Cos[c + d*x])^2) + ((2*a^2 - 7*b^2)*Cos[c + d*x]^4*Sin[c + d*x])/(2*a
^2*b^2*d*(b + a*Cos[c + d*x]))
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2891

```
Int[cos[(e_.) + (f_.)*(x_)]^4*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Simp[((a^2 - b^2)*Cos[e +
 f*x]*(a + b*Sin[e + f*x])^(m + 1)*(d*Sin[e + f*x])^(n + 1))/(a*b^2*d*f*(m
+ 1)), x] + (-Dist[1/(a^2*b^2*(m + 1)*(m + 2)), Int[(a + b*Sin[e + f*x])^(m
 + 2)*(d*Sin[e + f*x])^n*Simp[a^2*(n + 1)*(n + 3) - b^2*(m + n + 2)*(m + n
+ 3) + a*b*(m + 2)*Sin[e + f*x] - (a^2*(n + 2)*(n + 3) - b^2*(m + n + 2)*(m
 + n + 4))*Sin[e + f*x]^2, x], x], x] + Simp[((a^2*(n - m + 1) - b^2*(m + n
 + 2))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 2)*(d*Sin[e + f*x])^(n + 1))/
(a^2*b^2*d*f*(m + 1)*(m + 2)), x]) /; FreeQ[{a, b, d, e, f, n}, x] && NeQ[a
```

```
^2 - b^2, 0] && IntegersQ[2*m, 2*n] && LtQ[m, -1] &&  !LtQ[n, -1] && (LtQ[m
, -2] || EqQ[m + n + 4, 0])
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^4(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$

$$= -\frac{(a^2-b^2)\cos^4(c+dx)\sin(c+dx)}{2a^2bd(b+a\cos(c+dx))^2} + \frac{(2a^2-7b^2)\cos^4(c+dx)\sin(c+dx)}{2a^2b^2d(b+a\cos(c+dx))} + \int \frac{\cos^3(c+dx)(-6(\ldots}{\ldots}$$

$$= -\frac{(4a^2-15b^2)\cos^3(c+dx)\sin(c+dx)}{4a^3b^2d} - \frac{(a^2-b^2)\cos^4(c+dx)\sin(c+dx)}{2a^2bd(b+a\cos(c+dx))^2} + \frac{(2a^2-7b^2)\,c\ldots}{2a^2b^2d(\ldots}$$

$$= \frac{(3a^2-10b^2)\cos^2(c+dx)\sin(c+dx)}{2a^4bd} - \frac{(4a^2-15b^2)\cos^3(c+dx)\sin(c+dx)}{4a^3b^2d} - \frac{(a^2-b^2)\,c\ldots}{2a^2bd(\ldots}$$

$$= -\frac{3(7a^2-20b^2)\cos(c+dx)\sin(c+dx)}{8a^5d} + \frac{(3a^2-10b^2)\cos^2(c+dx)\sin(c+dx)}{2a^4bd} - \frac{(4a^2-15\ldots}{\ldots}$$

$$= \frac{b(13a^2-30b^2)\sin(c+dx)}{2a^6d} - \frac{3(7a^2-20b^2)\cos(c+dx)\sin(c+dx)}{8a^5d} + \frac{(3a^2-10b^2)\cos^2(c\ldots}{2a^4b\ldots}$$

$$= \frac{3(a^4-24a^2b^2+40b^4)x}{8a^7} + \frac{b(13a^2-30b^2)\sin(c+dx)}{2a^6d} - \frac{3(7a^2-20b^2)\cos(c+dx)\sin(c+\ldots}{8a^5d}$$

$$= \frac{3(a^4-24a^2b^2+40b^4)x}{8a^7} + \frac{b(13a^2-30b^2)\sin(c+dx)}{2a^6d} - \frac{3(7a^2-20b^2)\cos(c+dx)\sin(c+\ldots}{8a^5d}$$

$$= \frac{3(a^4-24a^2b^2+40b^4)x}{8a^7} - \frac{3b(2a^4-11a^2b^2+10b^4)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^7\sqrt{a-b}\sqrt{a+b}d} + \frac{b(13a^2-\ldots}{\ldots}$$

Mathematica [B] time = 9.26501, size = 1178, normalized size = 3.54

$$-\frac{6\left(8(c+dx)+\frac{2b(15a^4-20b^2a^2+8b^4)\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}}-\frac{3a(2a^4-7b^2a^2+4b^4)\sin(c+dx)}{(a-b)^2(a+b)^2(b+a\cos(c+dx))}+\frac{ab(3a^2-4b^2)\sin(c+dx)}{(a-b)(a+b)(b+a\cos(c+dx))^2}\right)}{a^3} + \frac{6\left(\frac{6ab\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{(b(a^2+\ldots}{\ldots}\right)}{(a-b)^2(a+\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((-6*(8*(c + d*x) + (2*b*(15*a^4 - 20*a^2*b^2 + 8*b^4)*ArcTanh[((-a + b)*Ta
n[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) + (a*b*(3*a^2 - 4*b^2)*
Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c + d*x])^2) - (3*a*(2*a^4 - 7*a^
2*b^2 + 4*b^4)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b + a*Cos[c + d*x]))))/a
^3 + (6*((6*a*b*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[
a^2 - b^2] + ((b*(a^2 + 2*b^2) + a*(2*a^2 + b^2)*Cos[c + d*x])*Sin[c + d*x]
)/(b + a*Cos[c + d*x])^2))/((a - b)^2*(a + b)^2) - (2*(-24*(a^2 - 8*b^2)*(c
 + d*x) + (6*b*(-35*a^6 + 140*a^4*b^2 - 168*a^2*b^4 + 64*b^6)*ArcTanh[((-a
+ b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(5/2) - 96*a*b*Sin[c +
 d*x] + (a*b*(-5*a^4 + 20*a^2*b^2 - 16*b^4)*Sin[c + d*x])/((a - b)*(a + b)*
(b + a*Cos[c + d*x])^2) + (a*(10*a^6 - 115*a^4*b^2 + 220*a^2*b^4 - 112*b^6)
*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b + a*Cos[c + d*x])) + 8*a^2*Sin[2*(c
+ d*x)]))/a^5 + ((12*b*(105*a^8 - 840*a^6*b^2 + 2016*a^4*b^4 - 1920*a^2*b^6
 + 640*b^8)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^
2)^(5/2) + (48*a^10*c - 960*a^8*b^2*c + 1776*a^6*b^4*c + 2976*a^4*b^6*c - 7
680*a^2*b^8*c + 3840*b^10*c + 48*a^10*d*x - 960*a^8*b^2*d*x + 1776*a^6*b^4*
```

```
d*x + 2976*a^4*b^6*d*x - 7680*a^2*b^8*d*x + 3840*b^10*d*x + 192*a*b*(a^2 -
b^2)^2*(a^4 - 20*a^2*b^2 + 40*b^4)*(c + d*x)*Cos[c + d*x] + 48*(a^3 - a*b^2
)^2*(a^4 - 20*a^2*b^2 + 40*b^4)*(c + d*x)*Cos[2*(c + d*x)] + 114*a^9*b*Sin[
c + d*x] + 788*a^7*b^3*Sin[c + d*x] - 5696*a^5*b^5*Sin[c + d*x] + 8640*a^3*
b^7*Sin[c + d*x] - 3840*a*b^9*Sin[c + d*x] - 36*a^10*Sin[2*(c + d*x)] + 122
1*a^8*b^2*Sin[2*(c + d*x)] - 5182*a^6*b^4*Sin[2*(c + d*x)] + 6880*a^4*b^6*S
in[2*(c + d*x)] - 2880*a^2*b^8*Sin[2*(c + d*x)] + 120*a^9*b*Sin[3*(c + d*x)
] - 560*a^7*b^3*Sin[3*(c + d*x)] + 760*a^5*b^5*Sin[3*(c + d*x)] - 320*a^3*b
^7*Sin[3*(c + d*x)] - 8*a^10*Sin[4*(c + d*x)] + 56*a^8*b^2*Sin[4*(c + d*x)]
 - 88*a^6*b^4*Sin[4*(c + d*x)] + 40*a^4*b^6*Sin[4*(c + d*x)] - 8*a^9*b*Sin[
5*(c + d*x)] + 16*a^7*b^3*Sin[5*(c + d*x)] - 8*a^5*b^5*Sin[5*(c + d*x)] + 2
*a^10*Sin[6*(c + d*x)] - 4*a^8*b^2*Sin[6*(c + d*x)] + 2*a^6*b^4*Sin[6*(c +
d*x)])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2))/a^7)/(256*d)
```

Maple [B] time = 0.086, size = 1227, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*sec(d*x+c))^3,x)
```

```
[Out] -18/d/a^5*arctan(tan(1/2*d*x+1/2*c))*b^2+30/d/a^7*arctan(tan(1/2*d*x+1/2*c)
)*b^4+11/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5-11/4/d/a^3
/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3-3/4/d/a^3/(1+tan(1/2*d*x+1
/2*c)^2)^4*tan(1/2*d*x+1/2*c)+3/4/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*
d*x+1/2*c)^7+11/d*b^4/a^5/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-
b)^2*tan(1/2*d*x+1/2*c)^3+6/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/
2*c)^7*b+26/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*b-60/d/a^
6/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*b^3+6/d/a^4/(1+tan(1/2*d*
x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*b-20/d/a^6/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(
1/2*d*x+1/2*c)*b^3+6/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)*b^
2-6/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*b^2-20/d/a^6/(1+t
an(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7*b^3+26/d/a^4/(1+tan(1/2*d*x+1/2
*c)^2)^4*tan(1/2*d*x+1/2*c)^5*b-6/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*
d*x+1/2*c)^5*b^2-60/d/a^6/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^5*b
^3+6/d/a^5/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3*b^2+3/4/d/a^3*ar
ctan(tan(1/2*d*x+1/2*c))+5/d*b^3/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/
2*c)^2*b-a-b)^2*tan(1/2*d*x+1/2*c)^3-10/d*b^5/a^6/(tan(1/2*d*x+1/2*c)^2*a-t
an(1/2*d*x+1/2*c)^2*b-a-b)^2*tan(1/2*d*x+1/2*c)^3+5/d*b^3/a^4/(tan(1/2*d*x+
1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*tan(1/2*d*x+1/2*c)-10/d*b^5/a^6/(t
an(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*tan(1/2*d*x+1/2*c)+6/d*
b^2/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*tan(1/2*d*x+1
/2*c)-11/d*b^4/a^5/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*ta
n(1/2*d*x+1/2*c)-6/d*b/a^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/
2*c)/((a+b)*(a-b))^(1/2))+33/d*b^3/a^5/((a+b)*(a-b))^(1/2)*arctanh((a-b)*ta
n(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-30/d*b^5/a^7/((a+b)*(a-b))^(1/2)*arct
anh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-6/d*b^2/a^3/(tan(1/2*d*x+
1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2*tan(1/2*d*x+1/2*c)^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.68988, size = 2331, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/8*(3*(a^8 - 25*a^6*b^2 + 64*a^4*b^4 - 40*a^2*b^6)*d*x*cos(d*x + c)^2 + 6
*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7)*d*x*cos(d*x + c) + 3*(a^6*b^2
 - 25*a^4*b^4 + 64*a^2*b^6 - 40*b^8)*d*x + 6*(2*a^4*b^3 - 11*a^2*b^5 + 10*b
^7 + (2*a^6*b - 11*a^4*b^3 + 10*a^2*b^5)*cos(d*x + c)^2 + 2*(2*a^5*b^2 - 11
*a^3*b^4 + 10*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c)
- (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin
(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) +
 (52*a^5*b^3 - 172*a^3*b^5 + 120*a*b^7 + 2*(a^8 - a^6*b^2)*cos(d*x + c)^5 -
 4*(a^7*b - a^5*b^3)*cos(d*x + c)^4 - 5*(a^8 - 3*a^6*b^2 + 2*a^4*b^4)*cos(d
*x + c)^3 + 2*(11*a^7*b - 31*a^5*b^3 + 20*a^3*b^5)*cos(d*x + c)^2 + (83*a^6
*b^2 - 263*a^4*b^4 + 180*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - a^9*
b^2)*d*cos(d*x + c)^2 + 2*(a^10*b - a^8*b^3)*d*cos(d*x + c) + (a^9*b^2 - a^
7*b^4)*d), 1/8*(3*(a^8 - 25*a^6*b^2 + 64*a^4*b^4 - 40*a^2*b^6)*d*x*cos(d*x
+ c)^2 + 6*(a^7*b - 25*a^5*b^3 + 64*a^3*b^5 - 40*a*b^7)*d*x*cos(d*x + c) +
3*(a^6*b^2 - 25*a^4*b^4 + 64*a^2*b^6 - 40*b^8)*d*x - 12*(2*a^4*b^3 - 11*a^2
*b^5 + 10*b^7 + (2*a^6*b - 11*a^4*b^3 + 10*a^2*b^5)*cos(d*x + c)^2 + 2*(2*a
^5*b^2 - 11*a^3*b^4 + 10*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt
(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (52*a^5*b^3
 - 172*a^3*b^5 + 120*a*b^7 + 2*(a^8 - a^6*b^2)*cos(d*x + c)^5 - 4*(a^7*b -
a^5*b^3)*cos(d*x + c)^4 - 5*(a^8 - 3*a^6*b^2 + 2*a^4*b^4)*cos(d*x + c)^3 +
2*(11*a^7*b - 31*a^5*b^3 + 20*a^3*b^5)*cos(d*x + c)^2 + (83*a^6*b^2 - 263*a
^4*b^4 + 180*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - a^9*b^2)*d*cos(d
*x + c)^2 + 2*(a^10*b - a^8*b^3)*d*cos(d*x + c) + (a^9*b^2 - a^7*b^4)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.49874, size = 788, normalized size = 2.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a^4 - 24*a^2*b^2 + 40*b^4)*(d*x + c)/a^7 - 24*(2*a^4*b - 11*a^2*b^3
 + 10*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*t
an(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2
 + b^2)*a^7) - 8*(6*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 5*a^2*b^3*tan(1/2*d*x
+ 1/2*c)^3 - 11*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 10*b^5*tan(1/2*d*x + 1/2*c)^
3 - 6*a^3*b^2*tan(1/2*d*x + 1/2*c) - 5*a^2*b^3*tan(1/2*d*x + 1/2*c) + 11*a*
b^4*tan(1/2*d*x + 1/2*c) + 10*b^5*tan(1/2*d*x + 1/2*c))/((a*tan(1/2*d*x + 1
/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2*a^6) + 2*(3*a^3*tan(1/2*d*x +
 1/2*c)^7 + 24*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 24*a*b^2*tan(1/2*d*x + 1/2*c)
^7 - 80*b^3*tan(1/2*d*x + 1/2*c)^7 + 11*a^3*tan(1/2*d*x + 1/2*c)^5 + 104*a^
2*b*tan(1/2*d*x + 1/2*c)^5 - 24*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 240*b^3*tan(
1/2*d*x + 1/2*c)^5 - 11*a^3*tan(1/2*d*x + 1/2*c)^3 + 104*a^2*b*tan(1/2*d*x
+ 1/2*c)^3 + 24*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 240*b^3*tan(1/2*d*x + 1/2*c)
^3 - 3*a^3*tan(1/2*d*x + 1/2*c) + 24*a^2*b*tan(1/2*d*x + 1/2*c) + 24*a*b^2*
tan(1/2*d*x + 1/2*c) - 80*b^3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^
2 + 1)^4*a^6))/d
```

3.230 $\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=267

$$\frac{b\left(11a^2-12b^2\right)\sin(c+dx)}{2a^4d\left(a^2-b^2\right)}+\frac{\left(3a^2-4b^2\right)\sin(c+dx)\cos^2(c+dx)}{2a^2d\left(a^2-b^2\right)(a\cos(c+dx)+b)}-\frac{\left(5a^2-6b^2\right)\sin(c+dx)\cos(c+dx)}{2a^3d\left(a^2-b^2\right)}-\frac{b\left(-19a^2b^2+\right.}{}$$

```
[Out] ((a^2 - 12*b^2)*x)/(2*a^5) - (b*(6*a^4 - 19*a^2*b^2 + 12*b^4)*ArcTanh[(Sqrt
[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(3/2)*(a + b)^(3/2)*d)
 + (b*(11*a^2 - 12*b^2)*Sin[c + d*x])/(2*a^4*(a^2 - b^2)*d) - ((5*a^2 - 6*b
^2)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*(a^2 - b^2)*d) + (Cos[c + d*x]^3*Sin[
c + d*x])/(2*a*d*(b + a*Cos[c + d*x])^2) + ((3*a^2 - 4*b^2)*Cos[c + d*x]^2*
Sin[c + d*x])/(2*a^2*(a^2 - b^2)*d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 0.940394, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2889, 3048, 3049, 3023, 2735, 2659, 208}

$$\frac{b\left(11a^2-12b^2\right)\sin(c+dx)}{2a^4d\left(a^2-b^2\right)}+\frac{\left(3a^2-4b^2\right)\sin(c+dx)\cos^2(c+dx)}{2a^2d\left(a^2-b^2\right)(a\cos(c+dx)+b)}-\frac{\left(5a^2-6b^2\right)\sin(c+dx)\cos(c+dx)}{2a^3d\left(a^2-b^2\right)}-\frac{b\left(-19a^2b^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```


```
[Out] ((a^2 - 12*b^2)*x)/(2*a^5) - (b*(6*a^4 - 19*a^2*b^2 + 12*b^4)*ArcTanh[(Sqrt
[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(3/2)*(a + b)^(3/2)*d)
 + (b*(11*a^2 - 12*b^2)*Sin[c + d*x])/(2*a^4*(a^2 - b^2)*d) - ((5*a^2 - 6*b
^2)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*(a^2 - b^2)*d) + (Cos[c + d*x]^3*Sin[
c + d*x])/(2*a*d*(b + a*Cos[c + d*x])^2) + ((3*a^2 - 4*b^2)*Cos[c + d*x]^2*
Sin[c + d*x])/(2*a^2*(a^2 - b^2)*d*(b + a*Cos[c + d*x]))
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2889

```
Int[cos[(e_.) + (f_.)*(x_)]^2*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_) +
(b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[(d*Sin[e + f*x])^n*(a
 + b*Sin[e + f*x])^m*(1 - Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e, f, m, n
}, x] && NeQ[a^2 - b^2, 0] && (IGtQ[m, 0] || IntegersQ[2*m, 2*n])
```

Rule 3048

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
-Simp[((c^2*C + A*d^2)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*(c + d*Sin[e + f
*x])^(n + 1))/(d*f*(n + 1)*(c^2 - d^2)), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)
), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(
```

```
b*d*m + a*c*(n + 1)) + c*C*(b*c*m + a*d*(n + 1)) - (A*d*(a*d*(n + 2) - b*c*
(n + 1)) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b*(A*d
^2*(m + n + 2) + C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x]
/; FreeQ[{a, b, c, d, e, f, A, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2
, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3049

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(a + b*Sin[e + f*x]
)^m*(c + d*Sin[e + f*x])^(n + 1))/(d*f*(m + n + 2)), x] + Dist[1/(d*(m + n
+ 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(
m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*c
- b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m + n
+ 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}, x
] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m,
0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3023

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> -Simp[(C*Cos
[e + f*x]*(a + b*Sin[e + f*x])^(m + 1))/(b*f*(m + 2)), x] + Dist[1/(b*(m +
2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m +
2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
  !LtQ[m, -1]
```

Rule 2735

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[(b*x)/d, x] - Dist[(b*c - a*d)/d, Int[1/(c + d*
Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cos^3(c+dx)\sin^2(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$

$$= -\int \frac{\cos^3(c+dx)\left(1-\cos^2(c+dx)\right)}{(-b-a\cos(c+dx))^3}\,dx$$

$$= \frac{\cos^3(c+dx)\sin(c+dx)}{2ad(b+a\cos(c+dx))^2} + \frac{\int \frac{\cos^2(c+dx)\left(3\left(a^2-b^2\right)-4\left(a^2-b^2\right)\cos^2(c+dx)\right)}{(-b-a\cos(c+dx))^2}dx}{2a\left(a^2-b^2\right)}$$

$$= \frac{\cos^3(c+dx)\sin(c+dx)}{2ad(b+a\cos(c+dx))^2} + \frac{\left(3a^2-4b^2\right)\cos^2(c+dx)\sin(c+dx)}{2a^2\left(a^2-b^2\right)d(b+a\cos(c+dx))} - \frac{\int \frac{\cos(c+dx)\left(2\left(3a^4-7a^2b^2+4b^4\right)\cdots\right.}{\cdots}}{\cdots}$$

$$= -\frac{\left(5a^2-6b^2\right)\cos(c+dx)\sin(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cos^3(c+dx)\sin(c+dx)}{2ad(b+a\cos(c+dx))^2} + \frac{\left(3a^2-4b^2\right)\cos^2(c+dx)\cdots}{2a^2\left(a^2-b^2\right)d(b+a\,\mathrm{co}\cdots}$$

$$= \frac{b\left(11a^2-12b^2\right)\sin(c+dx)}{2a^4\left(a^2-b^2\right)d} - \frac{\left(5a^2-6b^2\right)\cos(c+dx)\sin(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cos^3(c+dx)\sin(c+dx)}{2ad(b+a\cos(c+dx))^2}\cdots$$

$$= \frac{\left(a^2-12b^2\right)x}{2a^5} + \frac{b\left(11a^2-12b^2\right)\sin(c+dx)}{2a^4\left(a^2-b^2\right)d} - \frac{\left(5a^2-6b^2\right)\cos(c+dx)\sin(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cos^3(c\cdots}{2ad(b\cdots}$$

$$= \frac{\left(a^2-12b^2\right)x}{2a^5} + \frac{b\left(11a^2-12b^2\right)\sin(c+dx)}{2a^4\left(a^2-b^2\right)d} - \frac{\left(5a^2-6b^2\right)\cos(c+dx)\sin(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\cos^3(c\cdots}{2ad(b\cdots}$$

$$= \frac{\left(a^2-12b^2\right)x}{2a^5} - \frac{b\left(6a^4-19a^2b^2+12b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{3/2}(a+b)^{3/2}d} + \frac{b\left(11a^2-12b^2\right)\sin(c+\cdots}{2a^4\left(a^2-b^2\right)d}$$

Mathematica [A] time = 3.87882, size = 282, normalized size = 1.06

$$\frac{\frac{b^2\left(2\left(-13a^2b^2+a^4+12b^4\right)(c+dx)+\left(22a^3b-24ab^3\right)\sin(c+dx)+\left(17a^4-18a^2b^2\right)\sin(2(c+dx))\right)+4ab\left(-13a^2b^2+a^4+12b^4\right)(c+dx)\cos(c+dx)-2a^4\left(a^2-b^2\right)\sin(c+dx)\cos^3\cdots}{(a\cos(c+dx)+b)^2}}{4a^5d(a-b)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((4*b*(6*a^4 - 19*a^2*b^2 + 12*b^4)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqr
t[a^2 - b^2]])/Sqrt[a^2 - b^2] + (4*a*b*(a^4 - 13*a^2*b^2 + 12*b^4)*(c + d*
x)*Cos[c + d*x] - 2*a^4*(a^2 - b^2)*Cos[c + d*x]^3*Sin[c + d*x] + 2*a^2*(a^
2 - b^2)*Cos[c + d*x]^2*((a^2 - 12*b^2)*(c + d*x) + 4*a*b*Sin[c + d*x]) + b
^2*(2*(a^4 - 13*a^2*b^2 + 12*b^4)*(c + d*x) + (22*a^3*b - 24*a*b^3)*Sin[c +
 d*x] + (17*a^4 - 18*a^2*b^2)*Sin[2*(c + d*x)]))/(b + a*Cos[c + d*x])^2)/(4
*a^5*(a - b)*(a + b)*d)
```

Maple [B] time = 0.088, size = 729, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3+6/d/a^4/(1+tan(1/2*
d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3*b+6/d/a^4/(1+tan(1/2*d*x+1/2*c)^2)^2*t
an(1/2*d*x+1/2*c)*b-1/d/a^3/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)-1
2/d/a^5*arctan(tan(1/2*d*x+1/2*c))*b^2+1/d/a^3*arctan(tan(1/2*d*x+1/2*c))-6
/d*b^2/a^2/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)*tan(
1/2*d*x+1/2*c)^3-1/d*b^3/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b
-a-b)^2/(a+b)*tan(1/2*d*x+1/2*c)^3+6/d*b^4/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(
1/2*d*x+1/2*c)^2*b-a-b)^2/(a+b)*tan(1/2*d*x+1/2*c)^3+6/d*b^2/a^2/(tan(1/2*d
*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)*tan(1/2*d*x+1/2*c)-1/d*b^
3/a^3/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2/(a-b)*tan(1/2*d
*x+1/2*c)-6/d*b^4/a^4/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2
/(a-b)*tan(1/2*d*x+1/2*c)-6/d*b/a/(a^2-b^2)/((a+b)*(a-b))^(1/2)*arctanh((a-
b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))+19/d*b^3/a^3/(a^2-b^2)/((a+b)*(a
-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))-12/d*b^5/a
^5/(a^2-b^2)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a
-b))^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.48311, size = 2184, normalized size = 8.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^8 - 14*a^6*b^2 + 25*a^4*b^4 - 12*a^2*b^6)*d*x*cos(d*x + c)^2 + 4
*(a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 12*a*b^7)*d*x*cos(d*x + c) + 2*(a^6*b^2
 - 14*a^4*b^4 + 25*a^2*b^6 - 12*b^8)*d*x - (6*a^4*b^3 - 19*a^2*b^5 + 12*b^7
 + (6*a^6*b - 19*a^4*b^3 + 12*a^2*b^5)*cos(d*x + c)^2 + 2*(6*a^5*b^2 - 19*a
^3*b^4 + 12*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) -
(a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d
*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2
*(11*a^5*b^3 - 23*a^3*b^5 + 12*a*b^7 - (a^8 - 2*a^6*b^2 + a^4*b^4)*cos(d*x
+ c)^3 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*cos(d*x + c)^2 + (17*a^6*b^2 - 35*
a^4*b^4 + 18*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 2*a^9*b^2 + a^7*
b^4)*d*cos(d*x + c)^2 + 2*(a^10*b - 2*a^8*b^3 + a^6*b^5)*d*cos(d*x + c) + (
a^9*b^2 - 2*a^7*b^4 + a^5*b^6)*d), 1/2*((a^8 - 14*a^6*b^2 + 25*a^4*b^4 - 12
*a^2*b^6)*d*x*cos(d*x + c)^2 + 2*(a^7*b - 14*a^5*b^3 + 25*a^3*b^5 - 12*a*b^
7)*d*x*cos(d*x + c) + (a^6*b^2 - 14*a^4*b^4 + 25*a^2*b^6 - 12*b^8)*d*x - (6
*a^4*b^3 - 19*a^2*b^5 + 12*b^7 + (6*a^6*b - 19*a^4*b^3 + 12*a^2*b^5)*cos(d*
x + c)^2 + 2*(6*a^5*b^2 - 19*a^3*b^4 + 12*a*b^6)*cos(d*x + c))*sqrt(-a^2 +
b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c
```

```
))) + (11*a^5*b^3 - 23*a^3*b^5 + 12*a*b^7 - (a^8 - 2*a^6*b^2 + a^4*b^4)*cos
(d*x + c)^3 + 4*(a^7*b - 2*a^5*b^3 + a^3*b^5)*cos(d*x + c)^2 + (17*a^6*b^2
- 35*a^4*b^4 + 18*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 2*a^9*b^2 +
 a^7*b^4)*d*cos(d*x + c)^2 + 2*(a^10*b - 2*a^8*b^3 + a^6*b^5)*d*cos(d*x + c
) + (a^9*b^2 - 2*a^7*b^4 + a^5*b^6)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sin^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(sin(c + d*x)**2/(a + b*sec(c + d*x))**3, x)
```

Giac [B] time = 1.51067, size = 815, normalized size = 3.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] -1/2*(2*(6*a^4*b - 19*a^2*b^3 + 12*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*s
gn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/
sqrt(-a^2 + b^2)))/((a^7 - a^5*b^2)*sqrt(-a^2 + b^2)) - 2*(a^5*tan(1/2*d*x
+ 1/2*c)^7 + 4*a^4*b*tan(1/2*d*x + 1/2*c)^7 - 18*a^3*b^2*tan(1/2*d*x + 1/2*
c)^7 + 7*a^2*b^3*tan(1/2*d*x + 1/2*c)^7 + 18*a*b^4*tan(1/2*d*x + 1/2*c)^7 -
 12*b^5*tan(1/2*d*x + 1/2*c)^7 - 3*a^5*tan(1/2*d*x + 1/2*c)^5 - 4*a^4*b*tan
(1/2*d*x + 1/2*c)^5 - 14*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 37*a^2*b^3*tan(1/
2*d*x + 1/2*c)^5 + 18*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 36*b^5*tan(1/2*d*x + 1
/2*c)^5 + 3*a^5*tan(1/2*d*x + 1/2*c)^3 - 4*a^4*b*tan(1/2*d*x + 1/2*c)^3 + 1
4*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 37*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 - 18*a
*b^4*tan(1/2*d*x + 1/2*c)^3 - 36*b^5*tan(1/2*d*x + 1/2*c)^3 - a^5*tan(1/2*d
*x + 1/2*c) + 4*a^4*b*tan(1/2*d*x + 1/2*c) + 18*a^3*b^2*tan(1/2*d*x + 1/2*c
) + 7*a^2*b^3*tan(1/2*d*x + 1/2*c) - 18*a*b^4*tan(1/2*d*x + 1/2*c) - 12*b^5
*tan(1/2*d*x + 1/2*c))/((a^6 - a^4*b^2)*(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1
/2*d*x + 1/2*c)^4 - 2*b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) - (a^2 - 12*b^2)
*(d*x + c)/a^5)/d
```

3.231 $\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=376

$$\frac{3b^4\sin(c+dx)}{2d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{b^3\sin(c+dx)}{2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}+\frac{b^2\left(3a^2-b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{2b^3\left(3a^2-b\right.}{ad}$$

```
[Out] (-2*b^3*(3*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/
(a*(a - b)^(7/2)*(a + b)^(7/2)*d) - (2*a*b*(3*a^2 + b^2)*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - (b^3*(
a^2 + 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*(a - b
)^(7/2)*(a + b)^(7/2)*d) - Sin[c + d*x]/(2*(a + b)^3*d*(1 - Cos[c + d*x]))
+ Sin[c + d*x]/(2*(a - b)^3*d*(1 + Cos[c + d*x])) - (b^3*Sin[c + d*x])/(2*(
a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) + (3*b^4*Sin[c + d*x])/(2*(a^2 - b^2
)^3*d*(b + a*Cos[c + d*x])) + (b^2*(3*a^2 - b^2)*Sin[c + d*x])/((a^2 - b^2)
^3*d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 0.658311, antiderivative size = 376, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3872, 2897, 2648, 2664, 12, 2659, 208, 2754}

$$\frac{3b^4\sin(c+dx)}{2d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{b^3\sin(c+dx)}{2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)^2}+\frac{b^2\left(3a^2-b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)}-\frac{2b^3\left(3a^2-b\right.}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```


```
[Out] (-2*b^3*(3*a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/
(a*(a - b)^(7/2)*(a + b)^(7/2)*d) - (2*a*b*(3*a^2 + b^2)*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - (b^3*(
a^2 + 2*b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a*(a - b
)^(7/2)*(a + b)^(7/2)*d) - Sin[c + d*x]/(2*(a + b)^3*d*(1 - Cos[c + d*x]))
+ Sin[c + d*x]/(2*(a - b)^3*d*(1 + Cos[c + d*x])) - (b^3*Sin[c + d*x])/(2*(
a^2 - b^2)^2*d*(b + a*Cos[c + d*x])^2) + (3*b^4*Sin[c + d*x])/(2*(a^2 - b^2
)^3*d*(b + a*Cos[c + d*x])) + (b^2*(3*a^2 - b^2)*Sin[c + d*x])/((a^2 - b^2)
^3*d*(b + a*Cos[c + d*x]))
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cos(c+dx)\cot^2(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$

$$= \int \left(-\frac{1}{2(a-b)^3(-1-\cos(c+dx))} + \frac{1}{2(a+b)^3(1-\cos(c+dx))} + \frac{3a^2b^2-b^4}{a\left(a^2-b^2\right)^2(-b-a\cos$$

$$= -\frac{\int \frac{1}{-1-\cos(c+dx)}\,dx}{2(a-b)^3} + \frac{\int \frac{1}{1-\cos(c+dx)}\,dx}{2(a+b)^3} - \frac{b^3\int \frac{1}{(b+a\cos(c+dx))^3}\,dx}{a\left(a^2-b^2\right)} + \frac{\left(b^2\left(3a^2-b^2\right)\right)\int \frac{}{(-b-a\cos}}{a\left(a^2-b^2\right)^2}$$

$$= -\frac{\sin(c+dx)}{2(a+b)^3d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{2(a-b)^3d(1+\cos(c+dx))} - \frac{b^3\sin(c+dx)}{2\left(a^2-b^2\right)^2d(b+a\cos(c}$$

$$= -\frac{2ab\left(3a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin(c+dx)}{2(a+b)^3d(1-\cos(c+dx))} + \frac{\sin(c}{2(a-b)^3d(1}$$

$$= -\frac{2ab\left(3a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{\sin(c+dx)}{2(a+b)^3d(1-\cos(c+dx))} + \frac{\sin(c}{2(a-b)^3d(1}$$

$$= -\frac{2b^3\left(3a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{7/2}(a+b)^{7/2}d} - \frac{2ab\left(3a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} -$$

$$= -\frac{2b^3\left(3a^2-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a(a-b)^{7/2}(a+b)^{7/2}d} - \frac{2ab\left(3a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} -$$

Mathematica [A] time = 0.919154, size = 231, normalized size = 0.61

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b)\left(\frac{b^2\left(6a^2+b^2\right)\sin(c+dx)(a\cos(c+dx)+b)}{(a-b)^3(a+b)^3} + \frac{6ab\left(2a^2+3b^2\right)(a\cos(c+dx)+b)^2\tanh^{-1}\left(\frac{(b-a)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}} - \frac{b^3\sin(}{(a-b)^2(}\right.}{2d(a+b\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^3*((6*a*b*(2*a^2 + 3*b^2)*ArcTanh[((-a +
 b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2)/(a^2 - b^2)^
(7/2) - ((b + a*Cos[c + d*x])^2*Cot[(c + d*x)/2])/(a + b)^3 - (b^3*Sin[c +
d*x])/((a - b)^2*(a + b)^2) + (b^2*(6*a^2 + b^2)*(b + a*Cos[c + d*x])*Sin[c
 + d*x])/((a - b)^3*(a + b)^3) + ((b + a*Cos[c + d*x])^2*Tan[(c + d*x)/2])/
(a - b)^3))/(2*d*(a + b*Sec[c + d*x])^3)
```

Maple [A] time = 0.091, size = 234, normalized size = 0.6

$$\frac{1}{d}\left(\frac{1}{2a^3-6a^2b+6ab^2-2b^3}\tan\left(\frac{dx}{2}+\frac{c}{2}\right) + 2\frac{b}{(a-b)^3(a+b)^3}\left(\frac{\left(-3a^3b+5/2a^2b^2-1/2ab^3+b^4\right)(\tan(1/2dx+c/2}{\left((\tan(1/2dx+c/2))^2a-(\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(1/2/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c)+2*b/(a-b)^3/(a+b)^3*(
((-3*a^3*b+5/2*a^2*b^2-1/2*a*b^3+b^4)*tan(1/2*d*x+1/2*c)^3+(3*a^3*b+5/2*a^2
*b^2+1/2*a*b^3+b^4)*tan(1/2*d*x+1/2*c))/(tan(1/2*d*x+1/2*c)^2*a-tan(1/2*d*x
+1/2*c)^2*b-a-b)^2-3/2*(2*a^2+3*b^2)*a/((a+b)*(a-b))^(1/2)*arctanh((a-b)*ta
n(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))-1/2/(a+b)^3/tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.2484, size = 1854, normalized size = 4.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(22*a^4*b^3 - 14*a^2*b^5 - 8*b^7 - 2*(2*a^7 + 10*a^5*b^2 - 11*a^3*b^4
- a*b^6)*cos(d*x + c)^3 - 3*(2*a^3*b^3 + 3*a*b^5 + (2*a^5*b + 3*a^3*b^3)*co
s(d*x + c)^2 + 2*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c))*sqrt(a^2 - b^2)*log(
(2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*c
os(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*co
s(d*x + c) + b^2))*sin(d*x + c) + 2*(2*a^6*b - 17*a^4*b^3 + 13*a^2*b^5 + 2*
b^7)*cos(d*x + c)^2 + 2*(16*a^5*b^2 - 17*a^3*b^4 + a*b^6)*cos(d*x + c))/(((
a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*d*cos(d*x + c)^2 + 2*(a
^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9)*d*cos(d*x + c) + (a^8*b^2
 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10)*d)*sin(d*x + c)), 1/2*(11*a^4*
b^3 - 7*a^2*b^5 - 4*b^7 - (2*a^7 + 10*a^5*b^2 - 11*a^3*b^4 - a*b^6)*cos(d*x
 + c)^3 - 3*(2*a^3*b^3 + 3*a*b^5 + (2*a^5*b + 3*a^3*b^3)*cos(d*x + c)^2 + 2
*(2*a^4*b^2 + 3*a^2*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 +
 b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) + (2*a^
6*b - 17*a^4*b^3 + 13*a^2*b^5 + 2*b^7)*cos(d*x + c)^2 + (16*a^5*b^2 - 17*a^
3*b^4 + a*b^6)*cos(d*x + c))/(((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 +
a^2*b^8)*d*cos(d*x + c)^2 + 2*(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 +
a*b^9)*d*cos(d*x + c) + (a^8*b^2 - 4*a^6*b^4 + 6*a^4*b^6 - 4*a^2*b^8 + b^10
)*d)*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*sec(c + d*x))**3, x)
```

Giac [A] time = 1.35506, size = 521, normalized size = 1.39

$$\frac{\frac{6\left(2a^3b+3ab^3\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^6-3a^4b^2+3a^2b^4-b^6\right)\sqrt{-a^2+b^2}}+\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3-3a^2b+3ab^2-b^3}-\frac{2\left(6a^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5a^2b^3\tan\left(\frac{1}{2}dx+\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/2*(6*(2*a^3*b + 3*a*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b)
 + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2
)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) + tan(1/2*d*x +
1/2*c)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 2*(6*a^3*b^2*tan(1/2*d*x + 1/2*c)^
3 - 5*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + a*b^4*tan(1/2*d*x + 1/2*c)^3 - 2*b^5
*tan(1/2*d*x + 1/2*c)^3 - 6*a^3*b^2*tan(1/2*d*x + 1/2*c) - 5*a^2*b^3*tan(1/
2*d*x + 1/2*c) - a*b^4*tan(1/2*d*x + 1/2*c) - 2*b^5*tan(1/2*d*x + 1/2*c))/(
(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d
*x + 1/2*c)^2 - a - b)^2) - 1/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*tan(1/2*d*x
+ 1/2*c)))/d
```

3.232 $\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=515

$$\frac{3a^2b^4\sin(c+dx)}{2d\left(a^2-b^2\right)^4(a\cos(c+dx)+b)}-\frac{a^2b^3\sin(c+dx)}{2d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)^2}+\frac{a^2b^2\left(3a^2+b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)^4(a\cos(c+dx)+b)}-\frac{2ab^3\left(3a^2+b^2\right)}{d(a-}$$

```
[Out] (-2*a*b^3*(3*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]]
)/((a - b)^(9/2)*(a + b)^(9/2)*d) - (a*b^3*(a^2 + 2*b^2)*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(9/2)*(a + b)^(9/2)*d) - (2*a*b
*(3*a^4 + 8*a^2*b^2 + b^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a +
b]])/((a - b)^(9/2)*(a + b)^(9/2)*d) - Sin[c + d*x]/(12*(a + b)^3*d*(1 - Co
s[c + d*x])^2) - ((a - 2*b)*Sin[c + d*x])/(4*(a + b)^4*d*(1 - Cos[c + d*x])
) - Sin[c + d*x]/(12*(a + b)^3*d*(1 - Cos[c + d*x])) + Sin[c + d*x]/(12*(a
- b)^3*d*(1 + Cos[c + d*x])^2) + Sin[c + d*x]/(12*(a - b)^3*d*(1 + Cos[c +
d*x])) + ((a + 2*b)*Sin[c + d*x])/(4*(a - b)^4*d*(1 + Cos[c + d*x])) - (a^2
*b^3*Sin[c + d*x])/(2*(a^2 - b^2)^3*d*(b + a*Cos[c + d*x])^2) + (3*a^2*b^4*
Sin[c + d*x])/(2*(a^2 - b^2)^4*d*(b + a*Cos[c + d*x])) + (a^2*b^2*(3*a^2 +
b^2)*Sin[c + d*x])/((a^2 - b^2)^4*d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 0.774694, antiderivative size = 515, normalized size of antiderivative = 1., number of steps used = 20, number of rules used = 9, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3872, 2897, 2650, 2648, 2664, 2754, 12, 2659, 208}

$$\frac{3a^2b^4\sin(c+dx)}{2d\left(a^2-b^2\right)^4(a\cos(c+dx)+b)}-\frac{a^2b^3\sin(c+dx)}{2d\left(a^2-b^2\right)^3(a\cos(c+dx)+b)^2}+\frac{a^2b^2\left(3a^2+b^2\right)\sin(c+dx)}{d\left(a^2-b^2\right)^4(a\cos(c+dx)+b)}-\frac{2ab^3\left(3a^2+b^2\right)}{d(a-}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```


```
[Out] (-2*a*b^3*(3*a^2 + b^2)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]]
)/((a - b)^(9/2)*(a + b)^(9/2)*d) - (a*b^3*(a^2 + 2*b^2)*ArcTanh[(Sqrt[a -
b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(9/2)*(a + b)^(9/2)*d) - (2*a*b
*(3*a^4 + 8*a^2*b^2 + b^4)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a +
b]])/((a - b)^(9/2)*(a + b)^(9/2)*d) - Sin[c + d*x]/(12*(a + b)^3*d*(1 - Co
s[c + d*x])^2) - ((a - 2*b)*Sin[c + d*x])/(4*(a + b)^4*d*(1 - Cos[c + d*x])
) - Sin[c + d*x]/(12*(a + b)^3*d*(1 - Cos[c + d*x])) + Sin[c + d*x]/(12*(a
- b)^3*d*(1 + Cos[c + d*x])^2) + Sin[c + d*x]/(12*(a - b)^3*d*(1 + Cos[c +
d*x])) + ((a + 2*b)*Sin[c + d*x])/(4*(a - b)^4*d*(1 + Cos[c + d*x])) - (a^2
*b^3*Sin[c + d*x])/(2*(a^2 - b^2)^3*d*(b + a*Cos[c + d*x])^2) + (3*a^2*b^4*
Sin[c + d*x])/(2*(a^2 - b^2)^4*d*(b + a*Cos[c + d*x])) + (a^2*b^2*(3*a^2 +
b^2)*Sin[c + d*x])/((a^2 - b^2)^4*d*(b + a*Cos[c + d*x]))
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2897

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)*((a_)
 + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig[(d*sin[
e + f*x])^n*(a + b*sin[e + f*x])^m*(1 - sin[e + f*x]^2)^(p/2), x], x] /; Fr
eeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0] && IntegersQ[m, 2*n, p/2] && (
LtQ[m, -1] || (EqQ[m, -1] && GtQ[p, 0]))
```

Rule 2650

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Cos[c
 + d*x]*(a + b*Sin[c + d*x])^n)/(a*d*(2*n + 1)), x] + Dist[(n + 1)/(a*(2*n
+ 1)), Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] &
& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2664

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[
c + d*x]*(a + b*Sin[c + d*x])^(n + 1))/(d*(n + 1)*(a^2 - b^2)), x] + Dist[1
/((n + 1)*(a^2 - b^2)), Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1) - b
*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^
2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2754

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> -Simp[((b*c - a*d)*Cos[e + f*x]*(a + b*Sin[e + f
*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), I
nt[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(m +
 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a
*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2659

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx = -\int \frac{\cot^3(c+dx)\csc(c+dx)}{(-b-a\cos(c+dx))^3}\,dx$$

$$= \int \left(\frac{1}{4(a-b)^3(-1-\cos(c+dx))^2} + \frac{-a-2b}{4(a-b)^4(-1-\cos(c+dx))} + \frac{1}{4(a+b)^3(1-\cos(c+dx))}\right.$$

$$= \frac{\int \frac{1}{(-1-\cos(c+dx))^2}dx}{4(a-b)^3} + \frac{(a-2b)\int \frac{1}{1-\cos(c+dx)}dx}{4(a+b)^4} + \frac{\int \frac{1}{(1-\cos(c+dx))^2}dx}{4(a+b)^3} - \frac{(a+2b)\int \frac{1}{-1-\cos(c+dx)}}{4(a-b)^4}$$

$$= -\frac{\sin(c+dx)}{12(a+b)^3d(1-\cos(c+dx))^2} - \frac{(a-2b)\sin(c+dx)}{4(a+b)^4d(1-\cos(c+dx))} + \frac{\sin(c+dx)}{12(a-b)^3d(1+\cos(c+dx)}$$

$$= -\frac{2ab\left(3a^4+8a^2b^2+b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d} - \frac{\sin(c+dx)}{12(a+b)^3d(1-\cos(c+dx))^2} - \frac{(a}{4(a+}$$

$$= -\frac{2ab\left(3a^4+8a^2b^2+b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d} - \frac{\sin(c+dx)}{12(a+b)^3d(1-\cos(c+dx))^2} - \frac{(a}{4(a+}$$

$$= -\frac{2ab^3\left(3a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d} - \frac{2ab\left(3a^4+8a^2b^2+b^4\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+d\right.}{\sqrt{a+b}}\right.}{(a-b)^{9/2}(a+b)^{9/2}d}$$

$$= -\frac{2ab^3\left(3a^2+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d} - \frac{ab^3\left(a^2+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{9/2}(a+b)^{9/2}d} - \frac{2a}{}$$

Mathematica [A] time = 1.07206, size = 388, normalized size = 0.75

$$\frac{\sec^3(c+dx)(a\cos(c+dx)+b)\left(\csc^3(c+dx)\left(-154a^5b^2\cos(3(c+dx))+62a^5b^2\cos(5(c+dx))+110a^4b^3\cos(4(c+dx))\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*((96*a*b*(6*a^4 + 23*a^2*b^2 + 6*b^4)*ArcTanh[((-a +
b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2)/Sqrt[a^2 - b^
2] + (36*a^6*b + 154*a^4*b^3 + 424*a^2*b^5 + 16*b^7 - 2*a*(16*a^6 - 94*a^4*
b^2 - 35*a^2*b^4 + 8*b^6)*Cos[c + d*x] + 8*(2*a^6*b - 45*a^4*b^3 - 56*a^2*b
^5 - 6*b^7)*Cos[2*(c + d*x)] - 4*a^7*Cos[3*(c + d*x)] - 154*a^5*b^2*Cos[3*(
c + d*x)] - 205*a^3*b^4*Cos[3*(c + d*x)] + 48*a*b^6*Cos[3*(c + d*x)] - 20*a
^6*b*Cos[4*(c + d*x)] + 110*a^4*b^3*Cos[4*(c + d*x)] + 120*a^2*b^5*Cos[4*(c
 + d*x)] + 4*a^7*Cos[5*(c + d*x)] + 62*a^5*b^2*Cos[5*(c + d*x)] + 39*a^3*b^
4*Cos[5*(c + d*x)])*Csc[c + d*x]^3)*Sec[c + d*x]^3)/(96*(a^2 - b^2)^4*d*(a
+ b*Sec[c + d*x])^3)
```

Maple [A] time = 0.102, size = 328, normalized size = 0.6

$$\frac{1}{d}\left(\frac{1}{\left(8\,a^{3}-24\,a^{2}b+24\,ab^{2}-8\,b^{3}\right)(a-b)}\left(\frac{a}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}-\frac{b}{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}+3\,a\tan\left(1/2\,dx+c/2\right)+3\,b\tan\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+b*sec(d*x+c))^3,x)
```

```
[Out] 1/d*(1/8/(a^3-3*a^2*b+3*a*b^2-b^3)/(a-b)*(1/3*tan(1/2*d*x+1/2*c)^3*a-1/3*b*
tan(1/2*d*x+1/2*c)^3+3*a*tan(1/2*d*x+1/2*c)+3*b*tan(1/2*d*x+1/2*c))+2*a*b/(
a-b)^4/(a+b)^4*(((5/2*a^3*b^2+3*a*b^4-3*a^4*b-5/2*a^2*b^3)*tan(1/2*d*x+1/2*
c)^3+(5/2*a^3*b^2+3*a*b^4+3*a^4*b+5/2*a^2*b^3)*tan(1/2*d*x+1/2*c))/(tan(1/2
*d*x+1/2*c)^2*a-tan(1/2*d*x+1/2*c)^2*b-a-b)^2-1/2*(6*a^4+23*a^2*b^2+6*b^4)/
((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))-
1/24/(a+b)^3/tan(1/2*d*x+1/2*c)^3-1/8/(a+b)^4*(3*a-3*b)/tan(1/2*d*x+1/2*c))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [A] time = 2.72401, size = 3421, normalized size = 6.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [-1/12*(78*a^6*b^3 + 46*a^4*b^5 - 116*a^2*b^7 - 8*b^9 + 2*(4*a^9 + 58*a^7*b
^2 - 23*a^5*b^4 - 39*a^3*b^6)*cos(d*x + c)^5 - 10*(2*a^8*b - 13*a^6*b^3 - a
^4*b^5 + 12*a^2*b^7)*cos(d*x + c)^4 - 4*(3*a^9 + 55*a^7*b^2 - 8*a^5*b^4 - 5
6*a^3*b^6 + 6*a*b^8)*cos(d*x + c)^3 + 3*(6*a^5*b^3 + 23*a^3*b^5 + 6*a*b^7 -
 (6*a^7*b + 23*a^5*b^3 + 6*a^3*b^5)*cos(d*x + c)^4 - 2*(6*a^6*b^2 + 23*a^4*
b^4 + 6*a^2*b^6)*cos(d*x + c)^3 + (6*a^7*b + 17*a^5*b^3 - 17*a^3*b^5 - 6*a*
b^7)*cos(d*x + c)^2 + 2*(6*a^6*b^2 + 23*a^4*b^4 + 6*a^2*b^6)*cos(d*x + c))*
sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*
sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d
*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))*sin(d*x + c) + 4*(6*a^8*b - 56*a^6*b
^3 - 8*a^4*b^5 + 55*a^2*b^7 + 3*b^9)*cos(d*x + c)^2 + 10*(12*a^7*b^2 - a^5*
b^4 - 13*a^3*b^6 + 2*a*b^8)*cos(d*x + c))/(((a^12 - 5*a^10*b^2 + 10*a^8*b^4
 - 10*a^6*b^6 + 5*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^4 + 2*(a^11*b - 5*a^9*
b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x + c)^3 - (a^1
2 - 6*a^10*b^2 + 15*a^8*b^4 - 20*a^6*b^6 + 15*a^4*b^8 - 6*a^2*b^10 + b^12)*
d*cos(d*x + c)^2 - 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*
b^9 - a*b^11)*d*cos(d*x + c) - (a^10*b^2 - 5*a^8*b^4 + 10*a^6*b^6 - 10*a^4*
b^8 + 5*a^2*b^10 - b^12)*d)*sin(d*x + c)), -1/6*(39*a^6*b^3 + 23*a^4*b^5 -
```

```
58*a^2*b^7 - 4*b^9 + (4*a^9 + 58*a^7*b^2 - 23*a^5*b^4 - 39*a^3*b^6)*cos(d*x
 + c)^5 - 5*(2*a^8*b - 13*a^6*b^3 - a^4*b^5 + 12*a^2*b^7)*cos(d*x + c)^4 -
2*(3*a^9 + 55*a^7*b^2 - 8*a^5*b^4 - 56*a^3*b^6 + 6*a*b^8)*cos(d*x + c)^3 -
3*(6*a^5*b^3 + 23*a^3*b^5 + 6*a*b^7 - (6*a^7*b + 23*a^5*b^3 + 6*a^3*b^5)*co
s(d*x + c)^4 - 2*(6*a^6*b^2 + 23*a^4*b^4 + 6*a^2*b^6)*cos(d*x + c)^3 + (6*a
^7*b + 17*a^5*b^3 - 17*a^3*b^5 - 6*a*b^7)*cos(d*x + c)^2 + 2*(6*a^6*b^2 + 2
3*a^4*b^4 + 6*a^2*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b
^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*sin(d*x + c) + 2*(6*a^
8*b - 56*a^6*b^3 - 8*a^4*b^5 + 55*a^2*b^7 + 3*b^9)*cos(d*x + c)^2 + 5*(12*a
^7*b^2 - a^5*b^4 - 13*a^3*b^6 + 2*a*b^8)*cos(d*x + c))/(((a^12 - 5*a^10*b^2
 + 10*a^8*b^4 - 10*a^6*b^6 + 5*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^4 + 2*(a^
11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x
+ c)^3 - (a^12 - 6*a^10*b^2 + 15*a^8*b^4 - 20*a^6*b^6 + 15*a^4*b^8 - 6*a^2*
b^10 + b^12)*d*cos(d*x + c)^2 - 2*(a^11*b - 5*a^9*b^3 + 10*a^7*b^5 - 10*a^5
*b^7 + 5*a^3*b^9 - a*b^11)*d*cos(d*x + c) - (a^10*b^2 - 5*a^8*b^4 + 10*a^6*
b^6 - 10*a^4*b^8 + 5*a^2*b^10 - b^12)*d)*sin(d*x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^4(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*sec(d*x+c))**3,x)

[Out] Integral(csc(c + d*x)**4/(a + b*sec(c + d*x))**3, x)
```

Giac [A] time = 1.45734, size = 957, normalized size = 1.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sec(d*x+c))^3,x, algorithm="giac")

[Out] 1/24*(24*(6*a^5*b + 23*a^3*b^3 + 6*a*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)
*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/
sqrt(-a^2 + b^2)))/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*sqrt(-a
^2 + b^2)) + (a^6*tan(1/2*d*x + 1/2*c)^3 - 6*a^5*b*tan(1/2*d*x + 1/2*c)^3 +
 15*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 - 20*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 15
*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 - 6*a*b^5*tan(1/2*d*x + 1/2*c)^3 + b^6*tan(
1/2*d*x + 1/2*c)^3 + 9*a^6*tan(1/2*d*x + 1/2*c) - 36*a^5*b*tan(1/2*d*x + 1/
2*c) + 45*a^4*b^2*tan(1/2*d*x + 1/2*c) - 45*a^2*b^4*tan(1/2*d*x + 1/2*c) +
36*a*b^5*tan(1/2*d*x + 1/2*c) - 9*b^6*tan(1/2*d*x + 1/2*c))/(a^9 - 9*a^8*b
+ 36*a^7*b^2 - 84*a^6*b^3 + 126*a^5*b^4 - 126*a^4*b^5 + 84*a^3*b^6 - 36*a^2
*b^7 + 9*a*b^8 - b^9) - 24*(6*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 5*a^4*b^3*ta
n(1/2*d*x + 1/2*c)^3 + 5*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 6*a^2*b^5*tan(1/2
*d*x + 1/2*c)^3 - 6*a^5*b^2*tan(1/2*d*x + 1/2*c) - 5*a^4*b^3*tan(1/2*d*x +
1/2*c) - 5*a^3*b^4*tan(1/2*d*x + 1/2*c) - 6*a^2*b^5*tan(1/2*d*x + 1/2*c))/(
(a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(a*tan(1/2*d*x + 1/2*c)^2 -
 b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2) - (9*a*tan(1/2*d*x + 1/2*c)^2 - 9*b*t
an(1/2*d*x + 1/2*c)^2 + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)
*tan(1/2*d*x + 1/2*c)^3))/d
```

3.233 $\int \frac{(e\sin(c+dx))^{7/2}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=516

$$\frac{2e^4\left(-28a^2b^2+5a^4+21b^4\right)\sqrt{\sin(c+dx)}\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{21a^5d\sqrt{e\sin(c+dx)}}-\frac{be^{7/2}\left(a^2-b^2\right)^{5/4}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{9/2}d}-\frac{be^7\dots}{\dots}$$

```
[Out] -((b*(a^2 - b^2)^(5/4)*e^(7/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2
- b^2)^(1/4)*Sqrt[e])])/(a^(9/2)*d)) - (b*(a^2 - b^2)^(5/4)*e^(7/2)*ArcTanh
[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(9/2)*d) +
 (2*(5*a^4 - 28*a^2*b^2 + 21*b^4)*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt
[Sin[c + d*x]])/(21*a^5*d*Sqrt[e*Sin[c + d*x]]) + (b^2*(a^2 - b^2)^2*e^4*El
lipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d
*x]])/(a^5*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (b^2*(
a^2 - b^2)^2*e^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2
, 2]*Sqrt[Sin[c + d*x]])/(a^5*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[
c + d*x]]) + (2*e^3*(21*b*(a^2 - b^2) - a*(5*a^2 - 7*b^2)*Cos[c + d*x])*Sqr
t[e*Sin[c + d*x]])/(21*a^4*d) + (2*e*(7*b - 5*a*Cos[c + d*x])*(e*Sin[c + d*
x])^(5/2))/(35*a^2*d)
```

Rubi [A] time = 1.70264, antiderivative size = 516, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {3872, 2865, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{be^{7/2}\left(a^2-b^2\right)^{5/4}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{9/2}d}-\frac{be^{7/2}\left(a^2-b^2\right)^{5/4}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{9/2}d}+\frac{2e^3\sqrt{e\sin(c+dx)}\left(21b\left(a^2-b^2\right)\dots\right)}{21a^4\dots}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(7/2)/(a + b*Sec[c + d*x]),x]
```


```
[Out] -((b*(a^2 - b^2)^(5/4)*e^(7/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2
- b^2)^(1/4)*Sqrt[e])])/(a^(9/2)*d)) - (b*(a^2 - b^2)^(5/4)*e^(7/2)*ArcTanh
[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(9/2)*d) +
 (2*(5*a^4 - 28*a^2*b^2 + 21*b^4)*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt
[Sin[c + d*x]])/(21*a^5*d*Sqrt[e*Sin[c + d*x]]) + (b^2*(a^2 - b^2)^2*e^4*El
lipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d
*x]])/(a^5*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (b^2*(
a^2 - b^2)^2*e^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2
, 2]*Sqrt[Sin[c + d*x]])/(a^5*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[
c + d*x]]) + (2*e^3*(21*b*(a^2 - b^2) - a*(5*a^2 - 7*b^2)*Cos[c + d*x])*Sqr
t[e*Sin[c + d*x]])/(21*a^4*d) + (2*e*(7*b - 5*a*Cos[c + d*x])*(e*Sin[c + d*
x])^(5/2))/(35*a^2*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
```

```
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{7/2}}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)(e\sin(c+dx))^{7/2}}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{2e(7b-5a\cos(c+dx))(e\sin(c+dx))^{5/2}}{35a^2d} - \frac{\left(2e^2\right)\int \frac{\left(-ab+\frac{1}{2}\left(5a^2-7b^2\right)\cos(c+dx)\right)(e\sin(c+dx))^{3/2}}{-b-a\cos(c+dx)}\,dx}{7a^2}$$

$$= \frac{2e^3\left(21b\left(a^2-b^2\right)-a\left(5a^2-7b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{21a^4d} + \frac{2e(7b-5a\cos(c+dx))(e\,\text{si}}{35a^2d}$$

$$= \frac{2e^3\left(21b\left(a^2-b^2\right)-a\left(5a^2-7b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{21a^4d} + \frac{2e(7b-5a\cos(c+dx))(e\,\text{si}}{35a^2d}$$

$$= \frac{2e^3\left(21b\left(a^2-b^2\right)-a\left(5a^2-7b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{21a^4d} + \frac{2e(7b-5a\cos(c+dx))(e\,\text{si}}{35a^2d}$$

$$= \frac{2\left(5a^4-28a^2b^2+21b^4\right)e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21a^5d\sqrt{e\sin(c+dx)}} + \frac{2e^3\left(21b\left(a^2-b^2\right)-a\left(5a^2\right.\right.}{}$$

$$= \frac{2\left(5a^4-28a^2b^2+21b^4\right)e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21a^5d\sqrt{e\sin(c+dx)}} - \frac{b^2\left(a^2-b^2\right)^{3/2}e^4\Pi\left(\frac{2a}{a-\sqrt{a^2-b}}\right.}{a^5\left(a-\sqrt{a^2-}\right.}$$

$$= -\frac{b\left(a^2-b^2\right)^{5/4}e^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{9/2}d} - \frac{b\left(a^2-b^2\right)^{5/4}e^{7/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{9/2}d} + \frac{2\left(5a\right.}{}$$

Mathematica [C] time = 17.3005, size = 2049, normalized size = 3.97

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(7/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((b + a*Cos[c + d*x])*(-((23*a^2 - 28*b^2)*Cos[c + d*x])/(42*a^3) - (b*Cos[
2*(c + d*x)])/(5*a^2) + Cos[3*(c + d*x)]/(14*a))*Csc[c + d*x]^3*Sec[c + d*x
]*(e*Sin[c + d*x])^(7/2))/(d*(a + b*Sec[c + d*x])) - ((b + a*Cos[c + d*x])*
Sec[c + d*x]*(e*Sin[c + d*x])^(7/2)*((2*(-100*a^3 + 98*a*b^2)*Cos[c + d*x]^
2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[a]*Sqrt
[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Si
n[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[a]*(
-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^
2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]
]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2 - b^2)*AppellF1[1/4,
 -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c
 + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5
/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5
/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (-a^2
 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos
[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(89*a^2*b - 70*b^3)*Cos[c + d*x]*(b +
 a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2*ArcTan[1 - ((1 + I)*
Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt
[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 + I)*
Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] - Log[Sqrt
[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Si
n[c + d*x]]))/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/
4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])/(S
qrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d
*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, 1/2, 2, 9
/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (a^2 - b^2)*AppellF
1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[
c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c + d*x])*Sqrt
[1 - Sin[c + d*x]^2]) + ((-231*a^2*b + 210*b^3)*Cos[c + d*x]*Cos[2*(c + d*x
)]*(b + a*Sqrt[1 - Sin[c + d*x]^2])*(((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[1 -
((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2 - b^
2)^(3/4)) - ((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin
[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2 - b^2)^(3/4)) + ((1/4 - I/4)*
(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[
Sin[c + d*x]] + I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) - ((1/4 - I/
4)*(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sq
rt[Sin[c + d*x]] + I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) + (4*Sqrt
[Sin[c + d*x]])/a + (4*b*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Si
n[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*b*(a^2
 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*Appe
llF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] +
2*(2*a^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a
^2 - b^2)] + (a^2 - b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Si
n[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2
)))))/((b + a*Cos[c + d*x])*(1 - 2*Sin[c + d*x]^2)*Sqrt[1 - Sin[c + d*x]^2]
)))/(420*a^3*d*(a + b*Sec[c + d*x])*Sin[c + d*x]^(7/2))
```

Maple [B] time = 5.92, size = 1776, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/5/d*b*e/a^2*(e*sin(d*x+c))^(5/2)+2/d*b*e^3/a^2*(e*sin(d*x+c))^(1/2)-2/d*b
^3*e^3/a^4*(e*sin(d*x+c))^(1/2)+1/d*b*e^5*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e
^2+b^2*e^2)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-2/d*b^3*
e^5/a^2*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*sin(d*x+c))^
(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+1/d*b^5*e^5/a^4*(e^2*(a^2-b^2)/a^2)^(1/4)/
(-a^2*e^2+b^2*e^2)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+1
/2/d*b*e^5*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^
(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^
(1/4)))-1/d*b^3*e^5/a^2*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e
*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a
^2-b^2)/a^2)^(1/4)))+1/2/d*b^5*e^5/a^4*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+
b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c)
)^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+2/7/d/a*e^4*cos(d*x+c)^3/(e*sin(d*x+c))
^(1/2)*sin(d*x+c)-5/21/d/a*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(-sin(d*x+c)
+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)
^(1/2),1/2*2^(1/2))+4/3/d/a^3*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2*(-sin
(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*
x+c)+1)^(1/2),1/2*2^(1/2))-1/d/a^5*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4*
(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-s
in(d*x+c)+1)^(1/2),1/2*2^(1/2))-16/21/d/a*e^4*cos(d*x+c)/(e*sin(d*x+c))^(1/
2)*sin(d*x+c)+2/3/d/a^3*e^4*cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2*sin(d*x+c)+
1/2/d/a^2*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a^2-b^2)^(1/2)*(-sin(d*x
+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*
EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/d/a
^4*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^
(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*Ellipti
cPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/2/d/a^6*e^
4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^6/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)
*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((
-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/d/a^2*e^4/cos
(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2
*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(
d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/d/a^4*e^4/cos(d*x+c)
/(e*sin(d*x+c))^(1/2)*b^4/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*
x+c))^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+
1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/d/a^6*e^4/cos(d*x+c)/(e*s
in(d*x+c))^(1/2)*b^6/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))
^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1
/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^(7/2)/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(7/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \sin(dx+c))^{\frac{7}{2}}}{b \sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(7/2)/(b*sec(d*x + c) + a), x)
```

3.234 $\int \frac{(e\sin(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=430

$$\frac{be^{5/2}\left(a^2-b^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{7/2}d}-\frac{be^{5/2}\left(a^2-b^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{7/2}d}+\frac{2e^2\left(3a^2-5b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\right.}{5a^3d\sqrt{\sin(c+dx}}$$

```
[Out] (b*(a^2 - b^2)^(3/4)*e^(5/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 -
b^2)^(1/4)*Sqrt[e])])/(a^(7/2)*d) - (b*(a^2 - b^2)^(3/4)*e^(5/2)*ArcTanh[(S
qrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(7/2)*d) - (b
^2*(a^2 - b^2)*e^3*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)
/2, 2]*Sqrt[Sin[c + d*x]])/(a^4*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]
]) - (b^2*(a^2 - b^2)*e^3*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2
 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^4*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c
 + d*x]]) + (2*(3*a^2 - 5*b^2)*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*
Sin[c + d*x]])/(5*a^3*d*Sqrt[Sin[c + d*x]]) + (2*e*(5*b - 3*a*Cos[c + d*x])
*(e*Sin[c + d*x])^(3/2))/(15*a^2*d)
```

Rubi [A] time = 1.10903, antiderivative size = 430, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {3872, 2865, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{be^{5/2}\left(a^2-b^2\right)^{3/4}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{7/2}d}-\frac{be^{5/2}\left(a^2-b^2\right)^{3/4}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{7/2}d}+\frac{2e^2\left(3a^2-5b^2\right)E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\right.}{5a^3d\sqrt{\sin(c+dx}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(5/2)/(a + b*Sec[c + d*x]),x]
```


```
[Out] (b*(a^2 - b^2)^(3/4)*e^(5/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 -
b^2)^(1/4)*Sqrt[e])])/(a^(7/2)*d) - (b*(a^2 - b^2)^(3/4)*e^(5/2)*ArcTanh[(S
qrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(7/2)*d) - (b
^2*(a^2 - b^2)*e^3*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)
/2, 2]*Sqrt[Sin[c + d*x]])/(a^4*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]
]) - (b^2*(a^2 - b^2)*e^3*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2
 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^4*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c
 + d*x]]) + (2*(3*a^2 - 5*b^2)*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*
Sin[c + d*x]])/(5*a^3*d*Sqrt[Sin[c + d*x]]) + (2*e*(5*b - 3*a*Cos[c + d*x])
*(e*Sin[c + d*x])^(3/2))/(15*a^2*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
```

```
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{5/2}}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)(e\sin(c+dx))^{5/2}}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{2e(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^2d} - \frac{\left(2e^2\right)\int \frac{\left(-ab+\frac{1}{2}\left(3a^2-5b^2\right)\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{-b-a\cos(c+dx)}\,dx}{5a^2}$$

$$= \frac{2e(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^2d} + \frac{\left(\left(3a^2-5b^2\right)e^2\right)\int \sqrt{e\sin(c+dx)}\,dx}{5a^3} + \frac{\left(b\left(a^2-\right.\right.}{}$$

$$= \frac{2e(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^2d} + \frac{\left(b^2\left(a^2-b^2\right)e^3\right)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,d}{2a^4}$$

$$= \frac{2\left(3a^2-5b^2\right)e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^3d\sqrt{\sin(c+dx)}} + \frac{2e(5b-3a\cos(c+dx))(e\sin(c+dx)}{15a^2d}$$

$$= -\frac{b^2\left(a^2-b^2\right)e^3\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{a^4\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{b^2\left(a^2-b^2\right)e^3\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\right.}{a^4\left(a+\sqrt{a^2-b\right.}}$$

$$= \frac{b\left(a^2-b^2\right)^{3/4}e^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{7/2}d} - \frac{b\left(a^2-b^2\right)^{3/4}e^{5/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{7/2}d} - \frac{b^2\left(a^2\right.}{}$$

Mathematica [C] time = 14.8852, size = 853, normalized size = 1.98

$$(b+a\cos(c+dx))\sec(c+dx$$

$$\frac{(b+a\cos(c+dx))\csc^2(c+dx)\sec(c+dx)(e\sin(c+dx))^{5/2}\left(\frac{2b\sin(c+dx)}{3a^2}-\frac{\sin(2(c+dx))}{5a}\right)}{d(a+b\sec(c+dx))} - \frac{}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(5/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((b + a*Cos[c + d*x])*Sec[c + d*x]*(e*Sin[c + d*x])^(5/2)*(((-3*a^2 + 5*b^
2)*Cos[c + d*x]^2*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sq
```

```
rt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[
a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2]
*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[Sqrt
[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Si
n[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (a^2*S
in[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))*(b + a*Sqrt[1 - Sin[c + d*x
]^2]))/(12*a^(3/2)*(a^2 - b^2)*(b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) +
 (4*a*b*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[
c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c +
d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^
2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + (1
+ I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]]))/(Sq
rt[a]*(a^2 - b^2)^(1/4)) + (b*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (a
^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))/(3*(-a^2 + b^2)))*(b +
a*Sqrt[1 - Sin[c + d*x]^2]))/((b + a*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2]
)))/(5*a^2*d*(a + b*Sec[c + d*x])*Sin[c + d*x]^(5/2)) + ((b + a*Cos[c + d*x
])*Csc[c + d*x]^2*Sec[c + d*x]*(e*Sin[c + d*x])^(5/2)*((2*b*Sin[c + d*x])/(
3*a^2) - Sin[2*(c + d*x)]/(5*a)))/(d*(a + b*Sec[c + d*x]))
```

Maple [B] time = 4.602, size = 1195, normalized size = 2.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/3/d*b*e*(e*sin(d*x+c))^(3/2)/a^2+1/d*b*e^3/a^2/(e^2*(a^2-b^2)/a^2)^(1/4)*
arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-1/d*b^3*e^3/a^4/(e^2
*(a^2-b^2)/a^2)^(1/4)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4)
)-1/2/d*b*e^3/a^2/(e^2*(a^2-b^2)/a^2)^(1/4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(
a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+1/2/
d*b^3*e^3/a^4/(e^2*(a^2-b^2)/a^2)^(1/4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-
b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+2/5/d/a*
e^3*cos(d*x+c)^3/(e*sin(d*x+c))^(1/2)-6/5/d/a*e^3/cos(d*x+c)/(e*sin(d*x+c))
^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*Ellipt
icE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+3/5/d/a*e^3/cos(d*x+c)/(e*sin(d*x+c)
)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*Ellip
ticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2/5/d/a*e^3*cos(d*x+c)/(e*sin(d*x+c
))^(1/2)+2/d/a^3*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2*(-sin(d*x+c)+1)^(1
/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2)
,1/2*2^(1/2))-1/d/a^3*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2*(-sin(d*x+c)+
1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^
(1/2),1/2*2^(1/2))+1/2/d/a^3*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2*(-sin(
d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/
a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/
2/d/a^5*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4*(-sin(d*x+c)+1)^(1/2)*(2+2*
sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d
*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/2/d/a^3*e^3/cos(d*x+c
)/(e*sin(d*x+c))^(1/2)*b^2*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin
(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(
a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/d/a^5*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2
)*b^4*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2
-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2
*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^(5/2)/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(5/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(5/2)/(b*sec(d*x + c) + a), x)
```

3.235 $\int \frac{(e\sin(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=444

$$\frac{2e^2\left(a^2-3b^2\right)\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3a^3d\sqrt{e\sin(c+dx)}}-\frac{be^{3/2}\sqrt[4]{a^2-b^2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{5/2}d}-\frac{be^{3/2}\sqrt[4]{a^2-b^2}\tanh^{-1}\left(\cdots\right.}{a^{5/2}d}$$

```
[Out] -((b*(a^2 - b^2)^(1/4)*e^(3/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2
- b^2)^(1/4)*Sqrt[e])])/(a^(5/2)*d)) - (b*(a^2 - b^2)^(1/4)*e^(3/2)*ArcTanh
[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(5/2)*d) +
 (2*(a^2 - 3*b^2)*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/
(3*a^3*d*Sqrt[e*Sin[c + d*x]]) + (b^2*(a^2 - b^2)*e^2*EllipticPi[(2*a)/(a -
 Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^3*(a^2 - b
^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (b^2*(a^2 - b^2)*e^2*Elli
pticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x
]])/(a^3*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*e*(3*
b - a*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(3*a^2*d)
```

Rubi [A] time = 1.04423, antiderivative size = 444, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {3872, 2865, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{be^{3/2}\sqrt[4]{a^2-b^2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{5/2}d}-\frac{be^{3/2}\sqrt[4]{a^2-b^2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{5/2}d}+\frac{2e^2\left(a^2-3b^2\right)\sqrt{\sin(c+dx)}F\left(\frac{1}{2}\left(c+dx-\cdots\right.\right.}{3a^3d\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(3/2)/(a + b*Sec[c + d*x]),x]
```


```
[Out] -((b*(a^2 - b^2)^(1/4)*e^(3/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2
- b^2)^(1/4)*Sqrt[e])])/(a^(5/2)*d)) - (b*(a^2 - b^2)^(1/4)*e^(3/2)*ArcTanh
[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(5/2)*d) +
 (2*(a^2 - 3*b^2)*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/
(3*a^3*d*Sqrt[e*Sin[c + d*x]]) + (b^2*(a^2 - b^2)*e^2*EllipticPi[(2*a)/(a -
 Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^3*(a^2 - b
^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (b^2*(a^2 - b^2)*e^2*Elli
pticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x
]])/(a^3*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*e*(3*
b - a*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])/(3*a^2*d)
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
```

```
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{3/2}}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)(e\sin(c+dx))^{3/2}}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{2e(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^2d} - \frac{\left(2e^2\right)\int \frac{-ab+\frac{1}{2}\left(a^2-3b^2\right)\cos(c+dx)}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{3a^2}$$

$$= \frac{2e(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^2d} + \frac{\left(\left(a^2-3b^2\right)e^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3a^3} + \frac{\left(b\left(a^2-b^2\right)e^2\right)\int \frac{}{(-b-}$$

$$= \frac{2e(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^2d} + \frac{\left(b^2\sqrt{a^2-b^2}e^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{2a^3} + \frac{\left(b^2\right.}{}$$

$$= \frac{2\left(a^2-3b^2\right)e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3a^3d\sqrt{e\sin(c+dx)}} + \frac{2e(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^2d} + \frac{(2b}{}$$

$$= \frac{2\left(a^2-3b^2\right)e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3a^3d\sqrt{e\sin(c+dx)}} - \frac{b^2\sqrt{a^2-b^2}e^2\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right.}{a^3\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+}}$$

$$= -\frac{b\sqrt[4]{a^2-b^2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}d} - \frac{b\sqrt[4]{a^2-b^2}e^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}d} + \frac{2\left(a^2-3b^2\right)e^2F}{}$$

Mathematica [C] time = 16.4583, size = 1959, normalized size = 4.41

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(3/2)/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*(b + a*Cos[c + d*x])*Csc[c + d*x]*(e*Sin[c + d*x])^(3/2))/(3*a*d*(a + b
*Sec[c + d*x])) + ((b + a*Cos[c + d*x])*Sec[c + d*x]*(e*Sin[c + d*x])^(3/2)
*((4*a*Cos[c + d*x]^2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1 - (
Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqr
t[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2]
 - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]
+ Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*
x]] + a*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2
```

```
- b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*Appe
llF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] +
 2*(2*a^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/
(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2
*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x
]^2)))))/((b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) - (2*b*Cos[c + d*x]*(b
 + a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2*ArcTan[1 - ((1 + I
)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sq
rt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 + I
)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] - Log[Sq
rt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*
Sin[c + d*x]]))/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1,
5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])/
(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c +
 d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, 1/2, 2,
 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (a^2 - b^2)*Appel
lF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Si
n[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c + d*x])*Sq
rt[1 - Sin[c + d*x]^2]) + (3*b*Cos[c + d*x]*Cos[2*(c + d*x)]*(b + a*Sqrt[1
- Sin[c + d*x]^2])*(((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[1 - ((1 + I)*Sqrt[a]*
Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2 - b^2)^(3/4)) - ((1/2
 - I/2)*(a^2 - 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2
- b^2)^(1/4)])/(a^(3/2)*(a^2 - b^2)^(3/4)) + ((1/4 - I/4)*(a^2 - 2*b^2)*Log
[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I
*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) - ((1/4 - I/4)*(a^2 - 2*b^2)*
Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]]
+ I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) + (4*Sqrt[Sin[c + d*x]])/a
 + (4*b*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)]*Sin[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*b*(a^2 - b^2)*AppellF1[
1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Si
n[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1,
 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1
[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (a^2
 - b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos
[c + d*x])*(1 - 2*Sin[c + d*x]^2)*Sqrt[1 - Sin[c + d*x]^2])))/(6*a*d*(a + b
*Sec[c + d*x])*Sin[c + d*x]^(3/2))
```

Maple [B] time = 4.273, size = 1120, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 2/d*b*e/a^2*(e*sin(d*x+c))^(1/2)+1/d*b*e^3*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*
e^2+b^2*e^2)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-1/d*b^3
*e^3/a^2*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*sin(d*x+c))
^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+1/2/d*b*e^3*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a
^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin
(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))-1/2/d*b^3*e^3/a^2*(e^2*(a^2-b^2)
/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)
^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))-1/3/d/a*e^2/cos(d
*x+c)/(e*sin(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin
(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2/3/d/a*e^2*cos(
```

```
d*x+c)/(e*sin(d*x+c))^(1/2)*sin(d*x+c)+1/d/a^3*e^2/cos(d*x+c)/(e*sin(d*x+c)
)^(1/2)*b^2*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*E
llipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+1/2/d/a^2*e^2/cos(d*x+c)/(e*sin
(d*x+c))^(1/2)*b^2/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(
1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2
),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/d/a^4*e^2/cos(d*x+c)/(e*sin(d*x+
c))^(1/2)*b^4/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*
sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(
1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/d/a^2*e^2/cos(d*x+c)/(e*sin(d*x+c))^(
1/2)*b^2/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d
*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^
2-b^2)^(1/2)/a),1/2*2^(1/2))+1/2/d/a^4*e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*
b^4/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)
^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2
)^(1/2)/a),1/2*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^(3/2)/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(3/2)/(b*sec(d*x + c) + a), x)
```

3.236 $\int \frac{\sqrt{e\sin(c+dx)}}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=356

$$\frac{b\sqrt{e}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{3/2}d\sqrt[4]{a^2-b^2}}-\frac{b\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{3/2}d\sqrt[4]{a^2-b^2}}-\frac{b^2e\sqrt{\sin(c+dx)}\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{a^2d\left(a-\sqrt{a^2-b^2}\right)\sqrt{e\sin(c+dx)}}-\frac{b^2e\sqrt{\sin(c}}{a^2d\left(\right.}$$

```
[Out] (b*Sqrt[e]*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e]
)])/(a^(3/2)*(a^2 - b^2)^(1/4)*d) - (b*Sqrt[e]*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[
c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(3/2)*(a^2 - b^2)^(1/4)*d) - (b
^2*e*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Si
n[c + d*x]])/(a^2*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) - (b^2*e*El
lipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d
*x]])/(a^2*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*EllipticE[(c
- Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(a*d*Sqrt[Sin[c + d*x]])
```

Rubi [A] time = 0.76152, antiderivative size = 356, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {3872, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{b\sqrt{e}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{3/2}d\sqrt[4]{a^2-b^2}}-\frac{b\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{a^{3/2}d\sqrt[4]{a^2-b^2}}-\frac{b^2e\sqrt{\sin(c+dx)}\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{a^2d\left(a-\sqrt{a^2-b^2}\right)\sqrt{e\sin(c+dx)}}-\frac{b^2e\sqrt{\sin(c}}{a^2d\left(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sin[c + d*x]]/(a + b*Sec[c + d*x]),x]
```


```
[Out] (b*Sqrt[e]*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e]
)])/(a^(3/2)*(a^2 - b^2)^(1/4)*d) - (b*Sqrt[e]*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[
c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(3/2)*(a^2 - b^2)^(1/4)*d) - (b
^2*e*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Si
n[c + d*x]])/(a^2*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) - (b^2*e*El
lipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d
*x]])/(a^2*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*EllipticE[(c
- Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(a*d*Sqrt[Sin[c + d*x]])
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)\sqrt{e\sin(c+dx)}}{-b-a\cos(c+dx)}\,dx$$
$$= \frac{\int \sqrt{e\sin(c+dx)}\,dx}{a} + \frac{b\int \frac{\sqrt{e\sin(c+dx)}}{-b-a\cos(c+dx)}\,dx}{a}$$
$$= \frac{\left(b^2e\right)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{2a^2} - \frac{\left(b^2e\right)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{2a^2} + \frac{(be)\,\mathrm{Subst}\left(\right.}{}$$
$$= \frac{2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{ad\sqrt{\sin(c+dx)}} + \frac{(2be)\,\mathrm{Subst}\left(\int \frac{x^2}{(-a^2+b^2)e^2+a^2x^4}\,dx, x, \sqrt{e\sin(c+dx)}\right)}{d} +$$
$$= -\frac{b^2e\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{a^2\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{b^2e\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c}}{a^2\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$
$$= \frac{b\sqrt{e}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\sqrt[4]{a^2-b^2}d} - \frac{b\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\sqrt[4]{a^2-b^2}d} - \frac{b^2e\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{a^2\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c}}$$

Mathematica [C] time = 20.0788, size = 351, normalized size = 0.99

$$\frac{\sqrt{e\sin(c+dx)}\left(a\sqrt{\cos^2(c+dx)}+b\right)\left(8a^{5/2}\sin^{\frac{3}{2}}(c+dx)F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)+3\sqrt{2}b\left(b^2-a^2\right)^{3/4}\left(-\mathrm{lo}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Sin[c + d*x]]/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((b + a*Sqrt[Cos[c + d*x]^2])*Sqrt[e*Sin[c + d*x]]*(3*Sqrt[2]*b*(-a^2 + b^2
)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/
4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)]
 - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d
*x]] + a*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2
)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/
2, 1, 7/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(
3/2)))/(12*a^(3/2)*(a^2 - b^2)*d*(b + a*Cos[c + d*x])*Sqrt[Sin[c + d*x]])
```

Maple [B] time = 2.525, size = 919, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] 1/d*b*e/a^2/(e^2*(a^2-b^2)/a^2)^(1/4)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2
-b^2)/a^2)^(1/4))-1/2/d*b*e/a^2/(e^2*(a^2-b^2)/a^2)^(1/4)*ln(((e*sin(d*x+c)
)^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2
)^(1/4)))-1/2/d*e*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(
1/2)*b^2/a^2/((a^2-b^2)^(1/2)-a)/(a+(a^2-b^2)^(1/2))/cos(d*x+c)/(e*sin(d*x+
c))^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-a/((a^2-b^2)^(1/2)-a),1/2*2^(1/
```

```
2))*(a^2-b^2)^(1/2)+1/2/d*e*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*si
n(d*x+c)^(1/2)*b^2/a^2/((a^2-b^2)^(1/2)-a)/(a+(a^2-b^2)^(1/2))/cos(d*x+c)/(
e*sin(d*x+c))^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^2-b^2)^(1/2)),
1/2*2^(1/2))*(a^2-b^2)^(1/2)+2/d*e*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(
1/2)*sin(d*x+c)^(1/2)*b^2/a/((a^2-b^2)^(1/2)-a)/(a+(a^2-b^2)^(1/2))/cos(d*x
+c)/(e*sin(d*x+c))^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/d*e
*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*b^2/a/((a^2-
b^2)^(1/2)-a)/(a+(a^2-b^2)^(1/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*EllipticF
((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/2/d*e*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(
d*x+c))^(1/2)*sin(d*x+c)^(1/2)*b^2/a/((a^2-b^2)^(1/2)-a)/(a+(a^2-b^2)^(1/2)
)/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-a/((a^2
-b^2)^(1/2)-a),1/2*2^(1/2))-1/2/d*e*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^
(1/2)*sin(d*x+c)^(1/2)*b^2/a/((a^2-b^2)^(1/2)-a)/(a+(a^2-b^2)^(1/2))/cos(d*
x+c)/(e*sin(d*x+c))^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^2-b^2)^(
1/2)),1/2*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\sin(dx+c)}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*sin(d*x + c))/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\sin(c+dx)}}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(1/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(e*sin(c + d*x))/(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\sin(dx+c)}}{b\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*sin(d*x + c))/(b*sec(d*x + c) + a), x)
```

3.237 $\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx$

Optimal. Leaf size=370

$$\frac{2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{ad\sqrt{e\sin(c+dx)}}-\frac{b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{\sqrt{a}d\sqrt{e}\left(a^2-b^2\right)^{3/4}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{\sqrt{a}d\sqrt{e}\left(a^2-b^2\right)^{3/4}}+\frac{b^2\sqrt{\sin(c+dx)}\Pi}{ad\left(-a\sqrt{a^2-b\right.}}$$

```
[Out] -((b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(S
qrt[a]*(a^2 - b^2)^(3/4)*d*Sqrt[e])) - (b*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d
*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(Sqrt[a]*(a^2 - b^2)^(3/4)*d*Sqrt[e]) +
 (2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a*d*Sqrt[e*Sin[c
+ d*x]]) + (b^2*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2,
 2]*Sqrt[Sin[c + d*x]])/(a*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c +
 d*x]]) + (b^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2,
2]*Sqrt[Sin[c + d*x]])/(a*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c +
d*x]])
```

Rubi [A] time = 0.780844, antiderivative size = 370, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.44, Rules used = {3872, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{\sqrt{a}d\sqrt{e}\left(a^2-b^2\right)^{3/4}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{\sqrt{a}d\sqrt{e}\left(a^2-b^2\right)^{3/4}}+\frac{b^2\sqrt{\sin(c+dx)}\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{ad\left(-a\sqrt{a^2-b^2}+a^2-b^2\right)\sqrt{e\sin(c+dx)}}+\frac{b^2\sqrt{\sin(c+d}}{ad\left(a\sqrt{a^2\right.}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] -((b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(S
qrt[a]*(a^2 - b^2)^(3/4)*d*Sqrt[e])) - (b*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d
*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(Sqrt[a]*(a^2 - b^2)^(3/4)*d*Sqrt[e]) +
 (2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a*d*Sqrt[e*Sin[c
+ d*x]]) + (b^2*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2,
 2]*Sqrt[Sin[c + d*x]])/(a*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c +
 d*x]]) + (b^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2,
2]*Sqrt[Sin[c + d*x]])/(a*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c +
d*x]])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))\sqrt{e\sin(c+dx)}}\,dx = -\int \frac{\cos(c+dx)}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx$$

$$= \frac{\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{a} + \frac{b\int \frac{1}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{a}$$

$$= \frac{b^2\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{2a\sqrt{a^2-b^2}} + \frac{b^2\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}+a\sin(c+dx)\right)}\,dx}{2a\sqrt{a^2-b^2}}$$

$$= \frac{2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{ad\sqrt{e\sin(c+dx)}} + \frac{(2be)\,\mathrm{Subst}\left(\int \frac{1}{(-a^2+b^2)e^2+a^2x^4}\,dx, x,\right.}{d}$$

$$= \frac{2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{ad\sqrt{e\sin(c+dx)}} + \frac{b^2\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{}}{a\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin(}}$$

$$= -\frac{b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\left(a^2-b^2\right)^{3/4}d\sqrt{e}} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\left(a^2-b^2\right)^{3/4}d\sqrt{e}} + \frac{2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\right|}{ad\sqrt{e\sin(c}}$$

Mathematica [C] time = 5.94347, size = 546, normalized size = 1.48

$$2\sqrt{\sin(c+dx)}\left(a\sqrt{\cos^2(c+dx)}+b\right)\left(\frac{b\left(-\log\left(-\sqrt{2}\sqrt{a}\sqrt[4]{b^2-a^2}\sqrt{\sin(c+dx)}+\sqrt{b^2-a^2}+a\sin(c+dx)\right)+\log\left(\sqrt{2}\sqrt{a}\sqrt[4]{b^2-a^2}\sqrt{\sin(c+dx)}+\sqrt{b^2-a^2}+a\sin\right.\right.}{4\sqrt{2}\sqrt{a}\left(b^2-a^2\right)^{3/4}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sec[c + d*x])*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] (2*(b + a*Sqrt[Cos[c + d*x]^2])*Sqrt[Sin[c + d*x]]*((b*(-2*ArcTan[1 - (Sqrt
[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]
*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - S
qrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Lo
g[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]]
+ a*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2 - b^
2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 -
b^2)]*Sqrt[Cos[c + d*x]^2]*Sqrt[Sin[c + d*x]])/((-a^2 + b^2 + a^2*Sin[c + d
*x]^2)*(5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[
c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x
]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[5/4, 1/2, 1,
 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2))))
/(d*(b + a*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]])
```

Maple [B] time = 2.793, size = 937, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/2/d*b*e*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(
1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(
1/4)))+1/d*b*e*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*sin(d
*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-1/d/a*(-sin(d*x+c)+1)^(1/2)*(2+2*si
n(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*(a^2-b^2)/((a^2-b^2)^(1/2)-a)/(a+(a^2-b^2)
^(1/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2
*2^(1/2))+1/d*a*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/
2)/((a^2-b^2)^(1/2)-a)/(a+(a^2-b^2)^(1/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*
EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/2/d/a*(-sin(d*x+c)+1)^(1/2)*
(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/((a^2-b^2)^(1/2)-a)/(a+(a^2-b^2)^(1
/2))/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-a/((
a^2-b^2)^(1/2)-a),1/2*2^(1/2))*b^2-1/2/d/a*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d
*x+c))^(1/2)*sin(d*x+c)^(1/2)/((a^2-b^2)^(1/2)-a)/(a+(a^2-b^2)^(1/2))/cos(d
*x+c)/(e*sin(d*x+c))^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^2-b^2)^
(1/2)),1/2*2^(1/2))*b^2-1/2/d*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*
sin(d*x+c)^(1/2)/(a^2-b^2)^(1/2)/((a^2-b^2)^(1/2)-a)/(a+(a^2-b^2)^(1/2))/co
s(d*x+c)/(e*sin(d*x+c))^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-a/((a^2-b^2
)^(1/2)-a),1/2*2^(1/2))*b^2+1/2/d*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1
/2)*sin(d*x+c)^(1/2)/(a^2-b^2)^(1/2)/((a^2-b^2)^(1/2)-a)/(a+(a^2-b^2)^(1/2)
)/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^
2-b^2)^(1/2)),1/2*2^(1/2))*b^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e \sin(c+dx)}\,(a+b\sec(c+dx))}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*sin(c + d*x))*(a + b*sec(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)\sqrt{e\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*sqrt(e*sin(d*x + c))), x)
```

3.238 $\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=430

$$\frac{\sqrt{a}b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{de^{3/2}\left(a^2-b^2\right)^{5/4}}-\frac{\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{de^{3/2}\left(a^2-b^2\right)^{5/4}}-\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\left(a^2-b^2\right)\sqrt{\sin(c+dx)}}+\frac{2(b-a\cos(c+}{de\left(a^2-b^2\right)\sqrt{e\sin(}}$$

```
[Out] (Sqrt[a]*b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e]
)])/((a^2 - b^2)^(5/4)*d*e^(3/2)) - (Sqrt[a]*b*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[
c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/((a^2 - b^2)^(5/4)*d*e^(3/2)) + (2
*(b - a*Cos[c + d*x]))/((a^2 - b^2)*d*e*Sqrt[e*Sin[c + d*x]]) - (b^2*Ellipt
icPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]]
)/((a^2 - b^2)*(a - Sqrt[a^2 - b^2])*d*e*Sqrt[e*Sin[c + d*x]]) - (b^2*Ellip
ticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]
])/((a^2 - b^2)*(a + Sqrt[a^2 - b^2])*d*e*Sqrt[e*Sin[c + d*x]]) - (2*a*Elli
pticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)*d*e^2*Sqrt[
Sin[c + d*x]])
```

Rubi [A] time = 1.03685, antiderivative size = 430, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {3872, 2866, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{\sqrt{a}b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{de^{3/2}\left(a^2-b^2\right)^{5/4}}-\frac{\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{de^{3/2}\left(a^2-b^2\right)^{5/4}}-\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{de^2\left(a^2-b^2\right)\sqrt{\sin(c+dx)}}+\frac{2(b-a\cos(c+}{de\left(a^2-b^2\right)\sqrt{e\sin(}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (Sqrt[a]*b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e]
)])/((a^2 - b^2)^(5/4)*d*e^(3/2)) - (Sqrt[a]*b*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[
c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/((a^2 - b^2)^(5/4)*d*e^(3/2)) + (2
*(b - a*Cos[c + d*x]))/((a^2 - b^2)*d*e*Sqrt[e*Sin[c + d*x]]) - (b^2*Ellipt
icPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]]
)/((a^2 - b^2)*(a - Sqrt[a^2 - b^2])*d*e*Sqrt[e*Sin[c + d*x]]) - (b^2*Ellip
ticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]
])/((a^2 - b^2)*(a + Sqrt[a^2 - b^2])*d*e*Sqrt[e*Sin[c + d*x]]) - (2*a*Elli
pticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)*d*e^2*Sqrt[
Sin[c + d*x]])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
```

```
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
```

```
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{3/2}}\,dx = -\int \frac{\cos(c+dx)}{(-b-a\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx$$

$$= \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}} + \frac{2\int \frac{\left(ab+\frac{1}{2}a^2\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{-b-a\cos(c+dx)}\,dx}{(a^2-b^2)\,e^2}$$

$$= \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}} - \frac{a\int \sqrt{e\sin(c+dx)}\,dx}{(a^2-b^2)\,e^2} + \frac{(ab)\int \frac{\sqrt{e\sin(c+dx)}}{-b-a\cos(c+dx)}\,dx}{(a^2-b^2)\,e^2}$$

$$= \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}} + \frac{b^2\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{2\,(a^2-b^2)\,e} - \frac{b^2\int}{}$$

$$= \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}} - \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{(a^2-b^2)\,de^2\sqrt{\sin(c+dx)}} + \frac{(2a^2}{}$$

$$= \frac{2(b-a\cos(c+dx))}{(a^2-b^2)\,de\sqrt{e\sin(c+dx)}} - \frac{b^2\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{(a^2-b^2)\left(a-\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}$$

$$= \frac{\sqrt{a}b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{(a^2-b^2)^{5/4}de^{3/2}} - \frac{\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{(a^2-b^2)^{5/4}de^{3/2}} + \frac{2(b-a\cos(c}{(a^2-b^2)\,de\sqrt{e\sin}}$$

Mathematica [C] time = 14.2727, size = 834, normalized size = 1.94

$$-\frac{a(b+a\cos(c+dx))\sec(c+dx)\left(\frac{\left(8F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin^{\frac{3}{2}}(c+dx)a^{5/2}+3\sqrt{2}b\left(b^2-a^2\right)^{3/4}\left(2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{a}\sqrt{\sin(c+dx)}}{\sqrt[4]{b^2-a^2}}\right)-2\tan^{-1}\left(\right.\right.\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] -((a*(b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[c + d*x]^(3/2)*((Cos[c + d*x]^2*
(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c +
```

```
d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*
x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b
^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqr
t[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]) + 8*a
^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)]*Sin[c + d*x]^(3/2))*(b + a*Sqrt[1 - Sin[c + d*x]^2]))/(12*Sqrt[a]
*(a^2 - b^2)*(b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (4*b*Cos[c + d*x]
*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^
2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(
1/4)] - Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c
+ d*x]] + I*a*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 -
b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]]))/(Sqrt[a]*(a^2 - b^2)^(1
/4)) + (b*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(
a^2 - b^2)]*Sin[c + d*x]^(3/2))/(3*(-a^2 + b^2)))*(b + a*Sqrt[1 - Sin[c + d
*x]^2]))/((b + a*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/((a - b)*(a + b)
*d*(a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(3/2))) - (2*(b - a*Cos[c + d*x])*
(b + a*Cos[c + d*x])*Tan[c + d*x])/((-a^2 + b^2)*d*(a + b*Sec[c + d*x])*(e*
Sin[c + d*x])^(3/2))
```

Maple [B] time = 3.08, size = 1083, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/d/e*b/(a+b)/(a-b)/(e^2*(a^2-b^2)/a^2)^(1/4)*arctan((e*sin(d*x+c))^(1/2)/(
e^2*(a^2-b^2)/a^2)^(1/4))-1/2/d/e*b/(a+b)/(a-b)/(e^2*(a^2-b^2)/a^2)^(1/4)*l
n(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e
^2*(a^2-b^2)/a^2)^(1/4)))+2/d/e*b/(a^2-b^2)/(e*sin(d*x+c))^(1/2)-1/2/d*b^2/
e/(a^2-b^2)^(1/2)/(a+(a^2-b^2)^(1/2))/((a^2-b^2)^(1/2)-a)/cos(d*x+c)/(e*sin
(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2
)*EllipticPi((-sin(d*x+c)+1)^(1/2),-a/((a^2-b^2)^(1/2)-a),1/2*2^(1/2))+1/2/
d*b^2/e/(a^2-b^2)^(1/2)/(a+(a^2-b^2)^(1/2))/((a^2-b^2)^(1/2)-a)/cos(d*x+c)/
(e*sin(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c
)^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^2-b^2)^(1/2)),1/2*2^(1/2))
-1/2/d*b^2/e/(a^2-b^2)/(a+(a^2-b^2)^(1/2))/((a^2-b^2)^(1/2)-a)/cos(d*x+c)/(
e*sin(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)
^(1/2)*a*EllipticPi((-sin(d*x+c)+1)^(1/2),-a/((a^2-b^2)^(1/2)-a),1/2*2^(1/2
))-1/2/d*b^2/e/(a^2-b^2)/(a+(a^2-b^2)^(1/2))/((a^2-b^2)^(1/2)-a)/cos(d*x+c)
/(e*sin(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+
c)^(1/2)*a*EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^2-b^2)^(1/2)),1/2*2^(1/
2))-2/d*b^2/e/(a^2-b^2)/(a+(a^2-b^2)^(1/2))/((a^2-b^2)^(1/2)-a)/cos(d*x+c)/
(e*sin(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c
)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a+1/d*b^2/e/(a^2-b^2)/
(a+(a^2-b^2)^(1/2))/((a^2-b^2)^(1/2)-a)/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(-s
in(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(
d*x+c)+1)^(1/2),1/2*2^(1/2))*a+2/d*b^2/e/(a^2-b^2)/(a+(a^2-b^2)^(1/2))/((a^
2-b^2)^(1/2)-a)*cos(d*x+c)/(e*sin(d*x+c))^(1/2)*a
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \sec\left(dx + c\right) + a\right) \left(e \sin\left(dx + c\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*(e*sin(d*x + c))^(3/2)), x)
```

3.239 $\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=452

$$\frac{2a\sqrt{\sin(c+dx)}\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3de^2\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}-\frac{a^{3/2}b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{de^{5/2}\left(a^2-b^2\right)^{7/4}}-\frac{a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{de^{5/2}\left(a^2-b^2\right)^{7/4}}+\frac{ab^2\sqrt{\mathrm{si}}}{de^2\left(a^2-\right.}$$

```
[Out] -((a^(3/2)*b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[
e])])/((a^2 - b^2)^(7/4)*d*e^(5/2))) - (a^(3/2)*b*ArcTanh[(Sqrt[a]*Sqrt[e*S
in[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/((a^2 - b^2)^(7/4)*d*e^(5/2)) +
 (2*(b - a*Cos[c + d*x]))/(3*(a^2 - b^2)*d*e*(e*Sin[c + d*x])^(3/2)) + (2*a
*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*(a^2 - b^2)*d*e^2*
Sqrt[e*Sin[c + d*x]]) + (a*b^2*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c -
 Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)*(a^2 - b^2 - a*Sqrt[a^2
 - b^2])*d*e^2*Sqrt[e*Sin[c + d*x]]) + (a*b^2*EllipticPi[(2*a)/(a + Sqrt[a^
2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)*(a^2 - b
^2 + a*Sqrt[a^2 - b^2])*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Rubi [A] time = 1.05037, antiderivative size = 452, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {3872, 2866, 2867, 2642, 2641, 2702, 2807, 2805, 329, 212, 208, 205}

$$-\frac{a^{3/2}b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{de^{5/2}\left(a^2-b^2\right)^{7/4}}-\frac{a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{de^{5/2}\left(a^2-b^2\right)^{7/4}}+\frac{2a\sqrt{\sin(c+dx)}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3de^2\left(a^2-b^2\right)\sqrt{e\sin(c+dx)}}+\frac{ab^2\sqrt{\sin(c+}}{de^2\left(a^2-b^2\right)\left(-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] -((a^(3/2)*b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[
e])])/((a^2 - b^2)^(7/4)*d*e^(5/2))) - (a^(3/2)*b*ArcTanh[(Sqrt[a]*Sqrt[e*S
in[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/((a^2 - b^2)^(7/4)*d*e^(5/2)) +
 (2*(b - a*Cos[c + d*x]))/(3*(a^2 - b^2)*d*e*(e*Sin[c + d*x])^(3/2)) + (2*a
*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*(a^2 - b^2)*d*e^2*
Sqrt[e*Sin[c + d*x]]) + (a*b^2*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c -
 Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)*(a^2 - b^2 - a*Sqrt[a^2
 - b^2])*d*e^2*Sqrt[e*Sin[c + d*x]]) + (a*b^2*EllipticPi[(2*a)/(a + Sqrt[a^
2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)*(a^2 - b
^2 + a*Sqrt[a^2 - b^2])*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
```

```
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
```

```
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{5/2}}\,dx = -\int \frac{\cos(c+dx)}{(-b-a\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx$$

$$= \frac{2(b-a\cos(c+dx))}{3\left(a^2-b^2\right)de(e\sin(c+dx))^{3/2}} + \frac{2\int \frac{ab-\frac{1}{2}a^2\cos(c+dx)}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{3\left(a^2-b^2\right)e^2}$$

$$= \frac{2(b-a\cos(c+dx))}{3\left(a^2-b^2\right)de(e\sin(c+dx))^{3/2}} + \frac{a\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{3\left(a^2-b^2\right)e^2} + \frac{(ab)\int \frac{1}{(-b-a\cos(c+dx)}}{\left(a^2-b^2\right.}$$

$$= \frac{2(b-a\cos(c+dx))}{3\left(a^2-b^2\right)de(e\sin(c+dx))^{3/2}} + \frac{\left(ab^2\right)\int \frac{1}{\sqrt{e\sin(c+dx)}\left(\sqrt{a^2-b^2}-a\sin(c+dx)\right)}\,dx}{2\left(a^2-b^2\right)^{3/2}e^2}$$

$$= \frac{2(b-a\cos(c+dx))}{3\left(a^2-b^2\right)de(e\sin(c+dx))^{3/2}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3\left(a^2-b^2\right)de^2\sqrt{e\sin(c+dx)}} +$$

$$= \frac{2(b-a\cos(c+dx))}{3\left(a^2-b^2\right)de(e\sin(c+dx))^{3/2}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3\left(a^2-b^2\right)de^2\sqrt{e\sin(c+dx)}} -$$

$$= -\frac{a^{3/2}b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\left(a^2-b^2\right)^{7/4}de^{5/2}} - \frac{a^{3/2}b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\left(a^2-b^2\right)^{7/4}de^{5/2}} + \frac{2(b-}{3\left(a^2-b^2\right)}$$

Mathematica [C] time = 12.2987, size = 1233, normalized size = 2.73

$$-\frac{a(b+a\cos(c+dx))\sec(c+dx)\left(\frac{4b\cos(c+dx)\left(\sqrt{1-\sin^2(c+dx)}a+b\right)\left(\frac{5b\left(a^2-b^2\right)F_1\left(\frac{1}{4};\right.}{\sqrt{1-\sin^2(c+dx)}\left(2\left(2F_1\left(\frac{5}{4};\frac{1}{2},2;\frac{9}{4};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)a^2+\left(a^2-b^2\right)F_1\left(\frac{5}{4};\frac{3}{2},1;\frac{9}{4};\right.\right.\right.}\right.}{\right.}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] -(a*(b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[c + d*x]^(5/2)*((-2*a*Cos[c + d*x
]^2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[a]*Sq
```

```
rt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*Sqrt[
Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[a]
*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[Sqrt[-a^2 +
b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*
x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2 - b^2)*AppellF1[1/
4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin
[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1,
 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1
[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (-a
^2 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(
a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*C
os[c + d*x])*(1 - Sin[c + d*x]^2)) + (4*b*Cos[c + d*x]*(b + a*Sqrt[1 - Sin[
c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin
[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c +
 d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b
^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] - Log[Sqrt[a^2 - b^2] + (1
 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]]))/(a
^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^
2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c +
d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[
c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]
^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (a^2 - b^2)*AppellF1[5/4, 3/2, 1, 9
/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2
 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x
]^2])))/(3*(a - b)*(a + b)*d*(a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2)) -
 (2*(b - a*Cos[c + d*x])*(b + a*Cos[c + d*x])*Tan[c + d*x])/(3*(-a^2 + b^2)
*d*(a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(5/2))
```

Maple [A] time = 4.983, size = 681, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x)
```

```
[Out] 2/3/d*b/e/(a^2-b^2)/(e*sin(d*x+c))^(3/2)+1/2/d*b/e/(a+b)/(a-b)*a^2*(e^2*(a^
2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2
)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+1/d*b/e/(a+
b)/(a-b)*a^2*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*sin(d*x
+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+1/2/d/e^2/cos(d*x+c)/(e*sin(d*x+c))^(
1/2)/(a-b)/(a+b)*b^2/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))
^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1
/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/d/e^2/cos(d*x+c)/(e*sin(d*x+c)
)^(1/2)/(a-b)/(a+b)*b^2/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+
c))^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)
^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/3/d*a/e^2/cos(d*x+c)/(e*sin(d
*x+c))^(1/2)/(a^2-b^2)/(cos(d*x+c)^2-1)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+
c))^(1/2)*sin(d*x+c)^(5/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+2/3
/d*a/e^2*cos(d*x+c)/(e*sin(d*x+c))^(1/2)/(a^2-b^2)/(cos(d*x+c)^2-1)*sin(d*x
+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*(e*sin(d*x + c))^(5/2)), x)
```

3.240 $\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{7/2}}\,dx$

Optimal. Leaf size=511

$$\frac{a^{5/2}b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{de^{7/2}\left(a^2-b^2\right)^{9/4}}-\frac{a^{5/2}b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{de^{7/2}\left(a^2-b^2\right)^{9/4}}+\frac{2\left(5a^2b-a\left(3a^2+2b^2\right)\cos(c+dx)\right)}{5de^3\left(a^2-b^2\right)^2\sqrt{e\sin(c+dx)}}-\frac{2a\left(3a^2+2b^2\right)E\left(\frac{1}{2}\left(\right.\right.}{5de^4\left(a^2-\right.}$$

```
[Out] (a^(5/2)*b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e]
)])/((a^2 - b^2)^(9/4)*d*e^(7/2)) - (a^(5/2)*b*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[
c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/((a^2 - b^2)^(9/4)*d*e^(7/2)) + (2
*(b - a*Cos[c + d*x]))/(5*(a^2 - b^2)*d*e*(e*Sin[c + d*x])^(5/2)) + (2*(5*a
^2*b - a*(3*a^2 + 2*b^2)*Cos[c + d*x]))/(5*(a^2 - b^2)^2*d*e^3*Sqrt[e*Sin[c
 + d*x]]) - (a^2*b^2*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*
x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)^2*(a - Sqrt[a^2 - b^2])*d*e^3*Sqr
t[e*Sin[c + d*x]]) - (a^2*b^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c -
Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)^2*(a + Sqrt[a^2 - b^2])*
d*e^3*Sqrt[e*Sin[c + d*x]]) - (2*a*(3*a^2 + 2*b^2)*EllipticE[(c - Pi/2 + d*
x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*(a^2 - b^2)^2*d*e^4*Sqrt[Sin[c + d*x]])
```

Rubi [A] time = 1.37661, antiderivative size = 511, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {3872, 2866, 2867, 2640, 2639, 2701, 2807, 2805, 329, 298, 205, 208}

$$\frac{a^{5/2}b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{de^{7/2}\left(a^2-b^2\right)^{9/4}}-\frac{a^{5/2}b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt{e}\sqrt[4]{a^2-b^2}}\right)}{de^{7/2}\left(a^2-b^2\right)^{9/4}}+\frac{2\left(5a^2b-a\left(3a^2+2b^2\right)\cos(c+dx)\right)}{5de^3\left(a^2-b^2\right)^2\sqrt{e\sin(c+dx)}}-\frac{2a\left(3a^2+2b^2\right)E\left(\frac{1}{2}\left(\right.\right.}{5de^4\left(a^2-\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(7/2)),x]
```


```
[Out] (a^(5/2)*b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e]
)])/((a^2 - b^2)^(9/4)*d*e^(7/2)) - (a^(5/2)*b*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[
c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/((a^2 - b^2)^(9/4)*d*e^(7/2)) + (2
*(b - a*Cos[c + d*x]))/(5*(a^2 - b^2)*d*e*(e*Sin[c + d*x])^(5/2)) + (2*(5*a
^2*b - a*(3*a^2 + 2*b^2)*Cos[c + d*x]))/(5*(a^2 - b^2)^2*d*e^3*Sqrt[e*Sin[c
 + d*x]]) - (a^2*b^2*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*
x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)^2*(a - Sqrt[a^2 - b^2])*d*e^3*Sqr
t[e*Sin[c + d*x]]) - (a^2*b^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c -
Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)^2*(a + Sqrt[a^2 - b^2])*
d*e^3*Sqrt[e*Sin[c + d*x]]) - (2*a*(3*a^2 + 2*b^2)*EllipticE[(c - Pi/2 + d*
x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*(a^2 - b^2)^2*d*e^4*Sqrt[Sin[c + d*x]])
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
```

```
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))(e\sin(c+dx))^{7/2}}\,dx = -\int \frac{\cos(c+dx)}{(-b-a\cos(c+dx))(e\sin(c+dx))^{7/2}}\,dx$$

$$= \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} + \frac{2\int \frac{ab-\frac{3}{2}a^2\cos(c+dx)}{(-b-a\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx}{5\left(a^2-b^2\right)e^2}$$

$$= \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} + \frac{2\left(5a^2b-a\left(3a^2+2b^2\right)\cos(c+dx)\right)}{5\left(a^2-b^2\right)^2de^3\sqrt{e\sin(c+dx)}} + \frac{4\int}{}$$

$$= \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} + \frac{2\left(5a^2b-a\left(3a^2+2b^2\right)\cos(c+dx)\right)}{5\left(a^2-b^2\right)^2de^3\sqrt{e\sin(c+dx)}} + \frac{(a^3}{}$$

$$= \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} + \frac{2\left(5a^2b-a\left(3a^2+2b^2\right)\cos(c+dx)\right)}{5\left(a^2-b^2\right)^2de^3\sqrt{e\sin(c+dx)}} + \frac{(a^2}{}$$

$$= \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} + \frac{2\left(5a^2b-a\left(3a^2+2b^2\right)\cos(c+dx)\right)}{5\left(a^2-b^2\right)^2de^3\sqrt{e\sin(c+dx)}} - \frac{2a(}{}$$

$$= \frac{2(b-a\cos(c+dx))}{5\left(a^2-b^2\right)de(e\sin(c+dx))^{5/2}} + \frac{2\left(5a^2b-a\left(3a^2+2b^2\right)\cos(c+dx)\right)}{5\left(a^2-b^2\right)^2de^3\sqrt{e\sin(c+dx)}} - \frac{a^2b}{(}$$

$$= \frac{a^{5/2}b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\left(a^2-b^2\right)^{9/4}de^{7/2}} - \frac{a^{5/2}b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\left(a^2-b^2\right)^{9/4}de^{7/2}} + \frac{2(b-a\cos}{5\left(a^2-b^2\right)de(e}$$

Mathematica [C] time = 6.79531, size = 930, normalized size = 1.82

$$\frac{(b+a\cos(c+dx))\left(-\frac{2(b-a\cos(c+dx))\csc^3(c+dx)}{5\left(b^2-a^2\right)}-\frac{2\left(3\cos(c+dx)a^3-5ba^2+2b^2\cos(c+dx)a\right)\csc(c+dx)}{5\left(b^2-a^2\right)^2}\right)\sin^3(c+dx)\tan(c+dx)}{d(a+b\sec(c+dx))(e\sin(c+dx))^{7/2}} - \frac{a(b+a}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sec[c + d*x])*(e*Sin[c + d*x])^(7/2)),x]
```

```
[Out] -(a*(b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[c + d*x]^(7/2)*(((3*a^3 + 2*a*b^2
)*Cos[c + d*x]^2*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*Sqr
t[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt[a
]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*
Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[Sqrt[
-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin
[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (a^2*Si
n[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))*(b + a*Sqrt[1 - Sin[c + d*x]
^2]))/(12*a^(3/2)*(a^2 - b^2)*(b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) +
(2*(8*a^2*b + 2*b^3)*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt
[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*
Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt
[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] + Log[Sqrt[a^2
 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c
+ d*x]]))/(Sqrt[a]*(a^2 - b^2)^(1/4)) + (b*AppellF1[3/4, 1/2, 1, 7/4, Sin[c
 + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))/(3*(-a^2 +
 b^2)))*(b + a*Sqrt[1 - Sin[c + d*x]^2]))/((b + a*Cos[c + d*x])*Sqrt[1 - Si
n[c + d*x]^2])))/(5*(a - b)^2*(a + b)^2*d*(a + b*Sec[c + d*x])*(e*Sin[c + d
*x])^(7/2)) + ((b + a*Cos[c + d*x])*((-2*(-5*a^2*b + 3*a^3*Cos[c + d*x] + 2
*a*b^2*Cos[c + d*x])*Csc[c + d*x])/(5*(-a^2 + b^2)^2) - (2*(b - a*Cos[c + d
*x])*Csc[c + d*x]^3)/(5*(-a^2 + b^2)))*Sin[c + d*x]^3*Tan[c + d*x])/(d*(a +
 b*Sec[c + d*x])*(e*Sin[c + d*x])^(7/2))
```

Maple [B] time = 3.823, size = 1672, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(7/2),x)
```

```
[Out] 1/d*b/e^3*a^2/(a+b)^2/(a-b)^2/(e^2*(a^2-b^2)/a^2)^(1/4)*arctan((e*sin(d*x+c
))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-1/2/d*b/e^3*a^2/(a+b)^2/(a-b)^2/(e^2*(a
^2-b^2)/a^2)^(1/4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*
sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+2/d*b/e^3/(a+b)^2/(a-b)^2*a^2
/(e*sin(d*x+c))^(1/2)+2/5/d*b/e/(a-b)/(a+b)/(e*sin(d*x+c))^(5/2)-1/2/d/e^3*
b^2*a^2/(a+(a^2-b^2)^(1/2))/((a^2-b^2)^(1/2)-a)/(a-b)^2/(a+b)^2*sin(d*x+c)^
(1/2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c)
)^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-a/((a^2-b^2)^(1/2)-a),1/2*2^(1/2)
)*(a^2-b^2)^(1/2)+1/2/d/e^3*b^2*a^2/(a+(a^2-b^2)^(1/2))/((a^2-b^2)^(1/2)-a)
/(a-b)^2/(a+b)^2*sin(d*x+c)^(1/2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(-sin(d*x
+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(
a^2-b^2)^(1/2)),1/2*2^(1/2))*(a^2-b^2)^(1/2)-6/5/d/e^3*b^2*a^3/(a+(a^2-b^2)
^(1/2))/((a^2-b^2)^(1/2)-a)/(a-b)^2/(a+b)^2*sin(d*x+c)^(1/2)/cos(d*x+c)/(e*
sin(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*EllipticE((-
sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-4/5/d/e^3*b^4*a/(a+(a^2-b^2)^(1/2))/((a^2-
b^2)^(1/2)-a)/(a-b)^2/(a+b)^2*sin(d*x+c)^(1/2)/cos(d*x+c)/(e*sin(d*x+c))^(1
/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*EllipticE((-sin(d*x+c)+1)^
(1/2),1/2*2^(1/2))+3/5/d/e^3*b^2*a^3/(a+(a^2-b^2)^(1/2))/((a^2-b^2)^(1/2)-a
)/(a-b)^2/(a+b)^2*sin(d*x+c)^(1/2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(-sin(d*
x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^
(1/2))+2/5/d/e^3*b^4*a/(a+(a^2-b^2)^(1/2))/((a^2-b^2)^(1/2)-a)/(a-b)^2/(a+b
)^2*sin(d*x+c)^(1/2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*
(2+2*sin(d*x+c))^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/2/d/e
^3*b^2*a^3/(a+(a^2-b^2)^(1/2))/((a^2-b^2)^(1/2)-a)/(a-b)^2/(a+b)^2*sin(d*x+
```

```
c)^(1/2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x
+c))^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),-a/((a^2-b^2)^(1/2)-a),1/2*2^(1
/2))-1/2/d/e^3*b^2*a^3/(a+(a^2-b^2)^(1/2))/((a^2-b^2)^(1/2)-a)/(a-b)^2/(a+b
)^2*sin(d*x+c)^(1/2)/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*(-sin(d*x+c)+1)^(1/2)*
(2+2*sin(d*x+c))^(1/2)*EllipticPi((-sin(d*x+c)+1)^(1/2),a/(a+(a^2-b^2)^(1/2
)),1/2*2^(1/2))-6/5/d/e^3*b^2*a^3/(a+(a^2-b^2)^(1/2))/((a^2-b^2)^(1/2)-a)/(
a-b)^2/(a+b)^2/sin(d*x+c)^2*cos(d*x+c)^3/(e*sin(d*x+c))^(1/2)-4/5/d/e^3*b^4
*a/(a+(a^2-b^2)^(1/2))/((a^2-b^2)^(1/2)-a)/(a-b)^2/(a+b)^2/sin(d*x+c)^2*cos
(d*x+c)^3/(e*sin(d*x+c))^(1/2)+8/5/d/e^3*b^2*a^3/(a+(a^2-b^2)^(1/2))/((a^2-
b^2)^(1/2)-a)/(a-b)^2/(a+b)^2/sin(d*x+c)^2*cos(d*x+c)/(e*sin(d*x+c))^(1/2)+
2/5/d/e^3*b^4*a/(a+(a^2-b^2)^(1/2))/((a^2-b^2)^(1/2)-a)/(a-b)^2/(a+b)^2/sin
(d*x+c)^2*cos(d*x+c)/(e*sin(d*x+c))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))**(7/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)(e\sin(dx+c))^{\frac{7}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))/(e*sin(d*x+c))^(7/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)*(e*sin(d*x + c))^(7/2)), x)
```

3.241 $\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=1070

result too large to display

```
[Out] (-7*b^3*(a^2 - b^2)^(3/4)*e^(9/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a
^2 - b^2)^(1/4)*Sqrt[e])])/(2*a^(13/2)*d) + (2*b*(a^2 - b^2)^(7/4)*e^(9/2)*
ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(13/
2)*d) + (7*b^3*(a^2 - b^2)^(3/4)*e^(9/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*
x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(2*a^(13/2)*d) - (2*b*(a^2 - b^2)^(7/4)*
e^(9/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])]
)/(a^(13/2)*d) + (7*b^4*(a^2 - b^2)*e^5*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^
2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^7*(a - Sqrt[a^2 - b^2]
)*d*Sqrt[e*Sin[c + d*x]]) - (2*b^2*(a^2 - b^2)^2*e^5*EllipticPi[(2*a)/(a -
Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^7*(a - Sqrt
[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (7*b^4*(a^2 - b^2)*e^5*EllipticPi[(2
*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^
7*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) - (2*b^2*(a^2 - b^2)^2*e^5*
EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c +
 d*x]])/(a^7*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (14*e^4*Ellipt
icE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(15*a^2*d*Sqrt[Sin[c + d*x
]]) - (7*b^2*(3*a^2 - 5*b^2)*e^4*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Si
n[c + d*x]])/(5*a^6*d*Sqrt[Sin[c + d*x]]) - (4*b^2*(8*a^2 - 5*b^2)*e^4*Elli
pticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*a^6*d*Sqrt[Sin[c + d*
x]]) - (14*e^3*Cos[c + d*x]*(e*Sin[c + d*x])^(3/2))/(45*a^2*d) - (7*b^2*e^3
*(5*b - 3*a*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(15*a^5*d) + (4*b*e^3*(5*
(a^2 - b^2) + 3*a*b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(15*a^5*d) + (4*b
*e*(e*Sin[c + d*x])^(7/2))/(7*a^3*d) - (2*e*Cos[c + d*x]*(e*Sin[c + d*x])^(
7/2))/(9*a^2*d) + (b^2*e*(e*Sin[c + d*x])^(7/2))/(a^3*d*(b + a*Cos[c + d*x]
))
```

Rubi [A] time = 2.7946, antiderivative size = 1070, normalized size of antiderivative = 1., number of steps used = 35, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {3872, 2912, 2635, 2640, 2639, 2693, 2865, 2867, 2701, 2807, 2805, 329, 298, 205, 208, 2695}

$$-\frac{2b^2\left(a^2-b^2\right)^2\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}e^5}{a^7\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}+\frac{7b^4\left(a^2-b^2\right)\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+}}{2a^7\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(9/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-7*b^3*(a^2 - b^2)^(3/4)*e^(9/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a
^2 - b^2)^(1/4)*Sqrt[e])])/(2*a^(13/2)*d) + (2*b*(a^2 - b^2)^(7/4)*e^(9/2)*
ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(13/
2)*d) + (7*b^3*(a^2 - b^2)^(3/4)*e^(9/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*
x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(2*a^(13/2)*d) - (2*b*(a^2 - b^2)^(7/4)*
e^(9/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])]
)/(a^(13/2)*d) + (7*b^4*(a^2 - b^2)*e^5*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^
2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^7*(a - Sqrt[a^2 - b^2]
)*d*Sqrt[e*Sin[c + d*x]]) - (2*b^2*(a^2 - b^2)^2*e^5*EllipticPi[(2*a)/(a -
Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^7*(a - Sqrt
```

```
[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (7*b^4*(a^2 - b^2)*e^5*EllipticPi[(2
*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^
7*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) - (2*b^2*(a^2 - b^2)^2*e^5*
EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c +
 d*x]])/(a^7*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (14*e^4*Ellipt
icE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(15*a^2*d*Sqrt[Sin[c + d*x
]]) - (7*b^2*(3*a^2 - 5*b^2)*e^4*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Si
n[c + d*x]])/(5*a^6*d*Sqrt[Sin[c + d*x]]) - (4*b^2*(8*a^2 - 5*b^2)*e^4*Elli
pticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(5*a^6*d*Sqrt[Sin[c + d*
x]]) - (14*e^3*Cos[c + d*x]*(e*Sin[c + d*x])^(3/2))/(45*a^2*d) - (7*b^2*e^3
*(5*b - 3*a*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(15*a^5*d) + (4*b*e^3*(5*
(a^2 - b^2) + 3*a*b*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2))/(15*a^5*d) + (4*b
*e*(e*Sin[c + d*x])^(7/2))/(7*a^3*d) - (2*e*Cos[c + d*x]*(e*Sin[c + d*x])^(
7/2))/(9*a^2*d) + (b^2*e*(e*Sin[c + d*x])^(7/2))/(a^3*d*(b + a*Cos[c + d*x]
))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{9/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{9/2}}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \int \left(\frac{(e\sin(c+dx))^{9/2}}{a^2} + \frac{b^2(e\sin(c+dx))^{9/2}}{a^2(b+a\cos(c+dx))^2} - \frac{2b(e\sin(c+dx))^{9/2}}{a^2(b+a\cos(c+dx))}\right)dx$$

$$= \frac{\int (e\sin(c+dx))^{9/2}\,dx}{a^2} - \frac{(2b)\int \frac{(e\sin(c+dx))^{9/2}}{b+a\cos(c+dx)}\,dx}{a^2} + \frac{b^2\int \frac{(e\sin(c+dx))^{9/2}}{(b+a\cos(c+dx))^2}\,dx}{a^2}$$

$$= \frac{4be(e\sin(c+dx))^{7/2}}{7a^3d} - \frac{2e\cos(c+dx)(e\sin(c+dx))^{7/2}}{9a^2d} + \frac{b^2e(e\sin(c+dx))^{7/2}}{a^3d(b+a\cos(c+dx))} + \frac{\left(7e^2\right)\ \cdots}{\cdots}$$

$$= -\frac{14e^3\cos(c+dx)(e\sin(c+dx))^{3/2}}{45a^2d} - \frac{7b^2e^3(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^5d} + \frac{4be^3\cdots}{\cdots}$$

$$= -\frac{14e^3\cos(c+dx)(e\sin(c+dx))^{3/2}}{45a^2d} - \frac{7b^2e^3(5b-3a\cos(c+dx))(e\sin(c+dx))^{3/2}}{15a^5d} + \frac{4be^3\cdots}{\cdots}$$

$$= \frac{14e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{15a^2d\sqrt{\sin(c+dx)}} - \frac{14e^3\cos(c+dx)(e\sin(c+dx))^{3/2}}{45a^2d} - \frac{7b^2e^3(5b\cdots}{\cdots}$$

$$= \frac{14e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{15a^2d\sqrt{\sin(c+dx)}} - \frac{7b^2\left(3a^2-5b^2\right)e^4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin\cdots}}{5a^6d\sqrt{\sin(c+dx)}}$$

$$= \frac{7b^4\left(a^2-b^2\right)e^5\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{2a^7\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{2b^2\left(a^2-b^2\right)^2e^5\Pi\left(\frac{2a}{a-\sqrt{a^2-\cdots}}\cdots\right.}{a^7\left(a-\sqrt{a^2\cdots}\right.}$$

$$= -\frac{7b^3\left(a^2-b^2\right)^{3/4}e^{9/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{13/2}d} + \frac{2b\left(a^2-b^2\right)^{7/4}e^{9/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{13/2}d} + \cdots$$

Mathematica [C] time = 15.3528, size = 974, normalized size = 0.91

$$(b + a\cos(c+dx))^2\sec^2(c+dx)\left(\frac{(14a^4-159b^2a^2+165b^4)\left(8F_1\left(\frac{3}{4};-\frac{1}{2},1;\frac{7}{4};\sin^2(c+dx),\frac{a^2\sin^2(c+dx)}{a^2-b^2}\right)\sin^{\frac{3}{2}}(c+dx)a^{5/2}+3\sqrt{2}b\left(b^2-a^2\right)^{3/4}\left(2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt{\ldots}}{\ldots}\right.\right.\right.}{\ldots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(9/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*(e*Sin[c + d*x])^(9/2)*(((14*a^4 - 1
59*a^2*b^2 + 165*b^4)*Cos[c + d*x]^2*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*Arc
Tan[1 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan
[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a
^2 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c
 + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[S
in[c + d*x]] + a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin
[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))*(b + a*S
qrt[1 - Sin[c + d*x]^2]))/(12*a^(3/2)*(a^2 - b^2)*(b + a*Cos[c + d*x])*(1 -
 Sin[c + d*x]^2)) + (2*(-46*a^3*b + 66*a*b^3)*Cos[c + d*x]*(((1/8 + I/8)*(2
*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*Arc
Tan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[
a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin
[c + d*x]] + Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[S
in[c + d*x]] + I*a*Sin[c + d*x]]))/(Sqrt[a]*(a^2 - b^2)^(1/4)) + (b*AppellF
1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c
 + d*x]^(3/2))/(3*(-a^2 + b^2)))*(b + a*Sqrt[1 - Sin[c + d*x]^2]))/((b + a*
Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(30*a^5*d*(a + b*Sec[c + d*x])^2*
Sin[c + d*x]^(9/2)) + ((b + a*Cos[c + d*x])^2*Csc[c + d*x]^4*Sec[c + d*x]^2
*(e*Sin[c + d*x])^(9/2)*(-(b*(-37*a^2 + 56*b^2)*Sin[c + d*x])/(21*a^5) + (a
^2*b^2*Sin[c + d*x] - b^4*Sin[c + d*x])/(a^5*(b + a*Cos[c + d*x])) - ((19*a
^2 - 54*b^2)*Sin[2*(c + d*x)])/(90*a^4) - (b*Sin[3*(c + d*x)])/(7*a^3) + Si
n[4*(c + d*x)]/(36*a^2)))/(d*(a + b*Sec[c + d*x])^2)
```

Maple [B] time = 9.805, size = 3808, normalized size = 3.6

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(9/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 7/2/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^6/a^8*(-sin(d*x+c)+1)^(1/2)*(2+
2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin
(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-2/d*e^5/cos(d*x+c)/(e
*sin(d*x+c))^(1/2)*b^4/a^4/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))
^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+1/d*e^
5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^6/a^6/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(
2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*
2^(1/2))-1/2/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a^2/(a^2-b^2)*(-sin(
d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x
+c)+1)^(1/2),1/2*2^(1/2))+1/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a^2/(
```

```
a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*Elli
pticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+3/4/d*e^5/cos(d*x+c)/(e*sin(d*x+c)
)^(1/2)*b^8/a^8/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(
d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a
^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2
/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2
)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/
2)/a),1/2*2^(1/2))+7/4/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/a^4/(a^2-b
^2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b
^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2
^(1/2))-2/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^6/a^6/(a^2-b^2)*(-sin(d*x
+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*
EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/4/d
*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^8/a^8/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2
)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi(
(-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/2/d*e^5/cos(d*
x+c)/(e*sin(d*x+c))^(1/2)*b^2/a^4*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1
/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2)
,1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-5/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/
2)*b^4/a^6*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1
-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a
),1/2*2^(1/2))-5/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/a^6*(-sin(d*x+c)
+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*Ell
ipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+7/2/d*e^
5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^6/a^8*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*
x+c))^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+
1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+5/d*e^5/cos(d*x+c)/(e*sin(d*x
+c))^(1/2)/a^6*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2
)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^4-24/5/d*e^5/cos(d*x+c)/(e
*sin(d*x+c))^(1/2)/a^4*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x
+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2-1/2/d*e^5/cos(d*
x+c)/(e*sin(d*x+c))^(1/2)*b^2/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(
d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c
)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+7/4/d*e^5/cos(d*x+c)/(e*sin
(d*x+c))^(1/2)*b^4/a^4/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/
2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),
1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-2/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2
)*b^6/a^6/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)
^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2
)^(1/2)/a),1/2*2^(1/2))+2/d*e^5*sin(d*x+c)^2*cos(d*x+c)/(e*sin(d*x+c))^(1/2
)*b^4/a^2/(a^2-b^2)/(-cos(d*x+c)^2*a^2+b^2)-1/d*e^5*sin(d*x+c)^2*cos(d*x+c)
/(e*sin(d*x+c))^(1/2)*b^6/a^4/(a^2-b^2)/(-cos(d*x+c)^2*a^2+b^2)+3/2/d*e^5/c
os(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a^4*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c
))^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^
(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/d*e^5/cos(d*x+c)/(e*sin(d*x+c)
)^(1/2)*b^4/a^4/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(
d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/2/d*e^5/cos(d*x
+c)/(e*sin(d*x+c))^(1/2)*b^6/a^6/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d
*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-
10/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^6*(-sin(d*x+c)+1)^(1/2)*(2+2*sin
(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2)
)*b^4+48/5/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^4*(-sin(d*x+c)+1)^(1/2)*
(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2
*2^(1/2))*b^2-1/d*e^5*sin(d*x+c)^2*cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a^2
-b^2)/(-cos(d*x+c)^2*a^2+b^2)-14/15/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a
^2*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE(
(-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+7/15/d*e^5/cos(d*x+c)/(e*sin(d*x+c))^(1/
2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*Ellipt
icF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/d/a^3*e^5*b/(e^2*(a^2-b^2)/a^2)^(1
```

```
/4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/
2)-(e^2*(a^2-b^2)/a^2)^(1/4)))-8/3/d/a^5*e^3*b^3*(e*sin(d*x+c))^(3/2)-2/9/d
*e^5*cos(d*x+c)^5/(e*sin(d*x+c))^(1/2)/a^2+34/45/d*e^5*cos(d*x+c)^3/(e*sin(
d*x+c))^(1/2)/a^2-8/15/d*e^5*cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^2+4/7*b*e*(e
*sin(d*x+c))^(7/2)/a^3/d-15/2/d/a^5*e^5*b^3/(e^2*(a^2-b^2)/a^2)^(1/4)*arcta
n((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+15/4/d/a^5*e^5*b^3/(e^2*(
a^2-b^2)/a^2)^(1/4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e
*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+11/2/d/a^7*e^5*b^5/(e^2*(a^2
-b^2)/a^2)^(1/4)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-11/
4/d/a^7*e^5*b^5/(e^2*(a^2-b^2)/a^2)^(1/4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^
2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+4/3/d/
a^3*e^3*b*(e*sin(d*x+c))^(3/2)+1/d/a^3*e^5*b^3*(e*sin(d*x+c))^(3/2)/(-a^2*c
os(d*x+c)^2*e^2+b^2*e^2)-6/5/d*e^5*cos(d*x+c)^3/(e*sin(d*x+c))^(1/2)/a^4*b^
2+6/5/d*e^5*cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^4*b^2-1/d/a^5*e^5*b^5*(e*sin(
d*x+c))^(3/2)/(-a^2*cos(d*x+c)^2*e^2+b^2*e^2)+2/d/a^3*e^5*b/(e^2*(a^2-b^2)/
a^2)^(1/4)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(9/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \sin (dx + c))^{\frac{9}{2}}}{(b \sec (dx + c) + a)^2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(9/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(9/2)/(b*sec(d*x + c) + a)^2, x)
```

3.242 $\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=1101

result too large to display

```
[Out] (5*b^3*(a^2 - b^2)^(1/4)*e^(7/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^
2 - b^2)^(1/4)*Sqrt[e])])/(2*a^(11/2)*d) - (2*b*(a^2 - b^2)^(5/4)*e^(7/2)*A
rcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(11/2
)*d) + (5*b^3*(a^2 - b^2)^(1/4)*e^(7/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x
]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(2*a^(11/2)*d) - (2*b*(a^2 - b^2)^(5/4)*e
^(7/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])
/(a^(11/2)*d) + (10*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]]
)/(21*a^2*d*Sqrt[e*Sin[c + d*x]]) - (5*b^2*(a^2 - 3*b^2)*e^4*EllipticF[(c -
 Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*a^6*d*Sqrt[e*Sin[c + d*x]]) - (4*
b^2*(4*a^2 - 3*b^2)*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]]
)/(3*a^6*d*Sqrt[e*Sin[c + d*x]]) - (5*b^4*(a^2 - b^2)*e^4*EllipticPi[(2*a)/
(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^6*(a
^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*b^2*(a^2 - b^2)^
2*e^4*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(a^6*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]])
- (5*b^4*(a^2 - b^2)*e^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2
+ d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^6*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*S
qrt[e*Sin[c + d*x]]) + (2*b^2*(a^2 - b^2)^2*e^4*EllipticPi[(2*a)/(a + Sqrt[
a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^6*(a^2 - b^2 + a
*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) - (10*e^3*Cos[c + d*x]*Sqrt[e*Sin
[c + d*x]])/(21*a^2*d) - (5*b^2*e^3*(3*b - a*Cos[c + d*x])*Sqrt[e*Sin[c + d
*x]])/(3*a^5*d) + (4*b*e^3*(3*(a^2 - b^2) + a*b*Cos[c + d*x])*Sqrt[e*Sin[c
+ d*x]])/(3*a^5*d) + (4*b*e*(e*Sin[c + d*x])^(5/2))/(5*a^3*d) - (2*e*Cos[c
+ d*x]*(e*Sin[c + d*x])^(5/2))/(7*a^2*d) + (b^2*e*(e*Sin[c + d*x])^(5/2))/(
a^3*d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 2.93047, antiderivative size = 1101, normalized size of antiderivative = 1., number of steps used = 35, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {3872, 2912, 2635, 2642, 2641, 2693, 2865, 2867, 2702, 2807, 2805, 329, 212, 208, 205, 2695}

$$-\frac{5b^2\left(a^2-3b^2\right)F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}e^4}{3a^6d\sqrt{e\sin(c+dx)}}-\frac{4b^2\left(4a^2-3b^2\right)F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}e^4}{3a^6d\sqrt{e\sin(c+dx)}}+\frac{10F\left(\frac{1}{2}\left(c\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(7/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (5*b^3*(a^2 - b^2)^(1/4)*e^(7/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^
2 - b^2)^(1/4)*Sqrt[e])])/(2*a^(11/2)*d) - (2*b*(a^2 - b^2)^(5/4)*e^(7/2)*A
rcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(11/2
)*d) + (5*b^3*(a^2 - b^2)^(1/4)*e^(7/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x
]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(2*a^(11/2)*d) - (2*b*(a^2 - b^2)^(5/4)*e
^(7/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])
/(a^(11/2)*d) + (10*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]]
)/(21*a^2*d*Sqrt[e*Sin[c + d*x]]) - (5*b^2*(a^2 - 3*b^2)*e^4*EllipticF[(c -
 Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*a^6*d*Sqrt[e*Sin[c + d*x]]) - (4*
b^2*(4*a^2 - 3*b^2)*e^4*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]]
```

```
)/(3*a^6*d*Sqrt[e*Sin[c + d*x]]) - (5*b^4*(a^2 - b^2)*e^4*EllipticPi[(2*a)/
(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^6*(a
^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*b^2*(a^2 - b^2)^
2*e^4*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(a^6*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]])
- (5*b^4*(a^2 - b^2)*e^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2
+ d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^6*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*S
qrt[e*Sin[c + d*x]]) + (2*b^2*(a^2 - b^2)^2*e^4*EllipticPi[(2*a)/(a + Sqrt[
a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^6*(a^2 - b^2 + a
*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) - (10*e^3*Cos[c + d*x]*Sqrt[e*Sin
[c + d*x]])/(21*a^2*d) - (5*b^2*e^3*(3*b - a*Cos[c + d*x])*Sqrt[e*Sin[c + d
*x]])/(3*a^5*d) + (4*b*e^3*(3*(a^2 - b^2) + a*b*Cos[c + d*x])*Sqrt[e*Sin[c
+ d*x]])/(3*a^5*d) + (4*b*e*(e*Sin[c + d*x])^(5/2))/(5*a^3*d) - (2*e*Cos[c
+ d*x]*(e*Sin[c + d*x])^(5/2))/(7*a^2*d) + (b^2*e*(e*Sin[c + d*x])^(5/2))/(
a^3*d*(b + a*Cos[c + d*x]))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2865

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*(g
*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x]))/(b^2*f*(m + p)*(m + p + 1)), x] + Dist[(g^2*
(p - 1))/(b^2*(m + p)*(m + p + 1)), Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin
[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^2
*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1,
 0] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{7/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{7/2}}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \int \left(\frac{(e\sin(c+dx))^{7/2}}{a^2} + \frac{b^2(e\sin(c+dx))^{7/2}}{a^2(b+a\cos(c+dx))^2} - \frac{2b(e\sin(c+dx))^{7/2}}{a^2(b+a\cos(c+dx))}\right)dx$$

$$= \frac{\int (e\sin(c+dx))^{7/2}\,dx}{a^2} - \frac{(2b)\int \frac{(e\sin(c+dx))^{7/2}}{b+a\cos(c+dx)}\,dx}{a^2} + \frac{b^2\int \frac{(e\sin(c+dx))^{7/2}}{(b+a\cos(c+dx))^2}\,dx}{a^2}$$

$$= \frac{4be(e\sin(c+dx))^{5/2}}{5a^3d} - \frac{2e\cos(c+dx)(e\sin(c+dx))^{5/2}}{7a^2d} + \frac{b^2e(e\sin(c+dx))^{5/2}}{a^3d(b+a\cos(c+dx))} + \frac{(5e^2)\ \ldots}{}$$

$$= -\frac{10e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d} - \frac{5b^2e^3(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^5d} + \frac{4be^3\left(3\left(a^2\ldots\right.\right.}{}$$

$$= -\frac{10e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d} - \frac{5b^2e^3(3b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}{3a^5d} + \frac{4be^3\left(3\left(a^2\ldots\right.\right.}{}$$

$$= \frac{10e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21a^2d\sqrt{e\sin(c+dx)}} - \frac{10e^3\cos(c+dx)\sqrt{e\sin(c+dx)}}{21a^2d} - \frac{5b^2e^3(3b-a\ldots}{}$$

$$= \frac{10e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21a^2d\sqrt{e\sin(c+dx)}} - \frac{5b^2\left(a^2-3b^2\right)e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+\ldots}}{3a^6d\sqrt{e\sin(c+dx)}}$$

$$= \frac{10e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{21a^2d\sqrt{e\sin(c+dx)}} - \frac{5b^2\left(a^2-3b^2\right)e^4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+\ldots}}{3a^6d\sqrt{e\sin(c+dx)}}$$

$$= \frac{5b^3\sqrt[4]{a^2-b^2}e^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{11/2}d} - \frac{2b\left(a^2-b^2\right)^{5/4}e^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{11/2}d} + \frac{5b^3\sqrt[4]{\ldots}}{}$$

Mathematica [C] time = 16.6283, size = 2095, normalized size = 1.9

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(7/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*(-((23*a^2 - 84*b^2)*Cos[c + d*x])/(42*a^4) - (b^2*
(-a^2 + b^2))/(a^5*(b + a*Cos[c + d*x])) - (2*b*Cos[2*(c + d*x)])/(5*a^3) +
 Cos[3*(c + d*x)]/(14*a^2))*Csc[c + d*x]^3*Sec[c + d*x]^2*(e*Sin[c + d*x])^
(7/2))/(d*(a + b*Sec[c + d*x])^2) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*
(e*Sin[c + d*x])^(7/2)*((2*(50*a^4 - 273*a^2*b^2 + 105*b^4)*Cos[c + d*x]^2*
(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[a]*Sqrt[S
in[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[
c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[a]*(-a
^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2]
 + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]])
)/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2 - b^2)*AppellF1[1/4, -
1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c +
 d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4
, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4
, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (-a^2 +
 b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2
- b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c
 + d*x])*(1 - Sin[c + d*x]^2)) + (2*(-139*a^3*b + 210*a*b^3)*Cos[c + d*x]*(
b + a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2*ArcTan[1 - ((1 +
I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*S
qrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 +
I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] - Log[S
qrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a
*Sin[c + d*x]]))/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1,
 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])
/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c
+ d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, 1/2, 2
, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (a^2 - b^2)*Appe
llF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*S
in[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c + d*x])*S
qrt[1 - Sin[c + d*x]^2]) + ((231*a^3*b - 420*a*b^3)*Cos[c + d*x]*Cos[2*(c +
 d*x)]*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[
1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2
- b^2)^(3/4)) - ((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt
[Sin[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2 - b^2)^(3/4)) + ((1/4 - I
/4)*(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*S
qrt[Sin[c + d*x]] + I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) - ((1/4
- I/4)*(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4
)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) + (4*
Sqrt[Sin[c + d*x]])/a + (4*b*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^
2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*b*
(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)
/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*
AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)
] + 2*(2*a^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2
)/(a^2 - b^2)] + (a^2 - b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^
2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*
x]^2)))))/((b + a*Cos[c + d*x])*(1 - 2*Sin[c + d*x]^2)*Sqrt[1 - Sin[c + d*x
]^2])))/(210*a^5*d*(a + b*Sec[c + d*x])^2*Sin[c + d*x]^(7/2))
```

Maple [B] time = 9.819, size = 3412, normalized size = 3.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 4/d/a^3*e^3*b*(e*sin(d*x+c))^(1/2)-8/d/a^5*e^3*b^3*(e*sin(d*x+c))^(1/2)-5/d
*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^6*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x
+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^
4-1/2/d*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a^2/(a^2-b^2)*(-sin(d*x+c)+
1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^
(1/2),1/2*2^(1/2))+1/d*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/a^4/(a^2-b^2
)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((
-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/2/d*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)
*b^6/a^6/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^
(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-5/d*e^4/cos(d*x+c)/(e*si
n(d*x+c))^(1/2)*b^4/a^5/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+
c))^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)
^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+7/2/d*e^4/cos(d*x+c)/(e*sin(d*x
+c))^(1/2)*b^6/a^7/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(
1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2
),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-3/2/d*e^4/cos(d*x+c)/(e*sin(d*x+c))^
(1/2)*b^2/a^3/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*
sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(
1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+5/d*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b
^4/a^5/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x
+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-
b^2)^(1/2)/a),1/2*2^(1/2))-3/d*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^6/a^5/
(a^2-b^2)^(3/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/
2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1
/2)/a),1/2*2^(1/2))+5/4/d*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^8/a^7/(a^2-
b^2)^(3/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1
-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a
),1/2*2^(1/2))+1/2/d*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a/(a^2-b^2)^(3
/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b
^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2
^(1/2))-7/2/d*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^6/a^7/(a^2-b^2)^(1/2)*(
-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(
1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2
))-1/2/d*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a/(a^2-b^2)^(3/2)*(-sin(d*
x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)
*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+9/4/
d*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/a^3/(a^2-b^2)^(3/2)*(-sin(d*x+c)+
1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*Elli
pticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-5/21/d*e^
4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c)
)^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+4/d*e
^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^4*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c
))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2-
9/4/d*e^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/a^3/(a^2-b^2)^(3/2)*(-sin(d*x
+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*
EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/d*e
^4/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^6/a^5/(a^2-b^2)^(3/2)*(-sin(d*x+c)+1)^
(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*Ellipti
cPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-5/4/d*e^4/co
s(d*x+c)/(e*sin(d*x+c))^(1/2)*b^8/a^7/(a^2-b^2)^(3/2)*(-sin(d*x+c)+1)^(1/2)
*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((
-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/2/d*e^4/cos(d*x
+c)/(e*sin(d*x+c))^(1/2)*b^2/a^3/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2
*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(
d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-13/4/d/a^3*e^5*b^3*(e^
2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^
2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+9/4/d/
```

```
a^5*e^5*b^5*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))
^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)
^(1/4)))+2/d*e^4*cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^4*b^2*sin(d*x+c)+2/d/a*e
^5*b*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*sin(d*x+c))^(1/
2)/(e^2*(a^2-b^2)/a^2)^(1/4))-13/2/d/a^3*e^5*b^3*(e^2*(a^2-b^2)/a^2)^(1/4)/
(-a^2*e^2+b^2*e^2)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+9
/2/d/a^5*e^5*b^5*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*sin
(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+1/d/a*e^5*b*(e^2*(a^2-b^2)/a^2)^(
1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))
/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))-1/d*e^4*sin(d*x+c)*cos(d
*x+c)/(e*sin(d*x+c))^(1/2)*b^6/a^4/(a^2-b^2)/(-cos(d*x+c)^2*a^2+b^2)+2/d*e^
4*sin(d*x+c)*cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/a^2/(a^2-b^2)/(-cos(d*x+c)
^2*a^2+b^2)+2/7/d*e^4*cos(d*x+c)^3/(e*sin(d*x+c))^(1/2)/a^2*sin(d*x+c)-16/2
1/d*e^4*cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^2*sin(d*x+c)+1/d/a^3*e^5*b^3*(e*s
in(d*x+c))^(1/2)/(-a^2*cos(d*x+c)^2*e^2+b^2*e^2)-1/d/a^5*e^5*b^5*(e*sin(d*x
+c))^(1/2)/(-a^2*cos(d*x+c)^2*e^2+b^2*e^2)+4/5*b*e*(e*sin(d*x+c))^(5/2)/a^3
/d-1/d*e^4*sin(d*x+c)*cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a^2-b^2)/(-cos(d
*x+c)^2*a^2+b^2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(7/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^{\frac{7}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(7/2)/(b*sec(d*x + c) + a)^2, x)
```

3.243 $\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=850

$$\frac{3e^3\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}b^4}{2a^5\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}+\frac{3e^3\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}b^4}{2a^5\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}-\frac{3e^{5/2}\tan^{-1}\left(\sqrt{\right.}{2a^{9/2}\sqrt[4]{a}$$

```
[Out] (-3*b^3*e^(5/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sq
rt[e])])/(2*a^(9/2)*(a^2 - b^2)^(1/4)*d) + (2*b*(a^2 - b^2)^(3/4)*e^(5/2)*A
rcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(9/2)
*d) + (3*b^3*e^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1
/4)*Sqrt[e])])/(2*a^(9/2)*(a^2 - b^2)^(1/4)*d) - (2*b*(a^2 - b^2)^(3/4)*e^(
5/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(
a^(9/2)*d) + (3*b^4*e^3*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 +
 d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^5*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c
 + d*x]]) - (2*b^2*(a^2 - b^2)*e^3*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]),
(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^5*(a - Sqrt[a^2 - b^2])*d*Sqr
t[e*Sin[c + d*x]]) + (3*b^4*e^3*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c
- Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^5*(a + Sqrt[a^2 - b^2])*d*Sqrt
[e*Sin[c + d*x]]) - (2*b^2*(a^2 - b^2)*e^3*EllipticPi[(2*a)/(a + Sqrt[a^2 -
 b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^5*(a + Sqrt[a^2 - b^2
])*d*Sqrt[e*Sin[c + d*x]]) + (6*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e
*Sin[c + d*x]])/(5*a^2*d*Sqrt[Sin[c + d*x]]) - (7*b^2*e^2*EllipticE[(c - Pi
/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(a^4*d*Sqrt[Sin[c + d*x]]) + (4*b*e*(
e*Sin[c + d*x])^(3/2))/(3*a^3*d) - (2*e*Cos[c + d*x]*(e*Sin[c + d*x])^(3/2)
)/(5*a^2*d) + (b^2*e*(e*Sin[c + d*x])^(3/2))/(a^3*d*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 2.12648, antiderivative size = 850, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3872, 2912, 2635, 2640, 2639, 2693, 2867, 2701, 2807, 2805, 329, 298, 205, 208, 2695}

$$\frac{3e^3\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}b^4}{2a^5\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}+\frac{3e^3\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}b^4}{2a^5\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}-\frac{3e^{5/2}\tan^{-1}\left(\sqrt{\right.}{2a^{9/2}\sqrt[4]{a}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(5/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-3*b^3*e^(5/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sq
rt[e])])/(2*a^(9/2)*(a^2 - b^2)^(1/4)*d) + (2*b*(a^2 - b^2)^(3/4)*e^(5/2)*A
rcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(9/2)
*d) + (3*b^3*e^(5/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1
/4)*Sqrt[e])])/(2*a^(9/2)*(a^2 - b^2)^(1/4)*d) - (2*b*(a^2 - b^2)^(3/4)*e^(
5/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(
a^(9/2)*d) + (3*b^4*e^3*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 +
 d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^5*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c
 + d*x]]) - (2*b^2*(a^2 - b^2)*e^3*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]),
(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^5*(a - Sqrt[a^2 - b^2])*d*Sqr
t[e*Sin[c + d*x]]) + (3*b^4*e^3*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c
- Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^5*(a + Sqrt[a^2 - b^2])*d*Sqrt
[e*Sin[c + d*x]]) - (2*b^2*(a^2 - b^2)*e^3*EllipticPi[(2*a)/(a + Sqrt[a^2 -
```

```
 b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^5*(a + Sqrt[a^2 - b^2
])*d*Sqrt[e*Sin[c + d*x]]) + (6*e^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e
*Sin[c + d*x]])/(5*a^2*d*Sqrt[Sin[c + d*x]]) - (7*b^2*e^2*EllipticE[(c - Pi
/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(a^4*d*Sqrt[Sin[c + d*x]]) + (4*b*e*(
e*Sin[c + d*x])^(3/2))/(3*a^3*d) - (2*e*Cos[c + d*x]*(e*Sin[c + d*x])^(3/2)
)/(5*a^2*d) + (b^2*e*(e*Sin[c + d*x])^(3/2))/(a^3*d*(b + a*Cos[c + d*x]))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
```

```
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^{5/2}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{5/2}}{(-b-a\cos(c+dx))^2}\,dx$$
$$= \int \left(\frac{(e\sin(c+dx))^{5/2}}{a^2} + \frac{b^2(e\sin(c+dx))^{5/2}}{a^2(b+a\cos(c+dx))^2} - \frac{2b(e\sin(c+dx))^{5/2}}{a^2(b+a\cos(c+dx))}\right) dx$$
$$= \frac{\int (e\sin(c+dx))^{5/2}\,dx}{a^2} - \frac{(2b)\int \frac{(e\sin(c+dx))^{5/2}}{b+a\cos(c+dx)}\,dx}{a^2} + \frac{b^2\int \frac{(e\sin(c+dx))^{5/2}}{(b+a\cos(c+dx))^2}\,dx}{a^2}$$
$$= \frac{4be(e\sin(c+dx))^{3/2}}{3a^3d} - \frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5a^2d} + \frac{b^2e(e\sin(c+dx))^{3/2}}{a^3d(b+a\cos(c+dx))} + \frac{(3e^2)}{}$$
$$= \frac{4be(e\sin(c+dx))^{3/2}}{3a^3d} - \frac{2e\cos(c+dx)(e\sin(c+dx))^{3/2}}{5a^2d} + \frac{b^2e(e\sin(c+dx))^{3/2}}{a^3d(b+a\cos(c+dx))} - \frac{(3b^2e^2}{}$$
$$= \frac{6e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}} + \frac{4be(e\sin(c+dx))^{3/2}}{3a^3d} - \frac{2e\cos(c+dx)(e\sin(c+}{5a^2d}$$
$$= \frac{6e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{5a^2d\sqrt{\sin(c+dx)}} - \frac{7b^2e^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^4d\sqrt{\sin(c+dx)}} + \frac{4}{}$$
$$= \frac{3b^4e^3\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{2a^5\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{2b^2\left(a^2-b^2\right)e^3\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\right.\right.}{a^5\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e}}$$
$$= -\frac{3b^3e^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{9/2}\sqrt[4]{a^2-b^2}d} + \frac{2b\left(a^2-b^2\right)^{3/4}e^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{9/2}d} + \frac{3b^3e^{5/2}\tanh}{2a^{9/2}}$$

Mathematica [C] time = 14.9118, size = 886, normalized size = 1.04

$$(b+a\cos$$

$$\frac{(b+a\cos(c+dx))^2\csc^2(c+dx)\sec^2(c+dx)(e\sin(c+dx))^{5/2}\left(\frac{\sin(c+dx)b^2}{a^3(b+a\cos(c+dx))}+\frac{4\sin(c+dx)b}{3a^3}-\frac{\sin(2(c+dx))}{5a^2}\right)}{d(a+b\sec(c+dx))^2} -$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(5/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*(e*Sin[c + d*x])^(5/2)*(((-6*a^2 +
35*b^2)*Cos[c + d*x]^2*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[
2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*
Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sq
rt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log
[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
 a*Sin[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (
a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))*(b + a*Sqrt[1 - Sin[c
+ d*x]^2]))/(12*a^(3/2)*(a^2 - b^2)*(b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^
2)) + (28*a*b*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqr
t[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Si
n[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^
2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2]
```

```
 + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]]
))/(Sqrt[a]*(a^2 - b^2)^(1/4)) + (b*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]
^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))/(3*(-a^2 + b^2)))
*(b + a*Sqrt[1 - Sin[c + d*x]^2]))/((b + a*Cos[c + d*x])*Sqrt[1 - Sin[c + d
*x]^2])))/(10*a^3*d*(a + b*Sec[c + d*x])^2*Sin[c + d*x]^(5/2)) + ((b + a*Co
s[c + d*x])^2*Csc[c + d*x]^2*Sec[c + d*x]^2*(e*Sin[c + d*x])^(5/2)*((4*b*Si
n[c + d*x])/(3*a^3) + (b^2*Sin[c + d*x])/(a^3*(b + a*Cos[c + d*x])) - Sin[2
*(c + d*x)]/(5*a^2)))/(d*(a + b*Sec[c + d*x])^2)
```

Maple [B] time = 7.117, size = 2540, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 3/5/d*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2+2*si
n(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2
))-6/5/d*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2+2
*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(
1/2))+1/d*e^3*sin(d*x+c)^2*cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^2*b^4/(a^2-b^2
)/(-cos(d*x+c)^2*a^2+b^2)-5/2/d*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/a^6
*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)
^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1
/2))+1/2/d*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^4*b^4/(a^2-b^2)*(-sin(d*x+
c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+
1)^(1/2),1/2*2^(1/2))+3/2/d*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a^4*(-s
in(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/
2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))
-1/d*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^4*b^4/(a^2-b^2)*(-sin(d*x+c)+1)^
(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/
2),1/2*2^(1/2))-5/2/d*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/a^6*(-sin(d*x
+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*
EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/d*e
^3*sin(d*x+c)^2*cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a^2-b^2)/(-cos(d*x+c)^
2*a^2+b^2)+6/d*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a^4*(-sin(d*x+c)+1)^
(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/
2),1/2*2^(1/2))-3/d*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a^4*(-sin(d*x+c
)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1
)^(1/2),1/2*2^(1/2))+5/4/d*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^4*b^4/(a^2
-b^2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2
-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2
*2^(1/2))-3/4/d*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^6*b^6/(a^2-b^2)*(-sin
(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)
/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1
/2/d*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/(a^2-b^2)/a^2*(-sin(d*x+c)+1)^(1/2
)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi(
(-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))*b^2+5/4/d*e^3/co
s(d*x+c)/(e*sin(d*x+c))^(1/2)/a^4*b^4/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2+2*
sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d
*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-3/4/d*e^3/cos(d*x+c)/(e
*sin(d*x+c))^(1/2)/a^6*b^6/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))
^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1
/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/d*e^3/cos(d*x+c)/(e*sin(d*x+c)
)^(1/2)/(a^2-b^2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+
c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b
```

```
^2)^(1/2)/a),1/2*2^(1/2))*b^2-1/2/d*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^2
*b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2
)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+3/2/d*e^3/cos(d*x+c)/(e*sin(
d*x+c))^(1/2)*b^2/a^4*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+
c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b
^2)^(1/2)/a),1/2*2^(1/2))+1/d*e^3/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^2*b^2/(
a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*Elli
pticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+2/5*e^3*cos(d*x+c)^3/a^2/d/(e*sin(
d*x+c))^(1/2)+1/d/a^3*e^3*b^3*(e*sin(d*x+c))^(3/2)/(-a^2*cos(d*x+c)^2*e^2+b
^2*e^2)+2/d/a^3*e^3*b/(e^2*(a^2-b^2)/a^2)^(1/4)*arctan((e*sin(d*x+c))^(1/2)
/(e^2*(a^2-b^2)/a^2)^(1/4))-1/d/a^3*e^3*b/(e^2*(a^2-b^2)/a^2)^(1/4)*ln(((e*
sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^
2-b^2)/a^2)^(1/4)))-7/2/d/a^5*e^3*b^3/(e^2*(a^2-b^2)/a^2)^(1/4)*arctan((e*s
in(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+7/4/d/a^5*e^3*b^3/(e^2*(a^2-b^2
)/a^2)^(1/4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*
x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+4/3*b*e*(e*sin(d*x+c))^(3/2)/a^3/d-
2/5*e^3*cos(d*x+c)/a^2/d/(e*sin(d*x+c))^(1/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(5/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^{\frac{5}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(5/2)/(b*sec(d*x + c) + a)^2, x)
```

3.244 $\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=882

$$-\frac{e^2\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a^4\left(a^2-\sqrt{a^2-b^2}\,a-b^2\right)d\sqrt{e\sin(c+dx)}}-\frac{e^2\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a^4\left(a^2+\sqrt{a^2-b^2}\,a-b^2\right)d\sqrt{e\sin(c+dx)}}+\frac{e^{3/2}\tan^{-1}\left(\cdots\right.}{2a^{7/2}\left(a\cdots\right.}$$

```
[Out] (b^3*e^(3/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[
e])])/(2*a^(7/2)*(a^2 - b^2)^(3/4)*d) - (2*b*(a^2 - b^2)^(1/4)*e^(3/2)*ArcT
an[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(7/2)*d)
 + (b^3*e^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*S
qrt[e])])/(2*a^(7/2)*(a^2 - b^2)^(3/4)*d) - (2*b*(a^2 - b^2)^(1/4)*e^(3/2)*
ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(7/
2)*d) + (2*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*a^2*
d*Sqrt[e*Sin[c + d*x]]) - (5*b^2*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[
Sin[c + d*x]])/(a^4*d*Sqrt[e*Sin[c + d*x]]) - (b^4*e^2*EllipticPi[(2*a)/(a
- Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^4*(a^2
- b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*b^2*(a^2 - b^2)*e^2
*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
+ d*x]])/(a^4*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) - (b^
4*e^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(2*a^4*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]
) + (2*b^2*(a^2 - b^2)*e^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/
2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^4*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*S
qrt[e*Sin[c + d*x]]) + (4*b*e*Sqrt[e*Sin[c + d*x]])/(a^3*d) - (2*e*Cos[c +
d*x]*Sqrt[e*Sin[c + d*x]])/(3*a^2*d) + (b^2*e*Sqrt[e*Sin[c + d*x]])/(a^3*d*
(b + a*Cos[c + d*x]))
```

Rubi [A] time = 2.16782, antiderivative size = 882, normalized size of antiderivative = 1., number of steps used = 32, number of rules used = 15, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3872, 2912, 2635, 2642, 2641, 2693, 2867, 2702, 2807, 2805, 329, 212, 208, 205, 2695}

$$-\frac{e^2\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a^4\left(a^2-\sqrt{a^2-b^2}\,a-b^2\right)d\sqrt{e\sin(c+dx)}}-\frac{e^2\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}\,b^4}{2a^4\left(a^2+\sqrt{a^2-b^2}\,a-b^2\right)d\sqrt{e\sin(c+dx)}}+\frac{e^{3/2}\tan^{-1}\left(\cdots\right.}{2a^{7/2}\left(a\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (b^3*e^(3/2)*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[
e])])/(2*a^(7/2)*(a^2 - b^2)^(3/4)*d) - (2*b*(a^2 - b^2)^(1/4)*e^(3/2)*ArcT
an[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(7/2)*d)
 + (b^3*e^(3/2)*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*S
qrt[e])])/(2*a^(7/2)*(a^2 - b^2)^(3/4)*d) - (2*b*(a^2 - b^2)^(1/4)*e^(3/2)*
ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(7/
2)*d) + (2*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*a^2*
d*Sqrt[e*Sin[c + d*x]]) - (5*b^2*e^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[
Sin[c + d*x]])/(a^4*d*Sqrt[e*Sin[c + d*x]]) - (b^4*e^2*EllipticPi[(2*a)/(a
- Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^4*(a^2
- b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*b^2*(a^2 - b^2)*e^2
*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
```

```
+ d*x]])/(a^4*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) - (b^
4*e^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(2*a^4*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]
) + (2*b^2*(a^2 - b^2)*e^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/
2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^4*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*S
qrt[e*Sin[c + d*x]]) + (4*b*e*Sqrt[e*Sin[c + d*x]])/(a^3*d) - (2*e*Cos[c +
d*x]*Sqrt[e*Sin[c + d*x]])/(3*a^2*d) + (b^2*e*Sqrt[e*Sin[c + d*x]])/(a^3*d*
(b + a*Cos[c + d*x]))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2693

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + 1)), x] + Dist[(g^2*(p - 1))/(b*(m + 1)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Free
Q[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && In
tegersQ[2*m, 2*p]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2695

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1))/(b*f*(m + p)), x] + Dist[(g^2*(p - 1))/(b*(m + p)), Int[(g*Cos[
e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
0] && IntegersQ[2*m, 2*p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(e\sin(c+dx))^{3/2}}{(a+b\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)(e\sin(c+dx))^{3/2}}{(-b-a\cos(c+dx))^2}\,dx \\
&= \int \left(\frac{(e\sin(c+dx))^{3/2}}{a^2} + \frac{b^2(e\sin(c+dx))^{3/2}}{a^2(b+a\cos(c+dx))^2} - \frac{2b(e\sin(c+dx))^{3/2}}{a^2(b+a\cos(c+dx))}\right)dx \\
&= \frac{\int (e\sin(c+dx))^{3/2}\,dx}{a^2} - \frac{(2b)\int \frac{(e\sin(c+dx))^{3/2}}{b+a\cos(c+dx)}\,dx}{a^2} + \frac{b^2\int \frac{(e\sin(c+dx))^{3/2}}{(b+a\cos(c+dx))^2}\,dx}{a^2} \\
&= \frac{4be\sqrt{e\sin(c+dx)}}{a^3d} - \frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d} + \frac{b^2e\sqrt{e\sin(c+dx)}}{a^3d(b+a\cos(c+dx))} + \frac{e^2\int \frac{1}{\sqrt{e\sin(c+d}}}{3a^2} \\
&= \frac{4be\sqrt{e\sin(c+dx)}}{a^3d} - \frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d} + \frac{b^2e\sqrt{e\sin(c+dx)}}{a^3d(b+a\cos(c+dx))} - \frac{(b^2e^2)\int \frac{}{\sqrt{e\sin}}}{2a^4} \\
&= \frac{2e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3a^2d\sqrt{e\sin(c+dx)}} + \frac{4be\sqrt{e\sin(c+dx)}}{a^3d} - \frac{2e\cos(c+dx)\sqrt{e\sin(c+dx)}}{3a^2d} \\
&= \frac{2e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3a^2d\sqrt{e\sin(c+dx)}} - \frac{5b^2e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{a^4d\sqrt{e\sin(c+dx)}} + \frac{4be\sqrt{e\,s}}{a} \\
&= \frac{2e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{3a^2d\sqrt{e\sin(c+dx)}} - \frac{5b^2e^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{a^4d\sqrt{e\sin(c+dx)}} - \frac{2b^2\sqrt{a^2}}{} \\
&= \frac{b^3e^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{7/2}\left(a^2-b^2\right)^{3/4}d} - \frac{2b\sqrt[4]{a^2-b^2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{7/2}d} + \frac{b^3e^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin}}{\sqrt[4]{a^2-}}\right)}{2a^{7/2}\left(a^2-b^2\right)^{3/4}}
\end{aligned}
$$

Mathematica [C] time = 16.0484, size = 2012, normalized size = 2.28

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^(3/2)/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*((-2*Cos[c + d*x])/(3*a^2) + b^2/(a^3*(b + a*Cos[c
+ d*x])))*Csc[c + d*x]*Sec[c + d*x]^2*(e*Sin[c + d*x])^(3/2))/(d*(a + b*Sec
[c + d*x])^2) - ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*(e*Sin[c + d*x])^(3/
2)*((2*(-2*a^2 + 3*b^2)*Cos[c + d*x]^2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b
*(-2*ArcTan[1 - (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] +
2*ArcTan[1 + (Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log
[Sqrt[-a^2 + b^2] - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
 a*Sin[c + d*x]] + Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4
)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3
/4)) - (5*a*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Si
n[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5
*(a^2 - b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^
2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2
*Sin[c + d*x]^2)/(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin
[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*
(-1 + Sin[c + d*x]^2))))/((b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (8*
a*b*Cos[c + d*x]*(b + a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2
*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*Arc
Tan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[
a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin
```

```
[c + d*x]] - Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[S
in[c + d*x]] + I*a*Sin[c + d*x]]))/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*App
ellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*S
qrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1
/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*Ap
pellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]
 + (a^2 - b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^
2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b +
 a*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2]) - (6*a*b*Cos[c + d*x]*Cos[2*(c +
 d*x)]*(b + a*Sqrt[1 - Sin[c + d*x]^2])*(((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[
1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2
- b^2)^(3/4)) - ((1/2 - I/2)*(a^2 - 2*b^2)*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt
[Sin[c + d*x]])/(a^2 - b^2)^(1/4)])/(a^(3/2)*(a^2 - b^2)^(3/4)) + ((1/4 - I
/4)*(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*S
qrt[Sin[c + d*x]] + I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) - ((1/4
- I/4)*(a^2 - 2*b^2)*Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4
)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]])/(a^(3/2)*(a^2 - b^2)^(3/4)) + (4*
Sqrt[Sin[c + d*x]])/a + (4*b*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^
2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(5/2))/(5*(a^2 - b^2)) + (10*b*
(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)
/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*
AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)
] + 2*(2*a^2*AppellF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2
)/(a^2 - b^2)] + (a^2 - b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^
2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*
x]^2)))))/((b + a*Cos[c + d*x])*(1 - 2*Sin[c + d*x]^2)*Sqrt[1 - Sin[c + d*x
]^2])))/(6*a^3*d*(a + b*Sec[c + d*x])^2*Sin[c + d*x]^(3/2))
```

Maple [B] time = 7.666, size = 2282, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 4*b*e*(e*sin(d*x+c))^(1/2)/a^3/d+1/d/a^3*e^3*b^3*(e*sin(d*x+c))^(1/2)/(-a^2
*cos(d*x+c)^2*e^2+b^2*e^2)+2/d/a*e^3*b*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+
b^2*e^2)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-5/2/d/a^3*e
^3*b^3*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*sin(d*x+c))^(
1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+1/d/a*e^3*b*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2
*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d
*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))-5/4/d/a^3*e^3*b^3*(e^2*(a^2-b^2)/a
^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(
1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))-1/3/d*e^2/cos(d*x+c
)/(e*sin(d*x+c))^(1/2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin
(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-2/3/d*e^2*cos(d*
x+c)/(e*sin(d*x+c))^(1/2)/a^2*sin(d*x+c)+3/d*e^2/cos(d*x+c)/(e*sin(d*x+c))^
(1/2)*b^2/a^4*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)
*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+3/2/d*e^2/cos(d*x+c)/(e*sin(d
*x+c))^(1/2)*b^2/a^3/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))
^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1
/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-5/2/d*e^2/cos(d*x+c)/(e*sin(d*x+c)
)^(1/2)*b^4/a^5/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2
)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1
/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-3/2/d*e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/
2)*b^2/a^3/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin
```

```
(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(
a^2-b^2)^(1/2)/a),1/2*2^(1/2))+5/2/d*e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^
4/a^5/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+
c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b
^2)^(1/2)/a),1/2*2^(1/2))-1/d*e^2*sin(d*x+c)*cos(d*x+c)/(e*sin(d*x+c))^(1/2
)*b^2/(a^2-b^2)/(-cos(d*x+c)^2*a^2+b^2)+1/d*e^2*sin(d*x+c)*cos(d*x+c)/(e*si
n(d*x+c))^(1/2)/a^2*b^4/(a^2-b^2)/(-cos(d*x+c)^2*a^2+b^2)-1/2/d*e^2/cos(d*x
+c)/(e*sin(d*x+c))^(1/2)/a^2*b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d
*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+
1/2/d*e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^4*b^4/(a^2-b^2)*(-sin(d*x+c)+1)
^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1
/2),1/2*2^(1/2))-1/2/d*e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(3/2)/
a*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2
)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(
1/2))*b^2+7/4/d*e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^3*b^4/(a^2-b^2)^(3/2)
*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)
^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1
/2))-5/4/d*e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^5*b^6/(a^2-b^2)^(3/2)*(-si
n(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2
)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+
1/2/d*e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/(a^2-b^2)^(3/2)/a*(-sin(d*x+c)+1)
^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*Ellipt
icPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))*b^2-7/4/d*e
^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^3*b^4/(a^2-b^2)^(3/2)*(-sin(d*x+c)+1)^
(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*Ellipti
cPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+5/4/d*e^2/co
s(d*x+c)/(e*sin(d*x+c))^(1/2)/a^5*b^6/(a^2-b^2)^(3/2)*(-sin(d*x+c)+1)^(1/2)
*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((
-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(3/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^{\frac{3}{2}}}{(b\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^(3/2)/(b*sec(d*x + c) + a)^2, x)
```

3.245 $\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=809

$$-\frac{e\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}b^4}{2a^3\left(a^2-b^2\right)\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}-\frac{e\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}b^4}{2a^3\left(a^2-b^2\right)\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}+\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin}}{\sqrt[4]{a^2-}}\right.}{2a^{5/2}\left(a^2-b^2\right.}$$

```
[Out] (b^3*Sqrt[e]*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[
e])])/(2*a^(5/2)*(a^2 - b^2)^(5/4)*d) + (2*b*Sqrt[e]*ArcTan[(Sqrt[a]*Sqrt[e
*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(5/2)*(a^2 - b^2)^(1/4)*d)
 - (b^3*Sqrt[e]*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*S
qrt[e])])/(2*a^(5/2)*(a^2 - b^2)^(5/4)*d) - (2*b*Sqrt[e]*ArcTanh[(Sqrt[a]*S
qrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(5/2)*(a^2 - b^2)^(1/
4)*d) - (2*b^2*e*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2
, 2]*Sqrt[Sin[c + d*x]])/(a^3*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]])
 - (b^4*e*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sq
rt[Sin[c + d*x]])/(2*a^3*(a^2 - b^2)*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c +
 d*x]]) - (2*b^2*e*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)
/2, 2]*Sqrt[Sin[c + d*x]])/(a^3*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]
]) - (b^4*e*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*
Sqrt[Sin[c + d*x]])/(2*a^3*(a^2 - b^2)*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c
 + d*x]]) + (2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(a^2*
d*Sqrt[Sin[c + d*x]]) - (b^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c
+ d*x]])/(a^2*(a^2 - b^2)*d*Sqrt[Sin[c + d*x]]) + (b^2*(e*Sin[c + d*x])^(3/
2))/(a*(a^2 - b^2)*d*e*(b + a*Cos[c + d*x]))
```

Rubi [A] time = 1.83326, antiderivative size = 809, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {3872, 2912, 2640, 2639, 2694, 2867, 2701, 2807, 2805, 329, 298, 205, 208}

$$-\frac{e\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}b^4}{2a^3\left(a^2-b^2\right)\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}-\frac{e\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}b^4}{2a^3\left(a^2-b^2\right)\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}}+\frac{\sqrt{e}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin}}{\sqrt[4]{a^2-}}\right.}{2a^{5/2}\left(a^2-b^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Sin[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (b^3*Sqrt[e]*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[
e])])/(2*a^(5/2)*(a^2 - b^2)^(5/4)*d) + (2*b*Sqrt[e]*ArcTan[(Sqrt[a]*Sqrt[e
*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(5/2)*(a^2 - b^2)^(1/4)*d)
 - (b^3*Sqrt[e]*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*S
qrt[e])])/(2*a^(5/2)*(a^2 - b^2)^(5/4)*d) - (2*b*Sqrt[e]*ArcTanh[(Sqrt[a]*S
qrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(5/2)*(a^2 - b^2)^(1/
4)*d) - (2*b^2*e*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2
, 2]*Sqrt[Sin[c + d*x]])/(a^3*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]])
 - (b^4*e*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sq
rt[Sin[c + d*x]])/(2*a^3*(a^2 - b^2)*(a - Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c +
 d*x]]) - (2*b^2*e*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)
/2, 2]*Sqrt[Sin[c + d*x]])/(a^3*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]
]) - (b^4*e*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*
Sqrt[Sin[c + d*x]])/(2*a^3*(a^2 - b^2)*(a + Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c
 + d*x]]) + (2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(a^2*
```

```
d*Sqrt[Sin[c + d*x]]) - (b^2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c
+ d*x]])/(a^2*(a^2 - b^2)*d*Sqrt[Sin[c + d*x]]) + (b^2*(e*Sin[c + d*x])^(3/
2))/(a*(a^2 - b^2)*d*e*(b + a*Cos[c + d*x]))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
```

```
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^2(c+dx)\sqrt{e\sin(c+dx)}}{(-b-a\cos(c+dx))^2}\,dx$$

$$= \int \left(\frac{\sqrt{e\sin(c+dx)}}{a^2} + \frac{b^2\sqrt{e\sin(c+dx)}}{a^2(b+a\cos(c+dx))^2} - \frac{2b\sqrt{e\sin(c+dx)}}{a^2(b+a\cos(c+dx))}\right)dx$$

$$= \frac{\int \sqrt{e\sin(c+dx)}\,dx}{a^2} - \frac{(2b)\int \frac{\sqrt{e\sin(c+dx)}}{b+a\cos(c+dx)}\,dx}{a^2} + \frac{b^2\int \frac{\sqrt{e\sin(c+dx)}}{(b+a\cos(c+dx))^2}\,dx}{a^2}$$

$$= \frac{b^2(e\sin(c+dx))^{3/2}}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))} + \frac{b^2\int \frac{\left(-b-\frac{1}{2}a\cos(c+dx)\right)\sqrt{e\sin(c+dx)}}{b+a\cos(c+dx)}\,dx}{a^2\left(a^2-b^2\right)} + \frac{\left(b^2e\right)\int \frac{}{\sqrt{e\sin(c+dx}}$$

$$= \frac{2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{e\sin(c+dx)}}{a^2d\sqrt{\sin(c+dx)}} + \frac{b^2(e\sin(c+dx))^{3/2}}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))} - \frac{b^2\int \sqrt{e\sin(c}}{2a^2\left(a^2-\right.}$$

$$= -\frac{2b^2e\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{a^3\left(a-\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c+dx)}} - \frac{2b^2e\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right.}{a^3\left(a+\sqrt{a^2-b^2}\right)d\sqrt{e\sin(c}}$$

$$= \frac{2b\sqrt{e}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}\sqrt[4]{a^2-b^2}d} - \frac{2b\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}\sqrt[4]{a^2-b^2}d} - \frac{2b^2e\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}\right.\right.}{a^3\left(a-\sqrt{a^2-b^2}\right)d}$$

$$= \frac{2b\sqrt{e}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}\sqrt[4]{a^2-b^2}d} - \frac{2b\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}\sqrt[4]{a^2-b^2}d} - \frac{2b^2e\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}\right.\right.}{a^3\left(a-\sqrt{a^2-b^2}\right)d}$$

$$= \frac{b^3\sqrt{e}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{5/2}\left(a^2-b^2\right)^{5/4}d} + \frac{2b\sqrt{e}\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{5/2}\sqrt[4]{a^2-b^2}d} - \frac{b^3\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{5/2}\left(a^2-b^2\right)^{5/4}d}$$

Mathematica [C] time = 15.0446, size = 854, normalized size = 1.06

$$\frac{(b+a\cos(c+dx))\sec(c+dx)\sqrt{e\sin(c+dx)}\tan(c+dx)b^2}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\sqrt{e\sin(c+dx)}\left(\frac{(3b^2-2a}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Sin[c + d*x]]/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Sqrt[e*Sin[c + d*x]]*(((-2*a^2 + 3*b
^2)*Cos[c + d*x]^2*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*S
qrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt
[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2
]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[Sqr
t[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*S
in[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (a^2*
Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))*(b + a*Sqrt[1 - Sin[c + d*
x]^2]))/(12*a^(3/2)*(a^2 - b^2)*(b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2))
+ (4*a*b*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin
```

```
[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c +
 d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b
^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] + Log[Sqrt[a^2 - b^2] + (1
 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]]))/(S
qrt[a]*(a^2 - b^2)^(1/4)) + (b*AppellF1[3/4, 1/2, 1, 7/4, Sin[c + d*x]^2, (
a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))/(3*(-a^2 + b^2)))*(b +
 a*Sqrt[1 - Sin[c + d*x]^2]))/((b + a*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2
])))/(2*a*(-a + b)*(a + b)*d*(a + b*Sec[c + d*x])^2*Sqrt[Sin[c + d*x]]) + (
b^2*(b + a*Cos[c + d*x])*Sec[c + d*x]*Sqrt[e*Sin[c + d*x]]*Tan[c + d*x])/(a
*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2)
```

Maple [A] time = 6.344, size = 1563, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c))^2,x)
```

```
[Out] 1/d/a*e*b^3/(a^2-b^2)*(e*sin(d*x+c))^(3/2)/(-a^2*cos(d*x+c)^2*e^2+b^2*e^2)+
2/d/a*e*b/(a^2-b^2)/(e^2*(a^2-b^2)/a^2)^(1/4)*arctan((e*sin(d*x+c))^(1/2)/(
e^2*(a^2-b^2)/a^2)^(1/4))-3/2/d/a^3*e*b^3/(a^2-b^2)/(e^2*(a^2-b^2)/a^2)^(1/
4)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-1/d/a*e*b/(a^2-b^
2)/(e^2*(a^2-b^2)/a^2)^(1/4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(
1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+3/4/d/a^3*e*b^3/(a^
2-b^2)/(e^2*(a^2-b^2)/a^2)^(1/4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^
2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))-2/d*e/cos(d*x+c
)/(e*sin(d*x+c))^(1/2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin
(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+1/d*e/cos(d*x+c)
/(e*sin(d*x+c))^(1/2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(
d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+3/2/d*e/cos(d*x+c
)/(e*sin(d*x+c))^(1/2)/a^4*b^2*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)
*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/
(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/2/d*e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/
a^4*b^2*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1+(a
^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1
/2*2^(1/2))-1/d*e*sin(d*x+c)^2*cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a^2-b^2
)/(-cos(d*x+c)^2*a^2+b^2)+1/d*e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a^2/(a^
2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*Ellipt
icE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/2/d*e/cos(d*x+c)/(e*sin(d*x+c))^(1
/2)*b^2/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+
c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/2/d*e/cos(d*x+c)/(e
*sin(d*x+c))^(1/2)*b^2/a^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))
^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1
/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/4/d*e/cos(d*x+c)/(e*sin(d*x+c))^
(1/2)*b^4/a^4/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*
x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2
-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/d*e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/a^2
/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1
+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a
),1/2*2^(1/2))+3/4/d*e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/a^4/(a^2-b^2)*(-
sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1
/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2)
)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\sin(c+dx)}}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**(1/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(e*sin(c + d*x))/(a + b*sec(c + d*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e\sin(dx+c)}}{(b\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*sin(d*x + c))/(b*sec(d*x + c) + a)^2, x)
```

3.246 $\int \frac{1}{(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$

Optimal. Leaf size=838

$$\frac{3\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}b^4}{2a^2\left(a^2-b^2\right)\left(a^2-\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}}+\frac{3\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}b^4}{2a^2\left(a^2-b^2\right)\left(a^2+\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}}-\frac{3\tan^{-1}\left(\sqrt{a}\cdots\right.}{2a^{3/2}\left(a^2-\cdots\right.}$$

```
[Out] (-3*b^3*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])
/(2*a^(3/2)*(a^2 - b^2)^(7/4)*d*Sqrt[e]) - (2*b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[
c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(3/2)*(a^2 - b^2)^(3/4)*d*Sqrt[
e]) - (3*b^3*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt
[e])])/(2*a^(3/2)*(a^2 - b^2)^(7/4)*d*Sqrt[e]) - (2*b*ArcTanh[(Sqrt[a]*Sqrt
[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(3/2)*(a^2 - b^2)^(3/4)*
d*Sqrt[e]) + (2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^2*d
*Sqrt[e*Sin[c + d*x]]) + (b^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c +
 d*x]])/(a^2*(a^2 - b^2)*d*Sqrt[e*Sin[c + d*x]]) + (2*b^2*EllipticPi[(2*a)/
(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^2*(a^2
 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (3*b^4*EllipticPi[(2*
a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^2
*(a^2 - b^2)*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*b
^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[
c + d*x]])/(a^2*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (
3*b^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(2*a^2*(a^2 - b^2)*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*S
in[c + d*x]]) + (b^2*Sqrt[e*Sin[c + d*x]])/(a*(a^2 - b^2)*d*e*(b + a*Cos[c
+ d*x]))
```

Rubi [A] time = 1.92404, antiderivative size = 838, normalized size of antiderivative = 1., number of steps used = 27, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {3872, 2912, 2642, 2641, 2694, 2867, 2702, 2807, 2805, 329, 212, 208, 205}

$$\frac{3\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}b^4}{2a^2\left(a^2-b^2\right)\left(a^2-\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}}+\frac{3\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}b^4}{2a^2\left(a^2-b^2\right)\left(a^2+\sqrt{a^2-b^2}a-b^2\right)d\sqrt{e\sin(c+dx)}}-\frac{3\tan^{-1}\left(\sqrt{a}\cdots\right.}{2a^{3/2}\left(a^2-\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] (-3*b^3*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])
/(2*a^(3/2)*(a^2 - b^2)^(7/4)*d*Sqrt[e]) - (2*b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[
c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(3/2)*(a^2 - b^2)^(3/4)*d*Sqrt[
e]) - (3*b^3*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt
[e])])/(2*a^(3/2)*(a^2 - b^2)^(7/4)*d*Sqrt[e]) - (2*b*ArcTanh[(Sqrt[a]*Sqrt
[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(a^(3/2)*(a^2 - b^2)^(3/4)*
d*Sqrt[e]) + (2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^2*d
*Sqrt[e*Sin[c + d*x]]) + (b^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c +
 d*x]])/(a^2*(a^2 - b^2)*d*Sqrt[e*Sin[c + d*x]]) + (2*b^2*EllipticPi[(2*a)/
(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a^2*(a^2
 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (3*b^4*EllipticPi[(2*
a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*a^2
*(a^2 - b^2)*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (2*b
^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[
```

```
c + d*x]])/(a^2*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*Sin[c + d*x]]) + (
3*b^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(2*a^2*(a^2 - b^2)*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*Sqrt[e*S
in[c + d*x]]) + (b^2*Sqrt[e*Sin[c + d*x]])/(a*(a^2 - b^2)*d*e*(b + a*Cos[c
+ d*x]))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx = \int \frac{\cos^2(c+dx)}{(-b-a\cos(c+dx))^2\sqrt{e\sin(c+dx)}}\,dx$$

$$= \int \left(\frac{1}{a^2\sqrt{e\sin(c+dx)}} + \frac{b^2}{a^2(-b-a\cos(c+dx))^2\sqrt{e\sin(c+dx)}} + \frac{}{a^2(-b-a}\right.$$

$$= \frac{\int \frac{1}{\sqrt{e\sin(c+dx)}}\,dx}{a^2} + \frac{(2b)\int \frac{1}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{a^2} + \frac{b^2\int \frac{1}{(-b-a\cos(c+dx))^2}}{a^2}$$

$$= \frac{b^2\sqrt{e\sin(c+dx)}}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))} + \frac{b^2\int \frac{b-\frac{1}{2}a\cos(c+dx)}{(-b-a\cos(c+dx))\sqrt{e\sin(c+dx)}}\,dx}{a^2\left(a^2-b^2\right)} + \frac{b^2\int}{}$$

$$= \frac{2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{a^2d\sqrt{e\sin(c+dx)}} + \frac{b^2\sqrt{e\sin(c+dx)}}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))} +$$

$$= \frac{2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{a^2d\sqrt{e\sin(c+dx)}} + \frac{2b^2\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{a^2\left(a^2-b^2-a\sqrt{a^2-b^2}\right)d\sqrt{e\sin}}$$

$$= -\frac{2b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\left(a^2-b^2\right)^{3/4}d\sqrt{e}} - \frac{2b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\left(a^2-b^2\right)^{3/4}d\sqrt{e}} + \frac{2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+d\right.\right.}{a^2d\sqrt{e\sin}}$$

$$= -\frac{2b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\left(a^2-b^2\right)^{3/4}d\sqrt{e}} - \frac{2b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\left(a^2-b^2\right)^{3/4}d\sqrt{e}} + \frac{2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+d\right.\right.}{a^2d\sqrt{e\sin}}$$

$$= -\frac{3b^3\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2a^{3/2}\left(a^2-b^2\right)^{7/4}d\sqrt{e}} - \frac{2b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{a^{3/2}\left(a^2-b^2\right)^{3/4}d\sqrt{e}} - \frac{3b^3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e}}{\sqrt[4]{a}}\right.}{2a^{3/2}\left(a^2-b^2\right)^{7}}$$

Mathematica [C] time = 12.8795, size = 1246, normalized size = 1.49

$$\frac{(b+a\cos(c+dx))\sec(c+dx)\tan(c+dx)b^2}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2\sqrt{e\sin(c+dx)}} + \frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\sqrt{\sin(c+dx)}\left(\frac{2\left(b^2-2a^2\right)\left(\sqrt{1-\sin^2(c+dx)}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]]),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Sqrt[Sin[c + d*x]]*((2*(-2*a^2 + b^2
)*Cos[c + d*x]^2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1 - (Sqrt[
2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqrt[2]*
Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sq
rt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log
[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] +
 a*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2 - b^2
)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b
^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*AppellF1[
1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2
```

```
*a^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2
- b^2)] + (-a^2 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[
c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2))
)))/((b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (4*a*b*Cos[c + d*x]*(b +
a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2*ArcTan[1 - ((1 + I)*S
qrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[
a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[a^2 - b^2] - (1 + I)*S
qrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] - Log[Sqrt[
a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin
[c + d*x]]))/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4
, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqrt[Sin[c + d*x]])/(Sq
rt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2, 1, 5/4, Sin[c + d*
x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*AppellF1[5/4, 1/2, 2, 9/
4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + (a^2 - b^2)*AppellF1
[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c
 + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a*Cos[c + d*x])*Sqrt[
1 - Sin[c + d*x]^2])))/(2*a*(-a + b)*(a + b)*d*(a + b*Sec[c + d*x])^2*Sqrt[
e*Sin[c + d*x]]) + (b^2*(b + a*Cos[c + d*x])*Sec[c + d*x]*Tan[c + d*x])/(a*
(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2*Sqrt[e*Sin[c + d*x]])
```

Maple [A] time = 7.002, size = 1475, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x)
```

```
[Out] 1/d/a*e*b^3/(a^2-b^2)*(e*sin(d*x+c))^(1/2)/(-a^2*cos(d*x+c)^2*e^2+b^2*e^2)+
2/d*a*e*b/(a^2-b^2)*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*
sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-1/2/d/a*e*b^3/(a^2-b^2)*(e^2*(
a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^
2-b^2)/a^2)^(1/4))+1/d*a*e*b/(a^2-b^2)*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+
b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c)
)^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))-1/4/d/a*e*b^3/(a^2-b^2)*(e^2*(a^2-b^2)/
a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^
(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))-1/d/cos(d*x+c)/(e*
sin(d*x+c))^(1/2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+
c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))+3/2/d/cos(d*x+c)/(e*s
in(d*x+c))^(1/2)/a^3*b^2/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x
+c))^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1
)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-3/2/d/cos(d*x+c)/(e*sin(d*x+c)
)^(1/2)/a^3*b^2/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2
)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1
/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/d*sin(d*x+c)*cos(d*x+c)/(e*sin(d*x+c)
)^(1/2)*b^2/(a^2-b^2)/(-cos(d*x+c)^2*a^2+b^2)-1/2/d/cos(d*x+c)/(e*sin(d*x+c
))^(1/2)/a^2*b^2/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin
(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/2/d/cos(d*x+c)
/(e*sin(d*x+c))^(1/2)/a*b^2/(a^2-b^2)^(3/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(
d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c
)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+5/4/d/cos(d*x+c)/(e*sin(d*x
+c))^(1/2)/a^3*b^4/(a^2-b^2)^(3/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(
1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2
),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+1/2/d/cos(d*x+c)/(e*sin(d*x+c))^(1/2
)/a*b^2/(a^2-b^2)^(3/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*
x+c)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2
-b^2)^(1/2)/a),1/2*2^(1/2))-5/4/d/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/a^3*b^4/(
```

```
a^2-b^2)^(3/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2
)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/
2)/a),1/2*2^(1/2))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**2/(e*sin(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^2\sqrt{e\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^2*sqrt(e*sin(d*x + c))), x)
```

3.247 $\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=1054

result too large to display

```
[Out] (5*b^3*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/
(2*Sqrt[a]*(a^2 - b^2)^(9/4)*d*e^(3/2)) + (2*b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c
 + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(Sqrt[a]*(a^2 - b^2)^(5/4)*d*e^(3/2
)) - (5*b^3*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[
e])])/(2*Sqrt[a]*(a^2 - b^2)^(9/4)*d*e^(3/2)) - (2*b*ArcTanh[(Sqrt[a]*Sqrt[
e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(Sqrt[a]*(a^2 - b^2)^(5/4)*d
*e^(3/2)) - (2*Cos[c + d*x])/(a^2*d*e*Sqrt[e*Sin[c + d*x]]) + b^2/(a*(a^2 -
 b^2)*d*e*(b + a*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]]) + (4*b*(a - b*Cos[c +
d*x]))/(a^2*(a^2 - b^2)*d*e*Sqrt[e*Sin[c + d*x]]) + (b^2*(5*a*b - (3*a^2 +
2*b^2)*Cos[c + d*x]))/(a^2*(a^2 - b^2)^2*d*e*Sqrt[e*Sin[c + d*x]]) - (5*b^4
*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
+ d*x]])/(2*a*(a^2 - b^2)^2*(a - Sqrt[a^2 - b^2])*d*e*Sqrt[e*Sin[c + d*x]])
 - (2*b^2*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sq
rt[Sin[c + d*x]])/(a*(a^2 - b^2)*(a - Sqrt[a^2 - b^2])*d*e*Sqrt[e*Sin[c + d
*x]]) - (5*b^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2,
2]*Sqrt[Sin[c + d*x]])/(2*a*(a^2 - b^2)^2*(a + Sqrt[a^2 - b^2])*d*e*Sqrt[e*
Sin[c + d*x]]) - (2*b^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 +
 d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a*(a^2 - b^2)*(a + Sqrt[a^2 - b^2])*d*e*Sq
rt[e*Sin[c + d*x]]) - (2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*
x]])/(a^2*d*e^2*Sqrt[Sin[c + d*x]]) - (4*b^2*EllipticE[(c - Pi/2 + d*x)/2,
2]*Sqrt[e*Sin[c + d*x]])/(a^2*(a^2 - b^2)*d*e^2*Sqrt[Sin[c + d*x]]) - (b^2*
(3*a^2 + 2*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(a^2
*(a^2 - b^2)^2*d*e^2*Sqrt[Sin[c + d*x]])
```

Rubi [A] time = 2.69182, antiderivative size = 1054, normalized size of antiderivative = 1., number of steps used = 33, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {3872, 2912, 2636, 2640, 2639, 2694, 2866, 2867, 2701, 2807, 2805, 329, 298, 205, 208, 2696}

$$-\frac{5\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}b^4}{2a\left(a^2-b^2\right)^2\left(a-\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}-\frac{5\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}b^4}{2a\left(a^2-b^2\right)^2\left(a+\sqrt{a^2-b^2}\right)de\sqrt{e\sin(c+dx)}}+\frac{5\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin\cdots}}{\sqrt[4]{a^2-b\cdots}}\right.}{2\sqrt{a}\left(a^2-b^2\right)^{9\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] (5*b^3*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/
(2*Sqrt[a]*(a^2 - b^2)^(9/4)*d*e^(3/2)) + (2*b*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c
 + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(Sqrt[a]*(a^2 - b^2)^(5/4)*d*e^(3/2
)) - (5*b^3*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[
e])])/(2*Sqrt[a]*(a^2 - b^2)^(9/4)*d*e^(3/2)) - (2*b*ArcTanh[(Sqrt[a]*Sqrt[
e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/(Sqrt[a]*(a^2 - b^2)^(5/4)*d
*e^(3/2)) - (2*Cos[c + d*x])/(a^2*d*e*Sqrt[e*Sin[c + d*x]]) + b^2/(a*(a^2 -
 b^2)*d*e*(b + a*Cos[c + d*x])*Sqrt[e*Sin[c + d*x]]) + (4*b*(a - b*Cos[c +
d*x]))/(a^2*(a^2 - b^2)*d*e*Sqrt[e*Sin[c + d*x]]) + (b^2*(5*a*b - (3*a^2 +
2*b^2)*Cos[c + d*x]))/(a^2*(a^2 - b^2)^2*d*e*Sqrt[e*Sin[c + d*x]]) - (5*b^4
*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c
+ d*x]])/(2*a*(a^2 - b^2)^2*(a - Sqrt[a^2 - b^2])*d*e*Sqrt[e*Sin[c + d*x]])
```

```
 - (2*b^2*EllipticPi[(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sq
rt[Sin[c + d*x]])/(a*(a^2 - b^2)*(a - Sqrt[a^2 - b^2])*d*e*Sqrt[e*Sin[c + d
*x]]) - (5*b^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2,
2]*Sqrt[Sin[c + d*x]])/(2*a*(a^2 - b^2)^2*(a + Sqrt[a^2 - b^2])*d*e*Sqrt[e*
Sin[c + d*x]]) - (2*b^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 +
 d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(a*(a^2 - b^2)*(a + Sqrt[a^2 - b^2])*d*e*Sq
rt[e*Sin[c + d*x]]) - (2*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*
x]])/(a^2*d*e^2*Sqrt[Sin[c + d*x]]) - (4*b^2*EllipticE[(c - Pi/2 + d*x)/2,
2]*Sqrt[e*Sin[c + d*x]])/(a^2*(a^2 - b^2)*d*e^2*Sqrt[Sin[c + d*x]]) - (b^2*
(3*a^2 + 2*b^2)*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[e*Sin[c + d*x]])/(a^2
*(a^2 - b^2)^2*d*e^2*Sqrt[Sin[c + d*x]])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2640

```
Int[Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[b*Sin[c + d*
x]]/Sqrt[Sin[c + d*x]], Int[Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c, d},
x]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
```

```
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2701

```
Int[Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_
)]), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, Dist[(a*g)/(2*b), Int[1/(Sq
rt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (-Dist[(a*g)/(2*b), Int[
1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x] + Dist[(b*g)/f, Subst
[Int[Sqrt[x]/(g^2*(a^2 - b^2) + b^2*x^2), x], x, g*Cos[e + f*x]], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx = \int \frac{\cos^2(c+dx)}{(-b-a\cos(c+dx))^2(e\sin(c+dx))^{3/2}}\,dx$$

$$= \int \left(\frac{1}{a^2(e\sin(c+dx))^{3/2}} + \frac{b^2}{a^2(-b-a\cos(c+dx))^2(e\sin(c+dx))^{3/2}} + \ldots\right.$$

$$= \frac{\int \frac{1}{(e\sin(c+dx))^{3/2}}\,dx}{a^2} + \frac{(2b)\int \frac{1}{(-b-a\cos(c+dx))(e\sin(c+dx))^{3/2}}\,dx}{a^2} + \frac{b^2\int \frac{1}{(-b-a\cos(\ldots}}{\ldots}$$

$$= -\frac{2\cos(c+dx)}{a^2de\sqrt{e\sin(c+dx)}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}} + \ldots$$

$$= -\frac{2\cos(c+dx)}{a^2de\sqrt{e\sin(c+dx)}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}} + \ldots$$

$$= -\frac{2\cos(c+dx)}{a^2de\sqrt{e\sin(c+dx)}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}} + \ldots$$

$$= -\frac{2\cos(c+dx)}{a^2de\sqrt{e\sin(c+dx)}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}} + \ldots$$

$$= -\frac{2\cos(c+dx)}{a^2de\sqrt{e\sin(c+dx)}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))\sqrt{e\sin(c+dx)}} + \ldots$$

$$= \frac{2b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\left(a^2-b^2\right)^{5/4}de^{3/2}} - \frac{2b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\left(a^2-b^2\right)^{5/4}de^{3/2}} - \frac{2\cos(c+dx)}{a^2de\sqrt{e\sin(c+\ldots}}$$

$$= \frac{5b^3\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2\sqrt{a}\left(a^2-b^2\right)^{9/4}de^{3/2}} + \frac{2b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\sqrt{a}\left(a^2-b^2\right)^{5/4}de^{3/2}} - \frac{5b^3\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\ldots}}{\sqrt[4]{\ldots}}\right.}{2\sqrt{a}\left(a^2-b^2\right)\ldots}$$

Mathematica [C] time = 6.85948, size = 922, normalized size = 0.87

$$\frac{(b+a\cos(c+dx))^2\left(\frac{ab^2\sin(c+dx)}{(b^2-a^2)^2(b+a\cos(c+dx))}-\frac{2(\cos(c+dx)a^2-2ba+b^2\cos(c+dx))\csc(c+dx)}{(b^2-a^2)^2}\right)\tan^2(c+dx)}{d(a+b\sec(c+dx))^2(e\sin(c+dx))^{3/2}}-\frac{(b+a\cos(c+dx))^2\sec^2(\cdots}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^(3/2)),x]
```

```
[Out] -((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Sin[c + d*x]^(3/2)*(((2*a^3 + 3*a*b
^2)*Cos[c + d*x]^2*(3*Sqrt[2]*b*(-a^2 + b^2)^(3/4)*(2*ArcTan[1 - (Sqrt[2]*S
qrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - 2*ArcTan[1 + (Sqrt[2]*Sqrt
[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2] - Sqrt[2
]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]] + Log[Sqr
t[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*S
in[c + d*x]]) + 8*a^(5/2)*AppellF1[3/4, -1/2, 1, 7/4, Sin[c + d*x]^2, (a^2*
Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))*(b + a*Sqrt[1 - Sin[c + d*
x]^2]))/(12*a^(3/2)*(a^2 - b^2)*(b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2))
+ (2*(6*a^2*b + 4*b^3)*Cos[c + d*x]*(((1/8 + I/8)*(2*ArcTan[1 - ((1 + I)*Sq
rt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTan[1 + ((1 + I)*Sqrt[a
]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - Log[Sqrt[a^2 - b^2] - (1 + I)*Sq
rt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c + d*x]] + Log[Sqrt[a
^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[
c + d*x]]))/(Sqrt[a]*(a^2 - b^2)^(1/4)) + (b*AppellF1[3/4, 1/2, 1, 7/4, Sin
[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sin[c + d*x]^(3/2))/(3*(-a^2
 + b^2)))*(b + a*Sqrt[1 - Sin[c + d*x]^2]))/((b + a*Cos[c + d*x])*Sqrt[1 -
Sin[c + d*x]^2])))/(2*(a - b)^2*(a + b)^2*d*(a + b*Sec[c + d*x])^2*(e*Sin[c
 + d*x])^(3/2)) + ((b + a*Cos[c + d*x])^2*((-2*(-2*a*b + a^2*Cos[c + d*x] +
 b^2*Cos[c + d*x])*Csc[c + d*x])/(-a^2 + b^2)^2 + (a*b^2*Sin[c + d*x])/((-a
^2 + b^2)^2*(b + a*Cos[c + d*x])))*Tan[c + d*x]^2)/(d*(a + b*Sec[c + d*x])^
2*(e*Sin[c + d*x])^(3/2))
```

Maple [A] time = 7.915, size = 2263, normalized size = 2.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x)
```

```
[Out] 1/d*a/e*b^3/(a+b)^2/(a-b)^2*(e*sin(d*x+c))^(3/2)/(-a^2*cos(d*x+c)^2*e^2+b^2
*e^2)+2/d*a/e*b/(a+b)^2/(a-b)^2/(e^2*(a^2-b^2)/a^2)^(1/4)*arctan((e*sin(d*x
+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))-1/d*a/e*b/(a+b)^2/(a-b)^2/(e^2*(a^2-b
^2)/a^2)^(1/4)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(
d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+1/2/d/a/e*b^3/(a+b)^2/(a-b)^2/(e^
2*(a^2-b^2)/a^2)^(1/4)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4
))-1/4/d/a/e*b^3/(a+b)^2/(a-b)^2/(e^2*(a^2-b^2)/a^2)^(1/4)*ln(((e*sin(d*x+c
))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^
2)^(1/4)))+4/d*a/e*b/(a^2-b^2)^2/(e*sin(d*x+c))^(1/2)+3/2/d/e/cos(d*x+c)/(e
*sin(d*x+c))^(1/2)*b^2/(a+b)^2/(a-b)^2*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c
))^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^
```

```
(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/d/e/cos(d*x+c)/(e*sin(d*x+c)
)^(1/2)*b^4/(a+b)^2/(a-b)^2/a^2*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2
)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1
/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))+3/2/d/e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)
*b^2/(a+b)^2/(a-b)^2*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c
)^(1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^
2)^(1/2)/a),1/2*2^(1/2))-1/2/d/e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/(a+b)^
2/(a-b)^2/a^2*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)
/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2
)/a),1/2*2^(1/2))-1/d/e*sin(d*x+c)^2*cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a
+b)/(a-b)*a^2/(a^2-b^2)/(-cos(d*x+c)^2*a^2+b^2)+1/d/e/cos(d*x+c)/(e*sin(d*x
+c))^(1/2)*b^2/(a+b)/(a-b)/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))
^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))-1/2/d/
e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a+b)/(a-b)/(a^2-b^2)*(-sin(d*x+c)+1)
^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1
/2),1/2*2^(1/2))-1/2/d/e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a+b)/(a-b)/(a
^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1-(a
^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1
/2*2^(1/2))+3/4/d/e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/(a+b)/(a-b)/(a^2-b^
2)/a^2*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1-(a^
2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/
2*2^(1/2))-1/2/d/e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a+b)/(a-b)/(a^2-b^2
)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2
)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(
1/2))+3/4/d/e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/(a+b)/(a-b)/(a^2-b^2)/a^2
*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1+(a^2-b^2)
^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),1/2*2^(1
/2))+2/d/e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2
)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)+1)^(1/2),1
/2*2^(1/2))*a^2+2/d/e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/(a^2-b^2)^2*(-sin(d*x
+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticE((-sin(d*x+c)
+1)^(1/2),1/2*2^(1/2))*b^2-1/d/e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/(a^2-b^2)^
2*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((
-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2-1/d/e/cos(d*x+c)/(e*sin(d*x+c))^(1/2)
/(a^2-b^2)^2*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*
EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*b^2-2/d/e*cos(d*x+c)/(e*sin(d*
x+c))^(1/2)/(a^2-b^2)^2*a^2-2/d/e*cos(d*x+c)/(e*sin(d*x+c))^(1/2)/(a^2-b^2)
^2*b^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**2/(e*sin(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^2 (e\sin(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(3/2)), x)
```

3.248 $\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=1089

result too large to display

```
[Out] (-7*Sqrt[a]*b^3*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sq
rt[e])])/(2*(a^2 - b^2)^(11/4)*d*e^(5/2)) - (2*Sqrt[a]*b*ArcTan[(Sqrt[a]*Sq
rt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/((a^2 - b^2)^(7/4)*d*e^(5
/2)) - (7*Sqrt[a]*b^3*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(
1/4)*Sqrt[e])])/(2*(a^2 - b^2)^(11/4)*d*e^(5/2)) - (2*Sqrt[a]*b*ArcTanh[(Sq
rt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/((a^2 - b^2)^(7/4
)*d*e^(5/2)) - (2*Cos[c + d*x])/(3*a^2*d*e*(e*Sin[c + d*x])^(3/2)) + b^2/(a
*(a^2 - b^2)*d*e*(b + a*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2)) + (4*b*(a - b
*Cos[c + d*x]))/(3*a^2*(a^2 - b^2)*d*e*(e*Sin[c + d*x])^(3/2)) + (b^2*(7*a*
b - (5*a^2 + 2*b^2)*Cos[c + d*x]))/(3*a^2*(a^2 - b^2)^2*d*e*(e*Sin[c + d*x]
)^(3/2)) + (2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*a^2*d
*e^2*Sqrt[e*Sin[c + d*x]]) + (4*b^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(3*a^2*(a^2 - b^2)*d*e^2*Sqrt[e*Sin[c + d*x]]) + (b^2*(5*a^2
+ 2*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*a^2*(a^2 -
 b^2)^2*d*e^2*Sqrt[e*Sin[c + d*x]]) + (7*b^4*EllipticPi[(2*a)/(a - Sqrt[a^2
 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*(a^2 - b^2)^2*(a^2
- b^2 - a*Sqrt[a^2 - b^2])*d*e^2*Sqrt[e*Sin[c + d*x]]) + (2*b^2*EllipticPi[
(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a
^2 - b^2)*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*e^2*Sqrt[e*Sin[c + d*x]]) + (7*
b^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin
[c + d*x]])/(2*(a^2 - b^2)^2*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*e^2*Sqrt[e*S
in[c + d*x]]) + (2*b^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 +
d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)*(a^2 - b^2 + a*Sqrt[a^2 - b^2])
*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Rubi [A] time = 2.78089, antiderivative size = 1089, normalized size of antiderivative = 1., number of steps used = 33, number of rules used = 16, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.64, Rules used = {3872, 2912, 2636, 2642, 2641, 2694, 2866, 2867, 2702, 2807, 2805, 329, 212, 208, 205, 2696}

$$\frac{7\Pi\left(\frac{2a}{a-\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}b^4}{2\left(a^2-b^2\right)^2\left(a^2-\sqrt{a^2-b^2}a-b^2\right)de^2\sqrt{e\sin(c+dx)}}+\frac{7\Pi\left(\frac{2a}{a+\sqrt{a^2-b^2}};\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{\sin(c+dx)}b^4}{2\left(a^2-b^2\right)^2\left(a^2+\sqrt{a^2-b^2}a-b^2\right)de^2\sqrt{e\sin(c+dx)}}-\frac{7\sqrt{a}\,\mathrm{ta}}{2(}$$

Antiderivative was successfully verified.

```
[In] Int[1/((a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```


```
[Out] (-7*Sqrt[a]*b^3*ArcTan[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sq
rt[e])])/(2*(a^2 - b^2)^(11/4)*d*e^(5/2)) - (2*Sqrt[a]*b*ArcTan[(Sqrt[a]*Sq
rt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/((a^2 - b^2)^(7/4)*d*e^(5
/2)) - (7*Sqrt[a]*b^3*ArcTanh[(Sqrt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(
1/4)*Sqrt[e])])/(2*(a^2 - b^2)^(11/4)*d*e^(5/2)) - (2*Sqrt[a]*b*ArcTanh[(Sq
rt[a]*Sqrt[e*Sin[c + d*x]])/((a^2 - b^2)^(1/4)*Sqrt[e])])/((a^2 - b^2)^(7/4
)*d*e^(5/2)) - (2*Cos[c + d*x])/(3*a^2*d*e*(e*Sin[c + d*x])^(3/2)) + b^2/(a
*(a^2 - b^2)*d*e*(b + a*Cos[c + d*x])*(e*Sin[c + d*x])^(3/2)) + (4*b*(a - b
*Cos[c + d*x]))/(3*a^2*(a^2 - b^2)*d*e*(e*Sin[c + d*x])^(3/2)) + (b^2*(7*a*
b - (5*a^2 + 2*b^2)*Cos[c + d*x]))/(3*a^2*(a^2 - b^2)^2*d*e*(e*Sin[c + d*x]
)^(3/2)) + (2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*a^2*d
```

```
*e^2*Sqrt[e*Sin[c + d*x]]) + (4*b^2*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(3*a^2*(a^2 - b^2)*d*e^2*Sqrt[e*Sin[c + d*x]]) + (b^2*(5*a^2
+ 2*b^2)*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*a^2*(a^2 -
 b^2)^2*d*e^2*Sqrt[e*Sin[c + d*x]]) + (7*b^4*EllipticPi[(2*a)/(a - Sqrt[a^2
 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(2*(a^2 - b^2)^2*(a^2
- b^2 - a*Sqrt[a^2 - b^2])*d*e^2*Sqrt[e*Sin[c + d*x]]) + (2*b^2*EllipticPi[
(2*a)/(a - Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a
^2 - b^2)*(a^2 - b^2 - a*Sqrt[a^2 - b^2])*d*e^2*Sqrt[e*Sin[c + d*x]]) + (7*
b^4*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 + d*x)/2, 2]*Sqrt[Sin
[c + d*x]])/(2*(a^2 - b^2)^2*(a^2 - b^2 + a*Sqrt[a^2 - b^2])*d*e^2*Sqrt[e*S
in[c + d*x]]) + (2*b^2*EllipticPi[(2*a)/(a + Sqrt[a^2 - b^2]), (c - Pi/2 +
d*x)/2, 2]*Sqrt[Sin[c + d*x]])/((a^2 - b^2)*(a^2 - b^2 + a*Sqrt[a^2 - b^2])
*d*e^2*Sqrt[e*Sin[c + d*x]])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2642

```
Int[1/Sqrt[(b_)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[Sin[c + d*
x]]/Sqrt[b*Sin[c + d*x]], Int[1/Sqrt[Sin[c + d*x]], x], x] /; FreeQ[{b, c,
d}, x]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2694

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> -Simp[(b*(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*
x])^(m + 1))/(f*g*(a^2 - b^2)*(m + 1)), x] + Dist[1/((a^2 - b^2)*(m + 1)),
Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + p +
 2)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^2 - b^2,
 0] && LtQ[m, -1] && IntegersQ[2*m, 2*p]
```

Rule 2866

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[((g*C
```

```
os[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*(b*c - a*d - (a*c - b*d)*
Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*(a^2 - b^2)*(p +
 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x], x
], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ
[p, -1] && IntegerQ[2*m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2702

```
Int[1/(Sqrt[cos[(e_.) + (f_.)*(x_)]*(g_.)]*((a_) + (b_.)*sin[(e_.) + (f_.)*
(x_)])), x_Symbol] :> With[{q = Rt[-a^2 + b^2, 2]}, -Dist[a/(2*q), Int[1/(S
qrt[g*Cos[e + f*x]]*(q + b*Cos[e + f*x])), x], x] + (Dist[(b*g)/f, Subst[In
t[1/(Sqrt[x]*(g^2*(a^2 - b^2) + b^2*x^2)), x], x, g*Cos[e + f*x]], x] - Dis
t[a/(2*q), Int[1/(Sqrt[g*Cos[e + f*x]]*(q - b*Cos[e + f*x])), x], x])] /; F
reeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2807

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d*Sin[e
 + f*x])/(c + d)]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 2805

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2*EllipticPi[(2*b)/(a + b), (1*(e - Pi
/2 + f*x))/2, (2*d)/(c + d)])/(f*(a + b)*Sqrt[c + d]), x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2696

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[((g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])
^(m + 1)*(b - a*Sin[e + f*x]))/(f*g*(a^2 - b^2)*(p + 1)), x] + Dist[1/(g^2*
(a^2 - b^2)*(p + 1)), Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*(
a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; Fr
eeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ[
2*m, 2*p]
```

Rubi steps

$$\int \frac{1}{(a+b\sec(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx = \int \frac{\cos^2(c+dx)}{(-b-a\cos(c+dx))^2(e\sin(c+dx))^{5/2}}\,dx$$

$$= \int \left(\frac{1}{a^2(e\sin(c+dx))^{5/2}} + \frac{b^2}{a^2(-b-a\cos(c+dx))^2(e\sin(c+dx))^{5/2}} + \frac{}{a^2(-b}\right.$$

$$= \frac{\int \frac{1}{(e\sin(c+dx))^{5/2}}\,dx}{a^2} + \frac{(2b)\int \frac{1}{(-b-a\cos(c+dx))(e\sin(c+dx))^{5/2}}\,dx}{a^2} + \frac{b^2\int \frac{}{(-b-a\cos(c+d}}{a}$$

$$= -\frac{2\cos(c+dx)}{3a^2de(e\sin(c+dx))^{3/2}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))(e\sin(c+dx))^{3/2}}$$

$$= -\frac{2\cos(c+dx)}{3a^2de(e\sin(c+dx))^{3/2}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))(e\sin(c+dx))^{3/2}}$$

$$= -\frac{2\cos(c+dx)}{3a^2de(e\sin(c+dx))^{3/2}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))(e\sin(c+dx))^{3/2}}$$

$$= -\frac{2\cos(c+dx)}{3a^2de(e\sin(c+dx))^{3/2}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))(e\sin(c+dx))^{3/2}}$$

$$= -\frac{2\cos(c+dx)}{3a^2de(e\sin(c+dx))^{3/2}} + \frac{b^2}{a\left(a^2-b^2\right)de(b+a\cos(c+dx))(e\sin(c+dx))^{3/2}}$$

$$= -\frac{2\sqrt{a}b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\left(a^2-b^2\right)^{7/4}de^{5/2}} - \frac{2\sqrt{a}b\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\left(a^2-b^2\right)^{7/4}de^{5/2}} - \frac{2\cos(c}{3a^2de(e\sin(}$$

$$= -\frac{7\sqrt{a}b^3\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{2\left(a^2-b^2\right)^{11/4}de^{5/2}} - \frac{2\sqrt{a}b\tan^{-1}\left(\frac{\sqrt{a}\sqrt{e\sin(c+dx)}}{\sqrt[4]{a^2-b^2}\sqrt{e}}\right)}{\left(a^2-b^2\right)^{7/4}de^{5/2}} - \frac{7\sqrt{a}b^3\tanh}{2\left(a^2-\right.}$$

Mathematica [C] time = 15.2096, size = 1320, normalized size = 1.21

$$\frac{(b+a\cos(c+dx))^2\left(\frac{ab^2}{\left(b^2-a^2\right)^2(b+a\cos(c+dx))}-\frac{2\left(\cos(c+dx)a^2-2ba+b^2\cos(c+dx)\right)\csc^2(c+dx)}{3\left(b^2-a^2\right)^2}\right)\sin(c+dx)\tan^2(c+dx)}{d(a+b\sec(c+dx))^2(e\sin(c+dx))^{5/2}}-\frac{(b+a\cos}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2)),x]
```

```
[Out] -((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*Sin[c + d*x]^(5/2)*((2*(-2*a^3 - 5*
a*b^2)*Cos[c + d*x]^2*(b + a*Sqrt[1 - Sin[c + d*x]^2])*((b*(-2*ArcTan[1 - (
Sqrt[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] + 2*ArcTan[1 + (Sqr
t[2]*Sqrt[a]*Sqrt[Sin[c + d*x]])/(-a^2 + b^2)^(1/4)] - Log[Sqrt[-a^2 + b^2]
 - Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*x]] + a*Sin[c + d*x]]
+ Log[Sqrt[-a^2 + b^2] + Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(1/4)*Sqrt[Sin[c + d*
x]] + a*Sin[c + d*x]]))/(4*Sqrt[2]*Sqrt[a]*(-a^2 + b^2)^(3/4)) - (5*a*(a^2
- b^2)*AppellF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^
2 - b^2)]*Sqrt[Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]^2])/((5*(a^2 - b^2)*Appe
llF1[1/4, -1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] +
 2*(2*a^2*AppellF1[5/4, -1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/
(a^2 - b^2)] + (-a^2 + b^2)*AppellF1[5/4, 1/2, 1, 9/4, Sin[c + d*x]^2, (a^2
*Sin[c + d*x]^2)/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x
]^2)))))/((b + a*Cos[c + d*x])*(1 - Sin[c + d*x]^2)) + (2*(10*a^2*b + 4*b^3
)*Cos[c + d*x]*(b + a*Sqrt[1 - Sin[c + d*x]^2])*(((-1/8 + I/8)*Sqrt[a]*(2*A
rcTan[1 - ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] - 2*ArcTa
n[1 + ((1 + I)*Sqrt[a]*Sqrt[Sin[c + d*x]])/(a^2 - b^2)^(1/4)] + Log[Sqrt[a^
2 - b^2] - (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin[c + d*x]] + I*a*Sin[c
 + d*x]] - Log[Sqrt[a^2 - b^2] + (1 + I)*Sqrt[a]*(a^2 - b^2)^(1/4)*Sqrt[Sin
[c + d*x]] + I*a*Sin[c + d*x]]))/(a^2 - b^2)^(3/4) + (5*b*(a^2 - b^2)*Appel
lF1[1/4, 1/2, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)]*Sqr
t[Sin[c + d*x]])/(Sqrt[1 - Sin[c + d*x]^2]*(5*(a^2 - b^2)*AppellF1[1/4, 1/2
, 1, 5/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] + 2*(2*a^2*Appe
llF1[5/4, 1/2, 2, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)/(a^2 - b^2)] +
(a^2 - b^2)*AppellF1[5/4, 3/2, 1, 9/4, Sin[c + d*x]^2, (a^2*Sin[c + d*x]^2)
/(a^2 - b^2)])*Sin[c + d*x]^2)*(b^2 + a^2*(-1 + Sin[c + d*x]^2)))))/((b + a
*Cos[c + d*x])*Sqrt[1 - Sin[c + d*x]^2])))/(6*(a - b)^2*(a + b)^2*d*(a + b*
Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2)) + ((b + a*Cos[c + d*x])^2*((a*b^2)/
((-a^2 + b^2)^2*(b + a*Cos[c + d*x])) - (2*(-2*a*b + a^2*Cos[c + d*x] + b^2
*Cos[c + d*x])*Csc[c + d*x]^2)/(3*(-a^2 + b^2)^2))*Sin[c + d*x]*Tan[c + d*x
]^2)/(d*(a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^(5/2))
```

Maple [A] time = 9.019, size = 2159, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x)
```

```
[Out] 1/d*a/e*b^3/(a+b)^2/(a-b)^2*(e*sin(d*x+c))^(1/2)/(-a^2*cos(d*x+c)^2*e^2+b^2
*e^2)+2/d*a^3/e*b/(a+b)^2/(a-b)^2*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e
```

```
^2)*arctan((e*sin(d*x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+3/2/d*a/e*b^3/(a
+b)^2/(a-b)^2*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*arctan((e*sin(d*
x+c))^(1/2)/(e^2*(a^2-b^2)/a^2)^(1/4))+1/d*a^3/e*b/(a+b)^2/(a-b)^2*(e^2*(a^
2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(d*x+c))^(1/2)+(e^2*(a^2-b^2
)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^2)/a^2)^(1/4)))+3/4/d*a/e*b
^3/(a+b)^2/(a-b)^2*(e^2*(a^2-b^2)/a^2)^(1/4)/(-a^2*e^2+b^2*e^2)*ln(((e*sin(
d*x+c))^(1/2)+(e^2*(a^2-b^2)/a^2)^(1/4))/((e*sin(d*x+c))^(1/2)-(e^2*(a^2-b^
2)/a^2)^(1/4)))+4/3/d*a/e*b/(a^2-b^2)^2/(e*sin(d*x+c))^(3/2)+3/2/d/e^2/cos(
d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a+b)^2/(a-b)^2*a/(a^2-b^2)^(1/2)*(-sin(d*x
+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^(1/2)/a)*
EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/2))-1/2/d
/e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/(a+b)^2/(a-b)^2/a/(a^2-b^2)^(1/2)*
(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^2)^
(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^(1/
2))-3/2/d/e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a+b)^2/(a-b)^2*a/(a^2-b^
2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1+(
a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)/a),
1/2*2^(1/2))+1/2/d/e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/(a+b)^2/(a-b)^2/
a/(a^2-b^2)^(1/2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(
1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^
(1/2)/a),1/2*2^(1/2))-1/d/e^2*sin(d*x+c)*cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^
2/(a+b)/(a-b)*a^2/(a^2-b^2)/(-cos(d*x+c)^2*a^2+b^2)-1/2/d/e^2/cos(d*x+c)/(e
*sin(d*x+c))^(1/2)*b^2/(a+b)/(a-b)/(a^2-b^2)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin
(d*x+c))^(1/2)*sin(d*x+c)^(1/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2)
)-1/2/d/e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a+b)/(a-b)/(a^2-b^2)^(3/2)
*a*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1-(a^2-b^
2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1/2*2^
(1/2))+5/4/d/e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/(a+b)/(a-b)/(a^2-b^2)^
(3/2)/a*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/(1-(a
^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1-(a^2-b^2)^(1/2)/a),1
/2*2^(1/2))+1/2/d/e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^2/(a+b)/(a-b)/(a^2-
b^2)^(3/2)*a*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(1/2)/
(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^(1/2)
/a),1/2*2^(1/2))-5/4/d/e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)*b^4/(a+b)/(a-b)/
(a^2-b^2)^(3/2)/a*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(
1/2)/(1+(a^2-b^2)^(1/2)/a)*EllipticPi((-sin(d*x+c)+1)^(1/2),1/(1+(a^2-b^2)^
(1/2)/a),1/2*2^(1/2))+1/3/d/e^2/cos(d*x+c)/(e*sin(d*x+c))^(1/2)/(a^2-b^2)^2
/(cos(d*x+c)^2-1)*(-sin(d*x+c)+1)^(1/2)*(2+2*sin(d*x+c))^(1/2)*sin(d*x+c)^(
5/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^(1/2))*a^2+1/3/d/e^2/cos(d*x+c)/
(e*sin(d*x+c))^(1/2)/(a^2-b^2)^2/(cos(d*x+c)^2-1)*(-sin(d*x+c)+1)^(1/2)*(2+
2*sin(d*x+c))^(1/2)*sin(d*x+c)^(5/2)*EllipticF((-sin(d*x+c)+1)^(1/2),1/2*2^
(1/2))*b^2+2/3/d/e^2*cos(d*x+c)/(e*sin(d*x+c))^(1/2)/(a^2-b^2)^2/(cos(d*x+c
)^2-1)*sin(d*x+c)*a^2+2/3/d/e^2*cos(d*x+c)/(e*sin(d*x+c))^(1/2)/(a^2-b^2)^2
/(cos(d*x+c)^2-1)*sin(d*x+c)*b^2
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))**2/(e*sin(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(b\sec(dx+c)+a)^2(e\sin(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(d*x+c))^2/(e*sin(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate(1/((b*sec(d*x + c) + a)^2*(e*sin(d*x + c))^(5/2)), x)
```

3.249 $\int \sqrt{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=125

$$-\frac{2\cot(e+fx)\sqrt{-\frac{b(1-\sec(e+fx))}{a+b\sec(e+fx)}}\sqrt{\frac{b(\sec(e+fx)+1)}{a+b\sec(e+fx)}}(a+b\sec(e+fx))\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(e+fx)}}\right)|\frac{a-b}{a+b}\right)}{f\sqrt{a+b}}$$

```
[Out] (-2*Cot[e + f*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sec[e
+ f*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sec[e + f*x]))/(a + b*Sec[e + f*x
]))]*Sqrt[(b*(1 + Sec[e + f*x]))/(a + b*Sec[e + f*x])]*(a + b*Sec[e + f*x])
)/(Sqrt[a + b]*f)
```

Rubi [A] time = 0.0316882, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3780}

$$-\frac{2\cot(e+fx)\sqrt{-\frac{b(1-\sec(e+fx))}{a+b\sec(e+fx)}}\sqrt{\frac{b(\sec(e+fx)+1)}{a+b\sec(e+fx)}}(a+b\sec(e+fx))\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(e+fx)}}\right)|\frac{a-b}{a+b}\right)}{f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (-2*Cot[e + f*x]*EllipticPi[a/(a + b), ArcSin[Sqrt[a + b]/Sqrt[a + b*Sec[e
+ f*x]]], (a - b)/(a + b)]*Sqrt[-((b*(1 - Sec[e + f*x]))/(a + b*Sec[e + f*x
]))]*Sqrt[(b*(1 + Sec[e + f*x]))/(a + b*Sec[e + f*x])]*(a + b*Sec[e + f*x])
)/(Sqrt[a + b]*f)
```

Rule 3780

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*(a + b
*Csc[c + d*x])*Sqrt[(b*(1 + Csc[c + d*x]))/(a + b*Csc[c + d*x])]*Sqrt[-((b*
(1 - Csc[c + d*x]))/(a + b*Csc[c + d*x]))]*EllipticPi[a/(a + b), ArcSin[Rt[
a + b, 2]/Sqrt[a + b*Csc[c + d*x]]], (a - b)/(a + b)])/(d*Rt[a + b, 2]*Cot[
c + d*x]), x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(e+fx)}\,dx = -\frac{2\cot(e+fx)\Pi\left(\frac{a}{a+b};\sin^{-1}\left(\frac{\sqrt{a+b}}{\sqrt{a+b\sec(e+fx)}}\right)|\frac{a-b}{a+b}\right)\sqrt{-\frac{b(1-\sec(e+fx))}{a+b\sec(e+fx)}}\sqrt{\frac{b(1+\sec(e+fx))}{a+b\sec(e+fx)}}(a+b\sec(e}{\sqrt{a+b}f}$$

Mathematica [A] time = 0.276401, size = 153, normalized size = 1.22

$$-\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)+1}}\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}\sqrt{a+b\sec(e+fx)}\left((a-b)\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-b}{a+b}\right)+}{f(a\cos(e+fx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (-4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Co
s[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*((a - b)*EllipticF[ArcSin[Tan[(e
+ f*x)/2]], (a - b)/(a + b)] + 2*a*EllipticPi[-1, -ArcSin[Tan[(e + f*x)/2]]
, (a - b)/(a + b)])*Sqrt[a + b*Sec[e + f*x]])/(f*(b + a*Cos[e + f*x]))
```

Maple [A] time = 0.258, size = 215, normalized size = 1.7

$$-2\,\frac{\left(1+\cos\left(fx+e\right)\right)^{2}\left(-1+\cos\left(fx+e\right)\right)}{f\left(a\cos\left(fx+e\right)+b\right)\left(\sin\left(fx+e\right)\right)^{2}}\sqrt{\frac{a\cos\left(fx+e\right)+b}{\cos\left(fx+e\right)}}\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\sqrt{\frac{a\cos\left(fx+e\right)+b}{\left(a+b\right)\left(1+\cos\left(fx+e\right)\right)}}\left(\mathrm{Elli}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/f*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1+cos(f*x+e))^2*(Ellipti
cF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a-EllipticF((-1+cos(f*x+
e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b-2*a*EllipticPi((-1+cos(f*x+e))/sin(f*
x+e),-1,((a-b)/(a+b))^(1/2)))*(-1+cos(f*x+e))/(a*cos(f*x+e)+b)/sin(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sec\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec\left(fx+e\right)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a), x)
```

3.250 $\int \csc^2(e+fx)\sqrt{a+b\sec(e+fx)}\,dx$

Optimal. Leaf size=121

$$\frac{\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\sec(e+fx)}}{f}$$

```
[Out] (Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[e + f*x]))/(a - b))])/f - (Cot[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/f
```

Rubi [A] time = 0.114954, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3875, 3832}

$$\frac{\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{f}-\frac{\cot(e+fx)\sqrt{a+b\sec(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]],x]
```


```
[Out] (Sqrt[a + b]*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[e + f*x]))/(a - b))])/f - (Cot[e + f*x]*Sqrt[a + b*Sec[e + f*x]])/f
```


Rule 3875

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(Tan[e + f*x]*(a + b*Csc[e + f*x])^m)/f, x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^2(e+fx)\sqrt{a+b\sec(e+fx)}\,dx &= -\frac{\cot(e+fx)\sqrt{a+b\sec(e+fx)}}{f}+\frac{1}{2}b\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx\\ &= \frac{\sqrt{a+b}\cot(e+fx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f}\end{aligned}$$

Mathematica [A] time = 1.1934, size = 120, normalized size = 0.99

$$\frac{b\sqrt{\frac{a+b\sec(e+fx)}{(a+b)(\sec(e+fx)+1)}}\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-b}{a+b}\right)-\csc(e+fx)\sqrt{\frac{1}{\sec(e+fx)+1}}(a\cos(e+fx)+b)}{f\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (-((b + a*Cos[e + f*x])*Csc[e + f*x]*Sqrt[(1 + Sec[e + f*x])^(-1)]) + b*Ell
ipticF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sqrt[(a + b*Sec[e + f*x])
/((a + b)*(1 + Sec[e + f*x]))])/(f*Sqrt[(1 + Sec[e + f*x])^(-1)]*Sqrt[a + b
*Sec[e + f*x]])
```

Maple [B] time = 0.278, size = 264, normalized size = 2.2

$$-\frac{\left(-1+\cos\left(fx+e\right)\right)^{2}\left(1+\cos\left(fx+e\right)\right)^{2}}{f\left(a\cos\left(fx+e\right)+b\right)\left(\sin\left(fx+e\right)\right)^{5}}\left(\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\sqrt{\frac{a\cos\left(fx+e\right)+b}{(a+b)\left(1+\cos\left(fx+e\right)\right)}}\text{EllipticF}\left(\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f*(-1+cos(f*x+e))^2*((cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f
*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/
(a+b))^(1/2))*b*sin(f*x+e)*cos(f*x+e)+EllipticF((-1+cos(f*x+e))/sin(f*x+e),
((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+
e)+b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*b+cos(f*x+e)^2*a+b*cos(f*x+e))*(1+co
s(f*x+e))^2*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)/(a*cos(f*x+e)+b)/sin(f*x+
e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sec\left(fx+e\right)+a}\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*csc(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec\left(fx+e\right)+a}\csc\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*csc(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sec\left(e + fx\right)} \csc^2\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sec(e + f*x))*csc(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sec\left(fx + e\right) + a} \csc\left(fx + e\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(f*x + e) + a)*csc(f*x + e)^2, x)
```

3.251 $\int (a+b\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=309

$$\frac{2(2a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f}-\frac{2(a-b)\sqrt{a+b}\cot(e$$

```
[Out] (-2*(a - b)*Sqrt[a + b]*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[e + f*x]))/(a - b))])/f + (2*(2*a - b)*Sqrt[a + b]*Cot[e +
f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a -
b))])/f - (2*a*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a
 + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/f
```

Rubi [A] time = 0.230189, antiderivative size = 309, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3781, 3921, 3784, 3832, 4004}

$$\frac{2(2a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{f}-\frac{2(a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b}{\ldots}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[e + f*x]))/(a - b))])/f + (2*(2*a - b)*Sqrt[a + b]*Cot[e +
f*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a -
b))])/f - (2*a*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a
 + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]
))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/f
```

Rule 3781

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(3/2), x_Symbol] :> Int[(a^2 + b
*(2*a - b)*Csc[c + d*x])/Sqrt[a + b*Csc[c + d*x]], x] + Dist[b^2, Int[(Csc[
c + d*x]*(1 + Csc[c + d*x]))/Sqrt[a + b*Csc[c + d*x]], x], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
```

```
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int (a+b\sec(e+fx))^{3/2}\,dx = b^2\int \frac{\sec(e+fx)(1+\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}}\,dx + \int \frac{a^2+(2a-b)b\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$$

$$= -\frac{2(a-b)\sqrt{a+b}\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f}$$

$$= -\frac{2(a-b)\sqrt{a+b}\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{f}$$

Mathematica [C] time = 6.11337, size = 882, normalized size = 2.85

$$\frac{2b\cos(e+fx)\sin(e+fx)(a+b\sec(e+fx))^{3/2}}{f(b+a\cos(e+fx))} + \frac{2\left(-b^2\sqrt{\frac{b-a}{a+b}}\tan^5\left(\frac{1}{2}(e+fx)\right)+ab\sqrt{\frac{b-a}{a+b}}\tan^5\left(\frac{1}{2}(e+fx)\right)-2ab\sqrt{\frac{b}{a}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (2*b*Cos[e + f*x]*(a + b*Sec[e + f*x])^(3/2)*Sin[e + f*x])/(f*(b + a*Cos[e
+ f*x])) + (2*(a + b*Sec[e + f*x])^(3/2)*(a*b*Sqrt[(-a + b)/(a + b)]*Tan[(e
 + f*x)/2] + b^2*Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2] - 2*a*b*Sqrt[(-a +
 b)/(a + b)]*Tan[(e + f*x)/2]^3 + a*b*Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/
2]^5 - b^2*Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]^5 + (2*I)*a^2*EllipticPi
[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a
 + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2
]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + (2*I)*a^2*EllipticPi[-((a + b)/(a -
b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*T
an[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x
)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] - I*(a - b)*b*EllipticE[I*ArcSinh[S
```

```
qrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e +
 f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b
*Tan[(e + f*x)/2]^2)/(a + b)] - I*(a - b)^2*EllipticF[I*ArcSinh[Sqrt[(-a +
b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2
]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e +
f*x)/2]^2)/(a + b)]))/(Sqrt[(-a + b)/(a + b)]*f*(b + a*Cos[e + f*x])^(3/2)*
Sec[e + f*x]^(3/2)*Sqrt[(1 - Tan[(e + f*x)/2]^2)^(-1)]*(-1 + Tan[(e + f*x)/
2]^2)*(1 + Tan[(e + f*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b
*Tan[(e + f*x)/2]^2)/(1 + Tan[(e + f*x)/2]^2)])
```

Maple [B] time = 0.3, size = 1199, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(f*x+e))^(3/2),x)
```

```
[Out] 2/f*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*(1+cos(f*x+e))^2*(-1+cos(f*x+e))^
2*(cos(f*x+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^2
*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e))
)^(1/2)*sin(f*x+e)-2*cos(f*x+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)
/(a+b))^(1/2))*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/
(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*a*b-cos(f*x+e)*EllipticF((-1+cos(f*x+e))/s
in(f*x+e),((a-b)/(a+b))^(1/2))*b^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+
b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)+cos(f*x+e)*EllipticE((
-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(1+cos(f*x+e)))^
(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*a*b+cos(f*
x+e)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b^2*(cos(f*x
+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*s
in(f*x+e)-2*cos(f*x+e)*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b
))^(1/2))*a^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(
1+cos(f*x+e)))^(1/2)*sin(f*x+e)+(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*
(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e)
,((a-b)/(a+b))^(1/2))*a^2*sin(f*x+e)-2*EllipticF((-1+cos(f*x+e))/sin(f*x+e)
,((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x
+e)+b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*a*b-(cos(f*x+e)/(1+cos(f*x+e)))^(1/
2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e)
)/sin(f*x+e),((a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)+(cos(f*x+e)/(1+cos(f*x+e))
)^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f
*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a*b*sin(f*x+e)+(cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+
cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)-2*(cos(f*x+e)/(1
+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*Ellipti
cPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*a^2*sin(f*x+e)-cos(f
*x+e)^2*a*b+a*b*cos(f*x+e)-b^2*cos(f*x+e)+b^2)/sin(f*x+e)^5/(a*cos(f*x+e)+b
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \left(a + b \sec \left(e + f x\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sec \left(f x + e\right) + a\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2), x)
```

3.252 $\int \csc^2(e+fx)(a+b\sec(e+fx))^{3/2}\,dx$

Optimal. Leaf size=228

$$\frac{3(a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f}-\frac{\cot(e+fx)(a+b\sec(}{f}$$

```
[Out] (-3*(a - b)*Sqrt[a + b]*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[e + f*x]))/(a - b))])/f + (3*(a - b)*Sqrt[a + b]*Cot[e + f*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b)
)])/f - (Cot[e + f*x]*(a + b*Sec[e + f*x])^(3/2))/f
```

Rubi [A] time = 0.240174, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3875, 3829, 3832, 4004}

$$-\frac{\cot(e+fx)(a+b\sec(e+fx))^{3/2}}{f}+\frac{3(a-b)\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a}{a}\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-3*(a - b)*Sqrt[a + b]*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[e + f*x]))/(a - b))])/f + (3*(a - b)*Sqrt[a + b]*Cot[e + f*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b)
)])/f - (Cot[e + f*x]*(a + b*Sec[e + f*x])^(3/2))/f
```

Rule 3875

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(Tan[e + f*x]*(a + b*Csc[e + f*x])^m)/f, x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3829

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[a - b, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[b, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x],
x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \csc^2(e+fx)(a+b\sec(e+fx))^{3/2}\,dx = -\frac{\cot(e+fx)(a+b\sec(e+fx))^{3/2}}{f}+\frac{1}{2}(3b)\int \sec(e+fx)\sqrt{a+b\sec(e+f}$$

$$= -\frac{\cot(e+fx)(a+b\sec(e+fx))^{3/2}}{f}+\frac{1}{2}(3(a-b)b)\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$$

$$= -\frac{3(a-b)\sqrt{a+b}\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{\cdot}}{f}$$

Mathematica [A] time = 11.1196, size = 276, normalized size = 1.21

$$\frac{3b(a+b\sec(e+fx))^{3/2}\left(-\frac{(a+b)\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}\left(E\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)-\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-b}{a+b}\right)\right)}{\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)+1}}}-\tan\left(\frac{1}{2}(e+fx)\right.}{f\sqrt{\sec^2\left(\frac{1}{2}(e+fx)\right)}\sec^{\frac{3}{2}}(e+fx)\sqrt{\cos^2\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)}(a\cos(e+fx)+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (Cos[e + f*x]*(a + b*Sec[e + f*x])^(3/2)*((-b - a*Cos[e + f*x])*Csc[e + f*x
] + 3*b*Sin[e + f*x]))/(f*(b + a*Cos[e + f*x])) + (3*b*(a + b*Sec[e + f*x])
^(3/2)*(-(((a + b)*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*
(EllipticE[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)] - EllipticF[ArcSin[Ta
n[(e + f*x)/2]], (a - b)/(a + b)]))/Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])])
- (b + a*Cos[e + f*x])*Tan[(e + f*x)/2]))/(f*(b + a*Cos[e + f*x])^2*Sqrt[Se
c[(e + f*x)/2]^2]*Sec[e + f*x]^(3/2)*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]])
```

Maple [B] time = 0.333, size = 850, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/f*(-1+cos(f*x+e))^2*(3*cos(f*x+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),(
(a-b)/(a+b))^(1/2))*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e
)+b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*a*b+3*cos(f*x+e)*EllipticF((-1+cos(f*
x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)
*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-3*cos(f*x+e)*El
lipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(1+cos(
f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*a
```

```
*b-3*cos(f*x+e)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b
^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e
)))^(1/2)*sin(f*x+e)+3*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(
1/2))*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*
x+e)))^(1/2)*sin(f*x+e)*a*b+3*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a
*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),(
(a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)-3*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(
a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f
*x+e),((a-b)/(a+b))^(1/2))*a*b*sin(f*x+e)-3*(cos(f*x+e)/(1+cos(f*x+e)))^(1/
2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e)
)/sin(f*x+e),((a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)+a^2*cos(f*x+e)^2+3*cos(f*x
+e)^2*a*b-a*b*cos(f*x+e)+3*b^2*cos(f*x+e)-2*b^2)*(1+cos(f*x+e))^2*(1/cos(f*
x+e)*(a*cos(f*x+e)+b))^(1/2)/(a*cos(f*x+e)+b)/sin(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)*csc(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\csc\left(fx+e\right)^{2}\sec\left(fx+e\right)+a\csc\left(fx+e\right)^{2}\right)\sqrt{b\sec\left(fx+e\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*csc(f*x + e)^2*sec(f*x + e) + a*csc(f*x + e)^2)*sqrt(b*sec(f*x
+ e) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sec(f*x+e))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(3/2)*csc(f*x + e)^2, x)
```

3.253 $\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal. Leaf size=106

$$-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{af}$$

```
[Out] (-2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e
+ f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(a*f)
```

Rubi [A] time = 0.0217945, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3784}

$$-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{af}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (-2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e
+ f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(a*f)
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sec(e+fx)}}\,dx=-\frac{2\sqrt{a+b}\cot(e+fx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{af}$$

Mathematica [A] time = 0.222563, size = 140, normalized size = 1.32

$$-\frac{4\cos^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)+1}}\sec(e+fx)\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}\left(\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-b}{a+b}\right)+2\Pi\left(-1;-\text{si}\right.\right.}{f\sqrt{a+b\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (-4*Cos[(e + f*x)/2]^2*Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Sqrt[(b + a*Co
s[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*(EllipticF[ArcSin[Tan[(e + f*x)/2
]], (a - b)/(a + b)] + 2*EllipticPi[-1, -ArcSin[Tan[(e + f*x)/2]], (a - b)/
(a + b)])*Sec[e + f*x])/(f*Sqrt[a + b*Sec[e + f*x]])
```

Maple [A] time = 0.255, size = 178, normalized size = 1.7

$$-2\frac{\left(1+\cos\left(fx+e\right)\right)^{2}\left(-1+\cos\left(fx+e\right)\right)}{f\left(a\cos\left(fx+e\right)+b\right)\left(\sin\left(fx+e\right)\right)^{2}}\sqrt{\frac{a\cos\left(fx+e\right)+b}{\cos\left(fx+e\right)}}\sqrt{\frac{\cos\left(fx+e\right)}{1+\cos\left(fx+e\right)}}\sqrt{\frac{a\cos\left(fx+e\right)+b}{(a+b)\left(1+\cos\left(fx+e\right)\right)}}\left(\text{Elli}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] -2/f*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2
)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*(1+cos(f*x+e))^2*(Ellipti
cF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))-2*EllipticPi((-1+cos(f*x
+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2)))*(-1+cos(f*x+e))/(a*cos(f*x+e)+b)/s
in(f*x+e)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sec(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{b\sec\left(fx+e\right)+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*sec(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sec(f*x + e) + a), x)
```

3.254 $\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal. Leaf size=255

$$-\frac{\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f\sqrt{a+b}}+\frac{b^2\tan(e+fx)}{f\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}-$$

```
[Out] (Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a +
b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x
]))/(a - b))])/(Sqrt[a + b]*f) - (Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*
Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(Sqrt[a + b]*f) - Cot[e + f
*x]/(f*Sqrt[a + b*Sec[e + f*x]]) + (b^2*Tan[e + f*x])/((a^2 - b^2)*f*Sqrt[a
 + b*Sec[e + f*x]])
```

Rubi [A] time = 0.320415, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3875, 3833, 21, 3829, 3832, 4004}

$$\frac{b^2\tan(e+fx)}{f\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}-\frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}}-\frac{\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e}}{\sqrt{a+b}}\right.\right.}{f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a +
b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x
]))/(a - b))])/(Sqrt[a + b]*f) - (Cot[e + f*x]*EllipticF[ArcSin[Sqrt[a + b*
Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(Sqrt[a + b]*f) - Cot[e + f
*x]/(f*Sqrt[a + b*Sec[e + f*x]]) + (b^2*Tan[e + f*x])/((a^2 - b^2)*f*Sqrt[a
 + b*Sec[e + f*x]])
```

Rule 3875

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(Tan[e + f*x]*(a + b*Csc[e + f*x])^m)/f, x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3833

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a
, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
```

```
a + b*x])
```

Rule 3829

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[a - b, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[b, Int[(Csc[e + f*x]*(1 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x],
x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx = -\frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}} - \frac{1}{2}b\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx$$

$$= -\frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}} + \frac{b^2\tan(e+fx)}{\left(a^2-b^2\right)f\sqrt{a+b\sec(e+fx)}} + \frac{b\int \frac{\sec(e+fx)\left(-\frac{a}{2}-\frac{1}{2}b\sec(e+fx)\right)}{\sqrt{a+b\sec(e+fx)}}\,dx}{a^2-b^2}$$

$$= -\frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}} + \frac{b^2\tan(e+fx)}{\left(a^2-b^2\right)f\sqrt{a+b\sec(e+fx)}} - \frac{b\int \sec(e+fx)\sqrt{a+b\sec(e+fx)}}{2\left(a^2-b^2\right)}$$

$$= -\frac{\cot(e+fx)}{f\sqrt{a+b\sec(e+fx)}} + \frac{b^2\tan(e+fx)}{\left(a^2-b^2\right)f\sqrt{a+b\sec(e+fx)}} - \frac{b\int \frac{\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{2(a+b)} - \frac{b^2\int \frac{\sec(e+fx}{\sqrt{a+}}}{2\,(a}$$

$$= \frac{\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{\sqrt{a+b}f} - \frac{\cot(e+fx)F\left(\sin\right.}{}$$

Mathematica [A] time = 7.60996, size = 259, normalized size = 1.02

$$\frac{\sqrt{\sec(e+fx)}\left(\frac{b\left(-\frac{(a+b)\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}\left(E\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)-\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\frac{a-b}{a+b}\right)\right)}{\sqrt{\frac{\cos(e+fx)}{\cos(e+fx)+1}}}-\tan\left(\frac{1}{2}(e+fx)\right)(a\cos(e+fx)+b)\right)}{\left(b^2-a^2\right)\sqrt{\sec^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{\cos^2\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)}}+\frac{\csc(e+fx)(}{}\right.}{f\sqrt{a+b\sec(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[e + f*x]^2/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (Sqrt[Sec[e + f*x]]*(((b + a*Cos[e + f*x])*(-a + b*Cos[e + f*x])*Csc[e + f*
x])/((a^2 - b^2)*Sqrt[Sec[e + f*x]]) + (b*(-(((a + b)*Sqrt[(b + a*Cos[e + f
*x])/((a + b)*(1 + Cos[e + f*x]))]*(EllipticE[ArcSin[Tan[(e + f*x)/2]], (a
- b)/(a + b)] - EllipticF[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]))/Sqrt
[Cos[e + f*x]/(1 + Cos[e + f*x])]) - (b + a*Cos[e + f*x])*Tan[(e + f*x)/2])
)/((-a^2 + b^2)*Sqrt[Sec[(e + f*x)/2]^2]*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*
x]])))/(f*Sqrt[a + b*Sec[e + f*x]])
```

Maple [B] time = 0.32, size = 852, normalized size = 3.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e))^(1/2),x)
```

```
[Out] -1/f/(a-b)/(a+b)*(-1+cos(f*x+e))^2*(cos(f*x+e)*EllipticE((-1+cos(f*x+e))/si
n(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a
*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*a*b+cos(f*x+e)*EllipticE((-
1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*b^2*(cos(f*x+e)/(1+cos(f*x+e)
))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-cos(f*x
+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*(cos(f*x+e)/(
1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*sin(f*
x+e)*a*b-cos(f*x+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2
))*b^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f
*x+e)))^(1/2)*sin(f*x+e)+(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(
f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)
/(a+b))^(1/2))*a*b*sin(f*x+e)+(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a
*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),(
(a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)-EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a
-b)/(a+b))^(1/2))*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+
b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*a*b-(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(
1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))/si
n(f*x+e),((a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)+a^2*cos(f*x+e)^2-cos(f*x+e)^2*
a*b+a*b*cos(f*x+e)-b^2*cos(f*x+e))*(1+cos(f*x+e))^2*(1/cos(f*x+e)*(a*cos(f*
x+e)+b))^(1/2)/(a*cos(f*x+e)+b)/sin(f*x+e)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^2/sqrt(b*sec(f*x + e) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\csc\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(csc(f*x + e)^2/sqrt(b*sec(f*x + e) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc^2\left(e+fx\right)}{\sqrt{a+b\sec\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/sqrt(a + b*sec(e + f*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^2}{\sqrt{b\sec\left(fx+e\right)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/sqrt(b*sec(f*x + e) + a), x)
```

3.255 $\int \frac{1}{(a+b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=347

$$-\frac{2\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{af\sqrt{a+b}}+\frac{2b^2\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}$$

```
[Out] (2*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f
*x]))/(a - b))])/(a*Sqrt[a + b]*f) - (2*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[
a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(a*Sqrt[a + b]*f) - (
2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e +
f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(a^2*f) + (2*b^2*Tan[e + f*x])/(a*(
a^2 - b^2)*f*Sqrt[a + b*Sec[e + f*x]])
```

Rubi [A] time = 0.332869, antiderivative size = 347, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3785, 4058, 3921, 3784, 3832, 4004}

$$\frac{2b^2\tan(e+fx)}{af\left(a^2-b^2\right)\sqrt{a+b\sec(e+fx)}}-\frac{2\sqrt{a+b}\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[e + f*x])^(-3/2),x]
```

```
[Out] (2*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a
+ b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f
*x]))/(a - b))])/(a*Sqrt[a + b]*f) - (2*Cot[e + f*x]*EllipticF[ArcSin[Sqrt[
a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(a*Sqrt[a + b]*f) - (
2*Sqrt[a + b]*Cot[e + f*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[e +
f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(a^2*f) + (2*b^2*Tan[e + f*x])/(a*(
a^2 - b^2)*f*Sqrt[a + b*Sec[e + f*x]])
```

Rule 3785

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Simp[(b^2*Cot
[c + d*x]*(a + b*Csc[c + d*x])^(n + 1))/(a*d*(n + 1)*(a^2 - b^2)), x] + Dis
t[1/(a*(n + 1)*(a^2 - b^2)), Int[(a + b*Csc[c + d*x])^(n + 1)*Simp[(a^2 - b
^2)*(n + 1) - a*b*(n + 1)*Csc[c + d*x] + b^2*(n + 2)*Csc[c + d*x]^2, x], x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && LtQ[n, -1] && Intege
rQ[2*n]
```

Rule 4058

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[(Csc[e + f*x]*(1
 + Csc[e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A,
```

```
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3921

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 3784

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(2*Rt[a
 + b, 2]*Sqrt[(b*(1 - Csc[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[c + d*x]))
/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[c + d*x]]/Rt[a + b,
2]], (a + b)/(a - b)])/(a*d*Cot[c + d*x]), x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(a+b\sec(e+fx))^{3/2}}\,dx &= \frac{2b^2\tan(e+fx)}{a\left(a^2-b^2\right)f\sqrt{a+b\sec(e+fx)}} - \frac{2\int \frac{\frac{1}{2}\left(-a^2+b^2\right)+\frac{1}{2}ab\sec(e+fx)+\frac{1}{2}b^2\sec^2(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{a\left(a^2-b^2\right)} \\
&= \frac{2b^2\tan(e+fx)}{a\left(a^2-b^2\right)f\sqrt{a+b\sec(e+fx)}} - \frac{2\int \frac{\frac{1}{2}\left(-a^2+b^2\right)+\left(\frac{ab}{2}-\frac{b^2}{2}\right)\sec(e+fx)}{\sqrt{a+b\sec(e+fx)}}\,dx}{a\left(a^2-b^2\right)} - \frac{b^2\int \frac{\sec(e+fx)(1+\sec(e+\ldots}{\sqrt{a+b\sec(e+fx)}}}{a\left(a^2-b^2\right)} \\
&= \frac{2\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{a\sqrt{a+b}f} + \frac{2b^2\,\mathrm{ta}\ldots}{a\left(a^2-b^2\right)f\sqrt{\ldots}} \\
&= \frac{2\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{a\sqrt{a+b}f} - \frac{2\cot(e+fx)\ldots}{\ldots}
\end{aligned}
$$

Mathematica [C] time = 6.14205, size = 1249, normalized size = 3.6

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[e + f*x])^(-3/2),x]
```

```
[Out] ((b + a*Cos[e + f*x])^2*Sec[e + f*x]^2*((2*b*Sin[e + f*x])/(a*(-a^2 + b^2))
 + (2*b^2*Sin[e + f*x])/(a*(a^2 - b^2)*(b + a*Cos[e + f*x]))))/(f*(a + b*Se
c[e + f*x])^(3/2)) + (2*(b + a*Cos[e + f*x])^(3/2)*Sec[e + f*x]^(3/2)*Sqrt[
(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(1 + Tan[(e + f*x)/2]
^2)]*(a*b*Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2] + b^2*Sqrt[(-a + b)/(a +
b)]*Tan[(e + f*x)/2] - 2*a*b*Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]^3 + a*
b*Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]^5 - b^2*Sqrt[(-a + b)/(a + b)]*Ta
n[(e + f*x)/2]^5 - (2*I)*a^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[
(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x
)/2]^2]*Sqrt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)]
 + (2*I)*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)
]*Tan[(e + f*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[(a
+ b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] - (2*I)*a^2*Ell
ipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/
2]], (a + b)/(a - b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sqrt[
(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + (2*I)*b^2*
EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*
x)/2]], (a + b)/(a - b)]*Tan[(e + f*x)/2]^2*Sqrt[1 - Tan[(e + f*x)/2]^2]*Sq
rt[(a + b - a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] - I*(a -
b)*b*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/
(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(a + b
- a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)] + I*(a^2 + a*b - 2*
b^2)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/
(a - b)]*Sqrt[1 - Tan[(e + f*x)/2]^2]*(1 + Tan[(e + f*x)/2]^2)*Sqrt[(a + b
- a*Tan[(e + f*x)/2]^2 + b*Tan[(e + f*x)/2]^2)/(a + b)]))/(a*Sqrt[(-a + b)/
(a + b)]*(a^2 - b^2)*f*(a + b*Sec[e + f*x])^(3/2)*(-1 + Tan[(e + f*x)/2]^2)
*Sqrt[(1 + Tan[(e + f*x)/2]^2)/(1 - Tan[(e + f*x)/2]^2)]*(a*(-1 + Tan[(e +
f*x)/2]^2) - b*(1 + Tan[(e + f*x)/2]^2)))
```

Maple [B] time = 0.281, size = 1209, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sec(f*x+e))^(3/2),x)
```

```
[Out] 1/f/a/(a+b)/(a-b)*4^(1/2)*(1/cos(f*x+e)*(a*cos(f*x+e)+b))^(1/2)*(cos(f*x+e)
*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*a^2*(cos(f*x+e)/
(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*sin(f
*x+e)+cos(f*x+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*
(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))
^(1/2)*sin(f*x+e)*a*b-cos(f*x+e)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b
)/(a+b))^(1/2))*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)
/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)*a*b-cos(f*x+e)*EllipticE((-1+cos(f*x+e))/
sin(f*x+e),((a-b)/(a+b))^(1/2))*b^2*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a
+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)-2*cos(f*x+e)*Elliptic
Pi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*a^2*(cos(f*x+e)/(1+co
s(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*sin(f*x+e)
+2*sin(f*x+e)*cos(f*x+e)*EllipticPi((-1+cos(f*x+e))/sin(f*x+e),-1,((a-b)/(a
+b))^(1/2))*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+
cos(f*x+e)))^(1/2)*b^2+(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*
x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(
a+b))^(1/2))*a^2*sin(f*x+e)+EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+
b))^(1/2))*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+c
```

```
os(f*x+e)))^(1/2)*sin(f*x+e)*a*b-(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)
*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(f*x+e
),((a-b)/(a+b))^(1/2))*a*b*sin(f*x+e)-(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/
(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticE((-1+cos(f*x+e))/sin(
f*x+e),((a-b)/(a+b))^(1/2))*b^2*sin(f*x+e)-2*(cos(f*x+e)/(1+cos(f*x+e)))^(1
/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*EllipticPi((-1+cos(f*x+
e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*a^2*sin(f*x+e)+2*EllipticPi((-1+cos(
f*x+e))/sin(f*x+e),-1,((a-b)/(a+b))^(1/2))*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*
x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*b^2+cos(f*x+e)
^2*a*b-cos(f*x+e)^2*b^2-a*b*cos(f*x+e)+b^2*cos(f*x+e))/(a*cos(f*x+e)+b)/sin
(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)+a}}{b^{2}\sec\left(fx+e\right)^{2}+2\,ab\sec\left(fx+e\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)/(b^2*sec(f*x + e)^2 + 2*a*b*sec(f*x + e)
+ a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(a+b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral((a + b*sec(e + f*x))**(-3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(f*x + e) + a)^(-3/2), x)
```

3.256 $\int \frac{\csc^2(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx$

Optimal. Leaf size=318

$$-\frac{(3a-b)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,\text{EllipticF}\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right),\frac{a+b}{a-b}\right)}{f(a-b)(a+b)^{3/2}}+\frac{4ab^2\tan(e+fx)}{f\left(a^2-b^2\right)^2\sqrt{a+b\sec(e+}}$$

```
[Out] (4*a*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e +
 f*x]))/(a - b))])/((a - b)*(a + b)^(3/2)*f) - ((3*a - b)*Cot[e + f*x]*Elli
pticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/((a
 - b)*(a + b)^(3/2)*f) - Cot[e + f*x]/(f*(a + b*Sec[e + f*x])^(3/2)) + (b^2
*Tan[e + f*x])/((a^2 - b^2)*f*(a + b*Sec[e + f*x])^(3/2)) + (4*a*b^2*Tan[e
+ f*x])/((a^2 - b^2)^2*f*Sqrt[a + b*Sec[e + f*x]])
```

Rubi [A] time = 0.52453, antiderivative size = 318, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3875, 3833, 4003, 4005, 3832, 4004}

$$\frac{4ab^2\tan(e+fx)}{f\left(a^2-b^2\right)^2\sqrt{a+b\sec(e+fx)}}+\frac{b^2\tan(e+fx)}{f\left(a^2-b^2\right)(a+b\sec(e+fx))^{3/2}}-\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}}-\frac{(3a-b)\cot(e+fx)\sqrt{\frac{b(1-}{}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Sec[e + f*x])^(3/2),x]
```


```
[Out] (4*a*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (
a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e +
 f*x]))/(a - b))])/((a - b)*(a + b)^(3/2)*f) - ((3*a - b)*Cot[e + f*x]*Elli
pticF[ArcSin[Sqrt[a + b*Sec[e + f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(
b*(1 - Sec[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/((a
 - b)*(a + b)^(3/2)*f) - Cot[e + f*x]/(f*(a + b*Sec[e + f*x])^(3/2)) + (b^2
*Tan[e + f*x])/((a^2 - b^2)*f*(a + b*Sec[e + f*x])^(3/2)) + (4*a*b^2*Tan[e
+ f*x])/((a^2 - b^2)^2*f*Sqrt[a + b*Sec[e + f*x]])
```


Rule 3875

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(Tan[e + f*x]*(a + b*Csc[e + f*x])^m)/f, x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3833

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> -Simp[(b*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(
a^2 - b^2)), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*(a*(m + 1) - b*(m + 2)*Csc[e + f*x]), x], x] /; FreeQ[{a
, b, e, f}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Rule 4003

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> -Simp[((A*b - a*B)*Cot[e
+ f*x]*(a + b*Csc[e + f*x])^(m + 1))/(f*(m + 1)*(a^2 - b^2)), x] + Dist[1/(
(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[(a
*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ[{
a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -
1]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[(Csc[e + f*x]*(1 + Csc[
e + f*x]))/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 3832

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[(-2*Rt[a + b, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-
((b*(1 + Csc[e + f*x]))/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]
/Rt[a + b, 2]], (a + b)/(a - b)])/(b*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[(-2*(A*b - a*B)*Rt[
a + (b*B)/A, 2]*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[-((b*(1 + Csc[e +
 f*x]))/(a - b))]*EllipticE[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + (b*B)/A,
 2]], (a*A + b*B)/(a*A - b*B)])/(b^2*f*Cot[e + f*x]), x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{(a+b\sec(e+fx))^{3/2}}\,dx = -\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}} - \frac{1}{2}(3b)\int \frac{\sec(e+fx)}{(a+b\sec(e+fx))^{5/2}}\,dx$$

$$= -\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}} + \frac{b^2\tan(e+fx)}{\left(a^2-b^2\right)f(a+b\sec(e+fx))^{3/2}} + \frac{b\int \frac{\sec(e+fx)\left(-\frac{3a}{2}+\frac{1}{2}b\sec(e+\right.}{(a+b\sec(e+fx))^{3/2}}}{a^2-b^2}$$

$$= -\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}} + \frac{b^2\tan(e+fx)}{\left(a^2-b^2\right)f(a+b\sec(e+fx))^{3/2}} + \frac{4ab^2\tan(e+fx)}{\left(a^2-b^2\right)^2 f\sqrt{a+b\sec(e}}$$

$$= -\frac{\cot(e+fx)}{f(a+b\sec(e+fx))^{3/2}} + \frac{b^2\tan(e+fx)}{\left(a^2-b^2\right)f(a+b\sec(e+fx))^{3/2}} + \frac{4ab^2\tan(e+fx)}{\left(a^2-b^2\right)^2 f\sqrt{a+b\sec(e}}$$

$$= \frac{4a\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\sec(e+fx))}{a-b}}}{(a-b)(a+b)^{3/2}f} - \frac{(3a-b)\,c}{}$$

Mathematica [A] time = 7.7989, size = 259, normalized size = 0.81

$$-2b\left(3a^2+4ab+b^2\right)\cos^2\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)\sqrt{\frac{1}{\sec(e+fx)+1}}\sqrt{\frac{a\cos(e+fx)+b}{(a+b)(\cos(e+fx)+1)}}\text{EllipticF}\left(\sin^{-1}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right),\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sec[e + f*x])^(3/2),x]
```

```
[Out] (-((a - b)*((3*a - b)*b + a*(a - 3*b)*Cos[e + f*x])*Csc[e + f*x]) + 8*a*b*(
a + b)*Cos[(e + f*x)/2]^2*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f
*x]))]*EllipticE[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sec[e + f*x]*Sq
rt[(1 + Sec[e + f*x])^(-1)] - 2*b*(3*a^2 + 4*a*b + b^2)*Cos[(e + f*x)/2]^2*
Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*EllipticF[ArcSin[Ta
n[(e + f*x)/2]], (a - b)/(a + b)]*Sec[e + f*x]*Sqrt[(1 + Sec[e + f*x])^(-1)
])/((a^2 - b^2)^2*f*Sqrt[a + b*Sec[e + f*x]])
```

Maple [B] time = 0.258, size = 1065, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sec(f*x+e))^(3/2),x)
```

```
[Out] -1/2/f/(a-b)^2/(a+b)^2*(4*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b)
)^(1/2))*cos(f*x+e)*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(
a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*a^2*b+4*EllipticE((-1+cos(f*x+e))/sin
(f*x+e),((a-b)/(a+b))^(1/2))*cos(f*x+e)*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e
)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*a*b^2-3*EllipticF
((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*cos(f*x+e)*sin(f*x+e)*(cos
(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/
2)*a^2*b-4*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*cos(f*
x+e)*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)
/(1+cos(f*x+e)))^(1/2)*a*b^2-EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a
+b))^(1/2))*cos(f*x+e)*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b
)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*b^3+4*EllipticE((-1+cos(f*x+e))/si
n(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*
(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*a^2*b+4*EllipticE((-1+cos(f
*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*(cos(f*x+e)/(1+cos(f*x+e)
))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*a*b^2-3*EllipticF(
(-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*(cos(f*x+e)/(1+c
os(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*a^2*b-4*E
llipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*(cos(f*
x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))^(1/2)*
a*b^2-EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*
(cos(f*x+e)/(1+cos(f*x+e)))^(1/2)*(1/(a+b)*(a*cos(f*x+e)+b)/(1+cos(f*x+e)))
^(1/2)*b^3+cos(f*x+e)^2*a^3-4*a^2*b*cos(f*x+e)^2+3*cos(f*x+e)^2*a*b^2+3*cos
(f*x+e)*a^2*b-4*a*b^2*cos(f*x+e)+cos(f*x+e)*b^3)*(1/cos(f*x+e)*(a*cos(f*x+e
)+b))^(1/2)*4^(1/2)/(a*cos(f*x+e)+b)/sin(f*x+e)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\csc\left(fx+e\right)^2}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^2/(b*sec(f*x + e) + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sec\left(fx+e\right)+a}\csc\left(fx+e\right)^{2}}{b^{2}\sec\left(fx+e\right)^{2}+2\,ab\sec\left(fx+e\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(f*x + e) + a)*csc(f*x + e)^2/(b^2*sec(f*x + e)^2 + 2*a*
b*sec(f*x + e) + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc^{2}\left(e+fx\right)}{\left(a+b\sec\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sec(f*x+e))**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(a + b*sec(e + f*x))**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\csc\left(fx+e\right)^{2}}{\left(b\sec\left(fx+e\right)+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sec(f*x+e))^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/(b*sec(f*x + e) + a)^(3/2), x)
```

3.257 $\int (a+b\sec(c+dx))^3(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=249

$$\frac{3a^2b(e\sin(c+dx))^{m+1}\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)}+\frac{a^3\cos(c+dx)(e\sin(c+dx))^{m+1}\text{Hypergeom}}{de(m+1)\sqrt{\cos^2}}$$

```
[Out] (a^3*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (3*a^2*b
*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x]
)^(1 + m))/(d*e*(1 + m)) + (b^3*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2,
Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)) + (3*a*b^2*Sqrt[Cos
[c + d*x]^2]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*S
ec[c + d*x]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m))
```

Rubi [A] time = 0.386978, antiderivative size = 249, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3872, 2912, 2643, 2564, 364, 2577}

$$\frac{3a^2b(e\sin(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}+\frac{a^3\cos(c+dx)(e\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*(e*Sin[c + d*x])^m,x]
```

```
[Out] (a^3*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (3*a^2*b
*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x]
)^(1 + m))/(d*e*(1 + m)) + (b^3*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2,
Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)) + (3*a*b^2*Sqrt[Cos
[c + d*x]^2]*Hypergeometric2F1[3/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*S
ec[c + d*x]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m))
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^3(e\sin(c+dx))^m\,dx = -\int (-b-a\cos(c+dx))^3\sec^3(c+dx)(e\sin(c+dx))^m\,dx$$
$$= -\int \left(-a^3(e\sin(c+dx))^m - 3a^2b\sec(c+dx)(e\sin(c+dx))^m - 3ab^2\sec^2(c$$
$$= a^3\int (e\sin(c+dx))^m\,dx + \left(3a^2b\right)\int \sec(c+dx)(e\sin(c+dx))^m\,dx + \left(3ab\right.$$
$$= \frac{a^3\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} + \frac{3ab^2\sqrt{\cdots}}{\cdots}$$
$$= \frac{a^3\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} + \frac{3a^2b\,{}_2F\cdots}{\cdots}$$

Mathematica [A] time = 0.315501, size = 182, normalized size = 0.73

$$\frac{\tan(c+dx)(e\sin(c+dx))^m\left(b\left(3a^2\cos(c+dx)\text{Hypergeometric2F1}\left(1,\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)+b\left(3a\sqrt{\cos^2(c+dx}\right.\right.\right.}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*(e*Sin[c + d*x])^m,x]
```

```
[Out] ((a^3*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin
[c + d*x]^2] + b*(3*a^2*Cos[c + d*x]*Hypergeometric2F1[1, (1 + m)/2, (3 + m
)/2, Sin[c + d*x]^2] + b*(3*a*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3/2, (
1 + m)/2, (3 + m)/2, Sin[c + d*x]^2] + b*Cos[c + d*x]*Hypergeometric2F1[2,
(1 + m)/2, (3 + m)/2, Sin[c + d*x]^2])))*(e*Sin[c + d*x])^m*Tan[c + d*x])/(
d*(1 + m))
```

Maple [F] time = 1.569, size = 0, normalized size = 0.

$$\int (a + b \sec(dx + c))^3 (e \sin(dx + c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+b*sec(d*x+c))^3*(e*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c) + a)^3 (e \sin(dx + c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*(e*sin(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^3 \sec(dx + c)^3 + 3\,ab^2 \sec(dx + c)^2 + 3\,a^2 b \sec(dx + c) + a^3\right)(e \sin(dx + c))^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c
) + a^3)*(e*sin(d*x + c))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(e*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sec(dx + c) + a)^3 (e \sin(dx + c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(e*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^3*(e*sin(d*x + c))^m, x)
```

3.258 $\int (a+b\sec(c+dx))^2(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=190

$$\frac{a^2\sin(c+dx)\cos(c+dx)(e\sin(c+dx))^m\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}}+\frac{2ab(e\sin(c+dx))^{m+1}\text{H}}{}$$

```
[Out] (a^2*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*Sin[c + d*x]*(e*Sin[c + d*x])^m)/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (2*
a*b*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d
*x])^(1 + m))/(d*e*(1 + m)) + (b^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3
/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^m*Tan[c + d*x])/
(d*(1 + m))
```

Rubi [A] time = 0.839372, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3872, 2911, 2564, 364, 4398, 4401, 2643, 2577}

$$\frac{a^2\sin(c+dx)\cos(c+dx)(e\sin(c+dx))^m\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{d(m+1)\sqrt{\cos^2(c+dx)}}+\frac{2ab(e\sin(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin\right.}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^m,x]
```


```
[Out] (a^2*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]
^2]*Sin[c + d*x]*(e*Sin[c + d*x])^m)/(d*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (2*
a*b*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d
*x])^(1 + m))/(d*e*(1 + m)) + (b^2*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[3
/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^m*Tan[c + d*x])/
(d*(1 + m))
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2911

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_Symbol] :> Dist[(2*a*b)/d, I
nt[(g*Cos[e + f*x])^p*(d*Sin[e + f*x])^(n + 1), x], x] + Int[(g*Cos[e + f*x
])^p*(d*Sin[e + f*x])^n*(a^2 + b^2*Sin[e + f*x]^2), x] /; FreeQ[{a, b, d, e
, f, g, n, p}, x] && NeQ[a^2 - b^2, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 4398

```
Int[(u_.)*((a_)*(v_))^(p_), x_Symbol] :> With[{uu = ActivateTrig[u], vv = A
ctivateTrig[v]}, Dist[(a^IntPart[p]*(a*vv)^FracPart[p])/vv^FracPart[p], Int
[uu*vv^p, x], x]] /; FreeQ[{a, p}, x] &&  !IntegerQ[p] &&  !InertTrigFreeQ[
v]
```

Rule 4401

```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2(e\sin(c+dx))^m\,dx = \int (-b-a\cos(c+dx))^2\sec^2(c+dx)(e\sin(c+dx))^m\,dx$$
$$= (2ab)\int \sec(c+dx)(e\sin(c+dx))^m\,dx + \int \left(b^2+a^2\cos^2(c+dx)\right)\sec^2(c+d$$
$$= \frac{(2ab)\,\mathrm{Subst}\left(\int \frac{x^m}{1-\frac{x^2}{e^2}}\,dx, x, e\sin(c+dx)\right)}{de} + \left(\sin^{-m}(c+dx)(e\sin(c+dx))^m\right)\int$$
$$= \frac{2ab\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)} + \left(\sin^{-m}(c+dx)(e\sin($$
$$= \frac{2ab\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)} + \left(a^2\sin^{-m}(c+dx)(e\sin$$
$$= \frac{a^2\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)\sin(c+dx)(e\sin(c+dx))^m}{d(1+m)\sqrt{\cos^2(c+dx)}} + \frac{2}{}$$

Mathematica [A] time = 0.252578, size = 134, normalized size = 0.71

$$\frac{(e\sin(c+dx))^m\left(\sqrt{\cos^2(c+dx)}\tan(c+dx)\left(a^2\mathrm{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)+b^2\mathrm{Hypergeometric}\right.\right.}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*(e*Sin[c + d*x])^m,x]
```

```
[Out] ((e*Sin[c + d*x])^m*(2*a*b*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c
 + d*x]^2]*Sin[c + d*x] + Sqrt[Cos[c + d*x]^2]*(a^2*Hypergeometric2F1[1/2,
(1 + m)/2, (3 + m)/2, Sin[c + d*x]^2] + b^2*Hypergeometric2F1[3/2, (1 + m)/
2, (3 + m)/2, Sin[c + d*x]^2])*Tan[c + d*x]))/(d*(1 + m))
```

Maple [F] time = 1.267, size = 0, normalized size = 0.

$$\int (a + b\sec(dx + c))^2 (e\sin(dx + c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+b*sec(d*x+c))^2*(e*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx + c) + a)^2 (e\sin(dx + c))^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*(e*sin(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b^2\sec(dx + c)^2 + 2\,ab\sec(dx + c) + a^2\right)(e\sin(dx + c))^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)*(e*sin(d*x + c))^m
, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(e*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^2(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(e*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^2*(e*sin(d*x + c))^m, x)
```

3.259 $\int (a+b\sec(c+dx))(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=119

$$\frac{a\cos(c+dx)(e\sin(c+dx))^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{de(m+1)\sqrt{\cos^2(c+dx)}}+\frac{b(e\sin(c+dx))^{m+1}\text{Hypergeome}}{de(}$$

```
[Out] (a*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2
]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (b*Hyperge
ometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1 + m
))/(d*e*(1 + m))
```

Rubi [A] time = 0.157583, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3872, 2838, 2564, 364, 2643}

$$\frac{a\cos(c+dx)(e\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)\sqrt{\cos^2(c+dx)}}+\frac{b(e\sin(c+dx))^{m+1}\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{de(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*(e*Sin[c + d*x])^m,x]
```


```
[Out] (a*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2
]*(e*Sin[c + d*x])^(1 + m))/(d*e*(1 + m)*Sqrt[Cos[c + d*x]^2]) + (b*Hyperge
ometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1 + m
))/(d*e*(1 + m))
```


Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rubi steps

$$\int (a+b\sec(c+dx))(e\sin(c+dx))^m\,dx = -\int (-b-a\cos(c+dx))\sec(c+dx)(e\sin(c+dx))^m\,dx$$

$$= a\int (e\sin(c+dx))^m\,dx + b\int \sec(c+dx)(e\sin(c+dx))^m\,dx$$

$$= \frac{a\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} + \frac{b\,\mathrm{Subst}\left(\int \right.}{}$$

$$= \frac{a\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sin^2(c+dx)\right)(e\sin(c+dx))^{1+m}}{de(1+m)\sqrt{\cos^2(c+dx)}} + \frac{b\,{}_2F_1\left(1,\frac{1+m}{2}\right.}{}$$

Mathematica [A] time = 0.106581, size = 98, normalized size = 0.82

$$\frac{\tan(c+dx)(e\sin(c+dx))^m\left(a\sqrt{\cos^2(c+dx)}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)+b\cos(c+dx)\text{Hyperge}\right.}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*(e*Sin[c + d*x])^m,x]
```

```
[Out] ((a*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c
 + d*x]^2] + b*Cos[c + d*x]*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, Sin[
c + d*x]^2])*(e*Sin[c + d*x])^m*Tan[c + d*x])/(d*(1 + m))
```

Maple [F] time = 0.619, size = 0, normalized size = 0.

$$\int (a+b\sec(dx+c))(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+b*sec(d*x+c))*(e*sin(d*x+c))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c)+a)(e\sin(dx+c))^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (e\sin(c+dx))^m (a+b\sec(c+dx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(e*sin(d*x+c))**m,x)
```

```
[Out] Integral((e*sin(c + d*x))**m*(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)(e\sin(dx+c))^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(e*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

3.260 $\int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=232

$$\frac{\cos(c+dx)(e\sin(c+dx))^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{ade(m+1)\sqrt{\cos^2(c+dx)}}-\frac{be(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-\cdots}{2}}\cdots}{\cdots}$$

```
[Out] -((b*e*AppellF1[1 - m, (1 - m)/2, (1 - m)/2, 2 - m, -((a - b)/(b + a*Cos[c
+ d*x])), (a + b)/(b + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*Co
s[c + d*x])))^((1 - m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((1
 - m)/2)*(e*Sin[c + d*x])^(-1 + m))/(a^2*d*(1 - m))) + (Cos[c + d*x]*Hyperg
eometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1
+ m))/(a*d*e*(1 + m)*Sqrt[Cos[c + d*x]^2])
```

Rubi [A] time = 0.258745, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3872, 2867, 2643, 2703}

$$\frac{\cos(c+dx)(e\sin(c+dx))^{m+1}\,{}_2F_1\left(\frac{1}{2},\frac{m+1}{2};\frac{m+3}{2};\sin^2(c+dx)\right)}{ade(m+1)\sqrt{\cos^2(c+dx)}}-\frac{be(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\left(\frac{a(\cos(c+dx)+1)}{a\cos(c+dx)+b}\right)^{\frac{1-\cdots}{2}}\cdots}{a^2d(1\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x]),x]
```

```
[Out] -((b*e*AppellF1[1 - m, (1 - m)/2, (1 - m)/2, 2 - m, -((a - b)/(b + a*Cos[c
+ d*x])), (a + b)/(b + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*Co
s[c + d*x])))^((1 - m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((1
 - m)/2)*(e*Sin[c + d*x])^(-1 + m))/(a^2*d*(1 - m))) + (Cos[c + d*x]*Hyperg
eometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1
+ m))/(a*d*e*(1 + m)*Sqrt[Cos[c + d*x]^2])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2867

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((c_.) + (d_.)*sin[(e_.) + (f_.)*
(x_)]))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[
(g*Cos[e + f*x])^p, x], x] + Dist[(b*c - a*d)/b, Int[(g*Cos[e + f*x])^p/(a
+ b*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[a^2 - b
^2, 0]
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 2703

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1)*AppellF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b
*Sin[e + f*x]), (a - b)/(a + b*Sin[e + f*x])])/(b*f*(m + p)*(-((b*(1 - Sin[
e + f*x]))/(a + b*Sin[e + f*x])))^((p - 1)/2)*((b*(1 + Sin[e + f*x]))/(a +
b*Sin[e + f*x]))^((p - 1)/2)), x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] && !IGtQ[m + p + 1, 0]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^m}{a+b\sec(c+dx)}\,dx = -\int \frac{\cos(c+dx)(e\sin(c+dx))^m}{-b-a\cos(c+dx)}\,dx$$

$$= \frac{\int (e\sin(c+dx))^m\,dx}{a} + \frac{b\int \frac{(e\sin(c+dx))^m}{-b-a\cos(c+dx)}\,dx}{a}$$

$$= -\frac{beF_1\left(1-m;\frac{1-m}{2},\frac{1-m}{2};2-m;-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\frac{a(1+\cos(c+dx))}{b+a\cos(c+dx)}\right.}{a^2d(1-m)}$$

Mathematica [B] time = 5.62394, size = 687, normalized size = 2.96

$$\overline{d(a+b\sec(c+dx))\left(2m\tan^2\left(\frac{1}{2}(c+dx)\right)\left((a+b)\text{Hypergeometric2F1}\left(\frac{m+1}{2},m+1,\frac{m+3}{2},-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)-bF_1\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(-(b*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*
Tan[(c + d*x)/2]^2)/(a + b)]) + (a + b)*Hypergeometric2F1[(1 + m)/2, 1 + m,
 (3 + m)/2, -Tan[(c + d*x)/2]^2])*(e*Sin[c + d*x])^m*Tan[(c + d*x)/2])/(d*(
a + b*Sec[c + d*x])*((-(b*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*
x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + (a + b)*Hypergeometric2F1
[(1 + m)/2, 1 + m, (3 + m)/2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2 + 2*
m*Cot[c + d*x]*(-(b*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^
2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + (a + b)*Hypergeometric2F1[(1 +
m)/2, 1 + m, (3 + m)/2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2] + 2*m*(-(b*A
ppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c +
d*x)/2]^2)/(a + b)]) + (a + b)*Hypergeometric2F1[(1 + m)/2, 1 + m, (3 + m)/
2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2 + ((1 + m)*Sec[(c + d*x)/2]^2*(
-((a + b)^2*(Hypergeometric2F1[(1 + m)/2, 1 + m, (3 + m)/2, -Tan[(c + d*x)/
2]^2] - (Sec[(c + d*x)/2]^2)^(-1 - m))) + (2*b*((-a + b)*AppellF1[(3 + m)/2
, m, 2, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b
)] + (a + b)*m*AppellF1[(3 + m)/2, 1 + m, 1, (5 + m)/2, -Tan[(c + d*x)/2]^2
, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Tan[(c + d*x)/2]^2)/(3 + m)))/(a +
```

```
b)))
```

Maple [F] time = 0.575, size = 0, normalized size = 0.

$$\int \frac{(e \sin (dx + c))^m}{a + b \sec (dx + c)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c)),x)
```

```
[Out] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c)),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \sin (dx + c))^m}{b \sec (dx + c) + a} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e \sin (dx + c))^m}{b \sec (dx + c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((e*sin(d*x + c))^m/(b*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \sin (c + dx))^m}{a + b \sec (c + dx)} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((e*sin(c + d*x))**m/(a + b*sec(c + d*x)), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \sin (dx + c))^m}{b \sec (dx + c) + a} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a), x)
```

3.261 $\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=405

$$\frac{\cos(c+dx)(e\sin(c+dx))^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{a^2de(m+1)\sqrt{\cos^2(c+dx)}}+\frac{b^2e(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1}{}}}{}$$

```
[Out] (-2*b*e*AppellF1[1 - m, (1 - m)/2, (1 - m)/2, 2 - m, -((a - b)/(b + a*Cos[c
 + d*x])), (a + b)/(b + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*C
os[c + d*x])))^((1 - m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((
1 - m)/2)*(e*Sin[c + d*x])^(-1 + m))/(a^3*d*(1 - m)) + (b^2*e*AppellF1[2 -
m, (1 - m)/2, (1 - m)/2, 3 - m, -((a - b)/(b + a*Cos[c + d*x])), (a + b)/(b
 + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*Cos[c + d*x])))^((1 -
m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((1 - m)/2)*(e*Sin[c +
d*x])^(-1 + m))/(a^3*d*(2 - m)*(b + a*Cos[c + d*x])) + (Cos[c + d*x]*Hyperg
eometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1
+ m))/(a^2*d*e*(1 + m)*Sqrt[Cos[c + d*x]^2])
```

Rubi [A] time = 0.455521, antiderivative size = 405, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3872, 2912, 2643, 2703}

$$\frac{b^2e(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\left(\frac{a(\cos(c+dx)+1)}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}F_1\left(2-m;\frac{1-m}{2},\frac{1-m}{2};3-m;-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)}{a^3d(2-m)(a\cos(c+dx)+b)}-\frac{2be(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-2*b*e*AppellF1[1 - m, (1 - m)/2, (1 - m)/2, 2 - m, -((a - b)/(b + a*Cos[c
 + d*x])), (a + b)/(b + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*C
os[c + d*x])))^((1 - m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((
1 - m)/2)*(e*Sin[c + d*x])^(-1 + m))/(a^3*d*(1 - m)) + (b^2*e*AppellF1[2 -
m, (1 - m)/2, (1 - m)/2, 3 - m, -((a - b)/(b + a*Cos[c + d*x])), (a + b)/(b
 + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*Cos[c + d*x])))^((1 -
m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((1 - m)/2)*(e*Sin[c +
d*x])^(-1 + m))/(a^3*d*(2 - m)*(b + a*Cos[c + d*x])) + (Cos[c + d*x]*Hyperg
eometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])^(1
+ m))/(a^2*d*e*(1 + m)*Sqrt[Cos[c + d*x]^2])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2703

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1)*AppellF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b
*Sin[e + f*x]), (a - b)/(a + b*Sin[e + f*x])])/(b*f*(m + p)*(-((b*(1 - Sin[
e + f*x]))/(a + b*Sin[e + f*x])))^((p - 1)/2)*((b*(1 + Sin[e + f*x]))/(a +
b*Sin[e + f*x]))^((p - 1)/2)), x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^2}\,dx &= \int \frac{\cos^2(c+dx)(e\sin(c+dx))^m}{(-b-a\cos(c+dx))^2}\,dx \\
&= \int \left(\frac{(e\sin(c+dx))^m}{a^2}+\frac{b^2(e\sin(c+dx))^m}{a^2(b+a\cos(c+dx))^2}-\frac{2b(e\sin(c+dx))^m}{a^2(b+a\cos(c+dx))}\right)dx \\
&= \frac{\int (e\sin(c+dx))^m\,dx}{a^2}-\frac{(2b)\int \frac{(e\sin(c+dx))^m}{b+a\cos(c+dx)}\,dx}{a^2}+\frac{b^2\int \frac{(e\sin(c+dx))^m}{(b+a\cos(c+dx))^2}\,dx}{a^2} \\
&= -\frac{2beF_1\left(1-m;\frac{1-m}{2},\frac{1-m}{2};2-m;-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\frac{a(1+\cos(c\cdots}{b+a\cos(c\cdots}\right.}{a^3d(1-m)}
\end{aligned}$$

Mathematica [B] time = 14.9641, size = 1494, normalized size = 3.69

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(b*(-2*a^2 - a*b + b^2)*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d
*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 2*a*b^2*AppellF1[(1 + m)/
2, m, 2, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a +
b)] + (a - b)*(a + b)^2*Hypergeometric2F1[(1 + m)/2, 1 + m, (3 + m)/2, -Tan
[(c + d*x)/2]^2])*(Sec[(c + d*x)/2]^2)^m*Sin[c + d*x]^m*(e*Sin[c + d*x])^m*
Tan[(c + d*x)/2])/(a^2*(a - b)*(a + b)^2*d*(1 + m)*(a + b*Sec[c + d*x])^2*(
((b*(-2*a^2 - a*b + b^2)*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x
)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 2*a*b^2*AppellF1[(1 + m)/2,
 m, 2, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)
] + (a - b)*(a + b)^2*Hypergeometric2F1[(1 + m)/2, 1 + m, (3 + m)/2, -Tan[(
c + d*x)/2]^2])*(Sec[(c + d*x)/2]^2)^(1 + m)*Sin[c + d*x]^m)/(a^2*(a - b)*(
a + b)^2*(1 + m)) + (2*m*Cos[c + d*x]*(b*(-2*a^2 - a*b + b^2)*AppellF1[(1 +
 m)/2, m, 1, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(
a + b)] + 2*a*b^2*AppellF1[(1 + m)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)/2]^2,
 ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (a - b)*(a + b)^2*Hypergeometric2F
1[(1 + m)/2, 1 + m, (3 + m)/2, -Tan[(c + d*x)/2]^2])*(Sec[(c + d*x)/2]^2)^m
*Sin[c + d*x]^(-1 + m)*Tan[(c + d*x)/2])/(a^2*(a - b)*(a + b)^2*(1 + m)) +
(2*m*(b*(-2*a^2 - a*b + b^2)*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, -Tan[(c +
```

```
 d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 2*a*b^2*AppellF1[(1 + m
)/2, m, 2, (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a
+ b)] + (a - b)*(a + b)^2*Hypergeometric2F1[(1 + m)/2, 1 + m, (3 + m)/2, -T
an[(c + d*x)/2]^2])*(Sec[(c + d*x)/2]^2)^m*Sin[c + d*x]^m*Tan[(c + d*x)/2]^
2)/(a^2*(a - b)*(a + b)^2*(1 + m)) + (2*(Sec[(c + d*x)/2]^2)^m*Sin[c + d*x]
^m*Tan[(c + d*x)/2]*(b*(-2*a^2 - a*b + b^2)*(((a - b)*(1 + m)*AppellF1[1 +
(1 + m)/2, m, 2, 1 + (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)
/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/((a + b)*(3 + m)) - (m
*(1 + m)*AppellF1[1 + (1 + m)/2, 1 + m, 1, 1 + (3 + m)/2, -Tan[(c + d*x)/2]
^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2])/(3 + m)) + 2*a*b^2*((2*(a - b)*(1 + m)*AppellF1[1 + (1 + m)/2, m, 3, 1
+ (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec
[(c + d*x)/2]^2*Tan[(c + d*x)/2])/((a + b)*(3 + m)) - (m*(1 + m)*AppellF1[1
 + (1 + m)/2, 1 + m, 2, 1 + (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c
 + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3 + m)) + ((a
- b)*(a + b)^2*(1 + m)*Csc[(c + d*x)/2]*Sec[(c + d*x)/2]*(-Hypergeometric2F
1[(1 + m)/2, 1 + m, (3 + m)/2, -Tan[(c + d*x)/2]^2] + (1 + Tan[(c + d*x)/2]
^2)^(-1 - m)))/2))/(a^2*(a - b)*(a + b)^2*(1 + m))))
```

Maple [F] time = 0.314, size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^m}{(a+b\sec(dx+c))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^2,x)

[Out] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^m}{(b\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^2,x, algorithm="maxima")

[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e\sin(dx+c))^m}{b^2\sec(dx+c)^2+2\,ab\sec(dx+c)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^2,x, algorithm="fricas")

[Out] integral((e*sin(d*x + c))^m/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2)
, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((e*sin(c + d*x))**m/(a + b*sec(c + d*x))**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^m}{(b\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^2, x)
```

3.262 $\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=580

$$\frac{\cos(c+dx)(e\sin(c+dx))^{m+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{m+1}{2},\frac{m+3}{2},\sin^2(c+dx)\right)}{a^3de(m+1)\sqrt{\cos^2(c+dx)}}+\frac{3b^2e(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right.}{}$$

```
[Out] (-3*b*e*AppellF1[1 - m, (1 - m)/2, (1 - m)/2, 2 - m, -((a - b)/(b + a*Cos[c
 + d*x])), (a + b)/(b + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*C
os[c + d*x])))^((1 - m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((
1 - m)/2)*(e*Sin[c + d*x])^(-1 + m))/(a^4*d*(1 - m)) - (b^3*e*AppellF1[3 -
m, (1 - m)/2, (1 - m)/2, 4 - m, -((a - b)/(b + a*Cos[c + d*x])), (a + b)/(b
 + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*Cos[c + d*x])))^((1 -
m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((1 - m)/2)*(e*Sin[c +
d*x])^(-1 + m))/(a^4*d*(3 - m)*(b + a*Cos[c + d*x])^2) + (3*b^2*e*AppellF1[
2 - m, (1 - m)/2, (1 - m)/2, 3 - m, -((a - b)/(b + a*Cos[c + d*x])), (a + b
)/(b + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*Cos[c + d*x])))^((
1 - m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((1 - m)/2)*(e*Sin[
c + d*x])^(-1 + m))/(a^4*d*(2 - m)*(b + a*Cos[c + d*x])) + (Cos[c + d*x]*Hy
pergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])
^(1 + m))/(a^3*d*e*(1 + m)*Sqrt[Cos[c + d*x]^2])
```

Rubi [A] time = 0.591756, antiderivative size = 580, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3872, 2912, 2643, 2703}

$$\frac{3b^2e(e\sin(c+dx))^{m-1}\left(-\frac{a(1-\cos(c+dx))}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}\left(\frac{a(\cos(c+dx)+1)}{a\cos(c+dx)+b}\right)^{\frac{1-m}{2}}F_1\left(2-m;\frac{1-m}{2},\frac{1-m}{2};3-m;-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)}{a^4d(2-m)(a\cos(c+dx)+b)}-\frac{b^3e}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-3*b*e*AppellF1[1 - m, (1 - m)/2, (1 - m)/2, 2 - m, -((a - b)/(b + a*Cos[c
 + d*x])), (a + b)/(b + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*C
os[c + d*x])))^((1 - m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((
1 - m)/2)*(e*Sin[c + d*x])^(-1 + m))/(a^4*d*(1 - m)) - (b^3*e*AppellF1[3 -
m, (1 - m)/2, (1 - m)/2, 4 - m, -((a - b)/(b + a*Cos[c + d*x])), (a + b)/(b
 + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*Cos[c + d*x])))^((1 -
m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((1 - m)/2)*(e*Sin[c +
d*x])^(-1 + m))/(a^4*d*(3 - m)*(b + a*Cos[c + d*x])^2) + (3*b^2*e*AppellF1[
2 - m, (1 - m)/2, (1 - m)/2, 3 - m, -((a - b)/(b + a*Cos[c + d*x])), (a + b
)/(b + a*Cos[c + d*x])]*(-((a*(1 - Cos[c + d*x]))/(b + a*Cos[c + d*x])))^((
1 - m)/2)*((a*(1 + Cos[c + d*x]))/(b + a*Cos[c + d*x]))^((1 - m)/2)*(e*Sin[
c + d*x])^(-1 + m))/(a^4*d*(2 - m)*(b + a*Cos[c + d*x])) + (Cos[c + d*x]*Hy
pergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sin[c + d*x]^2]*(e*Sin[c + d*x])
^(1 + m))/(a^3*d*e*(1 + m)*Sqrt[Cos[c + d*x]^2])
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2912

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[m] && (G
tQ[m, 0] || IntegerQ[n])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 2703

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*(g*Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x
])^(m + 1)*AppellF1[-p - m, (1 - p)/2, (1 - p)/2, 1 - p - m, (a + b)/(a + b
*Sin[e + f*x]), (a - b)/(a + b*Sin[e + f*x])])/(b*f*(m + p)*(-((b*(1 - Sin[
e + f*x]))/(a + b*Sin[e + f*x])))^((p - 1)/2)*((b*(1 + Sin[e + f*x]))/(a +
b*Sin[e + f*x]))^((p - 1)/2)), x] /; FreeQ[{a, b, e, f, g, p}, x] && NeQ[a^
2 - b^2, 0] && ILtQ[m, 0] &&  !IGtQ[m + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^3}\,dx &= -\int \frac{\cos^3(c+dx)(e\sin(c+dx))^m}{(-b-a\cos(c+dx))^3}\,dx\\ &= -\int\left(-\frac{(e\sin(c+dx))^m}{a^3}+\frac{b^3(e\sin(c+dx))^m}{a^3(b+a\cos(c+dx))^3}-\frac{3b^2(e\sin(c+dx))^m}{a^3(b+a\cos(c+dx))^2}+\frac{3b(e\sin(c\right.\\ &= \frac{\int(e\sin(c+dx))^m\,dx}{a^3}-\frac{(3b)\int\frac{(e\sin(c+dx))^m}{b+a\cos(c+dx)}\,dx}{a^3}+\frac{\left(3b^2\right)\int\frac{(e\sin(c+dx))^m}{(b+a\cos(c+dx))^2}\,dx}{a^3}-\frac{b^3\int\frac{(e\sin(c+d}{(b+a\cos(c}}{a^3}\\ &= -\frac{3beF_1\left(1-m;\frac{1-m}{2},\frac{1-m}{2};2-m;-\frac{a-b}{b+a\cos(c+dx)},\frac{a+b}{b+a\cos(c+dx)}\right)\left(-\frac{a(1-\cos(c+dx))}{b+a\cos(c+dx)}\right)^{\frac{1-m}{2}}\left(\frac{a(1+\cos(c}{b+a\cos(c}\right.}{a^4d(1-m)}\end{aligned}$$

Mathematica [B] time = 19.103, size = 2904, normalized size = 5.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (-2*(Sec[(c + d*x)/2]^2)^m*Sin[c + d*x]^m*(e*Sin[c + d*x])^m*Tan[(c + d*x)/
2]*(-((105 + 71*m + 15*m^2 + m^3)*AppellF1[(1 + m)/2, 1 + m, 3, (3 + m)/2,
-Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + (1 + m)*Tan[(
c + d*x)/2]^2*(3*(35 + 12*m + m^2)*AppellF1[(3 + m)/2, 1 + m, 3, (5 + m)/2,
 -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (3 + m)*Tan[(
c + d*x)/2]^2*(-3*(7 + m)*AppellF1[(5 + m)/2, 1 + m, 3, (7 + m)/2, -Tan[(c
+ d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (5 + m)*AppellF1[(7 +
m)/2, 1 + m, 3, (9 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2
)/(a + b)]*Tan[(c + d*x)/2]^2))))/((a + b)^3*d*(1 + m)*(3 + m)*(5 + m)*(7 +
 m)*(a + b*Sec[c + d*x])^3*(-(((Sec[(c + d*x)/2]^2)^(1 + m)*Sin[c + d*x]^m*
```

```
(-((105 + 71*m + 15*m^2 + m^3)*AppellF1[(1 + m)/2, 1 + m, 3, (3 + m)/2, -Ta
n[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + (1 + m)*Tan[(c +
 d*x)/2]^2*(3*(35 + 12*m + m^2)*AppellF1[(3 + m)/2, 1 + m, 3, (5 + m)/2, -T
an[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (3 + m)*Tan[(c +
 d*x)/2]^2*(-3*(7 + m)*AppellF1[(5 + m)/2, 1 + m, 3, (7 + m)/2, -Tan[(c + d
*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (5 + m)*AppellF1[(7 + m)/
2, 1 + m, 3, (9 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(
a + b)]*Tan[(c + d*x)/2]^2))))/((a + b)^3*(1 + m)*(3 + m)*(5 + m)*(7 + m)))
 - (2*m*Cos[c + d*x]*(Sec[(c + d*x)/2]^2)^m*Sin[c + d*x]^(-1 + m)*Tan[(c +
d*x)/2]*(-((105 + 71*m + 15*m^2 + m^3)*AppellF1[(1 + m)/2, 1 + m, 3, (3 + m
)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + (1 + m)*
Tan[(c + d*x)/2]^2*(3*(35 + 12*m + m^2)*AppellF1[(3 + m)/2, 1 + m, 3, (5 +
m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (3 + m)*
Tan[(c + d*x)/2]^2*(-3*(7 + m)*AppellF1[(5 + m)/2, 1 + m, 3, (7 + m)/2, -Ta
n[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (5 + m)*AppellF1[
(7 + m)/2, 1 + m, 3, (9 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)
/2]^2)/(a + b)]*Tan[(c + d*x)/2]^2))))/((a + b)^3*(1 + m)*(3 + m)*(5 + m)*(
7 + m)) - (2*m*(Sec[(c + d*x)/2]^2)^m*Sin[c + d*x]^m*Tan[(c + d*x)/2]^2*(-(
(105 + 71*m + 15*m^2 + m^3)*AppellF1[(1 + m)/2, 1 + m, 3, (3 + m)/2, -Tan[(
c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + (1 + m)*Tan[(c + d*
x)/2]^2*(3*(35 + 12*m + m^2)*AppellF1[(3 + m)/2, 1 + m, 3, (5 + m)/2, -Tan[
(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (3 + m)*Tan[(c + d*
x)/2]^2*(-3*(7 + m)*AppellF1[(5 + m)/2, 1 + m, 3, (7 + m)/2, -Tan[(c + d*x)
/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (5 + m)*AppellF1[(7 + m)/2,
1 + m, 3, (9 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a +
 b)]*Tan[(c + d*x)/2]^2))))/((a + b)^3*(1 + m)*(3 + m)*(5 + m)*(7 + m)) - (
2*(Sec[(c + d*x)/2]^2)^m*Sin[c + d*x]^m*Tan[(c + d*x)/2]*(-((105 + 71*m + 1
5*m^2 + m^3)*((3*(a - b)*(1 + m)*AppellF1[1 + (1 + m)/2, 1 + m, 4, 1 + (3 +
 m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2])/((a + b)*(3 + m)) - ((1 + m)^2*AppellF1[1 + (1
+ m)/2, 2 + m, 3, 1 + (3 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x
)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3 + m))) + (1 + m)*S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2]*(3*(35 + 12*m + m^2)*AppellF1[(3 + m)/2,
 1 + m, 3, (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a
+ b)] + (3 + m)*Tan[(c + d*x)/2]^2*(-3*(7 + m)*AppellF1[(5 + m)/2, 1 + m, 3
, (7 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (
5 + m)*AppellF1[(7 + m)/2, 1 + m, 3, (9 + m)/2, -Tan[(c + d*x)/2]^2, ((a -
b)*Tan[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2]^2)) + (1 + m)*Tan[(c + d*x
)/2]^2*(3*(35 + 12*m + m^2)*((3*(a - b)*(3 + m)*AppellF1[1 + (3 + m)/2, 1 +
 m, 4, 1 + (5 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a
+ b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/((a + b)*(5 + m)) - ((1 + m)*(3
+ m)*AppellF1[1 + (3 + m)/2, 2 + m, 3, 1 + (5 + m)/2, -Tan[(c + d*x)/2]^2,
((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/
(5 + m)) + (3 + m)*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]*(-3*(7 + m)*AppellF1
[(5 + m)/2, 1 + m, 3, (7 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x
)/2]^2)/(a + b)] + (5 + m)*AppellF1[(7 + m)/2, 1 + m, 3, (9 + m)/2, -Tan[(c
 + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2]^2) + (
3 + m)*Tan[(c + d*x)/2]^2*((5 + m)*AppellF1[(7 + m)/2, 1 + m, 3, (9 + m)/2,
 -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2
]^2*Tan[(c + d*x)/2] - 3*(7 + m)*((3*(a - b)*(5 + m)*AppellF1[1 + (5 + m)/2
, 1 + m, 4, 1 + (7 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2
)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/((a + b)*(7 + m)) - ((1 + m
)*(5 + m)*AppellF1[1 + (5 + m)/2, 2 + m, 3, 1 + (7 + m)/2, -Tan[(c + d*x)/2
]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2])/(7 + m)) + (5 + m)*Tan[(c + d*x)/2]^2*((3*(a - b)*(7 + m)*AppellF1[1 +
 (7 + m)/2, 1 + m, 4, 1 + (9 + m)/2, -Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c +
 d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/((a + b)*(9 + m))
 - ((1 + m)*(7 + m)*AppellF1[1 + (7 + m)/2, 2 + m, 3, 1 + (9 + m)/2, -Tan[(
c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan
```

```
[(c + d*x)/2])/(9 + m))))))/((a + b)^3*(1 + m)*(3 + m)*(5 + m)*(7 + m))))
```

Maple [F] time = 0.395, size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^m}{(a+b\sec(dx+c))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^3,x)
```

```
[Out] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^3,x)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e\sin(dx+c))^m}{b^3\sec(dx+c)^3+3\,ab^2\sec(dx+c)^2+3\,a^2b\sec(dx+c)+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] integral((e*sin(d*x + c))^m/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 +
3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\sin(dx+c))^m}{(b\sec(dx+c)+a)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^3, x)
```

3.263 $\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left((a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m,x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m, x]
```

Rubi [A] time = 0.0676679, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m,x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx=\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx$$

Mathematica [A] time = 7.60683, size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^{3/2}(e\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m,x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^(3/2)*(e*Sin[c + d*x])^m, x]
```

Maple [A] time = 0.205, size = 0, normalized size = 0.

$$\int (a+b\sec(dx+c))^{\frac{3}{2}}(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c)+a)^{\frac{3}{2}}(e\sin(dx+c))^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(e*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^{\frac{3}{2}}(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(e*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^(3/2)*(e*sin(d*x + c))^m, x)
```

3.264 $\int \sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m,x\right)$$

```
[Out] Unintegrable[Sqrt[a + b*Sec[c + d*x]]*(e*Sin[c + d*x])^m, x]
```

Rubi [A] time = 0.0620886, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*(e*Sin[c + d*x])^m,x]
```

```
[Out] Defer[Int][Sqrt[a + b*Sec[c + d*x]]*(e*Sin[c + d*x])^m, x]
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m\,dx=\int \sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m\,dx$$

Mathematica [A] time = 0.513436, size = 0, normalized size = 0.

$$\int \sqrt{a+b\sec(c+dx)}(e\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*(e*Sin[c + d*x])^m,x]
```

```
[Out] Integrate[Sqrt[a + b*Sec[c + d*x]]*(e*Sin[c + d*x])^m, x]
```

Maple [A] time = 0.217, size = 0, normalized size = 0.

$$\int (e\sin(dx+c))^m\sqrt{a+b\sec(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)+a}\,(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sec(dx+c)+a}\,(e\sin(dx+c))^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sec(dx+c)+a}\,(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sec(d*x + c) + a)*(e*sin(d*x + c))^m, x)
```

3.265 $\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}},x\right)$$

```
[Out] Unintegrable[(e*Sin[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi [A] time = 0.0696136, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*Sin[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Defer[Int][(e*Sin[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]], x]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx = \int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Mathematica [A] time = 2.56369, size = 0, normalized size = 0.

$$\int \frac{(e\sin(c+dx))^m}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Sin[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] Integrate[(e*Sin[c + d*x])^m/Sqrt[a + b*Sec[c + d*x]], x]
```

Maple [A] time = 0.211, size = 0, normalized size = 0.

$$\int (e\sin(dx+c))^m \frac{1}{\sqrt{a+b\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
```

```
[Out] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \sin (dx + c))^m}{\sqrt{b \sec (dx + c) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(e \sin (dx + c))^m}{\sqrt{b \sec (dx + c) + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((e*sin(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \sin (c + dx))^m}{\sqrt{a + b \sec (c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((e*sin(c + d*x))**m/sqrt(a + b*sec(c + d*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \sin (dx + c))^m}{\sqrt{b \sec (dx + c) + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^m/sqrt(b*sec(d*x + c) + a), x)
```

3.266 $\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=27

$$\text{Unintegrable}\left(\frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}},x\right)$$

```
[Out] Unintegrable[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi [A] time = 0.0709144, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Defer[Int][(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2), x]
```

Rubi steps

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx = \int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Mathematica [A] time = 2.80099, size = 0, normalized size = 0.

$$\int \frac{(e\sin(c+dx))^m}{(a+b\sec(c+dx))^{3/2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] Integrate[(e*Sin[c + d*x])^m/(a + b*Sec[c + d*x])^(3/2), x]
```

Maple [A] time = 0.202, size = 0, normalized size = 0.

$$\int (e\sin(dx+c))^m (a+b\sec(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
```

```
[Out] int((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \sin (dx + c))^m}{(b \sec (dx + c) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b \sec (dx + c) + a}\, (e \sin (dx + c))^m}{b^2 \sec (dx + c)^2 + 2\, ab \sec (dx + c) + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sec(d*x + c) + a)*(e*sin(d*x + c))^m/(b^2*sec(d*x + c)^2 +
2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))**m/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e \sin (dx + c))^m}{(b \sec (dx + c) + a)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*sin(d*x+c))^m/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((e*sin(d*x + c))^m/(b*sec(d*x + c) + a)^(3/2), x)
```

3.267 $\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left((e\sin(c+dx))^m(a+b\sec(c+dx))^n,x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^n*(e*Sin[c + d*x])^m, x]
```

Rubi [A] time = 0.0439409, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n*(e*Sin[c + d*x])^m,x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*(e*Sin[c + d*x])^m, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx=\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx$$

Mathematica [A] time = 3.19461, size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^n(e\sin(c+dx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*(e*Sin[c + d*x])^m,x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^n*(e*Sin[c + d*x])^m, x]
```

Maple [A] time = 0.72, size = 0, normalized size = 0.

$$\int (a+b\sec(dx+c))^n\,(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x)
```

```
[Out] int((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n\,(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c)+a)^n\,(e\sin(dx+c))^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*(e*sin(d*x+c))**m,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n\,(e\sin(dx+c))^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*(e*sin(d*x+c))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*(e*sin(d*x + c))^m, x)
```

3.268 $\int (a+b\sec(c+dx))^n \sin^5(c+dx)\,dx$

Optimal. Leaf size=150

$$-\frac{2b^3(a+b\sec(c+dx))^{n+1}\text{Hypergeometric2F1}\left(4,n+1,n+2,\frac{b\sec(c+dx)}{a}+1\right)}{a^4d(n+1)}+\frac{b^5(a+b\sec(c+dx))^{n+1}\text{Hyperge}}{}$$

```
[Out] (b*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c
+ d*x])^(1 + n))/(a^2*d*(1 + n)) - (2*b^3*Hypergeometric2F1[4, 1 + n, 2 + n
, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c + d*x])^(1 + n))/(a^4*d*(1 + n)) + (
b^5*Hypergeometric2F1[6, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c
 + d*x])^(1 + n))/(a^6*d*(1 + n))
```

Rubi [A] time = 0.125513, antiderivative size = 150, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3874, 180, 65}

$$-\frac{2b^3(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(4,n+1;n+2;\frac{b\sec(c+dx)}{a}+1\right)}{a^4d(n+1)}+\frac{b^5(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(6,n+1;n+2;\frac{b\sec(c+dx)}{a}\right.}{a^6d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^5,x]
```

```
[Out] (b*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c
+ d*x])^(1 + n))/(a^2*d*(1 + n)) - (2*b^3*Hypergeometric2F1[4, 1 + n, 2 + n
, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c + d*x])^(1 + n))/(a^4*d*(1 + n)) + (
b^5*Hypergeometric2F1[6, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c
 + d*x])^(1 + n))/(a^6*d*(1 + n))
```

Rule 3874

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> -Dist[f^(-1), Subst[Int[((-1 + x)^((p - 1)/2)*(1 + x)^((p
- 1)/2)*(a + b*x)^m)/x^(p + 1), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, e,
 f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (a+b\sec(c+dx))^n \sin^5(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(-1+x)^2(1+x)^2(a-bx)^n}{x^6}\,dx, x, -\sec(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{(a-bx)^n}{x^6} - \frac{2(a-bx)^n}{x^4} + \frac{(a-bx)^n}{x^2}\right)dx, x, -\sec(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{(a-bx)^n}{x^6}\,dx, x, -\sec(c+dx)\right)}{d} - \frac{\text{Subst}\left(\int \frac{(a-bx)^n}{x^2}\,dx, x, -\sec(c+dx)\right)}{d}$$
$$= \frac{b\,{}_2F_1\left(2, 1+n; 2+n; 1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{a^2 d(1+n)} - \frac{2b^3\,{}_2F_1\left(4, 1+n; 2\right.}{}$$

Mathematica [B] time = 8.16185, size = 562, normalized size = 3.75

$$-\frac{\cos^6\left(\frac{1}{2}(c+dx)\right)\cos(c+dx)(a+b\sec(c+dx))^n\left(-10a\sec^6\left(\frac{1}{2}(c+dx)\right)\left(b\left(12a^2b(n-1)+24a^3-4ab^2\left(n^2-3n+2\right)-\right.\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^5,x]
```

```
[Out] -(Cos[(c + d*x)/2]^6*Cos[c + d*x]*(192*a^3*(-1 + n)*(b + a*Cos[c + d*x])^2
 - 240*a^3*(-1 + n)*(b + a*Cos[c + d*x])^2*Sec[(c + d*x)/2]^2 - 24*a^2*(2*a
 - b*(-4 + n))*(-1 + n)*(b + a*Cos[c + d*x])^2*Sec[(c + d*x)/2]^2 + 40*a^2*(
2*a - b*(-3 + n))*(-1 + n)*(b + a*Cos[c + d*x])^2*Sec[(c + d*x)/2]^4 + a*(1
 - n)*(96*a^2 + 4*a*b*(6 - 4*n) - 4*b^2*(12 - 7*n + n^2))*(b + a*Cos[c + d*
x])^2*Sec[(c + d*x)/2]^4 - 10*a*((-1 + n)*(-14*a^2 + 2*a*b*(-1 + n) + b^2*(
6 - 5*n + n^2))*(b + a*Cos[c + d*x])^2 + b*(24*a^3 + 12*a^2*b*(-1 + n) - 4*
a*b^2*(2 - 3*n + n^2) - b^3*(-6 + 11*n - 6*n^2 + n^3))*Hypergeometric2F1[2,
 1 - n, 2 - n, (a*Cos[c + d*x])/(b + a*Cos[c + d*x])])*Sec[(c + d*x)/2]^6 +
 ((-1 + n)*(-84*a^3 + 2*a^2*b*(18 - 7*n) + 4*a*b^2*(9 - 9*n + 2*n^2) + b^3*
(-24 + 26*n - 9*n^2 + n^3))*(b + a*Cos[c + d*x])^2 + b*(120*a^4 + 120*a^3*b
*(-1 + n) - 10*a*b^3*(-6 + 11*n - 6*n^2 + n^3) - b^4*(24 - 50*n + 35*n^2 -
10*n^3 + n^4))*Hypergeometric2F1[2, 1 - n, 2 - n, (a*Cos[c + d*x])/(b + a*C
os[c + d*x])])*Sec[(c + d*x)/2]^6)*(a + b*Sec[c + d*x])^n)/(120*a^4*d*(-1 +
 n)*(b + a*Cos[c + d*x]))
```

Maple [F] time = 0.753, size = 0, normalized size = 0.

$$\int (a+b\sec(dx+c))^n (\sin(dx+c))^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c)^5,x)
```

```
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c)^5,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n \sin(dx+c)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1\right)\left(b\sec\left(dx+c\right)+a\right)^{n}\sin\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="fricas")
```

```
[Out] integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(b*sec(d*x + c) + a)^n*sin
(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(dx+c\right)+a\right)^{n}\sin\left(dx+c\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^5,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^5, x)
```

3.269 $\int (a+b\sec(c+dx))^n \sin^3(c+dx)\,dx$

Optimal. Leaf size=121

$$\frac{b\left(6a^2-b^2\left(n^2-3n+2\right)\right)(a+b\sec(c+dx))^{n+1}\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{b\sec(c+dx)}{a}+1\right)}{6a^4d(n+1)}+\frac{\cos^3(c+dx)(2a$$

```
[Out] (b*(6*a^2 - b^2*(2 - 3*n + n^2))*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*
Sec[c + d*x])/a]*(a + b*Sec[c + d*x])^(1 + n))/(6*a^4*d*(1 + n)) + (Cos[c +
 d*x]^3*(a + b*Sec[c + d*x])^(1 + n)*(2*a - b*(2 - n)*Sec[c + d*x]))/(6*a^2
*d)
```

Rubi [A] time = 0.10479, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3874, 145, 65}

$$\frac{b\left(6a^2-b^2\left(n^2-3n+2\right)\right)(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{b\sec(c+dx)}{a}+1\right)}{6a^4d(n+1)}+\frac{\cos^3(c+dx)(2a-b(2-n)\sec(c+}{6a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^3,x]
```

```
[Out] (b*(6*a^2 - b^2*(2 - 3*n + n^2))*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*
Sec[c + d*x])/a]*(a + b*Sec[c + d*x])^(1 + n))/(6*a^4*d*(1 + n)) + (Cos[c +
 d*x]^3*(a + b*Sec[c + d*x])^(1 + n)*(2*a - b*(2 - n)*Sec[c + d*x]))/(6*a^2
*d)
```

Rule 3874

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> -Dist[f^(-1), Subst[Int[((-1 + x)^((p - 1)/2)*(1 + x)^((p
- 1)/2)*(a + b*x)^m)/x^(p + 1), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, e,
 f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 145

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.)*((e_) + (f_.)*(x_)
)*((g_.) + (h_.)*(x_)), x_Symbol] :> Simp[((b^3*c*e*g*(m + 2) - a^3*d*f*h*(
n + 2) - a^2*b*(c*f*h*m - d*(f*g + e*h)*(m + n + 3)) - a*b^2*(c*(f*g + e*h)
 + d*e*g*(2*m + n + 4)) + b*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1) - d*(
f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)))*x)*(
a + b*x)^(m + 1)*(c + d*x)^(n + 1))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), x]
 + Dist[(f*h)/b^2 - (d*(m + n + 3)*(a^2*d*f*h*(m - n) - a*b*(2*c*f*h*(m + 1
) - d*(f*g + e*h)*(n + 1)) + b^2*(c*(f*g + e*h)*(m + 1) - d*e*g*(m + n + 2)
)))/(b^2*(b*c - a*d)^2*(m + 1)*(m + 2)), Int[(a + b*x)^(m + 2)*(c + d*x)^n,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, h, m, n}, x] && (LtQ[m, -2] || (EqQ[
m + n + 3, 0] &&  !LtQ[n, -2]))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int (a+b\sec(c+dx))^n \sin^3(c+dx)\,dx = -\frac{\text{Subst}\left(\int \frac{(-1+x)(1+x)(a-bx)^n}{x^4}\,dx, x, -\sec(c+dx)\right)}{d}$$
$$= \frac{\cos^3(c+dx)(a+b\sec(c+dx))^{1+n}(2a-b(2-n)\sec(c+dx))}{6a^2d} - \frac{\left(6-\frac{b^2(1-n)(2}{a^2}\right.}{}$$
$$= \frac{b\left(6a^2-b^2\left(2-3n+n^2\right)\right)\,{}_2F_1\left(2,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+d}{6a^4d(1+n)}$$

Mathematica [A] time = 1.70088, size = 155, normalized size = 1.28

$$\frac{\cos(c+dx)(a+b\sec(c+dx))^n\left(-\frac{2b\left(b^2\left(n^2-3n+2\right)-6a^2\right)\text{Hypergeometric2F1}\left(2,1-n,2-n,\frac{a\cos(c+dx)}{a\cos(c+dx)+b}\right)}{a(n-1)}-\frac{2(2a-b(n-2))(a\cos(c+dx)+b)^2}{a}+8\right.}{12ad(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^3,x]
```

```
[Out] (Cos[c + d*x]*((-2*(2*a - b*(-2 + n))*(b + a*Cos[c + d*x])^2)/a + 8*Cos[(c
+ d*x)/2]^2*(b + a*Cos[c + d*x])^2 - (2*b*(-6*a^2 + b^2*(2 - 3*n + n^2))*Hy
pergeometric2F1[2, 1 - n, 2 - n, (a*Cos[c + d*x])/(b + a*Cos[c + d*x])])/(a
*(-1 + n)))*(a + b*Sec[c + d*x])^n)/(12*a*d*(b + a*Cos[c + d*x]))
```

Maple [F] time = 0.635, size = 0, normalized size = 0.

$$\int (a+b\sec(dx+c))^n (\sin(dx+c))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c)^3,x)
```

```
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n \sin(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos(dx+c)^2-1\right)(b\sec(dx+c)+a)^n\sin(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(b*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n \sin(dx+c)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^3, x)
```

3.270 $\int (a+b\sec(c+dx))^n\sin(c+dx)\,dx$

Optimal. Leaf size=48

$$\frac{b(a+b\sec(c+dx))^{n+1}\text{Hypergeometric2F1}\left(2,n+1,n+2,\frac{b\sec(c+dx)}{a}+1\right)}{a^2d(n+1)}$$

```
[Out] (b*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c
+ d*x])^(1 + n))/(a^2*d*(1 + n))
```

Rubi [A] time = 0.0394676, antiderivative size = 48, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3874, 65}

$$\frac{b(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(2,n+1;n+2;\frac{b\sec(c+dx)}{a}+1\right)}{a^2d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x],x]
```

```
[Out] (b*Hypergeometric2F1[2, 1 + n, 2 + n, 1 + (b*Sec[c + d*x])/a]*(a + b*Sec[c
+ d*x])^(1 + n))/(a^2*d*(1 + n))
```

Rule 3874

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> -Dist[f^(-1), Subst[Int[((-1 + x)^((p - 1)/2)*(1 + x)^((p
- 1)/2)*(a + b*x)^m)/x^(p + 1), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, e,
 f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int (a+b\sec(c+dx))^n\sin(c+dx)\,dx &= -\frac{\text{Subst}\left(\int\frac{(a-bx)^n}{x^2}\,dx,x,-\sec(c+dx)\right)}{d}\\ &= \frac{b\,{}_2F_1\left(2,1+n;2+n;1+\frac{b\sec(c+dx)}{a}\right)(a+b\sec(c+dx))^{1+n}}{a^2d(1+n)}\end{aligned}$$

Mathematica [A] time = 0.48009, size = 72, normalized size = 1.5

$$\frac{b\cos(c+dx)(a+b\sec(c+dx))^n\text{Hypergeometric2F1}\left(2,1-n,2-n,\frac{a\cos(c+dx)}{a\cos(c+dx)+b}\right)}{d(n-1)(a\cos(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x],x]
```

```
[Out] (b*Cos[c + d*x]*Hypergeometric2F1[2, 1 - n, 2 - n, (a*Cos[c + d*x])/(b + a*
Cos[c + d*x])]*(a + b*Sec[c + d*x])^n)/(d*(-1 + n)*(b + a*Cos[c + d*x]))
```

Maple [F] time = 0.319, size = 0, normalized size = 0.

$$\int (a+b\sec(dx+c))^n \sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c),x)
```

```
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n \sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\sec(dx+c)+a)^n \sin(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^n \sin(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n*sin(c + d*x), x)
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n \sin(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c), x)
```

3.271 $\int \csc(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal. Leaf size=115

$$\frac{(a+b\sec(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\sec(c+dx)}{a-b}\right)}{2d(n+1)(a-b)}-\frac{(a+b\sec(c+dx))^{n+1}\text{Hypergeometric2F1}}{2d(n+1)(a+b)}$$

```
[Out] (Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*(a + b*Se
c[c + d*x])^(1 + n))/(2*(a - b)*d*(1 + n)) - (Hypergeometric2F1[1, 1 + n, 2
 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(2*(a + b
)*d*(1 + n))
```

Rubi [A] time = 0.118902, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3874, 73, 712, 68}

$$\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a-b}\right)}{2d(n+1)(a-b)}-\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a+b}\right)}{2d(n+1)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*(a + b*Se
c[c + d*x])^(1 + n))/(2*(a - b)*d*(1 + n)) - (Hypergeometric2F1[1, 1 + n, 2
 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(2*(a + b
)*d*(1 + n))
```

Rule 3874

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> -Dist[f^(-1), Subst[Int[((-1 + x)^((p - 1)/2)*(1 + x)^((p
- 1)/2)*(a + b*x)^m)/x^(p + 1), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, e,
 f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 73

```
Int[((a_) + (b_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_)
)^(p_.), x_Symbol] :> Int[(a*c + b*d*x^2)^m*(e + f*x)^p, x] /; FreeQ[{a, b,
 c, d, e, f, m, n, p}, x] && EqQ[b*c + a*d, 0] && EqQ[n, m] && IntegerQ[m]
```

Rule 712

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \csc(c+dx)(a+b\sec(c+dx))^n\,dx = -\frac{\text{Subst}\left(\int \frac{(a-bx)^n}{(-1+x)(1+x)}\,dx, x, -\sec(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{(a-bx)^n}{-1+x^2}\,dx, x, -\sec(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{(a-bx)^n}{2(1-x)} - \frac{(a-bx)^n}{2(1+x)}\right)dx, x, -\sec(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a-bx)^n}{1-x}\,dx, x, -\sec(c+dx)\right)}{2d} + \frac{\text{Subst}\left(\int \frac{(a-bx)^n}{1+x}\,dx, x, -\sec(c+dx)\right)}{2d}$$
$$= \frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{2(a-b)d(1+n)} - \frac{{}_2F_1\left(1,1+n;2+n;\right.}{2}$$

Mathematica [A] time = 0.945822, size = 132, normalized size = 1.15

$$\frac{(a+b\sec(c+dx))^n\left(\text{Hypergeometric2F1}\left(1,-n,1-n,\frac{(a+b)\cos(c+dx)}{a\cos(c+dx)+b}\right) - 2^n\left(\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)(a\cos(c+dx)+b)}{b}\right)^{-n}\text{Hypergeon}\right.}{2dn}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] ((Hypergeometric2F1[1, -n, 1 - n, ((a + b)*Cos[c + d*x])/(b + a*Cos[c + d*x
])] - (2^n*Hypergeometric2F1[-n, -n, 1 - n, ((-a + b)*Cos[c + d*x]*Sec[(c +
 d*x)/2]^2)/(2*b)])/(((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/b)^n)*(a + b
*Sec[c + d*x])^n)/(2*d*n)
```

Maple [F] time = 0.241, size = 0, normalized size = 0.

$$\int \csc(dx+c)(a+b\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*sec(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)*(a+b*sec(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c)+a)^n\csc(dx+c),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n\csc(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c), x)
```

3.272 $\int \csc^3(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal. Leaf size=231

$$\frac{(a+b\sec(c+dx))^{n+1}\text{Hypergeometric2F1}\left(1,n+1,n+2,\frac{a+b\sec(c+dx)}{a-b}\right)}{4d(n+1)(a-b)}-\frac{(a+b\sec(c+dx))^{n+1}\text{Hypergeometric2}}{4d(n+1)(a-}$$

```
[Out] (Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*(a + b*Se
c[c + d*x])^(1 + n))/(4*(a - b)*d*(1 + n)) - (Hypergeometric2F1[1, 1 + n, 2
 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(4*(a + b
)*d*(1 + n)) + (b*Hypergeometric2F1[2, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(
a - b)]*(a + b*Sec[c + d*x])^(1 + n))/(4*(a - b)^2*d*(1 + n)) + (b*Hypergeo
metric2F1[2, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x
])^(1 + n))/(4*(a + b)^2*d*(1 + n))
```

Rubi [A] time = 0.1947, antiderivative size = 231, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3874, 180, 68, 712}

$$\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a-b}\right)}{4d(n+1)(a-b)}-\frac{(a+b\sec(c+dx))^{n+1}\,{}_2F_1\left(1,n+1;n+2;\frac{a+b\sec(c+dx)}{a+b}\right)}{4d(n+1)(a+b)}+\frac{b(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a - b)]*(a + b*Se
c[c + d*x])^(1 + n))/(4*(a - b)*d*(1 + n)) - (Hypergeometric2F1[1, 1 + n, 2
 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x])^(1 + n))/(4*(a + b
)*d*(1 + n)) + (b*Hypergeometric2F1[2, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(
a - b)]*(a + b*Sec[c + d*x])^(1 + n))/(4*(a - b)^2*d*(1 + n)) + (b*Hypergeo
metric2F1[2, 1 + n, 2 + n, (a + b*Sec[c + d*x])/(a + b)]*(a + b*Sec[c + d*x
])^(1 + n))/(4*(a + b)^2*d*(1 + n))
```

Rule 3874

```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m
_), x_Symbol] :> -Dist[f^(-1), Subst[Int[((-1 + x)^((p - 1)/2)*(1 + x)^((p
- 1)/2)*(a + b*x)^m)/x^(p + 1), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, e,
 f, m}, x] && IntegerQ[(p - 1)/2] && NeQ[a^2 - b^2, 0]
```

Rule 180

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_))^(q_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x
)^m*(c + d*x)^n*(e + f*x)^p*(g + h*x)^q, x], x] /; FreeQ[{a, b, c, d, e, f,
 g, h, m, n}, x] && IntegersQ[p, q]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 712

```
Int[((d_) + (e_.)*(x_))^(m_)/((a_) + (c_.)*(x_)^2), x_Symbol] :> Int[Expand
Integrand[(d + e*x)^m, 1/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e, m}, x] &
& NeQ[c*d^2 + a*e^2, 0] &&  !IntegerQ[m]
```

Rubi steps

$$\int \csc^3(c+dx)(a+b\sec(c+dx))^n\,dx = -\frac{\text{Subst}\left(\int \frac{x^2(a-bx)^n}{(-1+x)^2(1+x)^2}\,dx, x, -\sec(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{(a-bx)^n}{4(-1+x)^2}+\frac{(a-bx)^n}{4(1+x)^2}+\frac{(a-bx)^n}{2(-1+x^2)}\right)dx, x, -\sec(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{(a-bx)^n}{(-1+x)^2}\,dx, x, -\sec(c+dx)\right)}{4d} - \frac{\text{Subst}\left(\int \frac{(a-bx)^n}{(1+x)^2}\,dx, x, -\sec(c+dx)\right)}{4d}$$

$$= \frac{b\,{}_2F_1\left(2,1+n;2+n;\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{4(a-b)^2d(1+n)} + \frac{b\,{}_2F_1\left(2,1+n;2+n\right.}{4}$$

$$= \frac{b\,{}_2F_1\left(2,1+n;2+n;\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{4(a-b)^2d(1+n)} + \frac{b\,{}_2F_1\left(2,1+n;2+n\right.}{4}$$

$$= \frac{{}_2F_1\left(1,1+n;2+n;\frac{a+b\sec(c+dx)}{a-b}\right)(a+b\sec(c+dx))^{1+n}}{4(a-b)d(1+n)} - \frac{{}_2F_1\left(1,1+n;2+n;\frac{a-}{}\right.}{4(a}$$

Mathematica [B] time = 17.198, size = 710, normalized size = 3.07

$$\left(\frac{1}{1-\tan^2\left(\frac{1}{2}(c+dx)\right)}\right)^n\left(1-\tan^2\left(\frac{1}{2}(c+dx)\right)\right)^{-2n}\left(1-\tan^4\left(\frac{1}{2}(c+dx)\right)\right)^n\left(\cos(c+dx)\sec^2\left(\frac{1}{2}(c+dx)\right)\right)^n\left(\cos(c+dx)\sec^4\left(\frac{1}{2}(\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] ((Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(a + b*Sec[c + d*x])^n*((1 - Tan[(c +
d*x)/2]^2)^(-1))^n*(1 - Tan[(c + d*x)/2]^4)^n*(b + (a - a*Tan[(c + d*x)/2]^
2)/(1 + Tan[(c + d*x)/2]^2))^n*(2*(a + b + b*n)*Hypergeometric2F1[1, -n, 1
- n, ((a + b)*(-1 + Tan[(c + d*x)/2]^2))/(a*(-1 + Tan[(c + d*x)/2]^2) - b*(
1 + Tan[(c + d*x)/2]^2))]*(1 - Tan[(c + d*x)/2]^2)^n - (Cot[(c + d*x)/2]^2*
(2^(1 + n)*(a - b)*(1 + n)*(a + b + b*n)*Hypergeometric2F1[-n, -n, 1 - n, (
(a - b)*(-1 + Tan[(c + d*x)/2]^2))/(2*b)]*Tan[(c + d*x)/2]^2*(2 - 2*Tan[(c
+ d*x)/2]^2)^n + n*(1 - Tan[(c + d*x)/2]^2)^n*(a*(-1 + Tan[(c + d*x)/2]^2)
- b*(1 + Tan[(c + d*x)/2]^2))*(2^n*(a - b)*(1 + n)*(-1 + Tan[(c + d*x)/2]^2
)*((a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/b)^n - 2*a*Hyperge
ometric2F1[n, 1 + n, 2 + n, (a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)
/2]^2)/(2*b)]*Tan[(c + d*x)/2]^2*(-(((-1 + Tan[(c + d*x)/2]^2)*(-2*a*b*Tan[
(c + d*x)/2]^2 + a^2*(-1 + Tan[(c + d*x)/2]^2) + b^2*(1 + Tan[(c + d*x)/2]^
2)))/b^2))^n)))/(2^n*(a - b)*(1 + n)*((a + b - a*Tan[(c + d*x)/2]^2 + b*Tan
[(c + d*x)/2]^2)/b)^n)))/(8*(a + b)*d*n*(b + a*Cos[c + d*x])^n*(Cos[c + d*x
]*Sec[(c + d*x)/2]^4)^n*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^n*(1 - Tan[(c + d
*x)/2]^2)^(2*n))
```

Maple [F] time = 0.261, size = 0, normalized size = 0.

$$\int (\csc (dx+c))^{3} (a+b\sec (dx+c))^{n} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*sec(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^3*(a+b*sec(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (dx+c)+a)^{n} \csc (dx+c)^{3} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec (dx+c)+a)^{n} \csc (dx+c)^{3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*sec(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec (dx+c)+a)^{n} \csc (dx+c)^{3} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^3, x)
```

3.273 $\int (a+b\sec(c+dx))^n \sin^4(c+dx)\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\sin^4(c+dx)(a+b\sec(c+dx))^n,x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^4, x]
```

Rubi [A] time = 0.0398236, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (a+b\sec(c+dx))^n \sin^4(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^4,x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Sin[c + d*x]^4, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n \sin^4(c+dx)\,dx = \int (a+b\sec(c+dx))^n \sin^4(c+dx)\,dx$$

Mathematica [A] time = 14.292, size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^n \sin^4(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^4,x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^4, x]
```

Maple [A] time = 0.684, size = 0, normalized size = 0.

$$\int (a+b\sec(dx+c))^n (\sin(dx+c))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x)
```

```
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n \sin(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\cos\left(dx+c\right)^4 - 2\,\cos\left(dx+c\right)^2 + 1\right)\left(b\sec\left(dx+c\right)+a\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*(b*sec(d*x + c) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(dx+c\right)+a\right)^n \sin\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^4, x)
```

3.274 $\int (a+b\sec(c+dx))^n \sin^2(c+dx)\,dx$

Optimal. Leaf size=23

$$\text{Unintegrable}\left(\sin^2(c+dx)(a+b\sec(c+dx))^n, x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^2, x]
```

Rubi [A] time = 0.0398864, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (a+b\sec(c+dx))^n \sin^2(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^2,x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Sin[c + d*x]^2, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n \sin^2(c+dx)\,dx = \int (a+b\sec(c+dx))^n \sin^2(c+dx)\,dx$$

Mathematica [A] time = 3.77905, size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^n \sin^2(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^2,x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^2, x]
```

Maple [A] time = 0.588, size = 0, normalized size = 0.

$$\int (a+b\sec(dx+c))^n (\sin(dx+c))^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x)
```

```
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n \sin(dx+c)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\left(\cos\left(dx+c\right)^2-1\right)\left(b\sec\left(dx+c\right)+a\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*(b*sec(d*x + c) + a)^n, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(dx+c\right)+a\right)^n\sin\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^2, x)
```

3.275 $\int \csc^2(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal. Leaf size=136

$$\frac{\sqrt{2}bn\tan(c+dx)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},1-n;\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d(a+b)\sqrt{\sec(c+dx)+1}}-\frac{\cot(c+dx)(a}{}$$

```
[Out] -((Cot[c + d*x]*(a + b*Sec[c + d*x])^n)/d) + (Sqrt[2]*b*n*AppellF1[1/2, 1/2
, 1 - n, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*
Sec[c + d*x])^n*Tan[c + d*x])/((a + b)*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec
[c + d*x])/(a + b))^n)
```

Rubi [A] time = 0.164858, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3875, 3834, 139, 138}

$$\frac{\sqrt{2}bn\tan(c+dx)(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}F_1\left(\frac{1}{2};\frac{1}{2},1-n;\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)}{d(a+b)\sqrt{\sec(c+dx)+1}}-\frac{\cot(c+dx)(a}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^n,x]
```


```
[Out] -((Cot[c + d*x]*(a + b*Sec[c + d*x])^n)/d) + (Sqrt[2]*b*n*AppellF1[1/2, 1/2
, 1 - n, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*
Sec[c + d*x])^n*Tan[c + d*x])/((a + b)*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Sec
[c + d*x])/(a + b))^n)
```


Rule 3875

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)/cos[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Simp[(Tan[e + f*x]*(a + b*Csc[e + f*x])^m)/f, x] + Dist[b*m, I
nt[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x], x] /; FreeQ[{a, b, e, f,
m}, x]
```

Rule 3834

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Dist[Cot[e + f*x]/(f*Sqrt[1 + Csc[e + f*x]]*Sqrt[1 - Csc[e + f*x
]]), Subst[Int[(a + b*x)^m/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Csc[e + f*x]],
 x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0] &&  !IntegerQ[2*m]
```

Rule 139

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
((b*(e + f*x))/(b*e - a*f))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*((b*e
)/(b*e - a*f) + (b*f*x)/(b*e - a*f))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] &&  !IntegerQ[m] &&  !IntegerQ[n] &&  !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] &&  !GtQ[b/(b*e - a*f), 0]
```

Rule 138

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*AppellF1[m + 1, -n, -p, m + 2,
```

```
-((d*(a + b*x))/(b*c - a*d)), -((f*(a + b*x))/(b*e - a*f))])/(b*(m + 1)*(b/
(b*c - a*d))^n*(b/(b*e - a*f))^p), x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
 x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplerQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplerQ[e + f*x, a + b*x])
```

Rubi steps

$$\int \csc^2(c+dx)(a+b\sec(c+dx))^n\,dx = -\frac{\cot(c+dx)(a+b\sec(c+dx))^n}{d} + (bn)\int \sec(c+dx)(a+b\sec(c+dx))^{-1+n}$$

$$= -\frac{\cot(c+dx)(a+b\sec(c+dx))^n}{d} - \frac{(bn\tan(c+dx))\,\mathrm{Subst}\left(\int \frac{(a+bx)^{-1+n}}{\sqrt{1-x}\sqrt{1+x}}\,dx, x\right.}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+}}$$

$$= -\frac{\cot(c+dx)(a+b\sec(c+dx))^n}{d} - \frac{\left(bn(a+b\sec(c+dx))^n\left(-\frac{a+b\sec(c+dx)}{-a-b}\right)^{-n}\right.}{(a+b)d\sqrt{1-}}$$

$$= -\frac{\cot(c+dx)(a+b\sec(c+dx))^n}{d} + \frac{\sqrt{2}bnF_1\left(\frac{1}{2};\frac{1}{2},1-n;\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)\right.}{(a}$$

Mathematica [B] time = 18.4615, size = 3614, normalized size = 26.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] ((b + a*Cos[c + d*x])^n*Cot[(c + d*x)/2]*Csc[c + d*x]^2*Sec[c + d*x]^n*(a +
 b*Sec[c + d*x])^n*(-((AppellF1[-1/2, n, -n, 1/2, Tan[(c + d*x)/2]^2, ((a -
 b)*Tan[(c + d*x)/2]^2)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n)/(((b
+ a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b))^n) + (3*(a + b)*AppellF1[1/2
, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Tan
[(c + d*x)/2]^2)/(3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, (
(a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 2*n*((-a + b)*AppellF1[3/2, n, 1 - n
, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*
AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]
^2)/(a + b)])*Tan[(c + d*x)/2]^2)))/(2*d*(-((b + a*Cos[c + d*x])^n*Csc[(c +
 d*x)/2]^2*Sec[c + d*x]^n*(-((AppellF1[-1/2, n, -n, 1/2, Tan[(c + d*x)/2]^2
, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n
)/(((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b))^n) + (3*(a + b)*Appel
lF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a +
b)]*Tan[(c + d*x)/2]^2)/(3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/
2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 2*n*((-a + b)*AppellF1[3/2, n
, 1 - n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (
a + b)*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c +
d*x)/2]^2)/(a + b)])*Tan[(c + d*x)/2]^2)))/4 - (a*n*(b + a*Cos[c + d*x])^(-
1 + n)*Cot[(c + d*x)/2]*Sec[c + d*x]^n*Sin[c + d*x]*(-((AppellF1[-1/2, n, -
n, 1/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*(Cos[c +
d*x]*Sec[(c + d*x)/2]^2)^n)/(((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a +
 b))^n) + (3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)
*Tan[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2]^2)/(3*(a + b)*AppellF1[1/2,
n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 2*n
*((-a + b)*AppellF1[3/2, n, 1 - n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c
```

```
 + d*x)/2]^2)/(a + b)] + (a + b)*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x
)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Tan[(c + d*x)/2]^2)))/2 + (n
*(b + a*Cos[c + d*x])^n*Cot[(c + d*x)/2]*Sec[c + d*x]^(1 + n)*Sin[c + d*x]*
(-((AppellF1[-1/2, n, -n, 1/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2
]^2)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n)/(((b + a*Cos[c + d*x])*S
ec[(c + d*x)/2]^2)/(a + b))^n) + (3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan[(
c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2]^2)/(3
*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*
x)/2]^2)/(a + b)] + 2*n*((-a + b)*AppellF1[3/2, n, 1 - n, 5/2, Tan[(c + d*x
)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*AppellF1[3/2, 1 + n
, -n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Tan[(
c + d*x)/2]^2)))/2 + ((b + a*Cos[c + d*x])^n*Cot[(c + d*x)/2]*Sec[c + d*x]^
n*(-(((Cos[c + d*x]*Sec[(c + d*x)/2]^2)^n*(((a - b)*n*AppellF1[1/2, n, 1 -
n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2])/(a + b) - n*AppellF1[1/2, 1 + n, -n, 3/2, Tan[(
c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan
[(c + d*x)/2]))/(((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b))^n) - (n
*AppellF1[-1/2, n, -n, 1/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2
)/(a + b)]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(-1 + n)*(-(Sec[(c + d*x)/2]^2
*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(((b +
a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b))^n + n*AppellF1[-1/2, n, -n, 1/
2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*(Cos[c + d*x]*
Sec[(c + d*x)/2]^2)^n*(((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b))^(
-1 - n)*(-((a*Sec[(c + d*x)/2]^2*Sin[c + d*x])/(a + b)) + ((b + a*Cos[c + d
*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(a + b)) + (3*(a + b)*AppellF1[1/
2, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Se
c[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan
[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 2*n*((-a + b)*Appe
llF1[3/2, n, 1 - n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(
a + b)] + (a + b)*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a - b
)*Tan[(c + d*x)/2]^2)/(a + b)])*Tan[(c + d*x)/2]^2) + (3*(a + b)*Tan[(c + d
*x)/2]^2*(-((a - b)*n*AppellF1[3/2, n, 1 - n, 5/2, Tan[(c + d*x)/2]^2, ((a
- b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3*(
a + b)) + (n*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan
[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3))/(3*(a +
b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^
2)/(a + b)] + 2*n*((-a + b)*AppellF1[3/2, n, 1 - n, 5/2, Tan[(c + d*x)/2]^2
, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*AppellF1[3/2, 1 + n, -n,
5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)])*Tan[(c + d*
x)/2]^2) - (3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2, ((a - b
)*Tan[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2]^2*(2*n*((-a + b)*AppellF1[3
/2, n, 1 - n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)
] + (a + b)*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[
(c + d*x)/2]^2)/(a + b)])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + 3*(a + b)*(
-((a - b)*n*AppellF1[3/2, n, 1 - n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(
c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3*(a + b)) +
(n*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)
/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/3) + 2*n*Tan[(c + d*x)
/2]^2*((-a + b)*((3*(a - b)*(1 - n)*AppellF1[5/2, n, 2 - n, 7/2, Tan[(c + d
*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c +
 d*x)/2])/(5*(a + b)) + (3*n*AppellF1[5/2, 1 + n, 1 - n, 7/2, Tan[(c + d*x)
/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(c + d*
x)/2])/5) + (a + b)*((-3*(a - b)*n*AppellF1[5/2, 1 + n, 1 - n, 7/2, Tan[(c
+ d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2])/(5*(a + b)) + (3*(1 + n)*AppellF1[5/2, 2 + n, -n, 7/2, Tan[(c
+ d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2])/5))))/(3*(a + b)*AppellF1[1/2, n, -n, 3/2, Tan[(c + d*x)/2]^2,
 ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + 2*n*((-a + b)*AppellF1[3/2, n, 1 -
 n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b
```

```
)*AppellF1[3/2, 1 + n, -n, 5/2, Tan[(c + d*x)/2]^2, ((a - b)*Tan[(c + d*x)/
2]^2)/(a + b)])*Tan[(c + d*x)/2]^2)^2))/2))
```

Maple [F] time = 0.252, size = 0, normalized size = 0.

$$\int (\csc (dx + c))^2 (a + b \sec (dx + c))^n \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+b*sec(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^2*(a+b*sec(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sec (dx + c) + a)^n \csc (dx + c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b \sec (dx + c) + a)^n \csc (dx + c)^2 , x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*sec(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b \sec (dx + c) + a)^n \csc (dx + c)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^2, x)
```

3.276 $\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx$

Optimal. Leaf size=424

$$-\frac{\cot^3(c+dx)(\sec(c+dx)+1)^{3/2}(a+b\sec(c+dx))^n\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{-n}F_1\left(-\frac{3}{2};\frac{5}{2},-n;-\frac{1}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx}{a+b}\right.}{6\sqrt{2}d}$$

```
[Out] (-3*AppellF1[-1/2, 5/2, -n, 1/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]
))/(a + b)]*Cot[c + d*x]*Sqrt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^n)/(2*
Sqrt[2]*d*((a + b*Sec[c + d*x])/(a + b))^n) - (AppellF1[-3/2, 5/2, -n, -1/2
, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*Cot[c + d*x]^3*(1 +
 Sec[c + d*x])^(3/2)*(a + b*Sec[c + d*x])^n)/(6*Sqrt[2]*d*((a + b*Sec[c + d
*x])/(a + b))^n) + (AppellF1[1/2, 3/2, -n, 3/2, (1 - Sec[c + d*x])/2, (b*(1
 - Sec[c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^n*Tan[c + d*x])/(Sqrt[2]*d*
Sqrt[1 + Sec[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^n) + (AppellF1[1/2, 5
/2, -n, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c + d*x]))/(a + b)]*(a + b*S
ec[c + d*x])^n*Tan[c + d*x])/(2*Sqrt[2]*d*Sqrt[1 + Sec[c + d*x]]*((a + b*Se
c[c + d*x])/(a + b))^n)
```

Rubi [F] time = 0.0400988, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] Defer[Int][Csc[c + d*x]^4*(a + b*Sec[c + d*x])^n, x]
```

Rubi steps

$$\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx=\int \csc^4(c+dx)(a+b\sec(c+dx))^n\,dx$$

Mathematica [B] time = 23.726, size = 6403, normalized size = 15.1

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Sec[c + d*x])^n,x]
```

```
[Out] Result too large to show
```

Maple [F] time = 0.296, size = 0, normalized size = 0.

$$\int (\csc(dx+c))^4(a+b\sec(dx+c))^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*sec(d*x+c))^n,x)
```

```
[Out] int(csc(d*x+c)^4*(a+b*sec(d*x+c))^n,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n \csc(dx+c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left((b\sec(dx+c)+a)^n \csc(dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*sec(d*x+c))**n,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n \csc(dx+c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sec(d*x+c))^n,x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*csc(d*x + c)^4, x)
```

3.277 $\int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\sin^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^n,x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^(3/2), x]
```

Rubi [A] time = 0.0387082, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^(3/2),x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Sin[c + d*x]^(3/2), x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx = \int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx$$

Mathematica [A] time = 1.74944, size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^n \sin^{\frac{3}{2}}(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^(3/2),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^n*Sin[c + d*x]^(3/2), x]
```

Maple [A] time = 0.222, size = 0, normalized size = 0.

$$\int (a+b\sec(dx+c))^n (\sin(dx+c))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x)
```

```
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n \sin(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c)+a)^n \sin(dx+c)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int (b\sec(dx+c)+a)^n \sin(dx+c)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*sin(d*x + c)^(3/2), x)
```

3.278 $\int (a+b\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\sqrt{\sin(c+dx)}(a+b\sec(c+dx))^n,x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]], x]
```

Rubi [A] time = 0.0401989, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int (a+b\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]],x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]], x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx=\int (a+b\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx$$

Mathematica [A] time = 4.8539, size = 0, normalized size = 0.

$$\int (a+b\sec(c+dx))^n\sqrt{\sin(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]],x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^n*Sqrt[Sin[c + d*x]], x]
```

Maple [A] time = 0.189, size = 0, normalized size = 0.

$$\int \left(a+b\sec\left(dx+c\right)\right)^n\sqrt{\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x)
```

```
[Out] int((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sec\left(dx+c\right)+a\right)^n\sqrt{\sin\left(dx+c\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((b\sec(dx+c)+a)^n\sqrt{\sin(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n*sin(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int(b\sec(dx+c)+a)^n\sqrt{\sin(dx+c)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n*sin(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n*sqrt(sin(d*x + c)), x)
```

3.279 $\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]], x]
```

Rubi [A] time = 0.0404432, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]],x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]], x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx = \int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx$$

Mathematica [A] time = 2.58372, size = 0, normalized size = 0.

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]],x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^n/Sqrt[Sin[c + d*x]], x]
```

Maple [A] time = 0.184, size = 0, normalized size = 0.

$$\int (a+b\sec(dx+c))^n \frac{1}{\sqrt{\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x)
```

```
[Out] int((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^n}{\sqrt{\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{(b\sec(dx+c)+a)^n}{\sqrt{\sin(dx+c)}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((b*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

Sympy [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a+b\sec(c+dx))^n}{\sqrt{\sin(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n/sin(d*x+c)**(1/2),x)
```

```
[Out] Integral((a + b*sec(c + d*x))**n/sqrt(sin(c + d*x)), x)
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^n}{\sqrt{\sin(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n/sqrt(sin(d*x + c)), x)
```

3.280 $\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=25

$$\text{Unintegrable}\left(\frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)},x\right)$$

```
[Out] Unintegrable[(a + b*Sec[c + d*x])^n/Sin[c + d*x]^(3/2), x]
```

Rubi [A] time = 0.0408955, antiderivative size = 0, normalized size of antiderivative = 0., number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0., Rules used = {}

$$\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^n/Sin[c + d*x]^(3/2),x]
```

```
[Out] Defer[Int][(a + b*Sec[c + d*x])^n/Sin[c + d*x]^(3/2), x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx=\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$$

Mathematica [A] time = 2.78008, size = 0, normalized size = 0.

$$\int \frac{(a+b\sec(c+dx))^n}{\sin^{\frac{3}{2}}(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^n/Sin[c + d*x]^(3/2),x]
```

```
[Out] Integrate[(a + b*Sec[c + d*x])^n/Sin[c + d*x]^(3/2), x]
```

Maple [A] time = 0.188, size = 0, normalized size = 0.

$$\int (a+b\sec(dx+c))^n(\sin(dx+c))^{-\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x)
```

```
[Out] int((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x)
```

Maxima [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^n}{\sin(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n/sin(d*x + c)^(3/2), x)
```

Fricas [A] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(-\frac{(b\sec(dx+c)+a)^n\sqrt{\sin(dx+c)}}{\cos(dx+c)^2-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*sec(d*x + c) + a)^n*sqrt(sin(d*x + c))/(cos(d*x + c)^2 - 1), x
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**n/sin(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 0., size = 0, normalized size = 0.

$$\int \frac{(b\sec(dx+c)+a)^n}{\sin(dx+c)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^n/sin(d*x+c)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sec(d*x + c) + a)^n/sin(d*x + c)^(3/2), x)
```

3.281 $\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=190

$$\frac{2ae^2\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{e\csc(c+dx)}}{3d}-\frac{2ae^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d}-\frac{2ae^2\cot(c+dx)\sqrt{e}}{3d}$$

```
[Out] (-2*a*e^2*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(3*d) - (2*a*e^2*Csc[c + d*x]*
Sqrt[e*Csc[c + d*x]])/(3*d) + (a*e^2*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[
c + d*x]]*Sqrt[Sin[c + d*x]])/d + (a*e^2*ArcTanh[Sqrt[Sin[c + d*x]]]*Sqrt[e
*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d + (2*a*e^2*Sqrt[e*Csc[c + d*x]]*Ellipt
icF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d)
```

Rubi [A] time = 0.165207, antiderivative size = 190, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3878, 3872, 2838, 2564, 325, 329, 212, 206, 203, 2636, 2641}

$$-\frac{2ae^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d}-\frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d}+\frac{ae^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\tan^{-1}\left(\sqrt{\sin(c+\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x]),x]
```


```
[Out] (-2*a*e^2*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(3*d) - (2*a*e^2*Csc[c + d*x]*
Sqrt[e*Csc[c + d*x]])/(3*d) + (a*e^2*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[
c + d*x]]*Sqrt[Sin[c + d*x]])/d + (a*e^2*ArcTanh[Sqrt[Sin[c + d*x]]]*Sqrt[e
*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d + (2*a*e^2*Sqrt[e*Csc[c + d*x]]*Ellipt
icF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d)
```


Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))\,dx = \left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{a+a\sec(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\,dx$$

$$= -\left(\left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{(-a-a\cos(c+dx))\sec(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\,dx\right.$$

$$= \left(ae^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sin^{\frac{5}{2}}(c+dx)}\,dx + \left(ae^2\sqrt{e\csc(c+dx)}\right.$$

$$= -\frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} + \frac{1}{3}\left(ae^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int$$

$$= -\frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2ae^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d} + \frac{2ae^2}{}$$

$$= -\frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2ae^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d} + \frac{2ae^2}{}$$

$$= -\frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2ae^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d} + \frac{2ae^2}{}$$

$$= -\frac{2ae^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2ae^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d} + \frac{ae^2}{}$$

Mathematica [A] time = 1.43489, size = 135, normalized size = 0.71

$$-\frac{a(e\csc(c+dx))^{5/2}\left(4\sqrt{\sin(c+dx)}\sqrt{\csc(c+dx)}\text{EllipticF}\left(\frac{1}{4}(-2c-2dx+\pi),2\right)+4\cot\left(\frac{1}{2}(c+dx)\right)\sqrt{\csc(c+dx)}+3\right.}{6d\csc^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] -(a*(e*Csc[c + d*x])^(5/2)*(6*ArcTan[Sqrt[Csc[c + d*x]]] + 4*Cot[(c + d*x)/
2]*Sqrt[Csc[c + d*x]] + 3*Log[1 - Sqrt[Csc[c + d*x]]] - 3*Log[1 + Sqrt[Csc[
c + d*x]]] + 4*Sqrt[Csc[c + d*x]]*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[
Sin[c + d*x]]))/(6*d*Csc[c + d*x]^(5/2))
```

Maple [C] time = 0.335, size = 694, normalized size = 3.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] 1/6*a/d*2^(1/2)*(4*I*sin(d*x+c)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(
d*x+c))^(1/2),1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(
-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+
c))^(1/2)-3*I*sin(d*x+c)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c)
)^(1/2),1/2-1/2*I,1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/
2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(
d*x+c))^(1/2)-3*I*sin(d*x+c)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*
```

```
x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))
^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/
sin(d*x+c))^(1/2)+3*sin(d*x+c)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(
d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c
))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I
)/sin(d*x+c))^(1/2)-3*sin(d*x+c)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/si
n(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x
+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)
-I)/sin(d*x+c))^(1/2)+2*2^(1/2))*(-1+cos(d*x+c))*(cos(d*x+c)+1)^2*(e/sin(d*
x+c))^(5/2)/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(ae^2\csc\left(dx+c\right)^2\sec\left(dx+c\right)+ae^2\csc\left(dx+c\right)^2\right)\sqrt{e\csc\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*e^2*csc(d*x + c)^2*sec(d*x + c) + a*e^2*csc(d*x + c)^2)*sqrt(e*
csc(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(5/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(e\csc\left(dx+c\right)\right)^{\frac{5}{2}}\left(a\sec\left(dx+c\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*csc(d*x + c))^(5/2)*(a*sec(d*x + c) + a), x)
```

3.282 $\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=169

$$-\frac{2ae\sqrt{e\csc(c+dx)}}{d}-\frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d}-\frac{ae\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d}+\frac{ae\sqrt{\sin}}{}$$

```
[Out] (-2*a*e*Sqrt[e*Csc[c + d*x]])/d - (2*a*e*Cos[c + d*x]*Sqrt[e*Csc[c + d*x]])
/d - (a*e*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]
])/d + (a*e*ArcTanh[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d
*x]])/d - (2*a*e*Sqrt[e*Csc[c + d*x]]*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt
[Sin[c + d*x]])/d
```

Rubi [A] time = 0.16223, antiderivative size = 169, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3878, 3872, 2838, 2564, 325, 329, 298, 203, 206, 2636, 2639}

$$-\frac{2ae\sqrt{e\csc(c+dx)}}{d}-\frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d}-\frac{ae\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d}+\frac{ae\sqrt{\sin}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x]),x]
```


```
[Out] (-2*a*e*Sqrt[e*Csc[c + d*x]])/d - (2*a*e*Cos[c + d*x]*Sqrt[e*Csc[c + d*x]])
/d - (a*e*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]
])/d + (a*e*ArcTanh[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d
*x]])/d - (2*a*e*Sqrt[e*Csc[c + d*x]]*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt
[Sin[c + d*x]])/d
```


Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))\,dx = \left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{a+a\sec(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\,dx$$
$$= -\left(\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{(-a-a\cos(c+dx))\sec(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\,dx\right)$$
$$= \left(ae\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sin^{\frac{3}{2}}(c+dx)}\,dx + \left(ae\sqrt{e\csc(c+dx)}\sqrt{\right.$$
$$= -\frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \left(ae\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \sqrt{\sin}$$
$$= -\frac{2ae\sqrt{e\csc(c+dx)}}{d} - \frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{2ae\sqrt{e\csc(c+dx)}}{}$$
$$= -\frac{2ae\sqrt{e\csc(c+dx)}}{d} - \frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{2ae\sqrt{e\csc(c+dx)}}{}$$
$$= -\frac{2ae\sqrt{e\csc(c+dx)}}{d} - \frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{2ae\sqrt{e\csc(c+dx)}}{}$$
$$= -\frac{2ae\sqrt{e\csc(c+dx)}}{d} - \frac{2ae\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{ae\tan^{-1}\left(\sqrt{\sin(c\right.}{}$$

Mathematica [C] time = 1.25764, size = 146, normalized size = 0.86

$$\frac{a(e\csc(c+dx))^{3/2}\left(\frac{2\sin(2(c+dx))\csc^{\frac{3}{2}}(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},\csc^2(c+dx)\right)}{\sqrt{-\cot^2(c+dx)}} - 4(\cos(c+dx)+1)\sqrt{\csc(c+dx)} - \log\left(1-\right.\right.}{2d\csc^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*(e*Csc[c + d*x])^(3/2)*(2*ArcTan[Sqrt[Csc[c + d*x]]] - 4*(1 + Cos[c + d*
x])*Sqrt[Csc[c + d*x]] - Log[1 - Sqrt[Csc[c + d*x]]] + Log[1 + Sqrt[Csc[c +
 d*x]]] + (2*Csc[c + d*x]^(3/2)*Hypergeometric2F1[-1/4, 1/2, 3/4, Csc[c + d
*x]^2]*Sin[2*(c + d*x)])/Sqrt[-Cot[c + d*x]^2]))/(2*d*Csc[c + d*x]^(3/2))
```

Maple [C] time = 0.246, size = 1526, normalized size = 9.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] -1/2*a/d*2^(1/2)*(I*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*c
os(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x
+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I
)/sin(d*x+c))^(1/2)-I*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I
*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/si
n(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1
```

```
/2+1/2*I,1/2*2^(1/2))+I*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+
c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/
sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d
*x+c))^(1/2)-I*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*
x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c
))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1
/2)-cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*
x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c
))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1
/2)-cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*
x+c)-I)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*El
lipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/
2))-4*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(
d*x+c)-I)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*
EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+2*cos
(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)
/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticF
(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))-(-I*(-1+cos(d*
x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*Ell
ipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2
))*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)-(-I*(-1+cos(d*x+c))/sin(d
*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I
*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((I*cos(
d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)-4*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c
)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)
/sin(d*x+c))^(1/2)+2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-
sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(
d*x+c))^(1/2),1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)+4
*2^(1/2))*(e/sin(d*x+c))^(3/2)*sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left((ae\csc(dx+c)\sec(dx+c)+ae\csc(dx+c))\sqrt{e\csc(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral((a*e*csc(d*x + c)*sec(d*x + c) + a*e*csc(d*x + c))*sqrt(e*csc(d*x
+ c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(3/2)*(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\csc(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a), x)
```

3.283 $\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))\,dx$

Optimal. Leaf size=121

$$\frac{2a\sqrt{\sin(c+dx)}\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{e\csc(c+dx)}}{d}+\frac{a\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d}+\frac{a\sqrt{s}}{}$$

```
[Out] (a*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d +
(a*ArcTanh[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d +
 (2*a*Sqrt[e*Csc[c + d*x]]*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*
x]])/d
```

Rubi [A] time = 0.137354, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3878, 3872, 2838, 2564, 329, 212, 206, 203, 2641}

$$\frac{a\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d}+\frac{a\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d}+\frac{2a\sqrt{\sin(c+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

```
[Out] (a*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d +
(a*ArcTanh[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d +
 (2*a*Sqrt[e*Csc[c + d*x]]*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*
x]])/d
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))\,dx = \left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{a+a\sec(c+dx)}{\sqrt{\sin(c+dx)}}\,dx$$

$$= -\left(\left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{(-a-a\cos(c+dx))\sec(c+dx)}{\sqrt{\sin(c+dx)}}\,dx\right)$$

$$= \left(a\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx + \left(a\sqrt{e\csc(c+dx)}\sqrt{\sin\right.$$

$$= \frac{2a\sqrt{e\csc(c+dx)}F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d} + \frac{\left(a\sqrt{e\csc(c+dx)}\sqrt{\sin\right.}{}$$

$$= \frac{2a\sqrt{e\csc(c+dx)}F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d} + \frac{\left(2a\sqrt{e\csc(c+dx)}\sqrt{s\right.}{}$$

$$= \frac{2a\sqrt{e\csc(c+dx)}F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d} + \frac{\left(a\sqrt{e\csc(c+dx)}\sqrt{\sin\right.}{}$$

$$= \frac{a\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} + \frac{a\tanh^{-1}\left(\sqrt{\sin(c+d\right.}{}$$

Mathematica [A] time = 0.835445, size = 111, normalized size = 0.92

$$-\frac{a\sqrt{e\csc(c+dx)}\left(4\sqrt{\sin(c+dx)}\sqrt{\csc(c+dx)}\text{EllipticF}\left(\frac{1}{4}(-2c-2dx+\pi),2\right)+\log\left(1-\sqrt{\csc(c+dx)}\right)-\log\left(\sqrt{\csc(\right.\right.}{2d\sqrt{\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x]),x]
```

```
[Out] -(a*Sqrt[e*Csc[c + d*x]]*(2*ArcTan[Sqrt[Csc[c + d*x]]] + Log[1 - Sqrt[Csc[c
 + d*x]]] - Log[1 + Sqrt[Csc[c + d*x]]] + 4*Sqrt[Csc[c + d*x]]*EllipticF[(-
2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c + d*x]]))/(2*d*Sqrt[Csc[c + d*x]])
```

Maple [C] time = 0.209, size = 288, normalized size = 2.4

$$-\frac{a\sqrt{2}\left(-1+\cos\left(dx+c\right)\right)\left(\cos\left(dx+c\right)+1\right)^{2}}{2\,d\left(\sin\left(dx+c\right)\right)^{2}}\sqrt{\frac{e}{\sin\left(dx+c\right)}}\sqrt{\frac{i\cos\left(dx+c\right)+\sin\left(dx+c\right)-i}{\sin\left(dx+c\right)}}\sqrt{\frac{-i\cos\left(dx+c\right)+\sin\left(dx+\right.}{\sin\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(e*csc(d*x+c))^(1/2),x)
```

```
[Out] -1/2*a/d*2^(1/2)*(e/sin(d*x+c))^(1/2)*(-1+cos(d*x+c))*((I*cos(d*x+c)+sin(d*
x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*(
-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(I*EllipticPi(((I*cos(d*x+c)+sin(d*x+c
)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+I*EllipticPi(((I*cos(d*x+c)+s
in(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-EllipticPi(((I*cos(d*
x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+EllipticPi(((I*
cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2)))/sin(d*x+
c)^2*(cos(d*x+c)+1)^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{e\csc\left(dx+c\right)}\left(a\sec\left(dx+c\right)+a\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{e\csc\left(dx+c\right)}\left(a\sec\left(dx+c\right)+a\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int\sqrt{e\csc\left(c+dx\right)}\,dx+\int\sqrt{e\csc\left(c+dx\right)}\sec\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*csc(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(sqrt(e*csc(c + d*x)), x) + Integral(sqrt(e*csc(c + d*x))*sec(c
+ d*x), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{e \csc (dx+c)}(a \sec (dx+c)+a)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(e*csc(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a), x)
```

3.284 $\int \frac{a+a\sec(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx$

Optimal. Leaf size=122

$$-\frac{a\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{a\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

```
[Out] -((a*ArcTan[Sqrt[Sin[c + d*x]]])/(d*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]]
)) + (a*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d
*x]]) + (2*a*EllipticE[(c - Pi/2 + d*x)/2, 2])/(d*Sqrt[e*Csc[c + d*x]]*Sqrt
[Sin[c + d*x]])
```

Rubi [A] time = 0.148031, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3878, 3872, 2838, 2564, 329, 298, 203, 206, 2639}

$$-\frac{a\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{a\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{2aE\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/Sqrt[e*Csc[c + d*x]],x]
```


```
[Out] -((a*ArcTan[Sqrt[Sin[c + d*x]]])/(d*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]]
)) + (a*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d
*x]]) + (2*a*EllipticE[(c - Pi/2 + d*x)/2, 2])/(d*Sqrt[e*Csc[c + d*x]]*Sqrt
[Sin[c + d*x]])
```


Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+a\sec(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx &= \frac{\int (a+a\sec(c+dx))\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= -\frac{\int (-a-a\cos(c+dx))\sec(c+dx)\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{a\int \sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{a\int \sec(c+dx)\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{a\,\mathrm{Subst}\left(\int \frac{\sqrt{x}}{1-x^2}\,dx,x,\sin(c+dx)\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{(2a)\,\mathrm{Subst}\left(\int \frac{x^2}{1-x^4}\,dx,x,\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{a\,\mathrm{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}-\frac{a\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx,\right.}{d\sqrt{e\csc(c+dx)}}\\
&= -\frac{a\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{a\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{2aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right.}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+\right.}}
\end{aligned}
$$

Mathematica [C] time = 0.809023, size = 130, normalized size = 1.07

$$\frac{a\left(\sqrt{-\cot^2(c+dx)}\sqrt{\csc(c+dx)}\left(-\log\left(1-\sqrt{\csc(c+dx)}\right)+\log\left(\sqrt{\csc(c+dx)}+1\right)+2\tan^{-1}\left(\sqrt{\csc(c+dx)}\right)\right)-4\cot(c\right.}{2d\sqrt{-\cot^2(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/Sqrt[e*Csc[c + d*x]],x]
```

```
[Out] (a*(-4*Cot[c + d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Csc[c + d*x]^2] + Sqr
t[-Cot[c + d*x]^2]*Sqrt[Csc[c + d*x]]*(2*ArcTan[Sqrt[Csc[c + d*x]]] - Log[1
 - Sqrt[Csc[c + d*x]]] + Log[1 + Sqrt[Csc[c + d*x]]])))/(2*d*Sqrt[-Cot[c +
d*x]^2]*Sqrt[e*Csc[c + d*x]])
```

Maple [C] time = 0.239, size = 1503, normalized size = 12.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x)
```

```
[Out] -1/2*a/d*2^(1/2)*(I*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*((I*co
s(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(
d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2
-1/2*I,1/2*2^(1/2))-I*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*((I*
cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/si
n(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1
/2+1/2*I,1/2*2^(1/2))-(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*((I*
cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/si
n(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1
/2-1/2*I,1/2*2^(1/2))-(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*((I*
cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/si
n(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1
/2+1/2*I,1/2*2^(1/2))+4*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*((
I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/
sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),
1/2*2^(1/2))-2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*((I*cos(d*x
+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c
))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/
2))+I*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(
((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-I*((I
*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d
*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-((I*cos(d*x+c)
+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-I
*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d
*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-((I*cos(d*x+c)+sin(d*x+c)
-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)
+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/si
n(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+4*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d
*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+
c)+I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^
(1/2),1/2*2^(1/2))-2*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1
```

```
+cos(d*x+c))/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1
/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+2
*cos(d*x+c)*2^(1/2)-2*2^(1/2))/(e/sin(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sec(dx+c)+a}{\sqrt{e\csc(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(e*csc(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{e\csc(dx+c)}(a\sec(dx+c)+a)}{e\csc(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)/(e*csc(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \frac{1}{\sqrt{e\csc(c+dx)}}\,dx + \int \frac{\sec(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))**(1/2),x)
```

```
[Out] a*(Integral(1/sqrt(e*csc(c + d*x)), x) + Integral(sec(c + d*x)/sqrt(e*csc(c
 + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sec(dx+c)+a}{\sqrt{e\csc(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/sqrt(e*csc(d*x + c)), x)
```

3.285 $\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=182

$$\frac{2a\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3de\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}-\frac{2a}{de\sqrt{e\csc(c+dx)}}-\frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}}+\frac{a\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{a\tanh^{-1}}{de\sqrt{\sin(c+}}$$

```
[Out] (-2*a)/(d*e*Sqrt[e*Csc[c + d*x]]) - (2*a*Cos[c + d*x])/(3*d*e*Sqrt[e*Csc[c
+ d*x]]) + (a*ArcTan[Sqrt[Sin[c + d*x]]])/(d*e*Sqrt[e*Csc[c + d*x]]*Sqrt[Si
n[c + d*x]]) + (a*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*e*Sqrt[e*Csc[c + d*x]]*Sq
rt[Sin[c + d*x]]) + (2*a*EllipticF[(c - Pi/2 + d*x)/2, 2])/(3*d*e*Sqrt[e*Cs
c[c + d*x]]*Sqrt[Sin[c + d*x]])
```

Rubi [A] time = 0.172281, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3878, 3872, 2838, 2564, 321, 329, 212, 206, 203, 2635, 2641}

$$-\frac{2a}{de\sqrt{e\csc(c+dx)}}-\frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}}+\frac{a\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{a\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{2aF\left(\frac{1}{2}\right.}{3de\sqrt{\sin(c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/(e*Csc[c + d*x])^(3/2),x]
```

```
[Out] (-2*a)/(d*e*Sqrt[e*Csc[c + d*x]]) - (2*a*Cos[c + d*x])/(3*d*e*Sqrt[e*Csc[c
+ d*x]]) + (a*ArcTan[Sqrt[Sin[c + d*x]]])/(d*e*Sqrt[e*Csc[c + d*x]]*Sqrt[Si
n[c + d*x]]) + (a*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*e*Sqrt[e*Csc[c + d*x]]*Sq
rt[Sin[c + d*x]]) + (2*a*EllipticF[(c - Pi/2 + d*x)/2, 2])/(3*d*e*Sqrt[e*Cs
c[c + d*x]]*Sqrt[Sin[c + d*x]])
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] && !IntegerQ[p]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{3/2}}\,dx = \frac{\int (a+a\sec(c+dx))\sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= -\frac{\int (-a-a\cos(c+dx))\sec(c+dx)\sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= \frac{a\int \sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\int \sec(c+dx)\sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= -\frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a\int \frac{1}{\sqrt{\sin(c+dx)}}dx}{3e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\,\text{Subst}\left(\int \frac{x^{3/2}}{1-x^2}dx, x, \sin(c+dx)\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= -\frac{2a}{de\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{3de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{x}(1-\ldots}\right.}{de\sqrt{e\csc(c+\ldots}}$$
$$= -\frac{2a}{de\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{3de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{1-\ldots}\right.}{de\sqrt{e\csc(c+\ldots}}$$
$$= -\frac{2a}{de\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{2aF\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{3de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\,\text{Subst}\left(\int \frac{1}{1-x^2}\ldots\right.}{de\sqrt{e\csc(c+\ldots}}$$
$$= -\frac{2a}{de\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\tanh^{-1}\left(\sqrt{\sin\ldots}\right.}{de\sqrt{e\csc(c+dx)}\sqrt{\ldots}}$$

Mathematica [A] time = 10.7266, size = 135, normalized size = 0.74

$$-\frac{a\left(\frac{4\text{EllipticF}\left(\frac{1}{4}(-2c-2dx+\pi),2\right)}{\sqrt{\sin(c+dx)}} + 4\cos(c+dx) + 3\sqrt{\csc(c+dx)}\log\left(1-\sqrt{\csc(c+dx)}\right) - 3\sqrt{\csc(c+dx)}\log\left(\sqrt{\csc(c+dx)}+\ldots\right.\right.}{6de\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Csc[c + d*x])^(3/2),x]
```

```
[Out] -(a*(12 + 4*Cos[c + d*x] + 6*ArcTan[Sqrt[Csc[c + d*x]]]*Sqrt[Csc[c + d*x]]
+ 3*Sqrt[Csc[c + d*x]]*Log[1 - Sqrt[Csc[c + d*x]]] - 3*Sqrt[Csc[c + d*x]]*L
og[1 + Sqrt[Csc[c + d*x]]] + (4*EllipticF[(-2*c + Pi - 2*d*x)/4, 2])/Sqrt[S
in[c + d*x]]))/(6*d*e*Sqrt[e*Csc[c + d*x]])
```

Maple [C] time = 0.233, size = 710, normalized size = 3.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*csc(d*x+c))^(3/2),x)
```

```
[Out] -1/6*a/d*2^(1/2)*(3*I*sin(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I
*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d
*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)
-I)/sin(d*x+c))^(1/2)+3*I*sin(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+s
```

```
in(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*
x+c)-I)/sin(d*x+c))^(1/2)-4*I*sin(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c
)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)
/sin(d*x+c))^(1/2)-3*sin(d*x+c)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin
(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+
c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-
I)/sin(d*x+c))^(1/2)+3*sin(d*x+c)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/s
in(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*
x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c
)-I)/sin(d*x+c))^(1/2)+2*cos(d*x+c)^2*2^(1/2)+4*cos(d*x+c)*2^(1/2)-6*2^(1/2
))/(-1+cos(d*x+c))/(e/sin(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sec(dx+c)+a}{(e\csc(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)/(e*csc(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{e\csc(dx+c)}\,(a\sec(dx+c)+a)}{e^2\csc(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)/(e^2*csc(d*x + c)^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a\left(\int \frac{1}{(e\csc(c+dx))^{\frac{3}{2}}}\,dx+\int \frac{\sec(c+dx)}{(e\csc(c+dx))^{\frac{3}{2}}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))**(3/2),x)
```

```
[Out] a*(Integral((e*csc(c + d*x))**(-3/2), x) + Integral(sec(c + d*x)/(e*csc(c +
 d*x))**(3/2), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sec(dx+c)+a}{(e\csc(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/(e*csc(d*x + c))^(3/2), x)
```

3.286 $\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=197

$$-\frac{2a\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}}-\frac{2a\sin(c+dx)\cos(c+dx)}{5de^2\sqrt{e\csc(c+dx)}}-\frac{a\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{a\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

```
[Out] -((a*ArcTan[Sqrt[Sin[c + d*x]]])/(d*e^2*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d
*x]])) + (a*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*e^2*Sqrt[e*Csc[c + d*x]]*Sqrt[S
in[c + d*x]]) + (6*a*EllipticE[(c - Pi/2 + d*x)/2, 2])/(5*d*e^2*Sqrt[e*Csc[
c + d*x]]*Sqrt[Sin[c + d*x]]) - (2*a*Sin[c + d*x])/(3*d*e^2*Sqrt[e*Csc[c +
d*x]]) - (2*a*Cos[c + d*x]*Sin[c + d*x])/(5*d*e^2*Sqrt[e*Csc[c + d*x]])
```

Rubi [A] time = 0.172174, antiderivative size = 197, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3878, 3872, 2838, 2564, 321, 329, 298, 203, 206, 2635, 2639}

$$-\frac{2a\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}}-\frac{2a\sin(c+dx)\cos(c+dx)}{5de^2\sqrt{e\csc(c+dx)}}-\frac{a\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{a\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])/(e*Csc[c + d*x])^(5/2),x]
```


```
[Out] -((a*ArcTan[Sqrt[Sin[c + d*x]]])/(d*e^2*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d
*x]])) + (a*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*e^2*Sqrt[e*Csc[c + d*x]]*Sqrt[S
in[c + d*x]]) + (6*a*EllipticE[(c - Pi/2 + d*x)/2, 2])/(5*d*e^2*Sqrt[e*Csc[
c + d*x]]*Sqrt[Sin[c + d*x]]) - (2*a*Sin[c + d*x])/(3*d*e^2*Sqrt[e*Csc[c +
d*x]]) - (2*a*Cos[c + d*x]*Sin[c + d*x])/(5*d*e^2*Sqrt[e*Csc[c + d*x]])
```


Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2838

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[a, Int[(g*Cos
[e + f*x])^p*(d*Sin[e + f*x])^n, x], x] + Dist[b/d, Int[(g*Cos[e + f*x])^p*
(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{a+a\sec(c+dx)}{(e\csc(c+dx))^{5/2}}\,dx = \frac{\int (a+a\sec(c+dx))\sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{\int (-a-a\cos(c+dx))\sec(c+dx)\sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{a\int \sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\int \sec(c+dx)\sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{2a\cos(c+dx)\sin(c+dx)}{5de^2\sqrt{e\csc(c+dx)}} + \frac{(3a)\int \sqrt{\sin(c+dx)}\,dx}{5e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\,\mathrm{Subst}\left(\int \frac{x^{5/2}}{1-x^2}\,dx, x, \sin(\ldots\right.}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(\ldots}}$$

$$= \frac{6aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{2a\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)\sin(c+dx)}{5de^2\sqrt{e\csc(c+dx)}} + \frac{a\,\mathrm{Su}\ldots}{de^2\ldots}$$

$$= \frac{6aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{2a\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)\sin(c+dx)}{5de^2\sqrt{e\csc(c+dx)}} + \frac{(2a)\ldots}{\ldots}$$

$$= \frac{6aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{2a\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} - \frac{2a\cos(c+dx)\sin(c+dx)}{5de^2\sqrt{e\csc(c+dx)}} + \frac{a\,\mathrm{Su}\ldots}{d\ldots}$$

$$= -\frac{a\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{6aE\left(\frac{1}{2}\left(c-\frac{\pi}{2}+\ldots\right.\right.}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\ldots}}$$

Mathematica [C] time = 1.39265, size = 165, normalized size = 0.84

$$\frac{a\left(-72\cot(c+dx)\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},\csc^2(c+dx)\right)-2\sqrt{-\cot^2(c+dx)}\left(20\sin(c+dx)+6\sin(2(c+dx\ldots\right.\right.}{60de^2\sqrt{-\cot^2(c+d\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])/(e*Csc[c + d*x])^(5/2),x]
```

```
[Out] (a*(-72*Cot[c + d*x]*Hypergeometric2F1[-1/4, 1/2, 3/4, Csc[c + d*x]^2] - 2*
Sqrt[-Cot[c + d*x]^2]*(-30*ArcTan[Sqrt[Csc[c + d*x]]]*Sqrt[Csc[c + d*x]] +
15*Sqrt[Csc[c + d*x]]*(Log[1 - Sqrt[Csc[c + d*x]]] - Log[1 + Sqrt[Csc[c + d
*x]]]) + 20*Sin[c + d*x] + 6*Sin[2*(c + d*x)])))/(60*d*e^2*Sqrt[-Cot[c + d*
x]^2]*Sqrt[e*Csc[c + d*x]])
```

Maple [C] time = 0.227, size = 1565, normalized size = 7.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))/(e*csc(d*x+c))^(5/2),x)
```

```
[Out] 1/30*a/d*2^(1/2)*(-15*I*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-
(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin
(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+
c)-I)/sin(d*x+c))^(1/2)-15*I*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos
```

```
(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c
)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/
sin(d*x+c))^(1/2)+15*I*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)
+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^
(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1
/2*2^(1/2))+15*I*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d
*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x
+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/
2*I,1/2*2^(1/2))+18*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*c
os(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(
d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*
2^(1/2))+15*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c
)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^
(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1
/2*2^(1/2))+15*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*
x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I
)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/sin
(d*x+c))^(1/2)-36*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos
(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*
x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^
(1/2))+6*cos(d*x+c)^3*2^(1/2)+18*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I
*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*
x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x
+c))^(1/2)+15*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x
+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c)
)^(1/2),1/2+1/2*I,1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/
2)+15*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/s
in(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),
1/2-1/2*I,1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)-36*(-
I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c
))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/
2))*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)+10*cos(d*x+c)^2*2^(1/2)-
24*cos(d*x+c)*2^(1/2)+8*2^(1/2))/(e/sin(d*x+c))^(5/2)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\csc\left(dx+c\right)}\left(a\sec\left(dx+c\right)+a\right)}{e^{3}\csc\left(dx+c\right)^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)/(e^3*csc(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{a\sec\left(dx+c\right)+a}{\left(e\csc\left(dx+c\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))/(e*csc(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)/(e*csc(d*x + c))^(5/2), x)
```

3.287 $\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=270

$$\frac{7a^2e^2\sqrt{\sin(c+dx)}\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{e\csc(c+dx)}}{3d}-\frac{4a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d}-\frac{2a^2e^2\cot(c+dx)\sqrt{e}}{3d}$$

```
[Out] (-2*a^2*e^2*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(3*d) - (4*a^2*e^2*Csc[c + d
*x]*Sqrt[e*Csc[c + d*x]])/(3*d) - (2*a^2*e^2*Csc[c + d*x]*Sqrt[e*Csc[c + d*
x]]*Sec[c + d*x])/(3*d) + (2*a^2*e^2*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[
c + d*x]]*Sqrt[Sin[c + d*x]])/d + (2*a^2*e^2*ArcTanh[Sqrt[Sin[c + d*x]]]*Sq
rt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d + (7*a^2*e^2*Sqrt[e*Csc[c + d*x]]*
EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d) + (5*a^2*e^2*Sqr
t[e*Csc[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rubi [A] time = 0.333941, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {3878, 3872, 2873, 2636, 2641, 2564, 325, 329, 212, 206, 203, 2570, 2571}

$$-\frac{4a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d}-\frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d}+\frac{5a^2e^2\tan(c+dx)\sqrt{e\csc(c+dx)}}{3d}-\frac{2a^2e^2\csc(c+}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (-2*a^2*e^2*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(3*d) - (4*a^2*e^2*Csc[c + d
*x]*Sqrt[e*Csc[c + d*x]])/(3*d) - (2*a^2*e^2*Csc[c + d*x]*Sqrt[e*Csc[c + d*
x]]*Sec[c + d*x])/(3*d) + (2*a^2*e^2*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[
c + d*x]]*Sqrt[Sin[c + d*x]])/d + (2*a^2*e^2*ArcTanh[Sqrt[Sin[c + d*x]]]*Sq
rt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d + (7*a^2*e^2*Sqrt[e*Csc[c + d*x]]*
EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(3*d) + (5*a^2*e^2*Sqr
t[e*Csc[c + d*x]]*Tan[c + d*x])/(3*d)
```


Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int (e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2\,dx = \left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sin^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{(-a-a\cos(c+dx))^2\sec^2(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \left(\frac{a^2}{\sin^{\frac{5}{2}}(c+dx)}+\frac{2a^2\sec(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}+\frac{a^2\sec}{\sin^{\frac{5}{2}}}\right.$$

$$= \left(a^2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sin^{\frac{5}{2}}(c+dx)}\,dx + \left(a^2e^2\sqrt{e\csc(c+dx)}\right.$$

$$= -\frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}\sec(c+}{3d}$$

$$= -\frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{4a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2a^2e}{}$$

$$= -\frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{4a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2a^2e}{}$$

$$= -\frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{4a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2a^2e}{}$$

$$= -\frac{2a^2e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{4a^2e^2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3d} - \frac{2a^2e}{}$$

Mathematica [C] time = 3.74815, size = 195, normalized size = 0.72

$$-\frac{a^2e^2\cos^4\left(\frac{1}{2}(c+dx)\right)\tan(c+dx)\sqrt{e\csc(c+dx)}\sec^4\left(\frac{1}{2}\csc^{-1}(\csc(c+dx))\right)\left(7\sqrt{-\cot^2(c+dx)}\text{Hypergeometric2F1}\left(\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(a^2*e^2*Cos[(c + d*x)/2]^4*Sqrt[e*Csc[c + d*x]]*(-7 + 6*ArcTan[Sqrt[Csc[c
 + d*x]]]*Sqrt[Cos[c + d*x]^2]*Sqrt[Csc[c + d*x]] - 6*ArcTanh[Sqrt[Csc[c +
d*x]]]*Sqrt[Cos[c + d*x]^2]*Sqrt[Csc[c + d*x]] + 4*Csc[c + d*x]^2 + 4*Sqrt[
Cos[c + d*x]^2]*Csc[c + d*x]^2 + 7*Sqrt[-Cot[c + d*x]^2]*Hypergeometric2F1[
1/4, 1/2, 5/4, Csc[c + d*x]^2])*Sec[ArcCsc[Csc[c + d*x]]/2]^4*Tan[c + d*x])
/(3*d)
```

Maple [C] time = 0.272, size = 730, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/6*a^2/d*2^(1/2)*(-1+cos(d*x+c))*(5*I*sin(d*x+c)*cos(d*x+c)*EllipticF(((I*
cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-I*cos(d*x+c)+sin
(d*x+c)+I)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)
*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-6*I*sin(d*x+c)*cos(d*x+c)*EllipticPi
(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((-I
*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/si
n(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)-6*I*sin(d*x+c)*cos(d*
x+c)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/
2*2^(1/2))*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+s
in(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)+6*sin(
d*x+c)*cos(d*x+c)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)
,1/2-1/2*I,1/2*2^(1/2))*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*((I
*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))
^(1/2)-6*sin(d*x+c)*cos(d*x+c)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(
d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+
c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)+7*cos(d*x+c)*2^(1/2)-3*2^(1/2))*(e/sin(d*x+c))^(5/2)*(c
os(d*x+c)+1)^2/sin(d*x+c)/cos(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2e^2\csc\left(dx+c\right)^2\sec\left(dx+c\right)^2+2\,a^2e^2\csc\left(dx+c\right)^2\sec\left(dx+c\right)+a^2e^2\csc\left(dx+c\right)^2\right)\sqrt{e\csc\left(dx+c\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*e^2*csc(d*x + c)^2*sec(d*x + c)^2 + 2*a^2*e^2*csc(d*x + c)^2*
sec(d*x + c) + a^2*e^2*csc(d*x + c)^2)*sqrt(e*csc(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(5/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\csc(dx+c))^{\frac{5}{2}} (a\sec(dx+c)+a)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*csc(d*x + c))^(5/2)*(a*sec(d*x + c) + a)^2, x)
```

3.288 $\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=240

$$-\frac{4a^2e\sqrt{e\csc(c+dx)}}{d}-\frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d}-\frac{2a^2e\sec(c+dx)\sqrt{e\csc(c+dx)}}{d}-\frac{2a^2e\sqrt{\sin(c+dx)}\sqrt{e\csc(c}}{}$$

```
[Out] (-4*a^2*e*Sqrt[e*Csc[c + d*x]])/d - (2*a^2*e*Cos[c + d*x]*Sqrt[e*Csc[c + d*
x]])/d - (2*a^2*e*Sqrt[e*Csc[c + d*x]]*Sec[c + d*x])/d - (2*a^2*e*ArcTan[Sq
rt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d + (2*a^2*e*Arc
Tanh[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d - (5*a^
2*e*Sqrt[e*Csc[c + d*x]]*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]
])/d + (3*a^2*e*Sqrt[e*Csc[c + d*x]]*Sin[c + d*x]*Tan[c + d*x])/d
```

Rubi [A] time = 0.330845, antiderivative size = 240, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 13, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.52, Rules used = {3878, 3872, 2873, 2636, 2639, 2564, 325, 329, 298, 203, 206, 2570, 2571}

$$-\frac{4a^2e\sqrt{e\csc(c+dx)}}{d}-\frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d}-\frac{2a^2e\sec(c+dx)\sqrt{e\csc(c+dx)}}{d}-\frac{2a^2e\sqrt{\sin(c+dx)}\sqrt{e\csc(c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*a^2*e*Sqrt[e*Csc[c + d*x]])/d - (2*a^2*e*Cos[c + d*x]*Sqrt[e*Csc[c + d*
x]])/d - (2*a^2*e*Sqrt[e*Csc[c + d*x]]*Sec[c + d*x])/d - (2*a^2*e*ArcTan[Sq
rt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d + (2*a^2*e*Arc
Tanh[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/d - (5*a^
2*e*Sqrt[e*Csc[c + d*x]]*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]
])/d + (3*a^2*e*Sqrt[e*Csc[c + d*x]]*Sin[c + d*x]*Tan[c + d*x])/d
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2570

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[((b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m + 1))/(
a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Cos[e + f*x])^n
*(a*Sin[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -1
] && IntegersQ[2*m, 2*n]
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int (e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2\,dx = \left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sin^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{(-a-a\cos(c+dx))^2\sec^2(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \left(\frac{a^2}{\sin^{\frac{3}{2}}(c+dx)}+\frac{2a^2\sec(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}+\frac{a^2 \mathrm{s}}{\mathrm{si}}\right.$$

$$= \left(a^2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sin^{\frac{3}{2}}(c+dx)}\,dx + \left(a^2e\sqrt{e\csc(c+dx}\right.$$

$$= -\frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{2a^2e\sqrt{e\csc(c+dx)}\sec(c+dx)}{d} - \left(a^2\right.$$

$$= -\frac{4a^2e\sqrt{e\csc(c+dx)}}{d} - \frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{2a^2e\sqrt{e\csc(c+}}{}$$

$$= -\frac{4a^2e\sqrt{e\csc(c+dx)}}{d} - \frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{2a^2e\sqrt{e\csc(c+}}{}$$

$$= -\frac{4a^2e\sqrt{e\csc(c+dx)}}{d} - \frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{2a^2e\sqrt{e\csc(c+}}{}$$

$$= -\frac{4a^2e\sqrt{e\csc(c+dx)}}{d} - \frac{2a^2e\cos(c+dx)\sqrt{e\csc(c+dx)}}{d} - \frac{2a^2e\sqrt{e\csc(c+}}{}$$

Mathematica [C] time = 4.72268, size = 195, normalized size = 0.81

$$2a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)(e\csc(c+dx))^{3/2}\sec^4\left(\frac{1}{2}\csc^{-1}(\csc(c+dx))\right)\left(5\sqrt{-\cot^2(c+dx)}\sqrt{\csc(c+dx)}\text{Hyperg}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*Cos[(c + d*x)/2]^4*(e*Csc[c + d*x])^(3/2)*(3*ArcTan[Sqrt[Csc[c + d*x
]]]*Sqrt[Cos[c + d*x]^2] + 3*ArcTanh[Sqrt[Csc[c + d*x]]]*Sqrt[Cos[c + d*x]^
2] - 6*Sqrt[Csc[c + d*x]] - 6*Sqrt[Cos[c + d*x]^2]*Sqrt[Csc[c + d*x]] + 5*S
qrt[-Cot[c + d*x]^2]*Sqrt[Csc[c + d*x]]*Hypergeometric2F1[3/4, 3/2, 7/4, Cs
c[c + d*x]^2])*Sec[c + d*x]*Sec[ArcCsc[Csc[c + d*x]]/2]^4)/(3*d*Csc[c + d*x
]^(3/2))
```

Maple [C] time = 0.197, size = 1559, normalized size = 6.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/2*a^2/d*2^(1/2)*(2*I*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I
)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/
2),1/2-1/2*I,1/2*2^(1/2))-2*I*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^
(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*
x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c
))^(1/2),1/2+1/2*I,1/2*2^(1/2))-2*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+si
n(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d
*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-2*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c
)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/s
in(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-10*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))
/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(
d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-
I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+5*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+
sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(
d*x+c))^(1/2),1/2*2^(1/2))+2*I*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x
+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c)
)^(1/2),1/2-1/2*I,1/2*2^(1/2))-2*I*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin
(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*
x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
cos(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+s
in(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(
d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*cos(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)
+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/si
n(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-10*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*cos(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x
+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/
sin(d*x+c))^(1/2),1/2*2^(1/2))+5*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(
d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d
*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c
))^(1/2),1/2*2^(1/2))+9*cos(d*x+c)*2^(1/2)-2^(1/2))*(e/sin(d*x+c))^(3/2)*si
n(d*x+c)/cos(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2 e \csc(dx+c)\sec(dx+c)^2 + 2\,a^2 e \csc(dx+c)\sec(dx+c) + a^2 e \csc(dx+c)\right)\sqrt{e\csc(dx+c)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral((a^2*e*csc(d*x + c)*sec(d*x + c)^2 + 2*a^2*e*csc(d*x + c)*sec(d*x
+ c) + a^2*e*csc(d*x + c))*sqrt(e*csc(d*x + c)), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(3/2)*(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int (e\csc(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)*(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a)^2, x)
```

3.289 $\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2\,dx$

Optimal. Leaf size=154

$$\frac{3a^2\sqrt{\sin(c+dx)}\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{e\csc(c+dx)}}{d}+\frac{a^2\tan(c+dx)\sqrt{e\csc(c+dx)}}{d}+\frac{2a^2\sqrt{\sin(c+dx)}\sqrt{e\csc(}}{}$$

```
[Out] (2*a^2*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/
d + (2*a^2*ArcTanh[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*
x]])/d + (3*a^2*Sqrt[e*Csc[c + d*x]]*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[
Sin[c + d*x]])/d + (a^2*Sqrt[e*Csc[c + d*x]]*Tan[c + d*x])/d
```

Rubi [A] time = 0.262707, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3878, 3872, 2873, 2641, 2564, 329, 212, 206, 203, 2571}

$$\frac{a^2\tan(c+dx)\sqrt{e\csc(c+dx)}}{d}+\frac{2a^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d}+\frac{2a^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\,\text{t}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*a^2*ArcTan[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])/
d + (2*a^2*ArcTanh[Sqrt[Sin[c + d*x]]]*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*
x]])/d + (3*a^2*Sqrt[e*Csc[c + d*x]]*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[
Sin[c + d*x]])/d + (a^2*Sqrt[e*Csc[c + d*x]]*Tan[c + d*x])/d
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2\,dx = \left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{(a+a\sec(c+dx))^2}{\sqrt{\sin(c+dx)}}\,dx$$
$$= \left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{(-a-a\cos(c+dx))^2\sec^2(c+dx)}{\sqrt{\sin(c+dx)}}\,dx$$
$$= \left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \left(\frac{a^2}{\sqrt{\sin(c+dx)}}+\frac{2a^2\sec(c+dx)}{\sqrt{\sin(c+dx)}}+\frac{a^2\sec^2(c}{\sqrt{\sin(c}}\right.$$
$$= \left(a^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx + \left(a^2\sqrt{e\csc(c+dx)}\sqrt{\sin(\right.$$
$$= \frac{2a^2\sqrt{e\csc(c+dx)}F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d} + \frac{a^2\sqrt{e\csc(c+dx)}\tan(}{d}$$
$$= \frac{3a^2\sqrt{e\csc(c+dx)}F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d} + \frac{a^2\sqrt{e\csc(c+dx)}\tan(}{d}$$
$$= \frac{3a^2\sqrt{e\csc(c+dx)}F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)\sqrt{\sin(c+dx)}}{d} + \frac{a^2\sqrt{e\csc(c+dx)}\tan(}{d}$$
$$= \frac{2a^2\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}{d} + \frac{2a^2\tanh^{-1}\left(\sqrt{\sin(c+\right.}{}$$

Mathematica [C] time = 2.43572, size = 168, normalized size = 1.09

$$-\frac{2a^2\sin\left(\frac{1}{2}(c+dx)\right)\cos^5\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{e\csc(c+dx)}\sec^4\left(\frac{1}{2}\csc^{-1}(\csc(c+dx))\right)\left(3\sqrt{-\cot^2(c+dx)}\text{Hyperge}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-2*a^2*Cos[(c + d*x)/2]^5*Sqrt[e*Csc[c + d*x]]*(-1 + 2*ArcTan[Sqrt[Csc[c +
 d*x]]]*Sqrt[Cos[c + d*x]^2]*Sqrt[Csc[c + d*x]] - 2*ArcTanh[Sqrt[Csc[c + d*
x]]]*Sqrt[Cos[c + d*x]^2]*Sqrt[Csc[c + d*x]] + 3*Sqrt[-Cot[c + d*x]^2]*Hype
rgeometric2F1[1/4, 1/2, 5/4, Csc[c + d*x]^2])*Sec[c + d*x]*Sec[ArcCsc[Csc[c
 + d*x]]/2]^4*Sin[(c + d*x)/2])/d
```

Maple [C] time = 0.242, size = 729, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(e*csc(d*x+c))^(1/2),x)
```

```
[Out] 1/2*a^2/d*2^(1/2)*(-1+cos(d*x+c))*(-2*I*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d
*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi(((I*cos(d*x+c)+sin(
d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-2*I*((-I*cos(d*x+c)+sin(
d*x+c)+I)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*
(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi(((I*
cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+I*((-I*co
s(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d
```

```
*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*El
lipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+2*sin(d
*x+c)*cos(d*x+c)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),
1/2-1/2*I,1/2*2^(1/2))*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*((I*
cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^
(1/2)-2*sin(d*x+c)*cos(d*x+c)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d
*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c
))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))
/sin(d*x+c))^(1/2)+cos(d*x+c)*2^(1/2)-2^(1/2))*(cos(d*x+c)+1)^2*(e/sin(d*x+
c))^(1/2)/sin(d*x+c)^3/cos(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(a^2\sec(dx+c)^2+2a^2\sec(dx+c)+a^2\right)\sqrt{e\csc(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*sqrt(e*csc(d*x + c
)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int\sqrt{e\csc(c+dx)}\,dx+\int 2\sqrt{e\csc(c+dx)}\sec(c+dx)\,dx+\int\sqrt{e\csc(c+dx)}\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(e*csc(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(sqrt(e*csc(c + d*x)), x) + Integral(2*sqrt(e*csc(c + d*x))*s
ec(c + d*x), x) + Integral(sqrt(e*csc(c + d*x))*sec(c + d*x)**2, x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{e\csc(dx+c)}(a\sec(dx+c)+a)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(e*csc(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)^2, x)
```

3.290 $\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\csc(c+dx)}}\,dx$

Optimal. Leaf size=153

$$\frac{a^2\tan(c+dx)}{d\sqrt{e\csc(c+dx)}}-\frac{2a^2\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{2a^2\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{a^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

```
[Out] (-2*a^2*ArcTan[Sqrt[Sin[c + d*x]]])/(d*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*
x]]) + (2*a^2*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin
[c + d*x]]) + (a^2*EllipticE[(c - Pi/2 + d*x)/2, 2])/(d*Sqrt[e*Csc[c + d*x]
]*Sqrt[Sin[c + d*x]]) + (a^2*Tan[c + d*x])/(d*Sqrt[e*Csc[c + d*x]])
```

Rubi [A] time = 0.271704, antiderivative size = 153, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3878, 3872, 2873, 2639, 2564, 329, 298, 203, 206, 2571}

$$\frac{a^2\tan(c+dx)}{d\sqrt{e\csc(c+dx)}}-\frac{2a^2\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{2a^2\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{a^2E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{d\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/Sqrt[e*Csc[c + d*x]],x]
```

```
[Out] (-2*a^2*ArcTan[Sqrt[Sin[c + d*x]]])/(d*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*
x]]) + (2*a^2*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin
[c + d*x]]) + (a^2*EllipticE[(c - Pi/2 + d*x)/2, 2])/(d*Sqrt[e*Csc[c + d*x]
]*Sqrt[Sin[c + d*x]]) + (a^2*Tan[c + d*x])/(d*Sqrt[e*Csc[c + d*x]])
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] && !IntegerQ[p]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2571

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> -Simp[((b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m + 1))/
(a*b*f*(m + 1)), x] + Dist[(m + n + 2)/(a^2*(m + 1)), Int[(b*Sin[e + f*x])^
n*(a*Cos[e + f*x])^(m + 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && LtQ[m, -
1] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{\sqrt{e\csc(c+dx)}}\,dx = \frac{\int (a+a\sec(c+dx))^2\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int (-a-a\cos(c+dx))^2\sec^2(c+dx)\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \left(a^2\sqrt{\sin(c+dx)}+2a^2\sec(c+dx)\sqrt{\sin(c+dx)}+a^2\sec^2(c+dx)\sqrt{\sin(c+dx)}\right)dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{a^2\int\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a^2\int\sec^2(c+dx)\sqrt{\sin(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{(2a^2)\int\sec(c+dx)\sqrt{}}{\sqrt{e\csc(c+dx)}\sqrt{s}}$$

$$= \frac{2a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a^2\tan(c+dx)}{d\sqrt{e\csc(c+dx)}} - \frac{a^2\int\sqrt{\sin(c+dx)}\,dx}{2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{(2a^2)}{}$$

$$= \frac{a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a^2\tan(c+dx)}{d\sqrt{e\csc(c+dx)}} + \frac{(4a^2)\,\text{Subst}\left(\int\frac{x^2}{1-x^4}dx,x,\sqrt{\sin(c+d}\right.}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a^2\tan(c+dx)}{d\sqrt{e\csc(c+dx)}} + \frac{(2a^2)\,\text{Subst}\left(\int\frac{1}{1-x^2}dx,x,\sqrt{\sin(c+d}\right.}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= -\frac{2a^2\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{2a^2\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\right.}{d\sqrt{e\csc(c+dx)}\sqrt{\sin(c}}$$

Mathematica [C] time = 8.42174, size = 287, normalized size = 1.88

$$-\frac{\left(\cos\left(2\left(\frac{c}{2}+\frac{dx}{2}\right)\right)+1\right)^2\cos(c+dx)\left(\csc^2(c+dx)-1\right)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)(a\sec(c+dx)+a)^2\left(\frac{\sqrt{1-\sin^2(c+dx)}\sqrt{\csc(c+dx)}\text{Hyperg}}{\sqrt{1-\csc}}\right.}{2d\sqrt{1-\sin^2(c+dx)}\csc^{\frac{3}{2}}(c+dx)\sqrt{e\,\text{cs}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/Sqrt[e*Csc[c + d*x]],x]
```

```
[Out] -((1 + Cos[2*(c/2 + (d*x)/2)])^2*Cos[c + d*x]*(-1 + Csc[c + d*x]^2)*Sec[c/2
 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(-ArcTan[Sqrt[Csc[c + d*x]]] - ArcTanh
[Sqrt[Csc[c + d*x]]] - (2*Sqrt[Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]^2]*Hyper
geometric2F1[3/4, 3/2, 7/4, Csc[c + d*x]^2])/(3*Sqrt[1 - Sin[c + d*x]^2]) +
 (Sqrt[Csc[c + d*x]]*Hypergeometric2F1[-1/4, 3/2, 3/4, Csc[c + d*x]^2]*Sqrt
[1 - Sin[c + d*x]^2])/Sqrt[1 - Csc[c + d*x]^2]))/(2*d*(1 + Cos[2*(c/2 + (-c
 + ArcCsc[Csc[c + d*x]])/2)])^2*Csc[c + d*x]^(3/2)*Sqrt[e*Csc[c + d*x]]*Sqr
t[1 - Sin[c + d*x]^2])
```

Maple [C] time = 0.231, size = 1610, normalized size = 10.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x)
```

```
[Out] -1/2*a^2/d*2^(1/2)*(-2*I*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2
*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-
```

```
I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1
/2),1/2+1/2*I,1/2*2^(1/2))+2*I*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*
x+c)^2*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d
*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+
c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*co
s(d*x+c)^2*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-s
in(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(
d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
cos(d*x+c)^2*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)
-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin
(d*x+c))^(1/2),1/2*2^(1/2))+2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x
+c)^2*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*
x+c)-I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c)
)^(1/2),1/2*2^(1/2))-2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2*(
(I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)
/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2
),1/2-1/2*I,1/2*2^(1/2))-2*I*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+
c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c
)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^
(1/2),1/2+1/2*I,1/2*2^(1/2))+2*I*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d
*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((I*cos(d*x+c)
+sin(d*x+c)-I)/sin(d*x+c))^(1/2)-2*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin
(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*
x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin
(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x
+c))^(1/2),1/2*2^(1/2))+2*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*
(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I
)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2
),1/2*2^(1/2))-2*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(
d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)
-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/s
in(d*x+c))^(1/2)+2*cos(d*x+c)^2*2^(1/2)-cos(d*x+c)*2^(1/2)-2^(1/2))/cos(d*x
+c)/(e/sin(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^2}{\sqrt{e\csc(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(e*csc(d*x + c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\left(a^2\sec(dx+c)^2+2a^2\sec(dx+c)+a^2\right)\sqrt{e\csc(dx+c)}}{e\csc(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*sqrt(e*csc(d*x + c
))/(e*csc(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$a^2\left(\int \frac{1}{\sqrt{e\csc(c+dx)}}\,dx + \int \frac{2\sec(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx + \int \frac{\sec^2(c+dx)}{\sqrt{e\csc(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*csc(d*x+c))**(1/2),x)
```

```
[Out] a**2*(Integral(1/sqrt(e*csc(c + d*x)), x) + Integral(2*sec(c + d*x)/sqrt(e*
csc(c + d*x)), x) + Integral(sec(c + d*x)**2/sqrt(e*csc(c + d*x)), x))
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^2}{\sqrt{e\csc(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/sqrt(e*csc(d*x + c)), x)
```

3.291 $\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=222

$$-\frac{a^2\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3de\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}-\frac{4a^2}{de\sqrt{e\csc(c+dx)}}-\frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}}+\frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}}+\frac{2a^2\tan^{-1}\left(\sqrt{\sin(c+\right.}}{de\sqrt{\sin(c+dx)}\sqrt{e\csc(}}$$

```
[Out] (-4*a^2)/(d*e*Sqrt[e*Csc[c + d*x]]) - (2*a^2*Cos[c + d*x])/(3*d*e*Sqrt[e*Cs
c[c + d*x]]) + (a^2*Sec[c + d*x])/(d*e*Sqrt[e*Csc[c + d*x]]) + (2*a^2*ArcTa
n[Sqrt[Sin[c + d*x]]])/(d*e*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]]) + (2*a
^2*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*e*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]
]) - (a^2*EllipticF[(c - Pi/2 + d*x)/2, 2])/(3*d*e*Sqrt[e*Csc[c + d*x]]*Sqr
t[Sin[c + d*x]])
```

Rubi [A] time = 0.312573, antiderivative size = 222, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {3878, 3872, 2873, 2635, 2641, 2564, 321, 329, 212, 206, 203, 2566}

$$-\frac{4a^2}{de\sqrt{e\csc(c+dx)}}-\frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}}+\frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}}+\frac{2a^2\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{2a^2\tanh^{-1}\left(\sqrt{\sin(c+\right.}}{de\sqrt{\sin(c+dx)}\sqrt{e\csc(c}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Csc[c + d*x])^(3/2),x]
```

```
[Out] (-4*a^2)/(d*e*Sqrt[e*Csc[c + d*x]]) - (2*a^2*Cos[c + d*x])/(3*d*e*Sqrt[e*Cs
c[c + d*x]]) + (a^2*Sec[c + d*x])/(d*e*Sqrt[e*Csc[c + d*x]]) + (2*a^2*ArcTa
n[Sqrt[Sin[c + d*x]]])/(d*e*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]]) + (2*a
^2*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*e*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]
]) - (a^2*EllipticF[(c - Pi/2 + d*x)/2, 2])/(3*d*e*Sqrt[e*Csc[c + d*x]]*Sqr
t[Sin[c + d*x]])
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] && !IntegerQ[p]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{3/2}}\,dx = \frac{\int (a+a\sec(c+dx))^2\sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= \frac{\int (-a-a\cos(c+dx))^2\sec^2(c+dx)\sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= \frac{\int \left(a^2\sin^{\frac{3}{2}}(c+dx)+2a^2\sec(c+dx)\sin^{\frac{3}{2}}(c+dx)+a^2\sec^2(c+dx)\sin^{\frac{3}{2}}(c+dx)\right)dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= \frac{a^2\int \sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a^2\int \sec^2(c+dx)\sin^{\frac{3}{2}}(c+dx)\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{(2a^2)\int \sec(c+dx)\sin^{\frac{3}{2}}}{e\sqrt{e\csc(c+dx)}\sqrt{\sin}}$$
$$= -\frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}} + \frac{a^2\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{a^2\int \frac{1}{\sqrt{\sin(}}}{2e\sqrt{e\csc(c+dx}}$$
$$= -\frac{4a^2}{de\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}} - \frac{a^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{3de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+d}}$$
$$= -\frac{4a^2}{de\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}} - \frac{a^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{3de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+d}}$$
$$= -\frac{4a^2}{de\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}} - \frac{a^2F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{3de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+d}}$$
$$= -\frac{4a^2}{de\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+dx)}{3de\sqrt{e\csc(c+dx)}} + \frac{a^2\sec(c+dx)}{de\sqrt{e\csc(c+dx)}} + \frac{2a^2\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx}}$$

Mathematica [C] time = 7.61748, size = 164, normalized size = 0.74

$$\frac{2a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\tan(c+dx)\sqrt{e\csc(c+dx)}\sec^4\left(\frac{1}{2}\csc^{-1}(\csc(c+dx))\right)\left(-6\sqrt{\cos^2(c+dx)}\text{Hypergeometric2F1}\left(-\frac{1}{4},\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Csc[c + d*x])^(3/2),x]
```

```
[Out] (2*a^2*Cos[(c + d*x)/2]^4*Sqrt[e*Csc[c + d*x]]*Sec[ArcCsc[Csc[c + d*x]]/2]^
4*(3 - 6*Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[-1/4, 1, 3/4, Csc[c + d*x]^
2] + 3*Sqrt[-Cot[c + d*x]^2]*Hypergeometric2F1[1/4, 1/2, 5/4, Csc[c + d*x]^
2] + Sqrt[-Cot[c + d*x]^2]*Hypergeometric2F1[-3/4, 3/2, 1/4, Csc[c + d*x]^2
]*Sin[c + d*x]^2)*Tan[c + d*x])/(3*d*e^2)
```

Maple [C] time = 0.218, size = 763, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(3/2),x)
```

```
[Out] -1/6*a^2/d*2^(1/2)*(6*I*sin(d*x+c)*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c
))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin
(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*
x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-13*I*sin(d*x+c)*cos(d*x+c)*(-I*(-1+cos(d
*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(
I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d
*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+6*I*sin(d*x+c)*cos(d*x+c)*(-I*(-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)
*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+
sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))-6*sin(d*x+c)*cos(d*x
+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(
d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticPi(((
I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))+6*sin(d
*x+c)*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d
*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*
EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(
1/2))+2*cos(d*x+c)^3*2^(1/2)+10*cos(d*x+c)^2*2^(1/2)-15*cos(d*x+c)*2^(1/2)+
3*2^(1/2))/(-1+cos(d*x+c))/sin(d*x+c)/cos(d*x+c)/(e/sin(d*x+c))^(3/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^2}{(e\csc(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/(e*csc(d*x + c))^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2\sec(dx+c)^2+2\,a^2\sec(dx+c)+a^2\right)\sqrt{e\csc(dx+c)}}{e^2\csc(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*sqrt(e*csc(d*x + c
))/(e^2*csc(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*csc(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^2}{(e\csc(dx+c))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/(e*csc(d*x + c))^(3/2), x)
```

3.292 $\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=236

$$-\frac{4a^2\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}}+\frac{a^2\tan(c+dx)}{de^2\sqrt{e\csc(c+dx)}}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{5de^2\sqrt{e\csc(c+dx)}}-\frac{2a^2\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{2a^2\tan}{de^2\sqrt{\sin}}$$

```
[Out] (-2*a^2*ArcTan[Sqrt[Sin[c + d*x]]])/(d*e^2*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c
+ d*x]]) + (2*a^2*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*e^2*Sqrt[e*Csc[c + d*x]]*
Sqrt[Sin[c + d*x]]) - (9*a^2*EllipticE[(c - Pi/2 + d*x)/2, 2])/(5*d*e^2*Sqr
t[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]]) - (4*a^2*Sin[c + d*x])/(3*d*e^2*Sqrt[
e*Csc[c + d*x]]) - (2*a^2*Cos[c + d*x]*Sin[c + d*x])/(5*d*e^2*Sqrt[e*Csc[c
+ d*x]]) + (a^2*Tan[c + d*x])/(d*e^2*Sqrt[e*Csc[c + d*x]])
```

Rubi [A] time = 0.319696, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 12, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.48, Rules used = {3878, 3872, 2873, 2635, 2639, 2564, 321, 329, 298, 203, 206, 2566}

$$-\frac{4a^2\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}}+\frac{a^2\tan(c+dx)}{de^2\sqrt{e\csc(c+dx)}}-\frac{2a^2\sin(c+dx)\cos(c+dx)}{5de^2\sqrt{e\csc(c+dx)}}-\frac{2a^2\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{2a^2\tan}{de^2\sqrt{\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2/(e*Csc[c + d*x])^(5/2),x]
```


```
[Out] (-2*a^2*ArcTan[Sqrt[Sin[c + d*x]]])/(d*e^2*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c
+ d*x]]) + (2*a^2*ArcTanh[Sqrt[Sin[c + d*x]]])/(d*e^2*Sqrt[e*Csc[c + d*x]]*
Sqrt[Sin[c + d*x]]) - (9*a^2*EllipticE[(c - Pi/2 + d*x)/2, 2])/(5*d*e^2*Sqr
t[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]]) - (4*a^2*Sin[c + d*x])/(3*d*e^2*Sqrt[
e*Csc[c + d*x]]) - (2*a^2*Cos[c + d*x]*Sin[c + d*x])/(5*d*e^2*Sqrt[e*Csc[c
+ d*x]]) + (a^2*Tan[c + d*x])/(d*e^2*Sqrt[e*Csc[c + d*x]])
```


Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] && !IntegerQ[p]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 298

```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-(a/b
), 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[s/(2*b), Int[1/(r + s*x^2), x
], x] - Dist[s/(2*b), Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !G
tQ[a/b, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 2566

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(a*Sin[e + f*x])^(m - 1)*(b*Cos[e + f*x])^(n + 1)
)/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Sin[e + f*x]
)^(m - 2)*(b*Cos[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ
[m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2}{(e\csc(c+dx))^{5/2}}\,dx = \frac{\int (a+a\sec(c+dx))^2\sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int (-a-a\cos(c+dx))^2\sec^2(c+dx)\sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \left(a^2\sin^{\frac{5}{2}}(c+dx)+2a^2\sec(c+dx)\sin^{\frac{5}{2}}(c+dx)+a^2\sec^2(c+dx)\sin^{\frac{5}{2}}(c+dx)\right)dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{a^2\int \sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{a^2\int \sec^2(c+dx)\sin^{\frac{5}{2}}(c+dx)\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{\left(2a^2\right)\int \sec(c+dx)}{e^2\sqrt{e\csc(c+dx)}}$$

$$= -\frac{2a^2\cos(c+dx)\sin(c+dx)}{5de^2\sqrt{e\csc(c+dx)}} + \frac{a^2\tan(c+dx)}{de^2\sqrt{e\csc(c+dx)}} + \frac{\left(3a^2\right)\int \sqrt{\sin(c+dx)}\,dx}{5e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{}{2}$$

$$= -\frac{9a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{4a^2\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+dx)\sin(c+dx)}{5de^2\sqrt{e\csc(c+dx)}} +$$

$$= -\frac{9a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{4a^2\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+dx)\sin(c+dx)}{5de^2\sqrt{e\csc(c+dx)}} +$$

$$= -\frac{9a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{4a^2\sin(c+dx)}{3de^2\sqrt{e\csc(c+dx)}} - \frac{2a^2\cos(c+dx)\sin(c+dx)}{5de^2\sqrt{e\csc(c+dx)}} +$$

$$= -\frac{2a^2\tan^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{2a^2\tanh^{-1}\left(\sqrt{\sin(c+dx)}\right)}{de^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{9a^2E\left(\frac{1}{2}\left(c-\frac{\pi}{2}\right.\right.}{5de^2\sqrt{e\csc(c+dx)}}$$

Mathematica [C] time = 10.861, size = 152, normalized size = 0.64

$$\frac{2a^2\cos^4\left(\frac{1}{2}(c+dx)\right)\tan(c+dx)\sec^4\left(\frac{1}{2}\csc^{-1}(\csc(c+dx))\right)\left(3\sqrt{-\cot^2(c+dx)}\left(\sin^2(c+dx)\text{Hypergeometric2F1}\left(\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^2/(e*Csc[c + d*x])^(5/2),x]
```

```
[Out] (2*a^2*Cos[(c + d*x)/2]^4*Sec[ArcCsc[Csc[c + d*x]]/2]^4*(-10*Sqrt[Cos[c + d
*x]^2]*Hypergeometric2F1[-3/4, 1, 1/4, Csc[c + d*x]^2] + 3*Sqrt[-Cot[c + d*
x]^2]*(-10*Hypergeometric2F1[-1/4, 3/2, 3/4, Csc[c + d*x]^2] + Hypergeometr
ic2F1[-5/4, 3/2, -1/4, Csc[c + d*x]^2]*Sin[c + d*x]^2))*Tan[c + d*x]))/(15*d
*e^2*Sqrt[e*Csc[c + d*x]])
```

Maple [C] time = 0.239, size = 1600, normalized size = 6.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(5/2),x)
```

```
[Out] 1/30*a^2/d*2^(1/2)*(30*I*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)^2
*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+
sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((I*cos(d*x+c)+sin(d
*x+c)-I)/sin(d*x+c))^(1/2)-30*I*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d
*x+c)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*
x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((I*cos(d*x+c)+
sin(d*x+c)-I)/sin(d*x+c))^(1/2)+30*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x
+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+s
in(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(
d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+30*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/si
n(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x
+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*cos(d*x+c)+sin(d*x+c)-I)
/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-27*cos(d*x+c)^2*(-I*(-1+cos(d*x+c
))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*co
s(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c
)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+54*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin
(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+
c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/s
in(d*x+c))^(1/2),1/2*2^(1/2))-30*I*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*co
s(d*x+c)^2*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((I*c
os(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))*((I*cos(d*
x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)+30*I*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*cos(d*x+c)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(
((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))*((I*c
os(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)+6*2^(1/2)*cos(d*x+c)^4+30*(-I*(-1
+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(
d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi(((
I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2+1/2*I,1/2*2^(1/2))+30*(-I*
(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/s
in(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticPi
(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2-1/2*I,1/2*2^(1/2))-27*(
-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I
)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*Ellipti
cF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+54*(-I*(-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*
x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticE(((I*c
os(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+20*cos(d*x+c)^3*2^(1
/2)+6*cos(d*x+c)^2*2^(1/2)-47*cos(d*x+c)*2^(1/2)+15*2^(1/2))/sin(d*x+c)^3/c
os(d*x+c)/(e/sin(d*x+c))^(5/2)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\left(a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2\right)\sqrt{e\csc\left(dx+c\right)}}{e^3\csc\left(dx+c\right)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^2)*sqrt(e*csc(d*x + c
))/(e^3*csc(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2/(e*csc(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(a\sec(dx+c)+a)^2}{(e\csc(dx+c))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sec(d*x + c) + a)^2/(e*csc(d*x + c))^(5/2), x)
```

3.293 $\int \frac{(e\csc(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=155

$$\frac{4e^2\sqrt{\sin(c+dx)}\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{e\csc(c+dx)}}{21ad}-\frac{2e^2\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7ad}+\frac{2e^2\cot(c+dx)\csc^2(c-}{7ad}$$

```
[Out] (-4*e^2*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(21*a*d) + (2*e^2*Cot[c + d*x]*C
sc[c + d*x]^2*Sqrt[e*Csc[c + d*x]])/(7*a*d) - (2*e^2*Csc[c + d*x]^3*Sqrt[e*
Csc[c + d*x]])/(7*a*d) + (4*e^2*Sqrt[e*Csc[c + d*x]]*EllipticF[(c - Pi/2 +
d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*a*d)
```

Rubi [A] time = 0.223577, antiderivative size = 155, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3878, 3872, 2839, 2564, 30, 2567, 2636, 2641}

$$-\frac{2e^2\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7ad}+\frac{2e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7ad}-\frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{21ad}+\frac{4e^2\sqrt{\text{si}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Csc[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```


```
[Out] (-4*e^2*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(21*a*d) + (2*e^2*Cot[c + d*x]*C
sc[c + d*x]^2*Sqrt[e*Csc[c + d*x]])/(7*a*d) - (2*e^2*Csc[c + d*x]^3*Sqrt[e*
Csc[c + d*x]])/(7*a*d) + (4*e^2*Sqrt[e*Csc[c + d*x]]*EllipticF[(c - Pi/2 +
d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*a*d)
```


Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\csc(c+dx))^{5/2}}{a+a\sec(c+dx)}\,dx &= \left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{(a+a\sec(c+dx))\sin^{\frac{5}{2}}(c+dx)}\,dx \\
&= -\left(\left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos(c+dx)}{(-a-a\cos(c+dx))\sin^{\frac{5}{2}}(c+dx)}\,dx\right) \\
&= \frac{\left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos(c+dx)}{\sin^{\frac{9}{2}}(c+dx)}\,dx}{a} - \frac{\left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+d}{\sin^{\frac{9}{2}}(c+d}}{a} \\
&= \frac{2e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7ad} + \frac{\left(2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sin^{\frac{5}{2}}(c+dx)}}{7a} \\
&= -\frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{21ad} + \frac{2e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7ad} - \frac{2e^2\csc^3(c}{} \\
&= -\frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{21ad} + \frac{2e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7ad} - \frac{2e^2\csc^3(c}{}
\end{aligned}$$

Mathematica [A] time = 0.998386, size = 131, normalized size = 0.85

$$-\frac{\sin^{\frac{5}{2}}(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)(e\csc(c+dx))^{5/2}\left((\cos(c+dx)-2\cos(2(c+dx))-\cos(3(c+dx))+2\right.}{168ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Csc[c + d*x])^(5/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(Csc[(c + d*x)/2]^2*(e*Csc[c + d*x])^(5/2)*Sec[(c + d*x)/2]^4*((2 + Cos[c
+ d*x] - 2*Cos[2*(c + d*x)] - Cos[3*(c + d*x)])*EllipticF[(-2*c + Pi - 2*d*
x)/4, 2] + 2*(4 + 2*Cos[c + d*x] + Cos[2*(c + d*x)])*Sqrt[Sin[c + d*x]])*Si
n[c + d*x]^(5/2))/(168*a*d)
```

Maple [C] time = 0.224, size = 465, normalized size = 3.

$$-\frac{\sqrt{2}\left(-1+\cos\left(dx+c\right)\right)^{3}\left(\cos\left(dx+c\right)+1\right)^{2}}{21\,da\left(\sin\left(dx+c\right)\right)^{5}}\left(2\,i\sin\left(dx+c\right)\left(\cos\left(dx+c\right)\right)^{2}\sqrt{\frac{-i\left(-1+\cos\left(dx+c\right)\right)}{\sin\left(dx+c\right)}}\sqrt{\frac{i\cos\left(dx+c\right)+\sin\left(dx+c\right)}{\sin\left(dx+c\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/21/a/d*2^(1/2)*(-1+cos(d*x+c))^3*(2*I*sin(d*x+c)*cos(d*x+c)^2*(-I*(-1+co
s(d*x+c))/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*
((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+si
n(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+4*I*((-I*cos(d*x+c)+sin(d*x+c)+I
)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+
cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticF(((I*cos(d*x+c
)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+2*I*EllipticF(((I*cos(d*x+c)
+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-I*cos(d*x+c)+sin(d*x+c)+I)
/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-2*cos(d*x+c)^2*2^(1/2)-2*cos(d*x+c)
*2^(1/2)-3*2^(1/2))*(cos(d*x+c)+1)^2*(e/sin(d*x+c))^(5/2)/sin(d*x+c)^5
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\csc\left(dx+c\right)}e^{2}\csc\left(dx+c\right)^{2}}{a\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))*e^2*csc(d*x + c)^2/(a*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\csc(dx+c))^{\frac{5}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*csc(d*x + c))^(5/2)/(a*sec(d*x + c) + a), x)
```

3.294 $\int \frac{(e\csc(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=145

$$-\frac{2e\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{5ad}-\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{5ad}+\frac{2e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{5ad}-\frac{4e\sqrt{\sin(c+}}{}$$

```
[Out] (-4*e*Cos[c + d*x]*Sqrt[e*Csc[c + d*x]])/(5*a*d) + (2*e*Cot[c + d*x]*Csc[c
+ d*x]*Sqrt[e*Csc[c + d*x]])/(5*a*d) - (2*e*Csc[c + d*x]^2*Sqrt[e*Csc[c + d
*x]])/(5*a*d) - (4*e*Sqrt[e*Csc[c + d*x]]*EllipticE[(c - Pi/2 + d*x)/2, 2]*
Sqrt[Sin[c + d*x]])/(5*a*d)
```

Rubi [A] time = 0.229357, antiderivative size = 145, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3878, 3872, 2839, 2564, 30, 2567, 2636, 2639}

$$-\frac{2e\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{5ad}-\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{5ad}+\frac{2e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{5ad}-\frac{4e\sqrt{\sin(c+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Csc[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-4*e*Cos[c + d*x]*Sqrt[e*Csc[c + d*x]])/(5*a*d) + (2*e*Cot[c + d*x]*Csc[c
+ d*x]*Sqrt[e*Csc[c + d*x]])/(5*a*d) - (2*e*Csc[c + d*x]^2*Sqrt[e*Csc[c + d
*x]])/(5*a*d) - (4*e*Sqrt[e*Csc[c + d*x]]*EllipticE[(c - Pi/2 + d*x)/2, 2]*
Sqrt[Sin[c + d*x]])/(5*a*d)
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{(e\csc(c+dx))^{3/2}}{a+a\sec(c+dx)}\,dx &= \left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{(a+a\sec(c+dx))\sin^{\frac{3}{2}}(c+dx)}\,dx \\
&= -\left(\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos(c+dx)}{(-a-a\cos(c+dx))\sin^{\frac{3}{2}}(c+dx)}\,dx\right) \\
&= \frac{\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos(c+dx)}{\sin^{\frac{7}{2}}(c+dx)}\,dx}{a} - \frac{\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)}{\sin^{\frac{7}{2}}(c+dx)}}{a} \\
&= \frac{2e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{5ad} + \frac{\left(2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sin^{\frac{3}{2}}(c+dx)}\,dx}{5a} \\
&= -\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{5ad} + \frac{2e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{5ad} - \frac{2e\csc^2(c+dx}{5} \\
&= -\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{5ad} + \frac{2e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{5ad} - \frac{2e\csc^2(c+dx}{5}
\end{aligned}$$

Mathematica [C] time = 1.33974, size = 230, normalized size = 1.59

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)(e\csc(c+dx))^{3/2}\left(-\frac{6\tan(c+dx)\left(\sec^2\left(\frac{1}{2}(c+dx)\right)+4\sec(c)\cos(dx)\right)}{d} + \frac{8\sqrt{2}e^{i(c-dx)}\sqrt{\frac{ie^{i(c+dx)}}{-1+e^{2i(c+dx)}}}\sec(c+dx)\left(\left(1+e^{2ic}\right)e^{2idx}\sqrt{1-e^{2}}\right.}{\left(1+e^{2ic}\right)d}\right.}{15a(\sec(c+dx)+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Csc[c + d*x])^(3/2)/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(e*Csc[c + d*x])^(3/2)*((8*Sqrt[2]*E^(I*(c - d*x))*Sqrt
[(I*E^(I*(c + d*x)))/(-1 + E^((2*I)*(c + d*x)))]*(3 - 3*E^((2*I)*(c + d*x))
 + E^((2*I)*d*x)*(1 + E^((2*I)*c))*Sqrt[1 - E^((2*I)*(c + d*x))]*Hypergeome
tric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))])*Sec[c + d*x])/(d*(1 + E^((2*I)
*c))*Csc[c + d*x]^(3/2)) - (6*(4*Cos[d*x]*Sec[c] + Sec[(c + d*x)/2]^2)*Tan[
c + d*x])/d))/(15*a*(1 + Sec[c + d*x]))
```

Maple [C] time = 0.21, size = 781, normalized size = 5.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] -1/5/a/d*2^(1/2)*(-1+cos(d*x+c))*(4*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*
x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+
sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(
d*x+c))^(1/2),1/2*2^(1/2))-2*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(
1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x
+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))
^(1/2),1/2*2^(1/2))+8*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*((I*
cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/si
n(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/
2*2^(1/2))-4*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*((I*cos(d*x+c
)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))
^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2)
)+4*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticE(((I
*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))-2*((I*cos(d*x+c)+s
in(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-I*c
os(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+
c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))-2*cos(d*x+c)*2^(1/2)-3*2^(1/2))*(e/sin
(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\csc\left(dx+c\right)}e\csc\left(dx+c\right)}{a\sec\left(dx+c\right)+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))*e*csc(d*x + c)/(a*sec(d*x + c) + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\csc(dx+c))^{\frac{3}{2}}}{a\sec(dx+c)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((e*csc(d*x + c))^(3/2)/(a*sec(d*x + c) + a), x)
```

3.295 $\int \frac{\sqrt{e\csc(c+dx)}}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=105

$$\frac{4\sqrt{\sin(c+dx)}\,\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{e\csc(c+dx)}}{3ad}-\frac{2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3ad}+\frac{2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3ad}$$

```
[Out] (2*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(3*a*d) - (2*Csc[c + d*x]*Sqrt[e*Csc[
c + d*x]])/(3*a*d) + (4*Sqrt[e*Csc[c + d*x]]*EllipticF[(c - Pi/2 + d*x)/2,
2]*Sqrt[Sin[c + d*x]])/(3*a*d)
```

Rubi [A] time = 0.203063, antiderivative size = 105, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3878, 3872, 2839, 2564, 30, 2567, 2641}

$$-\frac{2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3ad}+\frac{2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3ad}+\frac{4\sqrt{\sin(c+dx)}F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)\sqrt{e\csc(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Csc[c + d*x]]/(a + a*Sec[c + d*x]),x]
```


```
[Out] (2*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(3*a*d) - (2*Csc[c + d*x]*Sqrt[e*Csc[
c + d*x]])/(3*a*d) + (4*Sqrt[e*Csc[c + d*x]]*EllipticF[(c - Pi/2 + d*x)/2,
2]*Sqrt[Sin[c + d*x]])/(3*a*d)
```


Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\sqrt{e\csc(c+dx)}}{a+a\sec(c+dx)}\,dx = \left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{(a+a\sec(c+dx))\sqrt{\sin(c+dx)}}\,dx$$

$$= -\left(\left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos(c+dx)}{(-a-a\cos(c+dx))\sqrt{\sin(c+dx)}}\,dx\right)$$

$$= \frac{\left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\,dx}{a} - \frac{\left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\,d}{a}$$

$$= \frac{2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3ad} + \frac{\left(2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3a} + \frac{\left(\sqrt{e\csc(c+\right.}{}$$

$$= \frac{2\cot(c+dx)\sqrt{e\csc(c+dx)}}{3ad} - \frac{2\csc(c+dx)\sqrt{e\csc(c+dx)}}{3ad} + \frac{4\sqrt{e\csc(c+dx)}F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+\right.\right.}{3ad}$$

Mathematica [A] time = 0.354748, size = 60, normalized size = 0.57

$$\frac{2(e\csc(c+dx))^{3/2}\left(-2\sin^{\frac{3}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{4}(-2c-2dx+\pi),2\right)+\cos(c+dx)-1\right)}{3ade}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Csc[c + d*x]]/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*(e*Csc[c + d*x])^(3/2)*(-1 + Cos[c + d*x] - 2*EllipticF[(-2*c + Pi - 2*d
*x)/4, 2]*Sin[c + d*x]^(3/2)))/(3*a*d*e)
```

Maple [C] time = 0.215, size = 320, normalized size = 3.1

$$\frac{\sqrt{2}\left(\cos(dx+c)+1\right)^2\left(-1+\cos(dx+c)\right)^2}{3\,da\left(\sin(dx+c)\right)^5}\sqrt{\frac{e}{\sin(dx+c)}}\left(2\,i\sqrt{\frac{-i\cos(dx+c)+\sin(dx+c)+i}{\sin(dx+c)}}\sqrt{\frac{i\cos(dx+c)+\sin}{\sin(dx+}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/3/a/d*2^(1/2)*(e/sin(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^2*(2*
I*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)
-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(
d*x+c)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2)
)+2*I*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))
*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-
I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)+cos(d
*x+c)*2^(1/2)-2^(1/2))/sin(d*x+c)^5
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e \csc(dx+c)}}{a \sec(dx+c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(e*csc(d*x + c))/(a*sec(d*x + c) + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{e \csc(dx+c)}}{a \sec(dx+c) + a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))/(a*sec(d*x + c) + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\sqrt{e \csc(c+dx)}}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(e*csc(c + d*x))/(sec(c + d*x) + 1), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sqrt{e \csc(dx+c)}}{a \sec(dx+c) + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] integrate(sqrt(e*csc(d*x + c))/(a*sec(d*x + c) + a), x)
```

3.296 $\int \frac{1}{\sqrt{e \csc(c+dx)}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=99

$$-\frac{2\csc(c+dx)}{ad\sqrt{e\csc(c+dx)}}+\frac{2\cot(c+dx)}{ad\sqrt{e\csc(c+dx)}}+\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{ad\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

```
[Out] (2*Cot[c + d*x])/(a*d*Sqrt[e*Csc[c + d*x]]) - (2*Csc[c + d*x])/(a*d*Sqrt[e*
Csc[c + d*x]]) + (4*EllipticE[(c - Pi/2 + d*x)/2, 2])/(a*d*Sqrt[e*Csc[c + d
*x]]*Sqrt[Sin[c + d*x]])
```

Rubi [A] time = 0.210942, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3878, 3872, 2839, 2564, 30, 2567, 2639}

$$-\frac{2\csc(c+dx)}{ad\sqrt{e\csc(c+dx)}}+\frac{2\cot(c+dx)}{ad\sqrt{e\csc(c+dx)}}+\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{ad\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] (2*Cot[c + d*x])/(a*d*Sqrt[e*Csc[c + d*x]]) - (2*Csc[c + d*x])/(a*d*Sqrt[e*
Csc[c + d*x]]) + (4*EllipticE[(c - Pi/2 + d*x)/2, 2])/(a*d*Sqrt[e*Csc[c + d
*x]]*Sqrt[Sin[c + d*x]])
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))}\,dx &= \frac{\int \frac{\sqrt{\sin(c+dx)}}{a+a\sec(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\ &= -\frac{\int \frac{\cos(c+dx)\sqrt{\sin(c+dx)}}{-a-a\cos(c+dx)}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\ &= \frac{\int \frac{\cos(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\,dx}{a\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}-\frac{\int \frac{\cos^2(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\,dx}{a\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\ &= \frac{2\cot(c+dx)}{ad\sqrt{e\csc(c+dx)}}+\frac{2\int \sqrt{\sin(c+dx)}\,dx}{a\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}+\frac{\text{Subst}\left(\int \frac{1}{x^{3/2}}\,dx,x,\sin\right.}{ad\sqrt{e\csc(c+dx)}\sqrt{\sin}}\\ &= \frac{2\cot(c+dx)}{ad\sqrt{e\csc(c+dx)}}-\frac{2\csc(c+dx)}{ad\sqrt{e\csc(c+dx)}}+\frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{ad\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\end{aligned}$$

Mathematica [C] time = 0.602061, size = 95, normalized size = 0.96

$$\frac{6(\cot(c+dx)-\csc(c+dx)+2i)-4\sqrt{1-e^{2i(c+dx)}}(\cot(c+dx)+i)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},e^{2i(c+dx)}\right)}{3ad\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] (6*(2*I + Cot[c + d*x] - Csc[c + d*x]) - 4*Sqrt[1 - E^((2*I)*(c + d*x))]*(I
 + Cot[c + d*x])*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))])/(3*
a*d*Sqrt[e*Csc[c + d*x]])
```

Maple [C] time = 0.214, size = 524, normalized size = 5.3

$$-\frac{\sqrt{2}}{ad\sin(dx+c)}\left(4\sqrt{\frac{-i(-1+\cos(dx+c))}{\sin(dx+c)}}\cos(dx+c)\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}}\sqrt{\frac{-i\cos(dx+c)+\sin(dx}{\sin(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x)
```

```
[Out] -1/a/d*2^(1/2)*(4*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*((I*cos(
d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*
x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^
(1/2))-2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*((I*cos(d*x+c)+si
n(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/
2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+4*
((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos
(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))-2*((I*cos(d*x+c)+sin(d
*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-I*cos(d
*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I
)/sin(d*x+c))^(1/2),1/2*2^(1/2))+cos(d*x+c)*2^(1/2)-2^(1/2))/(e/sin(d*x+c))
^(1/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e\csc(dx+c)}(a\sec(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\csc(dx+c)}}{ae\csc(dx+c)\sec(dx+c)+ae\csc(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))/(a*e*csc(d*x + c)*sec(d*x + c) + a*e*csc(d*x
+ c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sqrt{e\csc(c+dx)}\sec(c+dx)+\sqrt{e\csc(c+dx)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*csc(d*x+c))**(1/2),x)
```

```
[Out] Integral(1/(sqrt(e*csc(c + d*x))*sec(c + d*x) + sqrt(e*csc(c + d*x))), x)/a
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e\csc(dx+c)}(a\sec(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))/(e*csc(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)), x)
```

3.297 $\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=106

$$-\frac{4\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{3ade\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{2}{ade\sqrt{e\csc(c+dx)}}-\frac{2\cos(c+dx)}{3ade\sqrt{e\csc(c+dx)}}$$

```
[Out] 2/(a*d*e*Sqrt[e*Csc[c + d*x]]) - (2*Cos[c + d*x])/(3*a*d*e*Sqrt[e*Csc[c + d
*x]]) - (4*EllipticF[(c - Pi/2 + d*x)/2, 2])/(3*a*d*e*Sqrt[e*Csc[c + d*x]]*
Sqrt[Sin[c + d*x]])
```

Rubi [A] time = 0.227105, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3878, 3872, 2839, 2564, 30, 2569, 2641}

$$\frac{2}{ade\sqrt{e\csc(c+dx)}}-\frac{2\cos(c+dx)}{3ade\sqrt{e\csc(c+dx)}}-\frac{4F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{3ade\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] 2/(a*d*e*Sqrt[e*Csc[c + d*x]]) - (2*Cos[c + d*x])/(3*a*d*e*Sqrt[e*Csc[c + d
*x]]) - (4*EllipticF[(c - Pi/2 + d*x)/2, 2])/(3*a*d*e*Sqrt[e*Csc[c + d*x]]*
Sqrt[Sin[c + d*x]])
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))}\,dx &= \frac{\int \frac{\sin^{\frac{3}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= -\frac{\int \frac{\cos(c+dx)\sin^{\frac{3}{2}}(c+dx)}{-a-a\cos(c+dx)}\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{\int \frac{\cos(c+dx)}{\sqrt{\sin(c+dx)}}\,dx}{ae\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{\int \frac{\cos^2(c+dx)}{\sqrt{\sin(c+dx)}}\,dx}{ae\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= -\frac{2\cos(c+dx)}{3ade\sqrt{e\csc(c+dx)}} - \frac{2\int \frac{1}{\sqrt{\sin(c+dx)}}\,dx}{3ae\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{x}}\,d\right.}{ade\sqrt{e\csc(c+}}\\
&= \frac{2}{ade\sqrt{e\csc(c+dx)}} - \frac{2\cos(c+dx)}{3ade\sqrt{e\csc(c+dx)}} - \frac{4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{3ade\sqrt{e\csc(c+dx)}\sqrt{\sin(c+}}
\end{aligned}$$

Mathematica [A] time = 0.374567, size = 70, normalized size = 0.66

$$\frac{4\text{EllipticF}\left(\frac{1}{4}(-2c-2dx+\pi),2\right)-2\sqrt{\sin(c+dx)}(\cos(c+dx)-3)}{3ad\sin^{\frac{3}{2}}(c+dx)(e\csc(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (4*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] - 2*(-3 + Cos[c + d*x])*Sqrt[Sin[c +
 d*x]])/(3*a*d*(e*Csc[c + d*x])^(3/2)*Sin[c + d*x]^(3/2))
```

Maple [C] time = 0.196, size = 195, normalized size = 1.8

$$\frac{\sqrt{2}}{3\,da\,(-1+\cos(dx+c))\sin(dx+c)}\left(2\,i\text{EllipticF}\left(\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}},\frac{\sqrt{2}}{2}\right)\sqrt{\frac{-i\cos(dx+c)+\sin(dx}{\sin(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/3/a/d*2^(1/2)*(2*I*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/
2),1/2*2^(1/2))*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*((I*cos(d*x
+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*s
in(d*x+c)-cos(d*x+c)^2*2^(1/2)+4*cos(d*x+c)*2^(1/2)-3*2^(1/2))/(-1+cos(d*x+
c))/(e/sin(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\csc(dx+c))^{\frac{3}{2}}\,(a\sec(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate(1/((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\csc(dx+c)}}{ae^2\csc(dx+c)^2\sec(dx+c)+ae^2\csc(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))/(a*e^2*csc(d*x + c)^2*sec(d*x + c) + a*e^2*cs
c(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\csc(dx+c))^{\frac{3}{2}}\,(a\sec(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c)),x, algorithm="giac")

[Out] integrate(1/((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a)), x)
```

3.298 $\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=120

$$\frac{2\sin(c+dx)}{3ade^2\sqrt{e\csc(c+dx)}}-\frac{2\sin(c+dx)\cos(c+dx)}{5ade^2\sqrt{e\csc(c+dx)}}-\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{5ade^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

```
[Out] (-4*EllipticE[(c - Pi/2 + d*x)/2, 2])/(5*a*d*e^2*Sqrt[e*Csc[c + d*x]]*Sqrt[
Sin[c + d*x]]) + (2*Sin[c + d*x])/(3*a*d*e^2*Sqrt[e*Csc[c + d*x]]) - (2*Cos
[c + d*x]*Sin[c + d*x])/(5*a*d*e^2*Sqrt[e*Csc[c + d*x]])
```

Rubi [A] time = 0.222214, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3878, 3872, 2839, 2564, 30, 2569, 2639}

$$\frac{2\sin(c+dx)}{3ade^2\sqrt{e\csc(c+dx)}}-\frac{2\sin(c+dx)\cos(c+dx)}{5ade^2\sqrt{e\csc(c+dx)}}-\frac{4E\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{5ade^2\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (-4*EllipticE[(c - Pi/2 + d*x)/2, 2])/(5*a*d*e^2*Sqrt[e*Csc[c + d*x]]*Sqrt[
Sin[c + d*x]]) + (2*Sin[c + d*x])/(3*a*d*e^2*Sqrt[e*Csc[c + d*x]]) - (2*Cos
[c + d*x]*Sin[c + d*x])/(5*a*d*e^2*Sqrt[e*Csc[c + d*x]])
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))}\,dx &= \frac{\int \frac{\sin^{\frac{5}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= -\frac{\int \frac{\cos(c+dx)\sin^{\frac{5}{2}}(c+dx)}{-a-a\cos(c+dx)}\,dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= \frac{\int \cos(c+dx)\sqrt{\sin(c+dx)}\,dx}{ae^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{\int \cos^2(c+dx)\sqrt{\sin(c+dx)}\,dx}{ae^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}\\
&= -\frac{2\cos(c+dx)\sin(c+dx)}{5ade^2\sqrt{e\csc(c+dx)}} - \frac{2\int\sqrt{\sin(c+dx)}\,dx}{5ae^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{\text{Subst}(}{ade^2\sqrt{e}}\\
&= -\frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}+dx\right)\middle|2\right)}{5ade^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{2\sin(c+dx)}{3ade^2\sqrt{e\csc(c+dx)}} - \frac{2\cos(c+d}{5ade^2\sqrt{e}}
\end{aligned}$$

Mathematica [C] time = 0.874674, size = 100, normalized size = 0.83

$$\frac{8\sqrt{1-e^{2i(c+dx)}}(\cot(c+dx)+i)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},e^{2i(c+dx)}\right)+20\sin(c+dx)-6(\sin(2(c+dx))+4i)}{30ade^2\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (8*Sqrt[1 - E^((2*I)*(c + d*x))]*(I + Cot[c + d*x])*Hypergeometric2F1[1/2,
3/4, 7/4, E^((2*I)*(c + d*x))] + 20*Sin[c + d*x] - 6*(4*I + Sin[2*(c + d*x)
]))/(30*a*d*e^2*Sqrt[e*Csc[c + d*x]])
```

Maple [C] time = 0.227, size = 563, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/15/a/d*2^(1/2)*(-6*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*
cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin
(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2
*2^(1/2))+12*cos(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+
c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))
^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2)
)+3*cos(d*x+c)^3*2^(1/2)-6*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d
*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I
)/sin(d*x+c))^(1/2),1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(
1/2)+12*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)
/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)
,1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)-5*cos(d*x+c)^2
*2^(1/2)+3*cos(d*x+c)*2^(1/2)-2^(1/2))/(e/sin(d*x+c))^(5/2)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\csc(dx+c)}}{ae^3\csc(dx+c)^3\sec(dx+c)+ae^3\csc(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))/(a*e^3*csc(d*x + c)^3*sec(d*x + c) + a*e^3*cs
c(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\csc(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*csc(d*x + c))^(5/2)*(a*sec(d*x + c) + a)), x)
```

3.299 $\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=149

$$-\frac{4\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{21ade^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{2\cos^3(c+dx)}{7ade^3\sqrt{e\csc(c+dx)}}-\frac{2\cos(c+dx)}{21ade^3\sqrt{e\csc(c+dx)}}+\frac{2\sin^2(c+dx)}{5ade^3\sqrt{e\csc(c+dx)}}$$

```
[Out] (-2*Cos[c + d*x])/(21*a*d*e^3*Sqrt[e*Csc[c + d*x]]) + (2*Cos[c + d*x]^3)/(7
*a*d*e^3*Sqrt[e*Csc[c + d*x]]) - (4*EllipticF[(c - Pi/2 + d*x)/2, 2])/(21*a
*d*e^3*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]]) + (2*Sin[c + d*x]^2)/(5*a*d
*e^3*Sqrt[e*Csc[c + d*x]])
```

Rubi [A] time = 0.253985, antiderivative size = 149, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3878, 3872, 2839, 2564, 30, 2568, 2569, 2641}

$$\frac{2\cos^3(c+dx)}{7ade^3\sqrt{e\csc(c+dx)}}-\frac{2\cos(c+dx)}{21ade^3\sqrt{e\csc(c+dx)}}+\frac{2\sin^2(c+dx)}{5ade^3\sqrt{e\csc(c+dx)}}-\frac{4F\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right)\middle|2\right)}{21ade^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Csc[c + d*x])^(7/2)*(a + a*Sec[c + d*x])),x]
```


```
[Out] (-2*Cos[c + d*x])/(21*a*d*e^3*Sqrt[e*Csc[c + d*x]]) + (2*Cos[c + d*x]^3)/(7
*a*d*e^3*Sqrt[e*Csc[c + d*x]]) - (4*EllipticF[(c - Pi/2 + d*x)/2, 2])/(21*a
*d*e^3*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]]) + (2*Sin[c + d*x]^2)/(5*a*d
*e^3*Sqrt[e*Csc[c + d*x]])
```


Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] && !IntegerQ[p]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2839

```
Int[((cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(
n_.))/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[g^2/a, Int[
(g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^n, x], x] - Dist[g^2/(b*d), Int[(
g*Cos[e + f*x])^(p - 2)*(d*Sin[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d,
 e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] && !(In
```

```
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2568

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> -Simp[(a*(b*Cos[e + f*x])^(n + 1)*(a*Sin[e + f*x])^(m - 1)
)/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Cos[e + f*x])^n*(a
*Sin[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
 NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))}\,dx &= \frac{\int \frac{\sin^{\frac{7}{2}}(c+dx)}{a+a\sec(c+dx)}\,dx}{e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} \\
&= -\frac{\int \frac{\cos(c+dx)\sin^{\frac{7}{2}}(c+dx)}{-a-a\cos(c+dx)}\,dx}{e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} \\
&= \frac{\int \cos(c+dx)\sin^{\frac{3}{2}}(c+dx)\,dx}{ae^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{\int \cos^2(c+dx)\sin^{\frac{3}{2}}(c+dx)\,dx}{ae^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} \\
&= \frac{2\cos^3(c+dx)}{7ade^3\sqrt{e\csc(c+dx)}} - \frac{\int \frac{\cos^2(c+dx)}{\sqrt{\sin(c+dx)}}\,dx}{7ae^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{\text{Subst}\left(\int x^{3/2}\right.}{ade^3\sqrt{e\csc(c}} \\
&= -\frac{2\cos(c+dx)}{21ade^3\sqrt{e\csc(c+dx)}} + \frac{2\cos^3(c+dx)}{7ade^3\sqrt{e\csc(c+dx)}} + \frac{2\sin^2(c+dx)}{5ade^3\sqrt{e\csc(c+dx)}} - \\
&= -\frac{2\cos(c+dx)}{21ade^3\sqrt{e\csc(c+dx)}} + \frac{2\cos^3(c+dx)}{7ade^3\sqrt{e\csc(c+dx)}} - \frac{4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+d\right.\right.}{21ade^3\sqrt{e\csc(c+dx)}\sqrt{}}
\end{aligned}$$

Mathematica [A] time = 0.565914, size = 91, normalized size = 0.61

$$\frac{\sqrt{e\csc(c+dx)}\left(80\sqrt{\sin(c+dx)}\text{EllipticF}\left(\frac{1}{4}(-2c-2dx+\pi),2\right)+126\sin(c+dx)+10\sin(2(c+dx))-42\sin(3(c+\right.}{420ade^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Csc[c + d*x])^(7/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Sqrt[e*Csc[c + d*x]]*(80*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sqrt[Sin[c +
d*x]] + 126*Sin[c + d*x] + 10*Sin[2*(c + d*x)] - 42*Sin[3*(c + d*x)] + 15*S
in[4*(c + d*x)]))/(420*a*d*e^4)
```

Maple [C] time = 0.254, size = 221, normalized size = 1.5

$$\frac{\sqrt{2}}{105\,da\left(-1+\cos\left(dx+c\right)\right)\left(\sin\left(dx+c\right)\right)^{3}}\left(10\,i\sin\left(dx+c\right)\sqrt{\frac{-i\left(-1+\cos\left(dx+c\right)\right)}{\sin\left(dx+c\right)}}\sqrt{\frac{-i\cos\left(dx+c\right)+\sin\left(dx+c\right)+i}{\sin\left(dx+c\right)}}\mathrm{Ell}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] 1/105/a/d*2^(1/2)*(10*I*sin(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((
-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(
d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d
*x+c))^(1/2)+15*2^(1/2)*cos(d*x+c)^4-36*cos(d*x+c)^3*2^(1/2)+16*cos(d*x+c)^
2*2^(1/2)+26*cos(d*x+c)*2^(1/2)-21*2^(1/2))/(-1+cos(d*x+c))/(e/sin(d*x+c))^
(7/2)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{e\csc\left(dx+c\right)}}{ae^{4}\csc\left(dx+c\right)^{4}\sec\left(dx+c\right)+ae^{4}\csc\left(dx+c\right)^{4}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))/(a*e^4*csc(d*x + c)^4*sec(d*x + c) + a*e^4*cs
c(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))**(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\csc(dx+c))^{\frac{7}{2}}(a\sec(dx+c)+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate(1/((e*csc(d*x + c))^(7/2)*(a*sec(d*x + c) + a)), x)
```

3.300 $\int \frac{(e \csc(c+dx))^{5/2}}{(a+a \sec(c+dx))^2} dx$

Optimal. Leaf size=268

$$\frac{4e^2\sqrt{\sin(c+dx)}\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{e\csc(c+dx)}}{231a^2d}+\frac{4e^2\csc^5(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d}-\frac{4e^2\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d}$$

```
[Out] (-4*e^2*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(231*a^2*d) + (16*e^2*Cot[c + d*
x]*Csc[c + d*x]^2*Sqrt[e*Csc[c + d*x]])/(77*a^2*d) - (2*e^2*Cot[c + d*x]^3*
Csc[c + d*x]^2*Sqrt[e*Csc[c + d*x]])/(11*a^2*d) - (4*e^2*Csc[c + d*x]^3*Sqr
t[e*Csc[c + d*x]])/(7*a^2*d) - (2*e^2*Cot[c + d*x]*Csc[c + d*x]^4*Sqrt[e*Cs
c[c + d*x]])/(11*a^2*d) + (4*e^2*Csc[c + d*x]^5*Sqrt[e*Csc[c + d*x]])/(11*a
^2*d) + (4*e^2*Sqrt[e*Csc[c + d*x]]*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(231*a^2*d)
```

Rubi [A] time = 0.504997, antiderivative size = 268, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3878, 3872, 2875, 2873, 2567, 2636, 2641, 2564, 14}

$$\frac{4e^2\csc^5(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d}-\frac{4e^2\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d}-\frac{2e^2\cot^3(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d}-\frac{2e^2\cot}{}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Csc[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*e^2*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(231*a^2*d) + (16*e^2*Cot[c + d*
x]*Csc[c + d*x]^2*Sqrt[e*Csc[c + d*x]])/(77*a^2*d) - (2*e^2*Cot[c + d*x]^3*
Csc[c + d*x]^2*Sqrt[e*Csc[c + d*x]])/(11*a^2*d) - (4*e^2*Csc[c + d*x]^3*Sqr
t[e*Csc[c + d*x]])/(7*a^2*d) - (2*e^2*Cot[c + d*x]*Csc[c + d*x]^4*Sqrt[e*Cs
c[c + d*x]])/(11*a^2*d) + (4*e^2*Csc[c + d*x]^5*Sqrt[e*Csc[c + d*x]])/(11*a
^2*d) + (4*e^2*Sqrt[e*Csc[c + d*x]]*EllipticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[S
in[c + d*x]])/(231*a^2*d)
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] && !IntegerQ[p]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{(e\csc(c+dx))^{5/2}}{(a+a\sec(c+dx))^2}\,dx = \left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{(a+a\sec(c+dx))^2\sin^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)}{(-a-a\cos(c+dx))^2\sin^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{\left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sin^{\frac{13}{2}}(c+dx)}\,dx}{a^4}$$

$$= \frac{\left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \left(\frac{a^2\cos^2(c+dx)}{\sin^{\frac{13}{2}}(c+dx)} - \frac{2a^2\cos^3(c+dx)}{\sin^{\frac{13}{2}}(c+dx)} + \frac{a^2\cos^4(c+dx)}{\sin^{\frac{13}{2}}(c+dx)}\right)dx}{a^4}$$

$$= \frac{\left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)}{\sin^{\frac{13}{2}}(c+dx)}\,dx}{a^2} + \frac{\left(e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^4(c+}{\sin^{\frac{13}{2}}(c+}$$

$$= -\frac{2e^2\cot^3(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d} - \frac{2e^2\cot(c+dx)\csc^4(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d}$$

$$= \frac{16e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{77a^2d} - \frac{2e^2\cot^3(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{11a^2d} -$$

$$= -\frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{231a^2d} + \frac{16e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{77a^2d} - \frac{2e^2\cot^3(c}{}$$

$$= -\frac{4e^2\cot(c+dx)\sqrt{e\csc(c+dx)}}{231a^2d} + \frac{16e^2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{77a^2d} - \frac{2e^2\cot^3(c}{}$$

Mathematica [A] time = 1.09839, size = 115, normalized size = 0.43

$$-\frac{e^3\csc^2\left(\frac{1}{2}(c+dx)\right)\sec^6\left(\frac{1}{2}(c+dx)\right)\left(\sin^{\frac{11}{2}}(c+dx)\csc^4\left(\frac{1}{2}(c+dx)\right)\text{EllipticF}\left(\frac{1}{4}(-2c-2dx+\pi),2\right)+97\cos(c+dx)+\right.}{3696a^2d\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Csc[c + d*x])^(5/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(e^3*Csc[(c + d*x)/2]^2*Sec[(c + d*x)/2]^6*(52 + 97*Cos[c + d*x] + 4*Cos[2
*(c + d*x)] + Cos[3*(c + d*x)] + Csc[(c + d*x)/2]^4*EllipticF[(-2*c + Pi -
2*d*x)/4, 2]*Sin[c + d*x]^(11/2)))/(3696*a^2*d*Sqrt[e*Csc[c + d*x]])
```

Maple [C] time = 0.247, size = 609, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/231/a^2/d*2^(1/2)*(-1+cos(d*x+c))^4*(2*I*(-I*(-1+cos(d*x+c))/sin(d*x+c))^
(1/2)*sin(d*x+c)*cos(d*x+c)^3*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2
)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+
sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+6*I*(-I*(-1+cos(d*x+c))/sin(d*
```

```
x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)^2*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c)
)^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d
*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+6*I*((-I*cos(d*x+c)+sin(
d*x+c)+I)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*
(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticF(((I*c
os(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+2*I*EllipticF(((I*co
s(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*((-I*cos(d*x+c)+sin(d
*x+c)+I)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(
-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*sin(d*x+c)-2*cos(d*x+c)^3*2^(1/2)-4*co
s(d*x+c)^2*2^(1/2)-47*cos(d*x+c)*2^(1/2)-24*2^(1/2))*(cos(d*x+c)+1)^2*(e/si
n(d*x+c))^(5/2)/sin(d*x+c)^7
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\csc\left(dx+c\right)}e^{2}\csc\left(dx+c\right)^{2}}{a^{2}\sec\left(dx+c\right)^{2}+2\,a^{2}\sec\left(dx+c\right)+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))*e^2*csc(d*x + c)^2/(a^2*sec(d*x + c)^2 + 2*a^
2*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(5/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\left(e\csc\left(dx+c\right)\right)^{\frac{5}{2}}}{\left(a\sec\left(dx+c\right)+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*csc(d*x + c))^(5/2)/(a*sec(d*x + c) + a)^2, x)
```

3.301 $\int \frac{(e\csc(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=250

$$\frac{4e\csc^4(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d}-\frac{4e\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{5a^2d}-\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{15a^2d}-\frac{2e\cot^3(c+dx)\csc(c}{9a}$$

```
[Out] (-4*e*Cos[c + d*x]*Sqrt[e*Csc[c + d*x]])/(15*a^2*d) + (16*e*Cot[c + d*x]*Cs
c[c + d*x]*Sqrt[e*Csc[c + d*x]])/(45*a^2*d) - (2*e*Cot[c + d*x]^3*Csc[c + d
*x]*Sqrt[e*Csc[c + d*x]])/(9*a^2*d) - (4*e*Csc[c + d*x]^2*Sqrt[e*Csc[c + d*
x]])/(5*a^2*d) - (2*e*Cot[c + d*x]*Csc[c + d*x]^3*Sqrt[e*Csc[c + d*x]])/(9*
a^2*d) + (4*e*Csc[c + d*x]^4*Sqrt[e*Csc[c + d*x]])/(9*a^2*d) - (4*e*Sqrt[e*
Csc[c + d*x]]*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(15*a^2*
d)
```

Rubi [A] time = 0.492683, antiderivative size = 250, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3878, 3872, 2875, 2873, 2567, 2636, 2639, 2564, 14}

$$\frac{4e\csc^4(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d}-\frac{4e\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{5a^2d}-\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{15a^2d}-\frac{2e\cot^3(c+dx)\csc(c}{9a}$$

Antiderivative was successfully verified.

```
[In] Int[(e*Csc[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*e*Cos[c + d*x]*Sqrt[e*Csc[c + d*x]])/(15*a^2*d) + (16*e*Cot[c + d*x]*Cs
c[c + d*x]*Sqrt[e*Csc[c + d*x]])/(45*a^2*d) - (2*e*Cot[c + d*x]^3*Csc[c + d
*x]*Sqrt[e*Csc[c + d*x]])/(9*a^2*d) - (4*e*Csc[c + d*x]^2*Sqrt[e*Csc[c + d*
x]])/(5*a^2*d) - (2*e*Cot[c + d*x]*Csc[c + d*x]^3*Sqrt[e*Csc[c + d*x]])/(9*
a^2*d) + (4*e*Csc[c + d*x]^4*Sqrt[e*Csc[c + d*x]])/(9*a^2*d) - (4*e*Sqrt[e*
Csc[c + d*x]]*EllipticE[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(15*a^2*
d)
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{(e\csc(c+dx))^{3/2}}{(a+a\sec(c+dx))^2}dx = \left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{(a+a\sec(c+dx))^2\sin^{\frac{3}{2}}(c+dx)}dx$$

$$= \left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)}{(-a-a\cos(c+dx))^2\sin^{\frac{3}{2}}(c+dx)}dx$$

$$= \frac{\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sin^{\frac{11}{2}}(c+dx)}dx}{a^4}$$

$$= \frac{\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \left(\frac{a^2\cos^2(c+dx)}{\sin^{\frac{11}{2}}(c+dx)} - \frac{2a^2\cos^3(c+dx)}{\sin^{\frac{11}{2}}(c+dx)} + \frac{a^2\cos^4(c+dx)}{\sin^{\frac{11}{2}}(c+dx)}\right)dx}{a^4}$$

$$= \frac{\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)}{\sin^{\frac{11}{2}}(c+dx)}dx}{a^2} + \frac{\left(e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^4(c}{\sin^{\frac{11}{2}}(c}$$

$$= -\frac{2e\cot^3(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d} - \frac{2e\cot(c+dx)\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d}$$

$$= \frac{16e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{45a^2d} - \frac{2e\cot^3(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{9a^2d} -$$

$$= -\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{15a^2d} + \frac{16e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{45a^2d} - \frac{2e\cot^3(c+}{}$$

$$= -\frac{4e\cos(c+dx)\sqrt{e\csc(c+dx)}}{15a^2d} + \frac{16e\cot(c+dx)\csc(c+dx)\sqrt{e\csc(c+dx)}}{45a^2d} - \frac{2e\cot^3(c+}{}$$

Mathematica [C] time = 1.79105, size = 247, normalized size = 0.99

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)(e\csc(c+dx))^{3/2}\left(-\frac{2\tan(c+dx)\left((13\cos(c+dx)+8)\sec^4\left(\frac{1}{2}(c+dx)\right)+24\sec(c)\cos(dx)\right)}{d} + \frac{16\sqrt{2}e^{i(c-dx)}\sqrt{\frac{ie^{i(c+}}{-1+e^{2i(}}}}{}\right.}{45a^2(\sec(c+dx)+1)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(e*Csc[c + d*x])^(3/2)/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]^4*(e*Csc[c + d*x])^(3/2)*Sec[c + d*x]*((16*Sqrt[2]*E^(I*(
c - d*x))*Sqrt[(I*E^(I*(c + d*x)))/(-1 + E^((2*I)*(c + d*x)))]*(3 - 3*E^((2
*I)*(c + d*x)) + E^((2*I)*d*x)*(1 + E^((2*I)*c))*Sqrt[1 - E^((2*I)*(c + d*x
))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))])*Sec[c + d*x])/(d
*(1 + E^((2*I)*c))*Csc[c + d*x]^(3/2)) - (2*(24*Cos[d*x]*Sec[c] + (8 + 13*C
os[c + d*x])*Sec[(c + d*x)/2]^4)*Tan[c + d*x])/d))/(45*a^2*(1 + Sec[c + d*x
])^2)
```

Maple [C] time = 0.233, size = 1044, normalized size = 4.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/45/a^2/d*2^(1/2)*(-1+cos(d*x+c))^2*(12*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*cos(d*x+c)^3*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticE(
((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))*((I*cos(d*x+c)+
sin(d*x+c)-I)/sin(d*x+c))^(1/2)-6*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos
(d*x+c)^3*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*((I*cos(d*x+c)+si
n(d*x+c)-I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*
x+c))^(1/2),1/2*2^(1/2))+36*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1
/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+
c)+I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^
(1/2),1/2*2^(1/2))-18*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((
I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/
sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),
1/2*2^(1/2))+36*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*((I*cos(d*
x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+
c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1
/2))-18*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*((I*cos(d*x+c)+sin
(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2
)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+12*
((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+
c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos
(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))-6*((I*cos(d*x+c)+sin(d
*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-I*cos(d
*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I
)/sin(d*x+c))^(1/2),1/2*2^(1/2))-6*cos(d*x+c)^2*2^(1/2)-25*cos(d*x+c)*2^(1/
2)-14*2^(1/2))*(e/sin(d*x+c))^(3/2)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\csc\left(dx+c\right)}e\csc\left(dx+c\right)}{a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))*e*csc(d*x + c)/(a^2*sec(d*x + c)^2 + 2*a^2*se
c(d*x + c) + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{(e\csc(dx+c))^{\frac{3}{2}}}{(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate((e*csc(d*x + c))^(3/2)/(a*sec(d*x + c) + a)^2, x)
```

3.302 $\int \frac{\sqrt{e\csc(c+dx)}}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=201

$$\frac{20\sqrt{\sin(c+dx)}\operatorname{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)\sqrt{e\csc(c+dx)}}{21a^2d}+\frac{4\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d}-\frac{4\csc(c+dx)\sqrt{e\csc(c+d\ldots}}{3a^2d}$$

```
[Out] (16*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(21*a^2*d) - (2*Cot[c + d*x]^3*Sqrt[
e*Csc[c + d*x]])/(7*a^2*d) - (4*Csc[c + d*x]*Sqrt[e*Csc[c + d*x]])/(3*a^2*d
) - (2*Cot[c + d*x]*Csc[c + d*x]^2*Sqrt[e*Csc[c + d*x]])/(7*a^2*d) + (4*Csc
[c + d*x]^3*Sqrt[e*Csc[c + d*x]])/(7*a^2*d) + (20*Sqrt[e*Csc[c + d*x]]*Elli
pticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*a^2*d)
```

Rubi [A] time = 0.448139, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3878, 3872, 2875, 2873, 2567, 2636, 2641, 2564, 14}

$$\frac{4\csc^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d}-\frac{4\csc(c+dx)\sqrt{e\csc(c+dx)}}{3a^2d}-\frac{2\cot^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d}-\frac{2\cot(c+dx)\csc^2(c+dx)\ldots}{7a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[e*Csc[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```


```
[Out] (16*Cot[c + d*x]*Sqrt[e*Csc[c + d*x]])/(21*a^2*d) - (2*Cot[c + d*x]^3*Sqrt[
e*Csc[c + d*x]])/(7*a^2*d) - (4*Csc[c + d*x]*Sqrt[e*Csc[c + d*x]])/(3*a^2*d
) - (2*Cot[c + d*x]*Csc[c + d*x]^2*Sqrt[e*Csc[c + d*x]])/(7*a^2*d) + (4*Csc
[c + d*x]^3*Sqrt[e*Csc[c + d*x]])/(7*a^2*d) + (20*Sqrt[e*Csc[c + d*x]]*Elli
pticF[(c - Pi/2 + d*x)/2, 2]*Sqrt[Sin[c + d*x]])/(21*a^2*d)
```


Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
```

```
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{\sqrt{e\csc(c+dx)}}{(a+a\sec(c+dx))^2}\,dx = \left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{1}{(a+a\sec(c+dx))^2\sqrt{\sin(c+dx)}}\,dx$$
$$= \left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)}{(-a-a\cos(c+dx))^2\sqrt{\sin(c+dx)}}\,dx$$
$$= \frac{\left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sin^{\frac{9}{2}}(c+dx)}\,dx}{a^4}$$
$$= \frac{\left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \left(\frac{a^2\cos^2(c+dx)}{\sin^{\frac{9}{2}}(c+dx)} - \frac{2a^2\cos^3(c+dx)}{\sin^{\frac{9}{2}}(c+dx)} + \frac{a^2\cos^4(c+dx)}{\sin^{\frac{9}{2}}(c+dx)}\right)dx}{a^4}$$
$$= \frac{\left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^2(c+dx)}{\sin^{\frac{9}{2}}(c+dx)}\,dx}{a^2} + \frac{\left(\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}\right)\int \frac{\cos^4(c+dx)}{\sin^{\frac{9}{2}}(c+dx)}\,dx}{a^2}$$
$$= -\frac{2\cot^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d} - \frac{2\cot(c+dx)\csc^2(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d} - \frac{\left(2\sqrt{e\csc(c+d\right.}{}$$
$$= \frac{16\cot(c+dx)\sqrt{e\csc(c+dx)}}{21a^2d} - \frac{2\cot^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d} - \frac{2\cot(c+dx)\csc^2(c+dx)\sqrt{}}{7a^2d}$$
$$= \frac{16\cot(c+dx)\sqrt{e\csc(c+dx)}}{21a^2d} - \frac{2\cot^3(c+dx)\sqrt{e\csc(c+dx)}}{7a^2d} - \frac{4\csc(c+dx)\sqrt{e\csc(c+dx)}}{3a^2d}$$

Mathematica [A] time = 0.690762, size = 82, normalized size = 0.41

$$-\frac{4\csc^3(c+dx)\sqrt{e\csc(c+dx)}\left(5\sin^{\frac{7}{2}}(c+dx)\text{EllipticF}\left(\frac{1}{4}(-2c-2dx+\pi),2\right)+2\sin^4\left(\frac{1}{2}(c+dx)\right)(11\cos(c+dx)+8)\right)}{21a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[e*Csc[c + d*x]]/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-4*Csc[c + d*x]^3*Sqrt[e*Csc[c + d*x]]*(2*(8 + 11*Cos[c + d*x])*Sin[(c + d
*x)/2]^4 + 5*EllipticF[(-2*c + Pi - 2*d*x)/4, 2]*Sin[c + d*x]^(7/2)))/(21*a
^2*d)
```

Maple [C] time = 0.24, size = 474, normalized size = 2.4

$$-\frac{\sqrt{2}(\cos(dx+c)+1)^2(-1+\cos(dx+c))^3}{21\,da^2(\sin(dx+c))^7}\sqrt{\frac{e}{\sin(dx+c)}}\left(10\,i(\cos(dx+c))^2\sin(dx+c)\sqrt{\frac{-i(-1+\cos(dx+c))}{\sin(dx+c)}}\sqrt{\frac{i\,c}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/21/a^2/d*2^(1/2)*(e/sin(d*x+c))^(1/2)*(cos(d*x+c)+1)^2*(-1+cos(d*x+c))^3
*(10*I*cos(d*x+c)^2*sin(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((I*co
s(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(
d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*
2^(1/2))+20*I*cos(d*x+c)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I
*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c)
```

```
)^(1/2)*sin(d*x+c)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)
,1/2*2^(1/2))+10*I*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+c
os(d*x+c))/sin(d*x+c))^(1/2)*(-(I*cos(d*x+c)-sin(d*x+c)-I)/sin(d*x+c))^(1/2
)*sin(d*x+c)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2
^(1/2))+11*cos(d*x+c)^2*2^(1/2)-3*cos(d*x+c)*2^(1/2)-8*2^(1/2))/sin(d*x+c)^
7
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\csc\left(dx+c\right)}}{a^2\sec\left(dx+c\right)^2+2\,a^2\sec\left(dx+c\right)+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))/(a^2*sec(d*x + c)^2 + 2*a^2*sec(d*x + c) + a^
2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int\frac{\sqrt{e\csc\left(c+dx\right)}}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))**(1/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Integral(sqrt(e*csc(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)/a*
*2
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sqrt{e\csc\left(dx+c\right)}}{\left(a\sec\left(dx+c\right)+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*csc(d*x+c))^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(e*csc(d*x + c))/(a*sec(d*x + c) + a)^2, x)
```

3.303 $\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=199

$$\frac{4\csc^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}}-\frac{4\csc(c+dx)}{a^2d\sqrt{e\csc(c+dx)}}-\frac{2\cot^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}}-\frac{2\cot(c+dx)\csc^2(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}}+\frac{16\cot(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}}+\frac{}{5a^2}$$

```
[Out] (16*Cot[c + d*x])/(5*a^2*d*Sqrt[e*Csc[c + d*x]]) - (2*Cot[c + d*x]^3)/(5*a^
2*d*Sqrt[e*Csc[c + d*x]]) - (4*Csc[c + d*x])/(a^2*d*Sqrt[e*Csc[c + d*x]]) -
 (2*Cot[c + d*x]*Csc[c + d*x]^2)/(5*a^2*d*Sqrt[e*Csc[c + d*x]]) + (4*Csc[c
+ d*x]^3)/(5*a^2*d*Sqrt[e*Csc[c + d*x]]) + (28*EllipticE[(c - Pi/2 + d*x)/2
, 2])/(5*a^2*d*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])
```

Rubi [A] time = 0.468637, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3878, 3872, 2875, 2873, 2567, 2636, 2639, 2564, 14}

$$\frac{4\csc^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}}-\frac{4\csc(c+dx)}{a^2d\sqrt{e\csc(c+dx)}}-\frac{2\cot^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}}-\frac{2\cot(c+dx)\csc^2(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}}+\frac{16\cot(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}}+\frac{}{5a^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```


```
[Out] (16*Cot[c + d*x])/(5*a^2*d*Sqrt[e*Csc[c + d*x]]) - (2*Cot[c + d*x]^3)/(5*a^
2*d*Sqrt[e*Csc[c + d*x]]) - (4*Csc[c + d*x])/(a^2*d*Sqrt[e*Csc[c + d*x]]) -
 (2*Cot[c + d*x]*Csc[c + d*x]^2)/(5*a^2*d*Sqrt[e*Csc[c + d*x]]) + (4*Csc[c
+ d*x]^3)/(5*a^2*d*Sqrt[e*Csc[c + d*x]]) + (28*EllipticE[(c - Pi/2 + d*x)/2
, 2])/(5*a^2*d*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])
```


Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
```

```
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{1}{\sqrt{e\csc(c+dx)}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sqrt{\sin(c+dx)}}{(a+a\sec(c+dx))^2}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)\sqrt{\sin(c+dx)}}{(-a-a\cos(c+dx))^2}\,dx}{\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sin^{\frac{7}{2}}(c+dx)}\,dx}{a^4\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \left(\frac{a^2\cos^2(c+dx)}{\sin^{\frac{7}{2}}(c+dx)} - \frac{2a^2\cos^3(c+dx)}{\sin^{\frac{7}{2}}(c+dx)} + \frac{a^2\cos^4(c+dx)}{\sin^{\frac{7}{2}}(c+dx)}\right)dx}{a^4\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)}{\sin^{\frac{7}{2}}(c+dx)}\,dx}{a^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{\int \frac{\cos^4(c+dx)}{\sin^{\frac{7}{2}}(c+dx)}\,dx}{a^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{2\int}{a^2\sqrt{e\csc(c}}$$

$$= -\frac{2\cot^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{2\cot(c+dx)\csc^2(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{2\int \frac{1}{\sin^{\frac{3}{2}}(c+dx)}\,dx}{5a^2\sqrt{e\csc(c+dx)}\sqrt{\sin(}}$$

$$= \frac{16\cot(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{2\cot^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{2\cot(c+dx)\csc^2(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} + \frac{}{5a^2}$$

$$= \frac{16\cot(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{2\cot^3(c+dx)}{5a^2d\sqrt{e\csc(c+dx)}} - \frac{4\csc(c+dx)}{a^2d\sqrt{e\csc(c+dx)}} - \frac{2\cot(c+d}{5a^2d\sqrt{e}}$$

<u>**Mathematica**</u> **[C]** time = 1.61185, size = 252, normalized size = 1.27

$$\frac{4\cos^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\csc(c+dx)}\sec^2(c+dx)\left(-3\sqrt{\csc(c+dx)}\left((5\cos(2c)-23)\sec(c)\cos(dx)-2\left(5\sin(c)\sin(dx)+\sec\right.\right.\right.}{15a^2d(\sec(c+dx)+1)^2\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[e*Csc[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (4*Cos[(c + d*x)/2]^4*Sqrt[Csc[c + d*x]]*Sec[c + d*x]^2*((-28*Sqrt[2]*E^(I*
(c - d*x))*Sqrt[(I*E^(I*(c + d*x)))/(-1 + E^((2*I)*(c + d*x)))]*(3 - 3*E^((
2*I)*(c + d*x)) + E^((2*I)*d*x)*(1 + E^((2*I)*c))*Sqrt[1 - E^((2*I)*(c + d*
x))]*Hypergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))]))/(1 + E^((2*I)*
c)) - 3*Sqrt[Csc[c + d*x]]*((-23 + 5*Cos[2*c])*Cos[d*x]*Sec[c] - 2*(-10 + S
ec[(c + d*x)/2]^2 + 5*Sin[c]*Sin[d*x]))))/(15*a^2*d*Sqrt[e*Csc[c + d*x]]*(1
 + Sec[c + d*x])^2)
```

<u>**Maple**</u> **[C]** time = 0.236, size = 793, normalized size = 4.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x)
```

```
[Out] 1/5/a^2/d*2^(1/2)*(-1+cos(d*x+c))*(28*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(
d*x+c))^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c
)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/si
n(d*x+c))^(1/2),1/2*2^(1/2))-14*cos(d*x+c)^2*(-I*(-1+cos(d*x+c))/sin(d*x+c)
)^(1/2)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(
d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+
c))^(1/2),1/2*2^(1/2))+56*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*
((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I
)/sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2
),1/2*2^(1/2))-28*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*((I*cos(
d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*
x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^
(1/2))+28*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c)
)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*Ellipti
cE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))-14*((I*cos(d
*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)
*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+s
in(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))+5*cos(d*x+c)^2*2^(1/2)+cos(d*x+
c)*2^(1/2)-6*2^(1/2))/(e/sin(d*x+c))^(1/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e\csc(dx+c)}(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\csc(dx+c)}}{a^2e\csc(dx+c)\sec(dx+c)^2+2\,a^2e\csc(dx+c)\sec(dx+c)+a^2e\csc(dx+c)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))/(a^2*e*csc(d*x + c)*sec(d*x + c)^2 + 2*a^2*e*
csc(d*x + c)*sec(d*x + c) + a^2*e*csc(d*x + c)), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{1}{\sqrt{e\csc(c+dx)}\sec^2(c+dx)+2\sqrt{e\csc(c+dx)}\sec(c+dx)+\sqrt{e\csc(c+dx)}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+a*sec(d*x+c))**2/(e*csc(d*x+c))**(1/2),x)
```

[Out] Integral(1/(sqrt(e*csc(c + d*x))*sec(c + d*x)**2 + 2*sqrt(e*csc(c + d*x))*s
ec(c + d*x) + sqrt(e*csc(c + d*x))), x)/a**2

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{e \csc(dx+c)}(a \sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(a+a*sec(d*x+c))^2/(e*csc(d*x+c))^(1/2),x, algorithm="giac")

[Out] integrate(1/(sqrt(e*csc(d*x + c))*(a*sec(d*x + c) + a)^2), x)

3.304 $\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=213

$$-\frac{4\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{a^2de\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}+\frac{4\csc^2(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}}+\frac{4}{a^2de\sqrt{e\csc(c+dx)}}-\frac{4\cos(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}}-\frac{2\cot(c+d\dots}{3a^2de\sqrt{e\dots}}$$

```
[Out] 4/(a^2*d*e*Sqrt[e*Csc[c + d*x]]) - (4*Cos[c + d*x])/(3*a^2*d*e*Sqrt[e*Csc[c
 + d*x]]) - (2*Cos[c + d*x]*Cot[c + d*x]^2)/(3*a^2*d*e*Sqrt[e*Csc[c + d*x]]
) - (2*Cot[c + d*x]*Csc[c + d*x])/(3*a^2*d*e*Sqrt[e*Csc[c + d*x]]) + (4*Csc
[c + d*x]^2)/(3*a^2*d*e*Sqrt[e*Csc[c + d*x]]) - (4*EllipticF[(c - Pi/2 + d*
x)/2, 2])/(a^2*d*e*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])
```

Rubi [A] time = 0.475118, antiderivative size = 213, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3878, 3872, 2875, 2873, 2567, 2641, 2564, 14, 2569}

$$\frac{4\csc^2(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}}+\frac{4}{a^2de\sqrt{e\csc(c+dx)}}-\frac{4\cos(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}}-\frac{2\cot(c+dx)\csc(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}}-\frac{2\cos(c+dx)\cot^2(c\dots}{3a^2de\sqrt{e\csc(c+\dots}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] 4/(a^2*d*e*Sqrt[e*Csc[c + d*x]]) - (4*Cos[c + d*x])/(3*a^2*d*e*Sqrt[e*Csc[c
 + d*x]]) - (2*Cos[c + d*x]*Cot[c + d*x]^2)/(3*a^2*d*e*Sqrt[e*Csc[c + d*x]]
) - (2*Cot[c + d*x]*Csc[c + d*x])/(3*a^2*d*e*Sqrt[e*Csc[c + d*x]]) + (4*Csc
[c + d*x]^2)/(3*a^2*d*e*Sqrt[e*Csc[c + d*x]]) - (4*EllipticF[(c - Pi/2 + d*
x)/2, 2])/(a^2*d*e*Sqrt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]])
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
```

```
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{1}{(e\csc(c+dx))^{3/2}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sin^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)\sin^{\frac{3}{2}}(c+dx)}{(-a-a\cos(c+dx))^2}\,dx}{e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sin^{\frac{5}{2}}(c+dx)}\,dx}{a^4e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \left(\frac{a^2\cos^2(c+dx)}{\sin^{\frac{5}{2}}(c+dx)} - \frac{2a^2\cos^3(c+dx)}{\sin^{\frac{5}{2}}(c+dx)} + \frac{a^2\cos^4(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\right)dx}{a^4e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\,dx}{a^2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{\int \frac{\cos^4(c+dx)}{\sin^{\frac{5}{2}}(c+dx)}\,dx}{a^2e\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{}{a^2e}$$

$$= -\frac{2\cos(c+dx)\cot^2(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{2\cot(c+dx)\csc(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{2\int}{3a^2e\sqrt{e\csc(c}}$$

$$= -\frac{4\cos(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{2\cos(c+dx)\cot^2(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{2\cot(c+dx)\csc(}{3a^2de\sqrt{e\csc(c}}$$

$$= \frac{4}{a^2de\sqrt{e\csc(c+dx)}} - \frac{4\cos(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}} - \frac{2\cos(c+dx)\cot^2(c+dx)}{3a^2de\sqrt{e\csc(c+dx)}}$$

Mathematica [A] time = 0.536797, size = 101, normalized size = 0.47

$$\frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\left(12(\cos(c+dx)+1)\text{EllipticF}\left(\frac{1}{4}(-2c-2dx+\pi),2\right)+\sqrt{\sin(c+dx)}(10\cos(c+dx)-\cos(2(c+dx))\right.}{6a^2d\sin^{\frac{3}{2}}(c+dx)(e\csc(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Csc[c + d*x])^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (Sec[(c + d*x)/2]^2*(12*(1 + Cos[c + d*x])*EllipticF[(-2*c + Pi - 2*d*x)/4,
 2] + (15 + 10*Cos[c + d*x] - Cos[2*(c + d*x)])*Sqrt[Sin[c + d*x]]))/(6*a^2
*d*(e*Csc[c + d*x])^(3/2)*Sin[c + d*x]^(3/2))
```

Maple [C] time = 0.221, size = 327, normalized size = 1.5

$$-\frac{\sqrt{2}}{3\,da^2\left(\sin\left(dx+c\right)\right)^3}\left(6\,i\sqrt{\frac{-i\cos\left(dx+c\right)+\sin\left(dx+c\right)+i}{\sin\left(dx+c\right)}}\sqrt{\frac{i\cos\left(dx+c\right)+\sin\left(dx+c\right)-i}{\sin\left(dx+c\right)}}\sqrt{\frac{-i\left(-1+\cos\left(dx+c\right)\right.}{\sin\left(dx+c\right)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/3/a^2/d*2^(1/2)*(6*I*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*((I
*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))
^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x
```

```
+c))^(1/2),1/2*2^(1/2))+6*I*sin(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)
+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1
/2),1/2*2^(1/2))-cos(d*x+c)^3*2^(1/2)+6*cos(d*x+c)^2*2^(1/2)+3*cos(d*x+c)*2
^(1/2)-8*2^(1/2))/(e/sin(d*x+c))^(3/2)/sin(d*x+c)^3
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\csc(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] integrate(1/((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a)^2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\csc(dx+c)}}{a^2e^2\csc(dx+c)^2\sec(dx+c)^2+2\,a^2e^2\csc(dx+c)^2\sec(dx+c)+a^2e^2\csc(dx+c)^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))/(a^2*e^2*csc(d*x + c)^2*sec(d*x + c)^2 + 2*a^
2*e^2*csc(d*x + c)^2*sec(d*x + c) + a^2*e^2*csc(d*x + c)^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\csc(dx+c))^{\frac{3}{2}}(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*csc(d*x + c))^(3/2)*(a*sec(d*x + c) + a)^2), x)
```

3.305 $\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=215

$$\frac{4\csc(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}}-\frac{2\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}}+\frac{4\sin(c+dx)}{3a^2de^2\sqrt{e\csc(c+dx)}}-\frac{2\cos^2(c+dx)\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}}-\frac{12\sin(c+dx)\cos}{5a^2de^2\sqrt{e\csc(c}}$$

```
[Out] (-2*Cot[c + d*x])/(a^2*d*e^2*Sqrt[e*Csc[c + d*x]]) - (2*Cos[c + d*x]^2*Cot[
c + d*x])/(a^2*d*e^2*Sqrt[e*Csc[c + d*x]]) + (4*Csc[c + d*x])/(a^2*d*e^2*Sq
rt[e*Csc[c + d*x]]) - (44*EllipticE[(c - Pi/2 + d*x)/2, 2])/(5*a^2*d*e^2*Sq
rt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]]) + (4*Sin[c + d*x])/(3*a^2*d*e^2*Sqrt
[e*Csc[c + d*x]]) - (12*Cos[c + d*x]*Sin[c + d*x])/(5*a^2*d*e^2*Sqrt[e*Csc[
c + d*x]])
```

Rubi [A] time = 0.471265, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.36, Rules used = {3878, 3872, 2875, 2873, 2567, 2639, 2564, 14, 2569}

$$\frac{4\csc(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}}-\frac{2\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}}+\frac{4\sin(c+dx)}{3a^2de^2\sqrt{e\csc(c+dx)}}-\frac{2\cos^2(c+dx)\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}}-\frac{12\sin(c+dx)\cos}{5a^2de^2\sqrt{e\csc(c}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (-2*Cot[c + d*x])/(a^2*d*e^2*Sqrt[e*Csc[c + d*x]]) - (2*Cos[c + d*x]^2*Cot[
c + d*x])/(a^2*d*e^2*Sqrt[e*Csc[c + d*x]]) + (4*Csc[c + d*x])/(a^2*d*e^2*Sq
rt[e*Csc[c + d*x]]) - (44*EllipticE[(c - Pi/2 + d*x)/2, 2])/(5*a^2*d*e^2*Sq
rt[e*Csc[c + d*x]]*Sqrt[Sin[c + d*x]]) + (4*Sin[c + d*x])/(3*a^2*d*e^2*Sqrt
[e*Csc[c + d*x]]) - (12*Cos[c + d*x]*Sin[c + d*x])/(5*a^2*d*e^2*Sqrt[e*Csc[
c + d*x]])
```

Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2567

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(a*Cos[e + f*x])^(m - 1)*(b*Sin[e + f*x])^(n + 1))
/(b*f*(n + 1)), x] + Dist[(a^2*(m - 1))/(b^2*(n + 1)), Int[(a*Cos[e + f*x])
^(m - 2)*(b*Sin[e + f*x])^(n + 2), x], x] /; FreeQ[{a, b, e, f}, x] && GtQ[
m, 1] && LtQ[n, -1] && (IntegersQ[2*m, 2*n] || EqQ[m + n, 0])
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rubi steps

$$\int \frac{1}{(e\csc(c+dx))^{5/2}(a+a\sec(c+dx))^2}dx = \frac{\int \frac{\sin^{\frac{5}{2}}(c+dx)}{(a+a\sec(c+dx))^2}dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= \frac{\int \frac{\cos^2(c+dx)\sin^{\frac{5}{2}}(c+dx)}{(-a-a\cos(c+dx))^2}dx}{e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= \frac{\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sin^{\frac{3}{2}}(c+dx)}dx}{a^4e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= \frac{\int \left(\frac{a^2\cos^2(c+dx)}{\sin^{\frac{3}{2}}(c+dx)} - \frac{2a^2\cos^3(c+dx)}{\sin^{\frac{3}{2}}(c+dx)} + \frac{a^2\cos^4(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}\right)dx}{a^4e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$
$$= \frac{\int \frac{\cos^2(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}dx}{a^2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{\int \frac{\cos^4(c+dx)}{\sin^{\frac{3}{2}}(c+dx)}dx}{a^2e^2\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \frac{}{a^2}$$
$$= -\frac{2\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} - \frac{2\cos^2(c+dx)\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} - \frac{2\int\sqrt{\sin(c}}{a^2e^2\sqrt{e\csc(c+dx}}$$
$$= -\frac{2\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} - \frac{2\cos^2(c+dx)\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} - \frac{4E\left(\frac{1}{2}\left(c-\frac{\pi}{2}\right.\right.}{a^2de^2\sqrt{e\csc(c+d}}$$
$$= -\frac{2\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} - \frac{2\cos^2(c+dx)\cot(c+dx)}{a^2de^2\sqrt{e\csc(c+dx)}} + \frac{4\csc(c+dx)}{a^2de^2\sqrt{e\csc(c+d}}$$

Mathematica [C] time = 2.13475, size = 125, normalized size = 0.58

$$\frac{88\sqrt{1-e^{2i(c+dx)}}(\cot(c+dx)+i)\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{3}{4},\frac{7}{4},e^{2i(c+dx)}\right) - 123\cot(c+dx) + \csc(c+dx)(-264i\sin(c}{30a^2de^2\sqrt{e\csc(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Csc[c + d*x])^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (-123*Cot[c + d*x] + 88*Sqrt[1 - E^((2*I)*(c + d*x))]*(I + Cot[c + d*x])*Hy
pergeometric2F1[1/2, 3/4, 7/4, E^((2*I)*(c + d*x))] + Csc[c + d*x]*(140 - 2
0*Cos[2*(c + d*x)] + 3*Cos[3*(c + d*x)] - (264*I)*Sin[c + d*x]))/(30*a^2*d*
e^2*Sqrt[e*Csc[c + d*x]])
```

Maple [C] time = 0.239, size = 551, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] 1/15/a^2/d*2^(1/2)*(132*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*((
I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/
sin(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),
1/2*2^(1/2))-66*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*cos(d*x+c)*((I*cos(d*
```

```
x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+
c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1
/2))+3*cos(d*x+c)^3*2^(1/2)+132*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1
/2)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin
(d*x+c))^(1/2)*EllipticE(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2
*2^(1/2))-66*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*(-I*(-1+cos(d*x
+c))/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)+I)/sin(d*x+c))^(1/2)*Elli
pticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2),1/2*2^(1/2))-10*cos(d*
x+c)^2*2^(1/2)+33*cos(d*x+c)*2^(1/2)-26*2^(1/2))/(e/sin(d*x+c))^(5/2)/sin(d
*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\csc(dx+c)}}{a^2e^3\csc(dx+c)^3\sec(dx+c)^2+2\,a^2e^3\csc(dx+c)^3\sec(dx+c)+a^2e^3\csc(dx+c)^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))/(a^2*e^3*csc(d*x + c)^3*sec(d*x + c)^2 + 2*a^
2*e^3*csc(d*x + c)^3*sec(d*x + c) + a^2*e^3*csc(d*x + c)^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))**(5/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\csc(dx+c))^{\frac{5}{2}}(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")

[Out] integrate(1/((e*csc(d*x + c))^(5/2)*(a*sec(d*x + c) + a)^2), x)
```

3.306 $\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=172

$$\frac{52\text{EllipticF}\left(\frac{1}{2}\left(c+dx-\frac{\pi}{2}\right),2\right)}{21a^2de^3\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}}-\frac{4}{a^2de^3\sqrt{e\csc(c+dx)}}+\frac{2\cos^3(c+dx)}{7a^2de^3\sqrt{e\csc(c+dx)}}+\frac{26\cos(c+dx)}{21a^2de^3\sqrt{e\csc(c+dx)}}+\frac{4\sin}{5a^2de^3\sqrt{}}$$

```
[Out] -4/(a^2*d*e^3*Sqrt[e*Csc[c + d*x]]) + (26*Cos[c + d*x])/(21*a^2*d*e^3*Sqrt[
e*Csc[c + d*x]]) + (2*Cos[c + d*x]^3)/(7*a^2*d*e^3*Sqrt[e*Csc[c + d*x]]) +
(52*EllipticF[(c - Pi/2 + d*x)/2, 2])/(21*a^2*d*e^3*Sqrt[e*Csc[c + d*x]]*Sq
rt[Sin[c + d*x]]) + (4*Sin[c + d*x]^2)/(5*a^2*d*e^3*Sqrt[e*Csc[c + d*x]])
```

Rubi [A] time = 0.463165, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3878, 3872, 2875, 2873, 2569, 2641, 2564, 14}

$$-\frac{4}{a^2de^3\sqrt{e\csc(c+dx)}}+\frac{2\cos^3(c+dx)}{7a^2de^3\sqrt{e\csc(c+dx)}}+\frac{26\cos(c+dx)}{21a^2de^3\sqrt{e\csc(c+dx)}}+\frac{4\sin^2(c+dx)}{5a^2de^3\sqrt{e\csc(c+dx)}}+\frac{52F\left(\frac{1}{2}\left(c+dx\right.\right.}{21a^2de^3\sqrt{\sin(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((e*Csc[c + d*x])^(7/2)*(a + a*Sec[c + d*x])^2),x]
```


```
[Out] -4/(a^2*d*e^3*Sqrt[e*Csc[c + d*x]]) + (26*Cos[c + d*x])/(21*a^2*d*e^3*Sqrt[
e*Csc[c + d*x]]) + (2*Cos[c + d*x]^3)/(7*a^2*d*e^3*Sqrt[e*Csc[c + d*x]]) +
(52*EllipticF[(c - Pi/2 + d*x)/2, 2])/(21*a^2*d*e^3*Sqrt[e*Csc[c + d*x]]*Sq
rt[Sin[c + d*x]]) + (4*Sin[c + d*x]^2)/(5*a^2*d*e^3*Sqrt[e*Csc[c + d*x]])
```


Rule 3878

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*((g_.)*sec[(e_.) + (f_.)*(
x_)])^(p_), x_Symbol] :> Dist[g^IntPart[p]*(g*Sec[e + f*x])^FracPart[p]*Cos
[e + f*x]^FracPart[p], Int[(a + b*Csc[e + f*x])^m/Cos[e + f*x]^p, x], x] /;
 FreeQ[{a, b, e, f, g, m, p}, x] &&  !IntegerQ[p]
```

Rule 3872

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_.), x_Symbol] :> Int[((g*Cos[e + f*x])^p*(b + a*Sin[e + f*x])^m)/S
in[e + f*x]^m, x] /; FreeQ[{a, b, e, f, g, p}, x] && IntegerQ[m]
```

Rule 2875

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Dist[(a/g)^(2*
m), Int[((g*Cos[e + f*x])^(2*m + p)*(d*Sin[e + f*x])^n)/(a - b*Sin[e + f*x]
)^m, x], x] /; FreeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && I
LtQ[m, 0]
```

Rule 2873

```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_), x_Symbol] :> Int[ExpandTrig
[(g*cos[e + f*x])^p, (d*sin[e + f*x])^n*(a + b*sin[e + f*x])^m, x], x] /; F
```

```
reeQ[{a, b, d, e, f, g, n, p}, x] && EqQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Rule 2569

```
Int[(cos[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n
_), x_Symbol] :> Simp[(a*(b*Sin[e + f*x])^(n + 1)*(a*Cos[e + f*x])^(m - 1))
/(b*f*(m + n)), x] + Dist[(a^2*(m - 1))/(m + n), Int[(b*Sin[e + f*x])^n*(a*
Cos[e + f*x])^(m - 2), x], x] /; FreeQ[{a, b, e, f, n}, x] && GtQ[m, 1] &&
NeQ[m + n, 0] && IntegersQ[2*m, 2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 2564

```
Int[cos[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.), x_
Symbol] :> Dist[1/(a*f), Subst[Int[x^m*(1 - x^2/a^2)^((n - 1)/2), x], x, a*
Sin[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2] &&  !(In
tegerQ[(m - 1)/2] && LtQ[0, m, n])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \frac{1}{(e\csc(c+dx))^{7/2}(a+a\sec(c+dx))^2}\,dx = \frac{\int \frac{\sin^{\frac{7}{2}}(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)\sin^{\frac{7}{2}}(c+dx)}{(-a-a\cos(c+dx))^2}\,dx}{e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)(-a+a\cos(c+dx))^2}{\sqrt{\sin(c+dx)}}\,dx}{a^4e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \left(\frac{a^2\cos^2(c+dx)}{\sqrt{\sin(c+dx)}} - \frac{2a^2\cos^3(c+dx)}{\sqrt{\sin(c+dx)}} + \frac{a^2\cos^4(c+dx)}{\sqrt{\sin(c+dx)}}\right)dx}{a^4e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}}$$

$$= \frac{\int \frac{\cos^2(c+dx)}{\sqrt{\sin(c+dx)}}\,dx}{a^2e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} + \frac{\int \frac{\cos^4(c+dx)}{\sqrt{\sin(c+dx)}}\,dx}{a^2e^3\sqrt{e\csc(c+dx)}\sqrt{\sin(c+dx)}} - \cdots$$

$$= \frac{2\cos(c+dx)}{3a^2de^3\sqrt{e\csc(c+dx)}} + \frac{2\cos^3(c+dx)}{7a^2de^3\sqrt{e\csc(c+dx)}} + \frac{2\int \frac{1}{\sqrt{\sin(c+dx)}} \cdots}{3a^2e^3\sqrt{e\csc(c+dx)}\sqrt{\cdots}}$$

$$= \frac{26\cos(c+dx)}{21a^2de^3\sqrt{e\csc(c+dx)}} + \frac{2\cos^3(c+dx)}{7a^2de^3\sqrt{e\csc(c+dx)}} + \frac{4F\left(\frac{1}{2}\left(c-\frac{\pi}{2}+\cdots\right.\right.}{3a^2de^3\sqrt{e\csc(c+dx)}\cdots}$$

$$= -\frac{4}{a^2de^3\sqrt{e\csc(c+dx)}} + \frac{26\cos(c+dx)}{21a^2de^3\sqrt{e\csc(c+dx)}} + \frac{2\cos^3(c+dx)}{7a^2de^3\sqrt{e\csc(c+dx)}} \cdots$$

Mathematica [A] time = 2.27638, size = 94, normalized size = 0.55

$$\frac{\sqrt{\sin(c+dx)}\sqrt{e\csc(c+dx)}\left(\sqrt{\sin(c+dx)}(305\cos(c+dx)-84\cos(2(c+dx))+15\cos(3(c+dx))-756)-520\text{Elliptic}\right.}{210a^2de^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((e*Csc[c + d*x])^(7/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (Sqrt[e*Csc[c + d*x]]*(-520*EllipticF[(-2*c + Pi - 2*d*x)/4, 2] + (-756 + 3
05*Cos[c + d*x] - 84*Cos[2*(c + d*x)] + 15*Cos[3*(c + d*x)])*Sqrt[Sin[c + d
*x]])*Sqrt[Sin[c + d*x]])/(210*a^2*d*e^4)
```

Maple [C] time = 0.208, size = 221, normalized size = 1.3

$$-\frac{\sqrt{2}}{105\,da^2\left(-1+\cos(dx+c)\right)\left(\sin(dx+c)\right)^3}\left(130\,i\sin(dx+c)\sqrt{\frac{-i\left(-1+\cos(dx+c)\right)}{\sin(dx+c)}}\sqrt{\frac{i\cos(dx+c)+\sin(dx+c)-i}{\sin(dx+c)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x)
```

```
[Out] -1/105/a^2/d*2^(1/2)*(130*I*sin(d*x+c)*(-I*(-1+cos(d*x+c))/sin(d*x+c))^(1/2
)*((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1/2)*((-I*cos(d*x+c)+sin(d*x+c)
+I)/sin(d*x+c))^(1/2)*EllipticF(((I*cos(d*x+c)+sin(d*x+c)-I)/sin(d*x+c))^(1
/2),1/2*2^(1/2))-15*2^(1/2)*cos(d*x+c)^4+57*cos(d*x+c)^3*2^(1/2)-107*cos(d*
x+c)^2*2^(1/2)+233*cos(d*x+c)*2^(1/2)-168*2^(1/2))/(-1+cos(d*x+c))/(e/sin(d
*x+c))^(7/2)/sin(d*x+c)^3
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{e\csc(dx+c)}}{a^2e^4\csc(dx+c)^4\sec(dx+c)^2+2\,a^2e^4\csc(dx+c)^4\sec(dx+c)+a^2e^4\csc(dx+c)^4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(e*csc(d*x + c))/(a^2*e^4*csc(d*x + c)^4*sec(d*x + c)^2 + 2*a^
2*e^4*csc(d*x + c)^4*sec(d*x + c) + a^2*e^4*csc(d*x + c)^4), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))**(7/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(e\csc(dx+c))^{\frac{7}{2}}(a\sec(dx+c)+a)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*csc(d*x+c))^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] integrate(1/((e*csc(d*x + c))^(7/2)*(a*sec(d*x + c) + a)^2), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
   (flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                         #since this estimate of leaf count is bit lower than
```

```
                      #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=", expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```